Indira Gandhi National
Open University
School of Sciences

BPHCT-131
MECHANICS

Block

2

BASIC CONCEPTS OF MECHANICS

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BLOCK 2: BASIC CONCEPTS OF MECHANICS

In Block 1 of this course, you have studied concepts of vector algebra and ordinary differential equations that you will be using in this course and in your later physics courses. In this block, you will start your study of objects in motion by revising the basic concepts of **kinematics** and **dynamics** that you have studied in school physics. These include **Newton's laws of motion** and the concepts of **force**, **linear momentum**, **impulse**, **work** and **energy**. These concepts apply to a large variety of macroscopic objects in motion around us as well as to the motion of heavenly bodies. For example, we use these concepts to explain how and why objects like rain drops, falling balls, parachutes, satellites and planets, etc. move the way they do. There are numerous (direct and indirect) practical applications of these concepts in transport (motion of bicycles, cars, buses, trains, etc.), health (treatment of fractures, administration of IV fluids, draining of lungs, etc.), amusement parks (slides, swings, merry-go-rounds, giant wheels, joyrides, etc.), and so on.

You may have studied most of these concepts in your school level physics courses. However, you may find some concepts in this block to be new in their presentation. You should study them carefully and thoroughly. You should spend more time in working through the examples, SAQs and Terminal Questions given in each unit of this block. There are 6 units in this block.

In **Unit 5**, we discuss **Newton's laws of motion** which explain why objects move in the way they do. You will learn the concepts of **force, mass** and **linear momentum** which are bound together in Newton's laws of motion. You will learn how different the concept of force and the understanding of motion as given by Newton's laws of motion are from our everyday experiences and common sense notions and also how to use these concepts correctly. We also introduce the concept of **inertial frame of reference** in this unit. You will learn about some interesting applications of these laws and concepts in everyday phenomena involving simple systems. For example, you will learn why we need to wear seat belts in cars, why it is more difficult to move a massive object like a cupboard of books from rest as compared to a lighter object such as a book kept on a table, why you need to shake a ketchup bottle to make ketchup flow out from it, how we can sail a boat with a steady speed in windy weather, etc.

In **Unit 6**, we apply Newton's laws to objects moving along a straight line and in a plane. You will learn about the various forces around us such as normal force, friction, tension and spring force. You will also learn how to determine the net force on any object and its motion under the force exerted on it. You will also revise the concept of **equilibrium of forces**, which finds application in many mechanical systems and devices. So, in this unit, you will find answers to questions like why we tend to fall on slippery surfaces, why the cables of a suspension bridge are curved and not horizontal, why curves in roads are banked, etc.

In **Unit 7,** you will study the **law of gravitation**, and understand how it applies to extended bodies. You will also study its application to uniform circular motion in a vertical plane. You will learn about the force of gravity on the Earth, the factors which cause variation in acceleration due to gravity at different points on and around the Earth and the phenomenon of **weightlessness**. You will learn about a variety of applications such as geosynchronous satellites and how to calculate their height from the surface of the Earth, minimum speeds of carts on roller coasters and giant wheels so that people taking joy rides do not fall off them, and also learn why astronauts float in space stations.

In **Unit 8**, we revise the concepts of **linear momentum** and **impulse** and the **principle of conservation of linear momentum**. You will learn how speeds of colliding cars can be calculated, how the speed of rockets can be increased by attaching additional stages to them, why it hurts more when you fall on a hard floor than when you fall on a soft mattress, and so on.

In **Units 9 and 10,** we revise the concepts of work and energy. **Unit 9** begins by revising the concepts of **work done by constant forces** and **kinetic energy**. We also discuss the concepts of **work done by variable forces** and the **work-energy theorem**, which is another form of Newton's second law of motion and **power**. You will learn why we need to have speed limits while driving and why it is important to keep a minimum distance between two vehicles moving on the road, what power must be applied to keep a bicycle moving at a steady speed in the face of air resistance, and so on.

In **Unit 10**, we present the concept of **potential energy** so as to arrive at the **law of conservation of energy**. These concepts are important because they make it far easier to study complex mechanical phenomena, where the application of Newton's laws is difficult. For example, we can calculate the final speed of a diver as she enters the pool, the escape velocity of an object from the Earth, how much energy is transformed as heat when a box slides on a rough floor, etc.

In the next block, you will study the kinematics and dynamics of rotational/angular motion and motion under central forces. You will also apply the concepts you have studied in this block to the motion of many particle systems.

We hope you enjoy studying the basic concepts of mechanics explained in this block as well as their applications to situations around us. We wish you success.

Why do we need to shake the bottle for ketchup to flow out? Discover the answer in this unit!

UNIT 5

NEWTON'S LAWS OF MOTION AND FORCE

Structure

5.1 Introduction
Expected Learning Outcomes
5.2 Newton's First Law of Motion
Concept of Inertia
Force
Inertial Frame of Reference
5.3 Newton's Second Law of Motion
Principle of Superposition of Forces
Mass
5.4 Newton's Third Law of Motion
5.5 Summary
5.6 Terminal Questions
5.7 Solutions and Answers

STUDY GUIDE

In this unit, you will study **Newton's laws of motion** and the concepts of **force** and **mass.** In order to understand these concepts, **you should know very well the concepts of vector algebra and differential and integral calculus you have studied in Units 1 and 2 of Block 1 and your school mathematics courses**. You should revise these concepts before studying this unit.

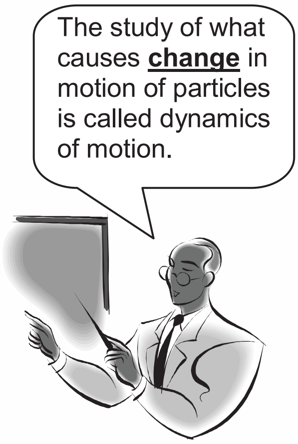
You have studied Newton's laws of motion in school physics. Here you will revise them. However, in this unit, our focus is on **building concepts related to Newton's laws and force**. In this unit, you will learn **how different the concept of force and the understanding of motion as given by Newton's laws of motion are from our everyday experiences and common sense notions**. Yet the beauty of these laws is that we can apply them to explain and predict the motion of macroscopic objects around us.

You should try to answer all conceptual questions given in all sections of this unit on your own. Each one of these should take you between 5 and 10 minutes to solve. If you take much more time than this to answer any question in a given section or if you find that your answer is incorrect, study that section again. In this way you will develop a correct understanding of the concepts explained here.

"I do not know what I may appear to the world, but to myself I seem to have been only like a boy playing on the seashore, and diverting myself in now and then finding a smoother pebble or a prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me."

Isaac Newton

5.1 INTRODUCTION



In your school physics courses, you have studied the *equations of kinematics* for particles *moving in a straight line* and *in a plane*. You have also learnt the concepts of position, displacement, velocity and acceleration needed for **describing** motion. These concepts help us describe *how* fast a particle moves at any given time and *by how much* it speeds up or slows down (kinematics). But you still have to learn **why** *an object slows down* or *speeds up as it moves*. *What causes change in the particle's speed or direction of motion*? This study is called the **dynamics** of motion.

These questions have puzzled some of the greatest minds in the history of physics: The answers were arrived at by Galileo (in part) and Newton (in full measure, when he was barely 22 years old). Newton developed Galileo's ideas and formulated what we know today as **Newtonian mechanics**. The most important feature of Galileo's and Newton's work is that they could think beyond the immediate everyday experiences and use imaginative thinking to arrive at their understanding.

For example, our experience shows that an object changes its velocity when we *push* or *pull* it: You can pull or push a car to move it from its state of rest. You change the speed and direction of a ball or a shuttlecock if you hit it with a bat or a racquet. A bus slows down when the driver applies brakes to it. But many objects change their speed and direction without any obvious push or pull or braking. For example, a ball rolling on a concrete floor comes to a stop; a bicycle comes to a stop if you stop pedalling it; a freely-falling ball thrown in the air speeds up as it comes down; etc. Sometimes an obvious push or pull cannot cause a change in the object's position: however hard you push a wall, it does not move!

These are our everyday experiences. All of us have spent years walking, running and doing many more things that involve motion such as throwing balls, pushing chairs, riding bicycles, etc. In the process, we have tended to develop our "common sense" ideas about motion and its causes. We need to change them while studying physics.

The genius of Galileo and Newton lies in developing **a set of laws** which relate the **change** in the velocity of particles, i.e., their **acceleration** to external interactions. These laws explain all our observations about the motion of macroscopic objects ranging from bodies on the Earth to heavenly bodies like stars and planets.

In order to study the dynamics of motion, we need to introduce the concepts of **force** and **mass**. These concepts are defined by Newton's laws of motion, which also introduce a quantity called **linear momentum**. In this unit, we revise **Newton's laws of motion** and the concepts of **force**, **mass** and **linear momentum**. We also introduce the concept of **inertial frame of reference**. You will learn how to apply these laws to study the motion of simple systems. In the next unit, you will learn about various forces around us. You will also learn how to apply Newton's laws of motion to a variety of situations in which these forces act.

REMEMBER: While studying this unit, you must always keep in mind that the concepts of Newtonian mechanics presented here are used to **model** the real physical phenomena. In Newton's world, all objects/bodies are to be treated as **point particles**.

But point particles do **not** exist in the real world. They are conceptual objects created by Newtonian theory. In this sense, the Newtonian world is an **idealised world constructed to describe the real physical world**. Moreover, the Newtonian mechanics as we study today was never presented in this form by Newton. It has been refined and extended by many physicists.

Also, **Newton's laws do not apply to all of nature**. Newtonian mechanics fails in the world of atoms, molecules and nuclei (which is described by quantum mechanics). It also does not hold in a world in which objects move with speeds close to the speed of light (which is described by the special theory of relativity).

NOTE

In your written work, always use an arrow above the letter you use to denote a vector, e.g., $\vec{r}$. Use a cap above the letter you use to denote a unit vector, e.g. $\hat{r}$.

Expected Learning Outcomes

After studying this unit, you should be able to:

- explain the concepts of force and mass on the basis of Newton's laws of motion;
- use words, diagrams and equations to describe the motion of a particle on which a given force is exerted; and
- apply Newton's laws of motion to simple problems.

5.2 NEWTON'S FIRST LAW OF MOTION

Do the following activities and think about the answers to the questions posed.

Activity

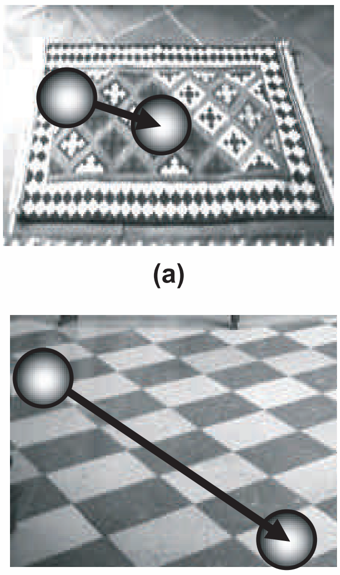
WHAT CAUSES THINGS TO MOVE?

a) Push an object (a ball, a marble, a can, roller skate, etc.) on the floor and push it again in a different direction as it is moving. What happens to it?

b) Push a ball or a marble on a rough surface such as a carpet. Push it on a smooth surface such as a smooth cemented, marbled, tiled or a polished floor or a mirror lying flat on the floor. What difference do you observe in the motion of the object in these cases?

c) Attach a stone to a string and swing it rapidly in a horizontal circle. At some instant let go of the string. What path does the stone follow thereafter? Note it as you will use it to answer Terminal Question 3.

What did you observe from the activities you did? Did you conclude from activity (a) that a 'push' or a 'pull' is needed to move an object? This understanding is based on our everyday experiences and makes a lot of sense. For example, a cart moves only when the bullock or horse starts pulling it; and when the animal stops pulling it, the cart comes to a stop. So we tend to think: **motion requires force**. This way of thinking arises from our experiences. Before Galileo's and Newton's work, most scientists believed that *the state of rest was the natural state for any object*. *Some external factor such as a push or a pull* (*called "force"*) *was needed to keep things moving. A moving object came to a stop because there was no force acting on it to keep it moving*.



(a)

(b)

Fig. 5.1: **The effect of friction-compared to a) a rough surface like that of a carpet, a ball rolls farther on b) a smoother floor.**

This thinking changed with Galileo's **experiments** with inclined planes. Galileo rolled objects with different initial speeds on different planes, horizontal, sloping up, sloping down, planes of different smoothness to arrive at his conclusions. Galileo, and later Newton, introduced another force, the **force of friction**, to explain the everyday experiences.

You may also have observed in activity (b) that as the surface grew smoother, the distance covered by the ball or the marble increased. Galileo said that it was the force of friction that caused objects to slow down. If we can reduce its effect, the object does not slow down as much and comes to a stop much later (Fig. 5.1). What would happen if friction were reduced to zero? *The ball should keep moving forever.*

This was the reasoning followed by Galileo who introduced the idea of friction. He reasoned that if there were no friction, the ball in his experiments with the inclined planes would keep moving at the same velocity forever! Thus, from his experiments and reasoning, Galileo concluded that **uniform motion does not require force.**

You may ask: How do we decide which is the better theory/model, the earlier one or Galileo's? The followers of the earlier thinking (that motion requires force) would require different explanations for different situations. Galileo's and Newton's idea that **uniform motion does not require force** encompasses a very wide range of experience and experiments. In fact, experiments allow us to distinguish the better theories from the not-so-good ones.

Galileo's out-of-the-box thinking lay in imagining a situation which was *free of all "forces, interactions, push or pulls"* and then see what happened to objects in motion. He came to the conclusion that

"*Any object in motion, if not obstructed, will continue to move with a constant speed along a horizontal line*."

This is called the **law of inertia** because **inertia means the tendency of an object to resist change** and continue doing what it was doing! In respect of motion, it means **the tendency of a body to stay at rest or keep moving at the same speed in the same direction, i.e., with the same constant velocity**. Newton's genius lay in recognizing the power of Galileo's idea of inertia and including it in his formulation of mechanics. Today, we know it as Newton's **first law of motion**. It introduces the concept of force as we know it today.

5.2.1 Concept of Inertia

You may like to do a simple activity for understanding the concept of inertia.

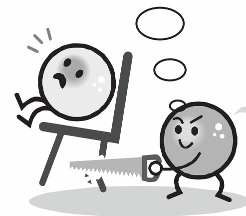
DEMONSTRATION OF THE LAW OF INERTIA

You will need a toy car, a small doll that can fit in the car, a brick or a block of wood (or any other object that can stop the car suddenly), and tape for this activity.

Put a brick or a block of wood at some place on the floor. Push the car on the floor towards the block where it should stop suddenly. If it does not, increase the height of the block. Now put the doll in the car and push it towards the block. Based on this activity, answer the questions given ahead.

Activity

Inertia: Objects tend to keep on doing what they are doing!



What did you observe when the car stopped suddenly at the block? Did the doll fall out? How can you prevent the doll from falling out? Tape the doll and push the car on the floor once again. What do you observe? Does the doll still fall?

What do you think was happening in this activity? Due to its inertia, the doll tended to keep moving even after the car came to a stop. That is why it had a tendency to fall. Taping the doll prevented its fall. **This is why seat belts are used in a car**: to prevent injuries to passengers when it has to stop or brake suddenly. With this experience of inertia, you can now learn Newton's first law.

NEWTON'S FIRST LAW OF MOTION

Consider a body on which no force is exerted. If it is at rest, the body will remain at rest. If the body is in uniform motion, that is, it is moving with constant velocity in a straight line, it will keep moving forever in the same straight line with the same constant velocity (i.e., with the same speed and in the same direction).

Newton's first law of motion gives us the definition of a free body/ free particle.

ALWAYS REMEMBER: A body/particle on which no force is exerted is defined to be a free body/free particle.

So we find that Newton's first law introduces the concept of ***force*** by *describing what happens when it is absent*. And this gives us a way of defining force.

5.2.2 Force

What is force in physics? From Newton's first law of motion, we can infer that *when a force is exerted on an object (by an agent), it changes the object's velocity*. This leads us to a formal definition of force.

FORCE

Force exerted on an object (by an agent) causes its velocity to change, that is, it causes the object to accelerate.

NOTE

In common usage, the words force, power, energy, push, actions are used to express the same meaning.

While watching the above scene in a boxing match, some people say, "What a *powerful* punch!" Others say, "What a *forceful* punch!" And some others say, "There's tremendous energy in the punch!"

But while using the concept of force in physics, you have to be more careful and precise.

Every force has an AGENT, which acts through contact or at a distance from the object. Every force describes the act of an agent on an object. A force cannot act independently without any agent.

Depending on the nature of the agent, forces may be classified as **contact forces** (involving direct contact) or the ones **due to long** or **short-range forces**. Push, pull, friction, tension are examples of contact forces and the force of gravitation is an example of a long-range force. The long-range forces such as the force of gravitation are exerted by each object on every other object in this universe. **To be exerted upon any object, such forces do not require a medium**.

You will learn about the force of gravitation in Unit 7.

You have learnt just now that a force produces acceleration in the body on which it is exerted. Since acceleration is a vector quantity, force should also be a vector quantity. In fact, it has been proved in experiments that force is a vector quantity. This means that when two or more forces act on an object we can find their **resultant** or the **net force** acting on the object using the laws of vector addition (read the margin remark). This *net force* has the same effect on the object as all individual forces acting together have on the object. Thus, Newton's first law is refined as follows.

You have learnt the laws of vector addition in Units 1 and 2 of Block 1 and solved many problems using them.

NEWTON'S FIRST LAW OF MOTION

If no NET force is exerted on a body that is moving, the body's velocity cannot change. If it is at rest, the body will remain at rest. If the body is moving, it will keep moving with a constant velocity (with the same speed and in the same direction).

A body can accelerate only if a non-zero NET external force acts on it.

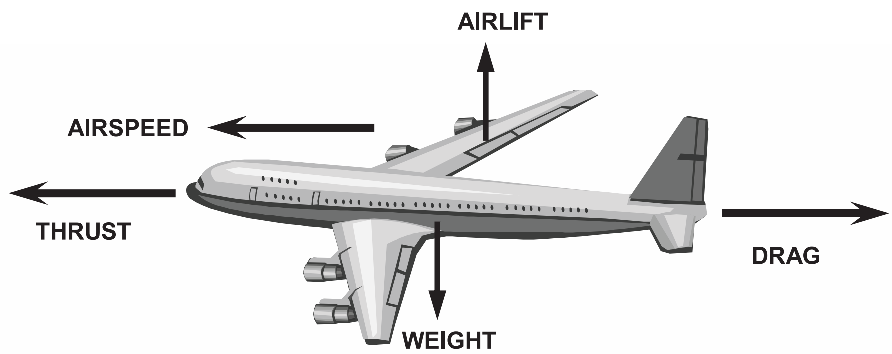
NOTE

In Newton's world, all objects/bodies are to be treated as **point particles**. But **REMEMBER**: point particles do not exist in the real world. They are conceptual objects created by Newtonian theory.

EXAMPLE 5.1: NEWTON'S FIRST LAW OF MOTION

a) Four forces are exerted on an aircraft in flight: airlift, weight of the aircraft, engine thrust and air drag. Suppose an aircraft is flying at a constant airspeed in the horizontal direction (see Fig. 5.2), i.e., its velocity is constant.

Since the aircraft is flying with a constant velocity, there is no net force on it: the engine thrust is equal and opposite to the air drag, and the airlift is equal and opposite to its weight. Now suppose the pilot increases the thrust of the engine, the thrust and air drag are no longer equal; the aircraft accelerates and the velocity increases in the horizontal direction. Thus, a net external force changes the velocity of the object. But the air drag ($\propto v^2$) increases with increased velocity.



Engine Thrust = Air drag and Airlift = Weight.
Plane flies at a constant velocity.

Fig. 5.2: Forces on the aircraft of Example 5.1a.

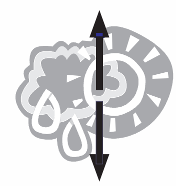
When the new air drag equals the thrust, the aircraft's acceleration becomes zero. It flies at a new constant velocity that is higher than the initial velocity and continues at this constant velocity.

b) When a raindrop falls to the ground, the two forces exerted on it are gravity and air drag (Fig. 5.3a). When its speed is zero, the air drag on it is zero (since it depends on speed).

As the raindrop falls, its speed increases under free-fall acceleration. Therefore, air drag on it increases and at some point becomes equal to the force of gravity. Then the net force on the raindrop is zero and it falls with a constant velocity. This constant velocity is called its **terminal velocity**.

c) We can tighten the head of a hammer by banging the bottom of its handle on the floor (Fig. 5.3b).

Due to inertia, the handle moves but the head of the hammer stays where it was.



Gravity = air drag
Raindrop falls at a constant velocity.

(a)

(b)

Fig. 5.3: Some applications of Newton's first law.

SAQ 1 – Newton's first law of motion

a) A ketchup bottle is turned upside down, thrust hard downwards and then halted to make ketchup flow out of it. Explain why.

b) Two cars travel on a straight road with constant speeds of $60\ \text{kmh}^{-1}$ and $55\ \text{kmh}^{-1}$, respectively. For which car is the net force larger?

Newton's laws do not hold in all frames of reference. They are valid in only special frames of reference known as **inertial frames of reference**. You may like to know: **What is an inertial frame of reference**?

5.2.3 Inertial Frame of Reference

The first law of Newton defines the inertial frame of reference.

INERTIAL FRAME OF REFERENCE

An INERTIAL FRAME OF REFERENCE is one in which Newton's first law (the law of inertia) holds. Thus, a frame of reference at rest or moving with a constant velocity is an inertial frame of reference.

Any other frame of reference at rest or in uniform translational motion (that is, motion in which the respective coordinate axes in the two frames always remain parallel to each other) relative to an inertial frame is also an inertial frame.

REMEMBER: A frame in accelerated motion relative to an inertial frame is NOT an inertial frame. It is a non-inertial frame.

Note that an inertial frame of reference is a non-accelerating frame of reference. Thus, **all frames of reference attached to bodies, which are accelerating are NON-INERTIAL FRAMES OF REFERENCE** (see Fig. 5.4).

(a) (b) (c)

Fig. 5.4: **The frame of reference attached to a) the cyclist is INERTIAL if she moves with a constant velocity with respect to ground; b) the planes are INERTIAL if they fly with constant velocity with respect to each other and with respect to the ground; c) a moving merry-go-round is NON-INERTIAL since the merry-go-round has centripetal acceleration (recall school physics for circular motion).**

From this definition of an inertial frame, a frame attached to the Earth is, strictly speaking, not an inertial frame since it has a centripetal acceleration due to the rotation of the Earth. But since its value is very small ($0.034\ \text{ms}^{-2}$**), we can neglect its effect.**

If we neglect the effect of the Earth's rotation or its motion around the Sun, we can take the Earth or ground as an inertial frame of reference.

The frame attached to any object at rest or moving with constant velocity with respect to the Earth or ground is an inertial frame of reference. Further, all frames of reference moving at constant velocity with respect to the Earth and each other are inertial frames of reference.

Using the definition of an inertial frame of reference, we can check if a frame of reference is inertial or not as follows:

TEST FOR INERTIAL FRAME OF REFERENCE

Take a body at rest or moving with a constant velocity with respect to the Earth at some instant of time. If the body remains at rest or keeps moving with a constant velocity <u>FOREVER</u> (at all later instants of time) with respect to the Earth, then the frame attached to it is an inertial frame of reference.

An observer making measurements in an inertial frame of reference is called an <u>INERTIAL OBSERVER</u>.

You should now be able to distinguish between inertial and non-inertial frames of reference. Try SAQ 2.

SAQ 2 – Inertial and non-inertial frames

Amongst the following frames, identify the inertial and non-inertial frames giving reasons for your answer. Frame (s) attached to:

a) a ball being swung in a circle,

b) a cyclist moving straight north at a constant speed with respect to the ground,

c) a bus moving in a straight line at a constant speed,

d) a cyclist taking a turn at constant speed,

e) a raindrop falling with terminal speed,

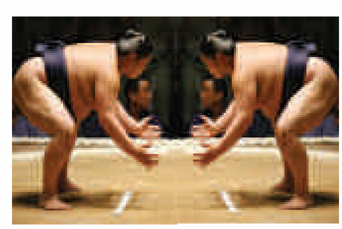
f) an aircraft flying due south at a constant speed and at a constant altitude,

g) a geostationary satellite moving about the Earth, and

h) two trains moving at constant velocity with respect to the Earth.

So far you have learnt about the concepts of inertia, force and inertial frame of reference. Now you may like to know: Can we visualise inertia as a physical property of an object, in the same way as we can visualise its length or volume? The answer is no, we cannot. However, it is related to an object's mass.

(a)



(b)

Fig. 5.5: **Inertia is related to the mass of an object.**

To understand how, consider a fat man and a small boy shown in Fig. 5.5a. Whom would we find it harder to move from their positions of rest? You can immediately say that the fat man would be harder to move. But suppose both were fat and the difference in their masses was small, as shown in Fig. 5.5b? Then who would be harder to move? This is related to the question: **<u>How much force</u> is needed to *<u>change</u>* an object's velocity or to move it from rest? What does it depend on?**

This is what Newton's second law tells us.

NOTE

In Eqs. (5.1 to 5.3), the force acting on the object is **external** to the object. These are called **external forces**. In contrast, there are **internal forces**, which one part of the object exerts on another part of the object or one particle exerts on the other in a system of particles. For example, in Fig. 5.4a, if we take the system to be that of the "boy and the bicycle", the force exerted on the bicycle by the boy and the force exerted on the boy by the bicycle are internal forces for this system. However, if we consider only the boy's motion, then the force on the boy exerted by the bicycle is an external force. Therefore**, we must always identify the system to which Newton's laws are applied**.

5.3 NEWTON'S SECOND LAW OF MOTION

We present the law as stated by Newton and then explain its meaning.

NEWTON'S SECOND LAW OF MOTION

The change of motion of an object is proportional to the force impressed on it; and it is made in the direction of the straight line in which the force is impressed.

What did Newton mean by "change of motion"? He introduced a new quantity, which was the product of mass (m) and velocity $(\vec{\mathbf{v}})$ of an object; it later came to be known as **linear momentum**, $\vec{\mathbf{p}}$:

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}} \tag{5.1}$$

By "change in motion", Newton meant the **rate of change of momentum with time**. From school mathematics, you know that the rate of change of any function with time is nothing but its derivative with respect to time. Thus, Newton's second law gives force as

$$\vec{\mathbf{F}} \propto \frac{d}{dt}(m\vec{\mathbf{v}}) \tag{5.2a}$$

or

$$\vec{\mathbf{F}} = k\frac{d}{dt}(m\vec{\mathbf{v}}) \tag{5.2b}$$

where $\vec{\mathbf{F}}$ is the force exerted and k is the constant of proportionality.

We first take up only those cases in which the **mass** (m) **of the object remains constant**. Then in Eq. (5.2b) only $\vec{\mathbf{v}}$ changes with time and we get

$$\vec{\mathbf{F}} = km\frac{d\vec{\mathbf{v}}}{dt} = km\vec{\mathbf{a}} \quad \text{and} \quad F = kma \tag{5.3}$$

where $\vec{\mathbf{a}}$ is the acceleration of the object.

Thus, an **external force** $\vec{\mathbf{F}}$ exerted on a body of constant mass m produces an acceleration $\vec{\mathbf{a}}$ in the body. It is given by Eq. (5.3).

We define **unit force as the force which produces unit acceleration when it acts on an object of unit mass**. For

$$F = 1 \text{ unit}, \; m = 1 \text{ unit and } a = 1 \text{ unit in Eq. (5.3)}, \; k = 1$$

Thus, Eqs. (5.2b and 5.3) take the form

Newton's second law

$$\vec{\mathbf{F}} = \frac{d\vec{\mathbf{p}}}{dt} = m\vec{\mathbf{a}} \quad \text{and} \quad F = ma \qquad \text{for constant mass} \tag{5.4}$$

A force of magnitude 1 newton produces an acceleration of $1\ \text{ms}^{-2}$ in an object of mass 1 kg:

$$1\,\text{N} = (1\,\text{kg})(1\,\text{ms}^{-2}) = 1\ \text{kg.ms}^{-2}$$

The concept of a <u>FORCE LAW</u> is an essential part of the concept of force. Do not regard $\vec{\mathbf{F}} = m\vec{\mathbf{a}}$ as a force law, which is just an equation of motion. Here $\vec{\mathbf{F}}$ is the force being exerted on a body and is given by a specific force law, e.g., the law of gravitation, Hooke's law, Coulomb's law for electrostatic forces, etc.

5.3.1 Principle of Superposition of Forces

Now suppose **several external forces** are exerted on a body. How does it move under the influence of these forces? The answer to this question is provided by the **principle of superposition of forces**. Let us explain this principle.

PRINCIPLE OF SUPERPOSITION OF FORCES

If several external forces $\vec{\mathbf{F}}_1, \vec{\mathbf{F}}_2, \vec{\mathbf{F}}_3$, etc. are exerted on an object at the same time, then its acceleration is the same as that produced in the body by a single force $\vec{\mathbf{F}}_{net}$, which is the vector sum of all those forces:

$$\vec{\mathbf{F}}_{net} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 + \vec{\mathbf{F}}_3 + \ldots = \sum_i \vec{\mathbf{F}}_i, \qquad i = 1, 2, 3, \ldots \tag{5.5}$$

The symbol Σ is a Greek letter pronounced as sigma. $\sum_i$ used in Eq. (5.5) denotes a sum over the index *i*.

For constant mass, Eq. (5.5) becomes

$$\vec{\mathbf{F}}_{net} = \frac{d\vec{\mathbf{p}}}{dt} = m\vec{\mathbf{a}} \qquad \text{and} \qquad F_{net} = ma \quad \text{for constant mass} \tag{5.6}$$

NOTE

In Eq. (5.6), $\vec{\mathbf{F}}_{net}$ is the net external force being exerted on the object: it is the vector sum or resultant of ALL external forces being exerted ON THE OBJECT.

Thus, **a net external force acting on an object produces a non-zero acceleration in it, which is given by Eq. (5.6) for constant mass.**

We can express Eq. (5.6) in component form in two-dimensional and three-dimensional coordinate systems as:

Newton's 2nd law: Component form

$$F_{net,x} = ma_x \quad \text{and} \quad F_{net,y} = ma_y \quad \text{for constant mass} \tag{5.7a}$$

$$F_{net,x} = ma_x,\ F_{net,y} = ma_y \text{ and } F_{net,z} = ma_z \quad \text{for constant mass} \tag{5.7b}$$

Newton's second law gives a **quantitative definition of force**. You can see that if the net force on an object is zero, its velocity is constant:

$$\vec{\mathbf{F}}_{net} = \vec{\mathbf{0}} \quad \Rightarrow \quad \vec{\mathbf{a}} = \vec{\mathbf{0}}, \text{ that is, } \vec{\mathbf{v}} = \text{constant} \tag{5.8a}$$

The **law holds only in inertial frames of reference** and tells us that

$$\vec{\mathbf{a}} \propto \vec{\mathbf{F}}_{net} \qquad \text{and} \qquad \vec{\mathbf{a}} = \frac{\vec{\mathbf{F}}_{net}}{m} \tag{5.8b}$$

NOTE

The acceleration of an object is in the direction of the net external force and is equal to the net external force divided by the mass of the object.

It also gives us a way of defining mass.

5.3.2 Mass

You may like to do an activity before studying the definition of mass. You will need three toy cars or three balls of different masses, a spring and a smooth chart paper. Do this activity and try to answer the questions related to it.

Activity

NEWTON'S SECOND LAW OF MOTION

Fix the chart paper on a table or on the floor. Hold the spring on a point on the chart paper so that one of its ends is supported against your palm at a fixed point. Label this point as P. Mark another point Q at some distance (less than the length of the spring) on a straight line from P. Hold one of the three cars or balls at the other end of the spring with your palm and push it so that the spring is compressed and that end of the spring is at point Q.

Now release the spring by removing your palm from the car/ball. What happens to the object you were holding? It moves on the table/floor. Practice releasing the object so that it travels in a straight line. Repeat the activity with the other cars/balls. What do you observe? Which car/ball speeds up more and which one less? How can you explain your observations?

What is common in all these situations? The spring is exerting an equal force on all objects since the distance PQ is the same. What is different in each case? It is the mass of the objects. Now answer this question: How is the mass of the object related to its acceleration for the same force being exerted on it?

If possible, repeat the activity with only one car/ball but with three springs of different stiffness (the stiffer the spring, the greater the force). Thus, mass is constant in this activity but the forces are different. What do you observe? How is the acceleration of the car/ball related to the force being exerted on it? How will you explain your observations?

NOTE

The word mass is commonly used around us and may mean a body's size, weight or density. Although these characteristics are confused with mass, they do not represent the mass of a body. **BE CAREFUL WHEN YOU USE THE CONCEPT OF MASS IN PHYSICS.**

From our everyday experience and from the activity you have done, you know that the same force produces different accelerations in bodies of different masses. This result has been tested in countless experiments conducted in inertial frames of reference. The idea is to

- apply equal force on an object having standard mass of 1 kg and another one having unknown mass, and
- measure the acceleration of the two objects in an inertial frame of reference.

Let m_s, a_s be the mass and acceleration of the standard object, respectively and m, a be the mass and acceleration of the unknown object, respectively.

Now you know that since equal force is exerted on the standard and unknown objects, we have

$$F = m_s a_s = ma \tag{5.9a}$$

$$\therefore \quad \frac{m}{m_s} = \frac{a_s}{a} \quad \text{or} \quad m = m_s\left(\frac{a_s}{a}\right) \qquad (5.9b)$$

For example, suppose $m_s = 1\text{ kg}$ and we apply a force of 1.0 N on both objects, and measure the acceleration of the object of mass m to be 0.2 ms^{-2}. From Eq. (5.4), the acceleration of the standard mass is 1.0 ms^{-2}. Thus, we get

$$m = 1.0\text{ kg}\left(\frac{1.0\,\text{ms}^{-2}}{0.2\text{ ms}^{-2}}\right) = 5.0\text{ kg}$$

What happens when we apply a different force, say, 10 N on the object? The acceleration of the standard mass is then $10.0\,\text{ms}^{-2}$. The acceleration of the unknown object is measured to be $2.0\,\text{ms}^{-2}$ and the value of the unknown mass is

$$m = 1.0\text{ kg}\left(\frac{10.0\,\text{ms}^{-2}}{2.0\text{ ms}^{-2}}\right) = 5.0\text{ kg}$$

This result is consistent with our earlier result. Many such experiments have given the same results. This helps us arrive at a reliable way of assigning a mass to a given object.

The mass defined by Eq. (5.9b) is called the **INERTIAL MASS**. Fig. 5.6 shows the masses of various objects.

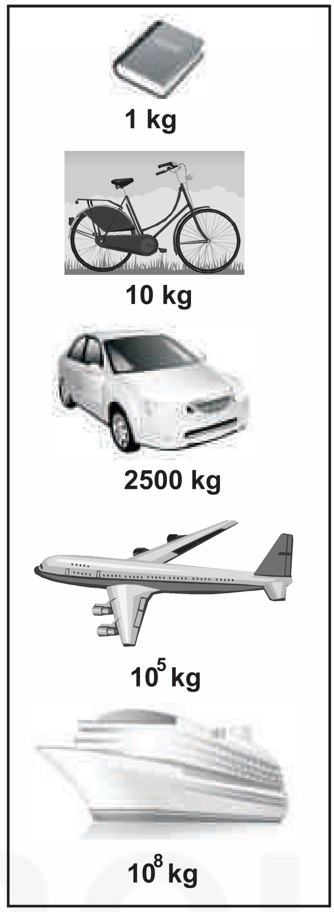


Fig. 5.6: **Masses of different objects**.

Let us revise the concept of mass explained in this section.

Recap

INERTIAL MASS OF AN OBJECT

Mass is an intrinsic characteristic of a body that relates the net external force acting on the body to the resulting acceleration of the body. For the **same net external force, a body having lesser mass has greater acceleration than a body having mass.**

For the **same net external force** $\vec{\mathbf{F}}$, $\quad m_1 < m_2 \Rightarrow \vec{\mathbf{a}}_1 > \vec{\mathbf{a}}_2$

We now take up two examples showing you how to apply Newton's laws.

EXAMPLE 5.2: NEWTON'S SECOND LAW OF MOTION

Determine the acceleration vector for the box of mass 5.0 kg in Fig. 5.7.

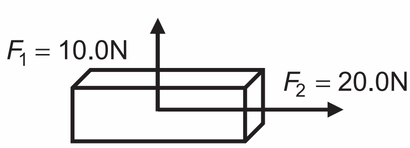


Fig. 5.7

SOLUTION ■ The **KEY IDEA** here is to obtain the resultant force and then apply Eq. (5.7a).

The magnitude of $\vec{\mathbf{a}}$ is

$$a = \sqrt{a_x^2 + a_y^2}$$

$$= \sqrt{(4)^2 + (2)^2}\ \text{ms}^{-2}$$

$$= 4.5\ \text{ms}^{-2}, \text{ and}$$

$$\theta = \tan^{-1}\left(\frac{2}{4}\right) = 27^\circ$$

is the angle $\vec{\mathbf{a}}$ makes with the x-axis. $\vec{\mathbf{a}}$ **is in the same direction as the net force**.

Let us choose the coordinate system such that the x-axis is along $\vec{\mathbf{F}}_2$ and the y-axis is along $\vec{\mathbf{F}}_1$. From the given data, the net force is:

$$\vec{\mathbf{F}} = F_2\,\hat{\mathbf{i}} + F_1\,\hat{\mathbf{j}} = (20.0\text{N})\,\hat{\mathbf{i}} + (10.0\text{N})\,\hat{\mathbf{j}} \text{ and}$$

$$\vec{\mathbf{a}} = \frac{F_2}{m}\,\hat{\mathbf{i}} + \frac{F_1}{m}\,\hat{\mathbf{j}} = \frac{20.0\text{ N}}{5.0\text{ kg}}\,\hat{\mathbf{i}} + \frac{10.0\text{ N}}{5.0\text{ kg}}\,\hat{\mathbf{j}} = 4.0\text{ms}^{-2}\,\hat{\mathbf{i}} + 2.0\text{ms}^{-2}\,\hat{\mathbf{j}}$$

Example 5.3: NEWTON'S SECOND LAW OF MOTION

The mass of an aircraft is 45,000 kg. It is flying in a straight line at a constant speed of $960.0\ \text{kmh}^{-1}$ (Fig. 5.8). The weight of the aircraft equals the airlift. The pilot increases the thrust of the engine to 90,000 N. Suppose the drag force equals the engine thrust in 20.0 s. What is the increased constant speed of the aircraft at that instant? What is the increase in its speed?

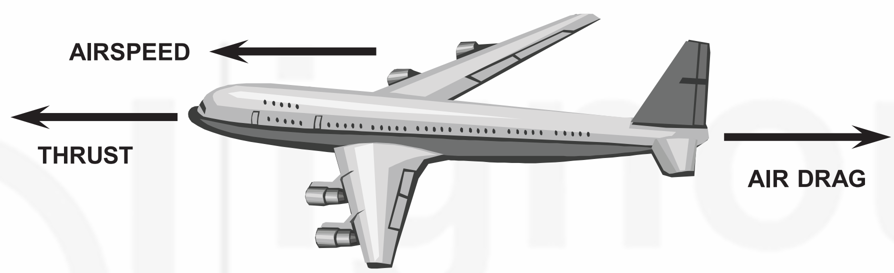


Fig. 5.8: **When engine thrust = air drag, the plane flies at a constant velocity**.

SOLUTION ■ The **KEY IDEA** here is to obtain the acceleration from Newton's second law of motion. Then we obtain the increased constant speed of the aircraft using the equations of kinematics for motion in a straight line.

After the engine thrust is increased, the acceleration of the aircraft is:

$$a = \frac{F_{thrust}}{m} = \frac{90{,}000\text{N}}{45{,}000\text{kg}} = 2.0\ \text{ms}^{-2}$$

When the thrust and drag force become equal in 20.0 s, the plane travels at a constant velocity (Newton's first law). The increased speed is given by the kinematical equation $v = v_0 + at$:

$$v = \left(\frac{960.0}{3.6}\right)\text{ms}^{-1} + 2.0\ \text{ms}^{-2} \times 20.0\ \text{s} \approx 307\ \text{ms}^{-1} = 1105\ \text{kmh}^{-1}$$

The increase in the aircraft's speed is

$$(1105 - 960.0)\ \text{kmh}^{-1} = 145\ \text{kmh}^{-1}$$

You should now solve simple problems based on these two laws.

SAQ 3 – Newton's second law of motion

a) When a child pushes a carom board striker of mass 20 g, it moves on the board in a straight line with an initial speed of $2.0\ \text{ms}^{-1}$. A constant force of friction $F = 0.04\,\text{N}$ is exerted opposite to the motion of the striker. How far will the striker move before it comes to rest?

b) A constant horizontal force is exerted on a large box. As a result, the box moves across a horizontal floor at a constant speed v_0. The magnitude of the constant force exerted on the box is

i) the same as the weight of the box.

ii) greater than the weight of the box.

iii) the same as the total force that resists the motion of the box.

iv) greater than the total force that resists the motion of the box.

v) greater than both the weight of the box and the total force that resists its motion.

c) If the constant horizontal force exerted on the box in part (b) is doubled to push it on the same horizontal floor, the box then moves

i) with a constant speed that is double the speed v_0 in part (b).

ii) with a constant speed that is greater than the speed v_0 in part (b), but not necessarily as great.

iii) for a while with a speed that is constant and greater than the speed v_0 in part (b), then with a speed that increases thereafter.

iv) for a while with an increasing speed, then with a constant speed thereafter.

v) with a continuously increasing speed.

5.4 NEWTON'S THIRD LAW OF MOTION

Imagine that you are moving quickly on the road and bump into someone coming from the opposite direction. No doubt, you feel a force due to this impact. What about the other person? The other person too feels some force. Thus, while you are applying force at that person, that person too is applying force on you. In other words, there is not just one force but a **pair of forces** involved though **both forces are acting on different objects**. Newton was the first to recognize that **all forces occur in pairs**. **There is no such force as an isolated or single force, existing all by itself**. However, **the forces in the pair are not exerted on the same object**. *Each force in the pair is exerted on a different object*.

> Newton's third law is also called the "action-reaction" law because it is also stated as follows:
>
> **"For every action (force), there is an equal and opposite reaction."**

This is evident when we ask: **How does one apply force on an object**? From our everyday experience, we see that some **agent** makes this possible. For example, you move a bag by pushing or pulling it: Your hand is the agent

interacting with the bag. We say that **pairs of forces arise from interactions between bodies**. This is what Newton's third law tells us.

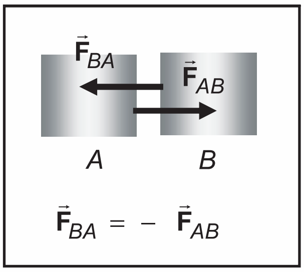


Fig. 5.9: **Newton's third law. Note the order of subscripts to the force**. $\vec{F}_{AB}$ **is the force <u>ON</u>** *A* **<u>BY</u>** *B*.

NEWTON'S THIRD LAW OF MOTION

When two bodies *A* **and** *B* **interact, the force that body** *A* **exerts on the body** *B* **is equal in magnitude and opposite in direction to the force that body** *B* **exerts on body** *A*.

If $\vec{F}_{BA}$ is the force exerted **<u>on</u>** body *B* **<u>by</u>** body *A* and $\vec{F}_{AB}$ is the force exerted **<u>on</u>** body *A* **<u>by</u>** body *B* (Fig. 5.9), then

$$\vec{F}_{BA} = -\vec{F}_{AB} \quad \text{and} \quad \left|\vec{F}_{BA}\right| = \left|\vec{F}_{AB}\right| \qquad (5.10)$$

The pair of forces or interactions also called **action-reaction forces** may be due to actual contact of the bodies or due to long-range forces.

Newton's third law deals with two forces, action and reaction:

- **The action-reaction pair of forces can never be on the same body. <u>THESE FORCES ACT ON DIFFERENT BODIES</u>. Therefore, these do not cancel each other out.**
- **Since each force is applied on a *different* body, they do not produce the same acceleration if the bodies have different masses.**
- **Both forces in the pair must be the same kind of forces (gravitational, electrostatic, frictional, normal, etc.).**
- **Both forces are *instantaneously* equal and opposite, never one after the other. Therefore, it does not matter which one of the two forces is called action and which one reaction.**

For example, if a person pushes a large stationary box, the box pushes back the person with exactly the same force. Whether or not the box starts moving has nothing to do with *the force the* ***box exerts on the person***. The box moves only if there is a **net non-zero force** being exerted on it.

REMEMBER: Each of the forces in the pair of interaction forces in Eq. (5.10) obeys both Newton's first and second laws of motion.

Fig. 5.10 shows some pairs of interaction forces or action-reaction pairs. Let us identify the pair of forces in the **first picture in column 1** of Fig. 5.10. These are: the force $\vec{F}_{21}$ exerted **on** animal 2 **by** animal 1 and $\vec{F}_{12}$ **on** animal 1 **by** animal 2. You may like to label the other action-reaction pairs shown in Fig. 5.10.

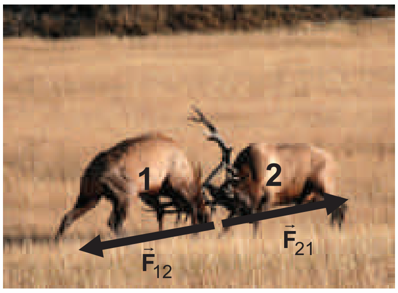
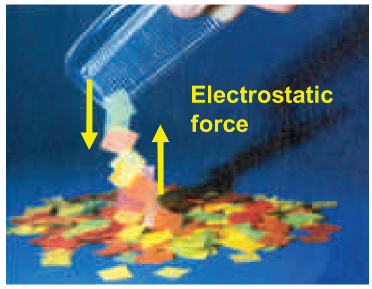
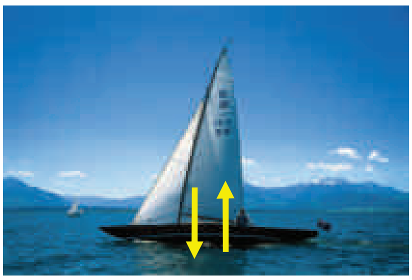
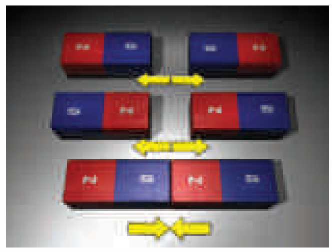
Examples of contact forces	Examples of long-range forces
 (a)	 ©
 (b)	 Magnetic force (d)

Fig. 5.10: **The pair of action-reaction forces in the case of contact forces (column 1) and long-range forces (column 2).**

SAQ 4 – Newton's third law of motion

Identify and suitably label the pair of forces shown for each situation in Figs. 5.10b to d. Express them in the form of Eq. (5.10).

Let us now consider some applications of Newton's third law.

Example 5.4: NEWTON'S THIRD LAW OF MOTION

The Earth attracts an apple with a force of magnitude *F* (Fig. 5.11). What is the magnitude of the force with which the apple attracts the Earth? We see the apple moving towards the Earth. Why do we not see the Earth moving towards the apple?

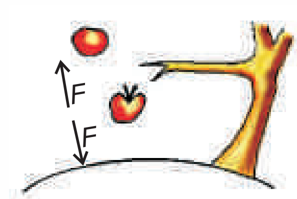


Fig. 5.11: **Diagram for Example 5.4.**

SOLUTION ■ The **KEY IDEA** here is to apply Newton's third law of motion along with the second law.

The apple also attracts the Earth with a force of magnitude *F*. The acceleration of the apple and the Earth are:

$$a_{apple} = \frac{F}{m_{apple}} \qquad \text{and} \qquad a_{Earth} = \frac{F}{m_{Earth}}$$

Since $m_{apple} \approx 0.2$ kg and $m_{Earth} = 6 \times 10^{24}$ kg, m_{Earth} is much larger than m_{apple}. Therefore, the acceleration of the Earth is much smaller ($\approx 10^{-24}$ times that of the apple). Therefore, we cannot see the movement of the Earth towards the apple.

You may like to try an SAQ to check your understanding of Newton's third law.

SAQ 5 – **Newton's third law of motion**

A large truck collides head-on with a small compact car. During the collision:

i) the truck exerts a greater amount of force on the car than the car exerts on the truck.

ii) the car exerts a greater amount of force on the truck than the truck exerts on the car.

iii) neither exerts a force on the other, the car gets smashed simply because it gets in the way of the truck.

iv) the truck exerts a force on the car but the car does not exert a force on the truck.

v) the truck exerts the same amount of force on the car as the car exerts on the truck.

We end this section by repeating the **concept of force** as understood by considering all Newton's laws together. According to these laws:

1. A particle on which the net resultant force is zero remains at rest or moves with a uniform velocity. Such a particle is defined to be a FREE PARTICLE.

2. A **non-zero resultant force** governed by a specific force law acting on a particle **produces accelerated motion**, i.e., it must give rise to a change in its velocity. **The force applied on the particle is equal to its rate of change of linear momentum** and the **change in linear momentum has the same direction as the applied force**. Suppose a non-zero force is exerted on a particle and still the body is either at rest or is moving with uniform velocity. This means that some other equal and opposite force is also being exerted on it so that the resultant force on it is zero.

3. All forces occur in pairs. **There is no such force as an isolated or single force, existing all by itself**.

We now sum up what you have learnt in this unit.

5.5 SUMMARY

Concept	Description
Newton's laws of motion	■ **FIRST LAW:** An object will remain at rest or keep moving with a constant velocity (same speed and direction) unless a net external force is exerted on it by some agent: $\vec{\mathbf{v}}_O = \text{constant} \;\; \textit{unless} \;\; \vec{\mathbf{F}}_{net} \textit{ on } O \neq \vec{\mathbf{0}}$

Newton's laws of motion

- **SECOND LAW:** The net external force exerted on an object by some agent is proportional to the rate of change of its linear momentum ($\vec{\mathbf{p}} = m\vec{\mathbf{v}}$):

$$\vec{\mathbf{F}}_{net} = \frac{d\vec{\mathbf{p}}}{dt}$$

This is the **equation of motion** for the object. For an object or a system of constant mass, it becomes:

$$\vec{\mathbf{F}}_{net} = m\vec{\mathbf{a}} \quad \text{and} \quad F_{net} = ma$$

- **THIRD LAW:** The forces that two interacting objects exert on each other are always equal in magnitude and opposite in direction:

$$\vec{\mathbf{F}}_{on\ A\ by\ B} = -\vec{\mathbf{F}}_{on\ B\ by\ A}$$

Inertial frame of reference

- An **INERTIAL FRAME OF REFERENCE** is one in which the law of inertia holds. Thus, a frame of reference at rest or moving with a constant velocity is an inertial frame of reference.

Force

- **The FORCE exerted on an object (by an agent) causes its velocity to change, that is, it causes the object to accelerate. Every force has an <u>AGENT</u>, which acts through contact or through long or short range interactions with the object. The concept of a** force law is an essential part of the concept of a force**.**

Inertial mass

- **INERTIAL MASS** is an intrinsic characteristic of an object **that relates the net external force acting on the body to the resulting acceleration of the body. For the same net external force, a body having lesser mass has greater acceleration than a body having greater mass.** Inertial mass is the measure of an object's inertia.

Principle of superposition

- If several external forces $\vec{\mathbf{F}}_1, \vec{\mathbf{F}}_2, \vec{\mathbf{F}}_3,$ etc. are exerted on an object at the same time, then its acceleration is the same as that produced in the body by a single force $\vec{\mathbf{F}}_{net}$, which is the vector sum of all those forces:

$$\vec{\mathbf{F}}_{net} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 + \vec{\mathbf{F}}_3 + \ldots = \sum_i \vec{\mathbf{F}}_i, \qquad i = 1, 2, 3, \ldots$$

5.6 TERMINAL QUESTIONS

1. While riding a bicycle, a child falls forward and off the bike if the bike stops suddenly on hitting a curb or rock or some other object. Explain why.
2. State, giving reason, whether each of the following statements is true or false or partially true:
   a) In the absence of forces, every object remains at rest.
   b) An external force is always a push or a pull exerted by an agent in direct *contact* with the object.
   c) A constant force produces a constant velocity.

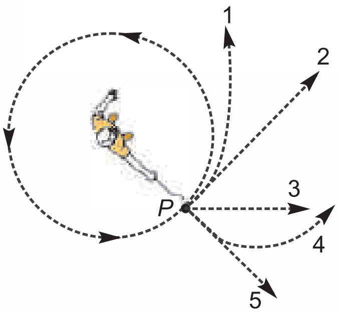


Fig. 5.12: **Diagram for Terminal Question 3.**

d) A *long-range force* must be transmitted by a medium, such as a rope connecting the object and the agent. Therefore, long-range forces cannot act on an object in a vacuum.

e) The engine of a spacecraft moving in outer space fails. It slows down and stops.

3. A steel ball is attached to a string and is swung in a circular path in a horizontal plane as shown in Fig. 5.12. At point *P*, the string suddenly breaks near the ball. If you observe these events directly from above, which of the paths 1 to 5 would the ball most closely follow after the string breaks?

4. In which of the following situations is the net force on the body zero?

   a) a truck driving in a straight line at a constant speed;

   b) a car moving in a circular path at a constant speed.

5. Consider a car at rest. Select the correct option from (a) and (b) to complete the following sentence: We can conclude that the downward gravitational pull of Earth on the car and the upward normal force of Earth on it are equal and opposite because

   a) the two forces form an interaction pair,

   b) the net force on the car is zero,

6. Is a net force being exerted on an object moving with (a) a constant acceleration of $2.0\,\text{ms}^{-2}$ or (b) a constant velocity of $2.0\,\text{ms}^{-1}$? Explain.

7. Newton's second law tells us that an object accelerates when a net force is exerted on it. Must the object accelerate when two or more forces are exerted on it at the same time? Explain.

8. A stone is thrown from the top of a building. As the stone falls, is the net force acting on it zero if we neglect air resistance? Explain.

9. A sail boat moves due to a force of 900 N exerted eastwards by the wind but faces resistance of 400 N due to water flowing due west. What is the acceleration of the boat given that its mass is 500 kg?

10. A car of mass 1500 kg is travelling due west at a speed of 55.0 kmh^{-1}. It comes to a stop in 10.0 s when the driver applies brakes. Determine the magnitude of the average force exerted on the car by using Newton's second law and the equations of kinematics.

5.7 SOLUTIONS AND ANSWERS

Self-Assessment Questions

1. a) When the bottle comes to rest, the ketchup in the bottle continues to move due to inertia and flows out of the bottle.

   b) There is no net force on either car because both are moving with constant speeds.

2. a) Non-inertial since the ball has finite acceleration in circular motion.

b) Inertial since the cyclist moves with constant velocity with respect to the ground.

c) Inertial since the bus moves with constant velocity with respect to the ground.

d) Non-inertial since the cyclist accelerates while turning.

e) Inertial since the raindrop moves with constant velocity with respect to the ground.

f) Inertial since the aircraft flies with constant velocity with respect to the ground.

g) Non-inertial since the satellite has a centripetal acceleration.

h) Inertial since the trains move with constant velocity with respect to the Earth.

3. a) Let us take the *x*-axis to be along the direction of motion of the striker. The force on the striker is $\vec{\mathbf{F}} = -0.04\,\text{N}\,\hat{\mathbf{i}}$. The acceleration of the striker is $\vec{\mathbf{a}} = \dfrac{\vec{\mathbf{F}}}{m} = -\dfrac{0.04\text{ N}}{0.02\text{ kg}}\,\hat{\mathbf{i}} = -2.0\,\text{ms}^{-2}\,\hat{\mathbf{i}}$

To determine the distance the striker moves before it comes to rest, we use the following equation of kinematics: $v^2 - u^2 = 2as$ with $u = 2.0\text{ ms}^{-1}, v = 0, a = -2.0\text{ ms}^{-2}$.

$$\therefore \qquad s = \frac{(2.0\text{ ms}^{-1})^2}{2 \times (2.0\text{ ms}^{-2})} = 1.0\,\text{m}$$

b) (iii). Since the box is moving with constant speed, the net force on it is zero. Hence, the magnitude of the constant force exerted on the box is the same as the total force that resists its motion.

c) (v). Since the applied force is doubled, it is greater than the resisting force in part (b). Hence, a net force acts on the box. Therefore, the box accelerates and moves with a continuously increasing speed.

4. See Fig. 5.13.

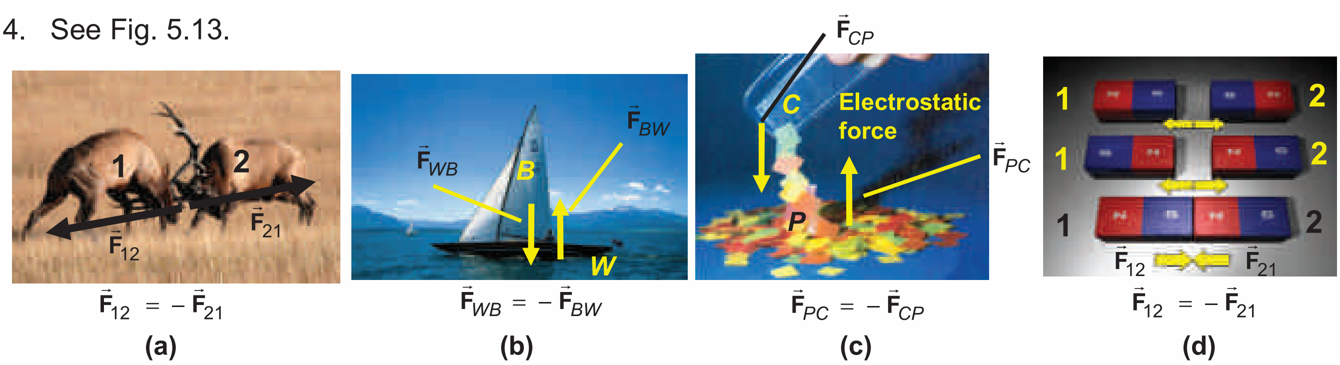


Fig. 5.13: The labelled pair of action-reaction forces in the case of a) and b) contact forces; c) and d) long-range forces.

5. (v) since these are action-reaction forces.

Terminal Questions

1. The bicycle comes to rest when it stops, but due to inertia the child continues to move forward and falls off.

2. a) Partially true. If the object is at rest in the absence of force, it will continue to stay at rest. If the object is moving with a uniform velocity, it will keep moving forever in a straight line with the same velocity.

   b) False. The agent may be at a distance from the object, for example, the gravitational force, which is a long-range force.

   c) False. A constant force produces constant acceleration.

   d) False. Long-range forces do not require a medium.

   e) False. Since the net force on the spacecraft becomes zero, it continues to move with a constant speed since friction is negligible in outer space.

3. The ball follows path 2 at a tangent to the circular path.

4. The net force is zero in (a) because the truck is moving in a straight line with constant speed, i.e., its velocity is constant.

5. (b) The force of gravity exerted by the Earth on the car is equal and opposite to the normal force exerted by the Earth on the car. So the net force on the car is zero.

6. The net force is being exerted on the object moving with a constant acceleration of $2.0\ \text{ms}^{-2}$.

7. The object may or may not accelerate for the following reasons:

   The object will accelerate whenever the net force given by the resultant of the forces is non-zero.

   If the resultant of the forces on the object is zero, it will not accelerate.

8. The net force on the stone is not zero since gravity is acting on it.

9. The net force on the sail boat is (900 – 400) N or 500 N eastwards. The acceleration of the boat is $a = \dfrac{500\,\text{N}}{500\ \text{kg}} = 1\text{ms}^{-2}$ eastwards.

10. Using the equation $v = u - at$,

    where $v = 0,\ u = 55.0\,\text{km h}^{-1} = 15.3\,\text{ms}^{-1}$ and $t = 10.0\,\text{s}$,

    we get $$a = \frac{15.3\ \text{ms}^{-1}}{10.0\,\text{s}} = 1.53\ \text{ms}^{-2}$$

    The average force exerted on the car is

    $$F = 1500\ \text{kg} \times 1.53\,\text{ms}^{-2} = 2295\,\text{N} = 2.30 \times 10^{3}\,\text{N}$$

    We have retained the answer up to 3 significant digits.

Why are the cables of a suspension bridge curved and not horizontal? You will find the answer in this unit!

UNIT 6

APPLYING NEWTON'S LAWS

Structure

6.1 Introduction
  Expected Learning Outcomes
6.2 Forces around Us
  Normal Force
  Friction
  Tension
  Spring Force
  Fundamental Forces in Nature
6.3 Applying Newton's Laws of Motion
  Drawing Free-body Diagrams
  Objects in Equilibrium
  Objects not in Equilibrium
6.4 Uniform Circular Motion
6.5 Summary
6.6 Terminal Questions
6.7 Solutions and Answers

STUDY GUIDE

In this unit, you will learn how to apply Newton's laws of motion to a variety of situations around us in which forces are present. In order to study this unit well, **you should know very well the concepts of vector algebra from Units 1 and 2, the calculus learnt in school mathematics as well as the concepts explained in Unit 5**. You may like to review these concepts before studying this unit.

You should study the methods of solving problems discussed in this unit carefully. You have also to learn how to draw the free-body diagram for an object whose motion is being analysed. Try to do all solved examples on your own and solve all problems. This will help you apply Newton's laws without making mistakes.

"Imagination is more important than knowledge. For knowledge is limited to all we now know and understand, while imagination embraces the entire world, and all there will ever be to know and understand."

Albert Einstein

6.1 INTRODUCTION

In Unit 5, you have studied Newton's laws of motion and the concepts of force and mass. You have learnt that in the equation $\vec{\mathbf{F}}_{net} = m\vec{\mathbf{a}}$, $\vec{\mathbf{F}}_{net}$ is the force being exerted on an object by some agent and has to be given by a **force law**. You may now like to learn: **What are the different types of forces and force laws for which you can apply Newton's laws to predict motion of particles**?

NOTE

In your written work, always use an arrow above the letter you use to denote a vector, e.g., $\vec{r}$. Use a cap above the letter you use to denote a unit vector, e.g. $\hat{r}$.

You are familiar with the forces of friction and tension, and the spring force that we discuss in this unit. These forces result from the contact between two interacting objects and are sometimes classified as "**contact**" forces. We shall describe them briefly in Sec. 6.2. In Sec 6.3 of this unit, you will learn how to solve the equation of motion given by Newton's second law for a variety of objects in motion. We shall consider motion in a straight line and in a plane, which you have also studied in school physics. Finally, in Sec. 6.4 we apply Newton's laws to uniform circular motion.

In the next unit, you will study about the force of gravitation, which acts on all macroscopic objects in the universe and gravity which acts on all macroscopic objects on the Earth. We shall also discuss the concept of weight.

Expected Learning Outcomes

After studying this unit, you should be able to:

- classify forces as fundamental and non-fundamental forces;
- draw free-body diagram for studying the motion of an object;
- apply Newton's laws of motion to simple problems; and
- solve problems related to uniform circular motion.

6.2 FORCES AROUND US

We classify forces in nature into two categories: **fundamental** and **non-fundamental**. The **fundamental forces**, which we discuss in Sec. 6.2.5, are truly unique in the sense that all other forces around us (the **non-fundamental** forces) can be explained in terms of them. For describing the motion of macroscopic particles, it helps to classify the forces as contact forces and long-range forces (see Table 6.1).

The explanation for the forces of deformation such as stress and shear comes from the physics of solids.

Examples of forces acting on extended bodies are **gravitation**, **fluid pressure** and **viscous force**, **solid pressure** and **friction**, **localized contact**, **stress** and **shear**. You will learn about the force of gravitation in Unit 7.

We now consider the following contact forces around us: **Normal force**, **force of friction** (static and kinetic), **tension, forces exerted in pushing and pulling objects, and the spring force**.

Table 6.1: Forces around us.

Contact forces	Long range forces
• ***Contact with solids*** – fixed contact with strings, springs and rigid things, – sliding contact, and – contact on impact. • ***Contact with fluids*** – buoyant, and drag.	• Gravitational (Newtonian), • Electrostatic (Coulomb), and • Magnetic (dipole).

6.2.1 Normal Force

You may like to do a simple activity for understanding the concept of normal force. Press a mattress or any soft surface (that of a soft toy) with your palm. What happens? The soft surface deforms and pushes back on your palm.

According to Newton's third law, the force your palm exerts on the surface is *equal and opposite* to the force exerted by the soft surface on your palm. You can feel this force: Try pushing a hard surface such as a table or a wall with your finger. If you push very hard, you feel pain in your finger because the table/wall exerts equal and opposite force on your finger.

The force exerted by the surface is called the **normal force**. It is called so because it is directed **perpendicular to the surface** (here *normal means perpendicular and not ordinary*). If you press the surface *at an angle*, you can resolve the force you exert into two components: one *parallel* and another *perpendicular* to the surface. We call the **component of the force *perpendicular* to the surfaces in contact as the normal force.**

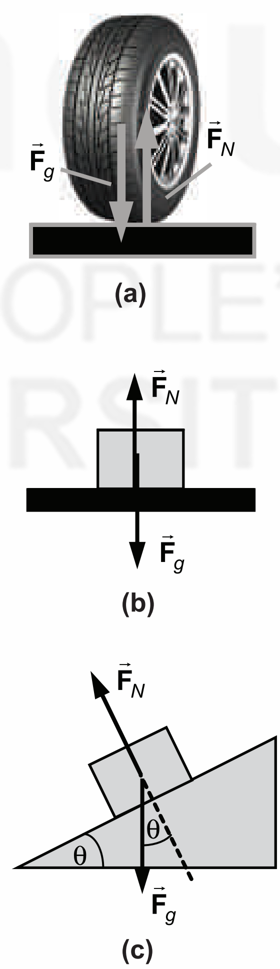


Fig. 6.1: Examples of the normal force.

NORMAL FORCE

When an object presses against another object with a **force having a component perpendicular to the surface** of the other object, the other object (even a rigid one) deforms. **It exerts an equal and opposite NORMAL FORCE on the first object.** The normal force is **PERPENDICULAR TO THE SURFACES IN CONTACT**.

Fig. 6.1 shows three examples of normal force. The tyre exerts a force on the ground (force due to gravity, $\vec{F}_g$) and the ground exerts an equal and opposite *normal force* (say, $\vec{F}_N$) on the tyre (Fig. 6.1a). The block on the horizontal surface exerts a force due to gravity ($\vec{F}_g$) on the surface and the surface exerts an equal and opposite normal force ($\vec{F}_N$) on the block (Fig. 6.1b). In both these cases,

$$F_N = F_g \tag{6.1}$$

In Fig. 6.1c, the block is placed on an incline, and the normal force is equal and opposite to the perpendicular component of $\vec{\mathbf{F}}_g$, i.e., $F_g \cos\theta$:

$$F_N = F_g \cos\theta \quad (6.2)$$

You have studied about the force of gravity in your school physics. It is the force exerted by the Earth on every object on it.

6.2.2 Friction

In Sec. 6.2.1, you have learnt that when an object is in contact with a surface or another object, the **normal force acts PERPENDICULAR to the surfaces of contact**. When the object *moves* or *attempts to move* along the surface, another force is exerted on the object that is **PARALLEL to the surfaces in contact**. This force is called the **frictional force**, the **force of friction** or simply **friction** (Fig. 6.2).

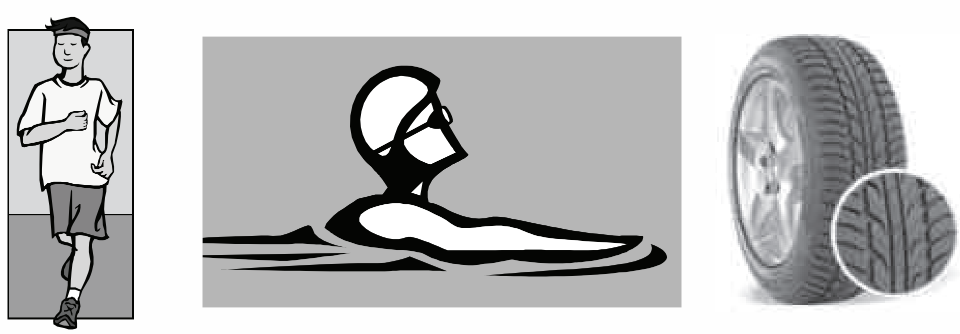
FRICTION

When an object moves or attempts to move along the surface of another object, the **FORCE OF FRICTION** is exerted on the object. It is **directed along the surfaces in contact**, **opposite to the direction in which an object moves or attempts to move**.

Friction arises due to the inter-atomic and inter-molecular interactions on the surface of materials in contact.

Forces of friction are all around us (Fig. 6.2). In most situations, we need to reduce friction; otherwise, they can cause objects in motion to stop. In fact, a great deal of human effort goes into minimising friction. For example, oil between the piston and cylinder walls of a vehicle's engine or grease between parts of any machine reduces friction between them and also their wear and tear.

There are situations in which friction is essential. For example, we would not be able to walk if there were no friction between our feet and the ground. That is why we tend to slip on wet or icy surfaces (because friction is much less on them). Water running between the surface of a tyre and the road reduces friction between them and increases the chances of skidding. That is why the raised treads on a tyre are designed to maintain friction; the spaces in the treads provide channels for water to move out of the way as the tyre rolls over a wet road (Fig. 6.2c). This maintains friction between the tyres and the road and reduces the risk of skidding.



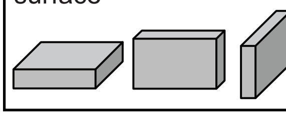
(a) (b) (c)

Fig. 6.2: **Friction is everywhere around us – a) It helps us in walking and we slip if there is no friction; b) we overcome it while swimming; c) it helps us drive on a wet road. As the treaded tyre rolls over a wet road, water flows more easily to the tread's outer edge diverting water away from the regions where the tyre is in contact with the road.**

There are two types of forces of friction: **static friction** and **kinetic friction**. **Static friction is exerted between surfaces <u>at rest</u> with respect to each other**. For example, suppose you push a heavy box and it does not move. This means that there is some force on the box which is equal and opposite to the force with which you push it. This is **<u>static friction</u>**. Now suppose you push harder and the box still does not move. This means that the static friction also increases in magnitude. As you increase the force with which you push the box, a stage comes when the box just starts to move. This would be the **maximum force of static friction** on the box.

NOTE

The maximum force of static friction would be the same, no matter which side of the object is in contact with the surface



The magnitude of the force of static friction can have any value from zero up to a maximum value, depending on the force exerted on any object. The maximum force of static friction is equal to the smallest force needed to move an object and is given by

$$F_s^{max} = \mu_s F_N \tag{6.3}$$

Maximum force of static friction

where μ_s is called the **coefficient of static friction** and F_N is the **magnitude of the normal force** given by Eq. (6.1) or (6.2).

NOTE

The coefficients of static and kinetic friction are dimensionless as these are ratios of forces. So they have no units. Their values depend on the type of material of which the surfaces in contact are made.

Once two surfaces start sliding over one another, a type of force of friction known as the <u>force of kinetic friction</u> is exerted between them. It opposes the relative sliding motion between the two surfaces.

While pushing objects across the floor, you may have noticed that it takes less force to keep them moving. Therefore, we can infer that the force of kinetic friction is less than the force of static friction. It is given by

$$F_k = \mu_k F_N \tag{6.4}$$

Maximum force of kinetic friction

where μ_k is called the **coefficient of kinetic friction** and F_N is the *magnitude of the normal force*.

When one body is in contact with another, always look for

- **normal forces <u>perpendicular</u> to the surface of contact, and**
- **forces of friction <u>along</u> the surfaces in contact.**

6.2.3 Tension

Forces are often exerted by means of ropes, cables or strings attached to an object to pull them (Fig. 6.3). The force with which the rope, string or cable pulls on the object, is called the **tension force** or simply **tension**.

TENSION

TENSION is the force exerted on an object by the pull of a rope, cable, string or any other cord attached to it. **It is directed away from the object and along the cord**. It is called the tension force because the cord is pulled taut and is said to be under tension. **The *tension in the cord* is the magnitude of the force on the object**. It is non-zero if the rope, cable, string is taut and zero if it is slack.

Fig. 6.3: Tension force. In this case, it is the dog pulling the boy!

A rope or a string is often said to be mass-less, i.e., its mass is assumed to be negligible. In such cases, **no net force is required to accelerate the string**. Thus, **the entire tension force applied to one end of a MASS-LESS string, is exerted on the object at its other end**. If, however, the string had some mass, part of the tension force would be used to accelerate it and the force exerted on the object would be less than the tension force exerted on the object.

6.2.4 Spring Force

Springs are used in many things around us such as toys, cars, spring balance. From school physics, you know that when a spring is compressed or stretched by an object attached to it, the spring exerts a force upon the object. This is called the **spring force**. The spring force tries to restore the object to its rest or equilibrium position. That is why it is also called the **restoring force**. For most springs, the magnitude of the spring force is proportional to the distance by which the spring is compressed or stretched.

Consider the spring-mass system shown in Fig. 6.4. One end of the spring is attached to a fixed wall and the other end to a block that can freely slide on a horizontal surface. A **spring force** due to the spring is exerted on the block. If we stretch the spring by pulling the block to the right, the spring force $\vec{\mathbf{F}}$ is exerted on the block in the left direction (Fig. 6.4b). If we compress the spring by pushing the block to the left, the spring force is exerted on the block in the right direction (Fig. 6.4c).

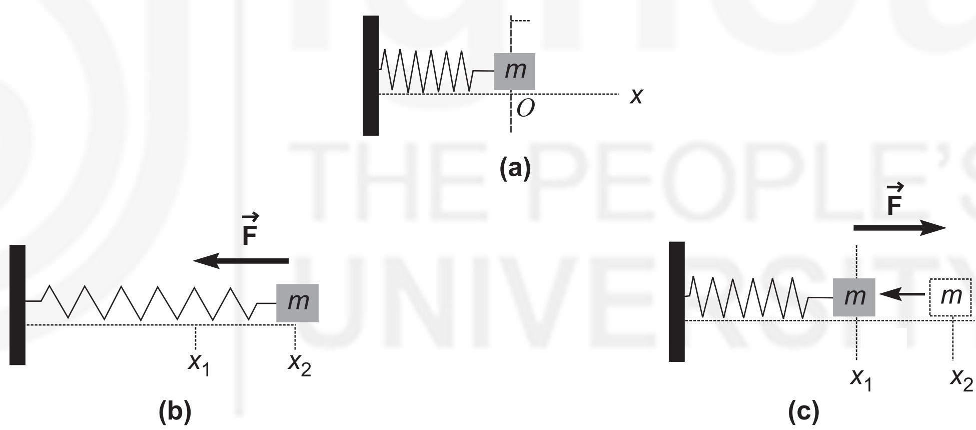


Fig. 6.4: The spring force in a spring-mass system – a) The origin of the x-axis is at the point at which the spring is connected to the block; b) the block is stretched from $x = x_1$ to $x = x_2$; c) the block is compressed from $x = x_2$ to $x = x_1$.

Hooke's law is valid as long as the elastic material of the spring stays within its *elastic limit.* As long as a spring stays within its elastic limit, we can write $F = -kx$.

Experimentally, the magnitude of the spring force has been found to be proportional to the distance by which the spring is stretched or compressed. So, if $\vec{\mathbf{x}}$ is the displacement of the spring, then the spring force is given by

$$\vec{\mathbf{F}} = -k\,\vec{\mathbf{x}} \qquad (6.5a)$$

The negative sign indicates that the spring force always points opposite to the direction of displacement of the free end of the spring. This is also known as Hooke's law for an ideal 1-D spring. For the choice of axis shown in Fig. 6.4a, we can write

$$\vec{\mathbf{F}} = -kx\,\hat{\mathbf{i}} \quad \text{or} \quad F = -kx \qquad (6.5b)$$

The constant k is called the **spring constant** (or **force constant**). It is always positive and is a measure of the stiffness of the spring: the larger k is, the stiffer the spring and the greater the spring force. Let us review the concept of spring force.

Recap

SPRING FORCE

When a spring is compressed or stretched by an object attached to it, the spring exerts a force upon the object. This is called the **spring force**. The spring force tries to restore the object to its rest or equilibrium position. The magnitude of the spring force is proportional to the distance by which the spring is stretched or compressed. So, if $\vec{\mathbf{x}}$ is the displacement of an ideal spring, the spring force is given by

$$\vec{\mathbf{F}} = -k\,\vec{\mathbf{x}} \tag{6.5a}$$

The constant k is called the **spring constant** (or **force constant**).

Finally, we briefly discuss the fundamental forces in nature. But before that, you may like to identify the forces for a few concrete situations.

SAQ 1 – **Forces around us**

In Fig. 6.5, draw the forces being exerted on the cart, the boat and the sled.

Fig. 6.5

6.2.5 Fundamental Forces in Nature

The three fundamental forces known to us are:

- gravitational force,
- electroweak force (which includes the forces of electromagnetism and the weak nuclear force), and
- strong force.

Strictly speaking, we do not refer to non-contact forces in nature as action-at-a-distance forces any more. These are all **force fields**, a concept you will learn about in the higher level physics electives. In this course, we shall be concerned mainly with the **force of gravitation**. You will learn about it along with the concepts of gravity and weight in Unit 7. We present the characteristics of the fundamental forces in Table 6.2.

Table 6.2: Some Characteristics of the Three Fundamental Forces

Force	Relative strength	Range	Importance
Gravitational	10^{-39}	**Infinite**	**Exerted between all macroscopic bodies in the universe.**
Electroweak: *Electromagnetic*	10^{-2}	**Infinite**	**Controls everyday phenomena such as friction, tension, normal force, etc.**
Weak nuclear	10^{-5}	10^{-15} cm	**Nuclear transmutations, beta decay, etc.**
Strong nuclear	**1**	10^{-13} cm	**Holds nucleons together**

The word macroscopic literally means "visible to the naked eye". However, even though we cannot see static charges, we can use Newton's laws to determine the paths of charges, for example, the paths of electrons in a cathode ray oscilloscope.

REMEMBER: Newton's laws of motion are the fundamental laws of mechanics which apply to the motion of any macroscopic particle. These laws establish a relation between the motion of the particle and the forces exerted on it. The net force in the equation of motion $(\vec{\mathbf{F}}_{net} = m\vec{\mathbf{a}})$ can be any of the forces such as friction, tension, spring force, etc. about which you have learnt in this section. We now take examples of each of these forces and show you how to apply Newton's laws of motion to solve a variety of problems.

6.3 APPLYING NEWTON'S LAWS OF MOTION

We begin by explaining the general problem solving strategy for applying Newton's laws to objects in motion. You should follow the steps given in the box to solve such problems.

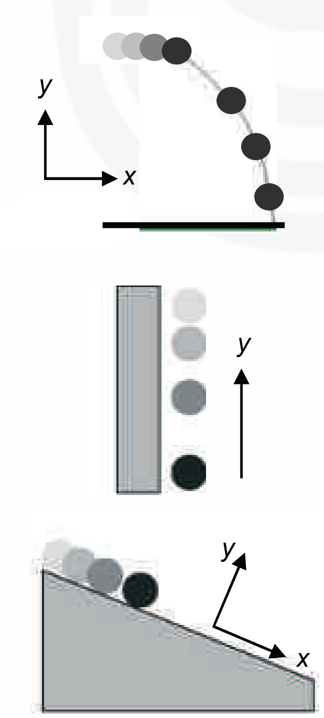


Fig. 6.6: **The coordinate axes can be chosen in different ways depending on the problem**.

PROBLEM SOLVING STRATEGY

For each problem:

1. **Identify all objects** and **forces** (interactions or agents and their types) being exerted on the objects. If possible, draw a simple diagram showing all key features stated in the problem.
2. Draw the **free-body diagram** for the object(s) to which you will apply Newton's laws of motion.
3. Select the inertial frame of reference and write down the laws of motion applicable to the problem.
4. Draw the coordinate axes on the free-body diagram. If the direction of the acceleration is known, choose *x*-axis along that direction. You can choose different frames of reference for each problem (see Fig. 6.6). However, **all of them should be inertial frames**.
5. Write down Newton's second law **in component form** for each body and solve for unknown quantities.
6. Compare your results with what you expect from reasoning and see whether they make sense. You can also check the results by studying special cases for which the answers may be known to you.

Before you start using the problem-solving strategy explained here and apply Newton's laws to motion of objects, you must **understand** the following points:

- Newtonian mechanics as we study today was never presented completely by Newton. It was initiated by Galileo and has been refined and extended by many physicists. Newtonian mechanics holds in a conceptual Newtonian world **constructed** to describe the real physical world. There is a sharp distinction between the real world and the Newtonian world.
- In the Newtonian world, **all objects are treated as (point) particles** (see the Note in the margin). The motion of these particles is analysed in **inertial frames of reference.** The concepts of velocity, acceleration, force, force law as discussed in Unit 5 and so far in this unit apply to particles in this world. Newton's laws hold for **all inertial observers**, that is, observers at rest or moving with a constant velocity with respect to each other.
- Newtonian mechanics is used to **model** real life phenomena in the physical world in terms of motion of particles being acted upon by certain forces.

NOTE

The Newtonian world is populated by (point) particles. Extended bodies in this world are reducible to the particles that compose them. But remember, point particles do not exist in the physical world; they are **conceptual objects** created by Newtonian theory. A rigorous formulation begins with forces **on** and **by** particles and later defines the force on a body as the sum of forces on its particles.

Newton's second law as given here cannot be applied directly to an extended body unless the body is modelled as a particle.

With this general understanding, you can now solve problems using Newton's laws. Before we consider applications of Newton's laws, you should learn how to draw free-body diagrams for any problem.

6.3.1 Drawing Free-body Diagrams

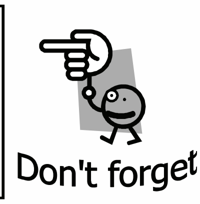
The procedure for drawing a free-body diagram is as follows:

DRAWING A FREE-BODY DIAGRAM

For each problem:

1. Take each body separately in the problem to which you want to apply Newton's laws. Represent the body as a point particle in the diagram.
2. Identify **all** forces being exerted on the body.
3. Draw a force vector for each force being exerted **ON** the body: Place the tail of the vector on the point particle or on the point on which it is being exerted on the body. Draw the vector in the direction of the force. Try to keep the length of the arrow proportional to the magnitude of the force. Label each force vector.
4. Draw all contact forces being exerted on the object as well as the non-contact (e.g., long-range gravitational force) forces on it.

While drawing the free-body diagram, include all forces exerted ON the object. Do not include any internal forces present in the system. DO NOT INCLUDE any forces exerted BY *the object on any other body.*



The free-body diagram is a very useful technique for keeping track of the forces acting on a body. It is also called a **force-diagram**. We now illustrate how to draw a free-body diagram.

EXAMPLE 6.1: DRAWING FREE-BODY DIGRAMS

A box of mass m is pulled on a rough floor by a mass-less string, which makes an angle of 30° with the horizontal (Fig. 6.7a). Draw the free-body diagram for the box. The coefficient of kinetic friction between the box and the floor is μ_k.

SOLUTION ■ The **KEY IDEA** here is to treat the box as a particle, identify all forces exerted **on** the box and show them in the free-body diagram. The forces exerted **on** the box are:

– tension $(\vec{\mathbf{T}})$ in the string,

– weight $(\vec{\mathbf{W}} = m\vec{\mathbf{g}})$ of the box,

– normal force $\vec{\mathbf{F}}_N$ and

– the force of kinetic friction $(\vec{\mathbf{F}}_k)$ between the box and the floor. It acts opposite to the direction of motion of the box.

Let m be the mass of the box. You know from school physics that $\vec{\mathbf{g}}$ is the acceleration due to gravity and acts vertically downward.

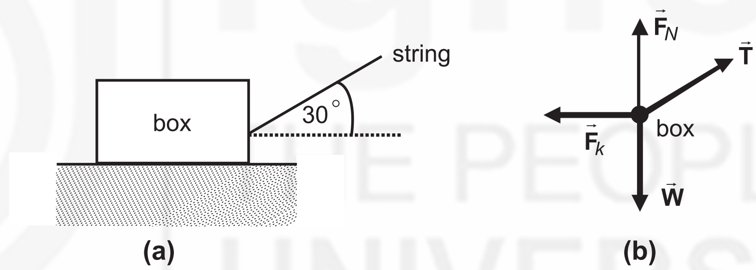


Fig. 6.7: **a) A box being pulled by a string on a rough floor; b) the free-body diagram for the box.**

In Fig. 6.7b, we have drawn the free-body diagram for the box. **Note** that the small circle represents the box as a point particle. **Note also** that we have identified **all** forces **ON** the box. Then we have drawn vectors representing them.

You may now like to draw free-body diagrams for some objects.

SAQ 2 – Drawing free-body diagrams

Draw free-body diagrams for the cart, the boat and the sled shown in Fig. 6.5 of SAQ 1.

We now apply Newton's laws of motion to a variety of systems in which different types of contact forces are being exerted for two basic situations:

when an object on which forces are being exerted is **in equilibrium** and when it is **not in equilibrium**.

6.3.2 Objects in Equilibrium

We first define equilibrium and then take a few examples of stationary and moving objects in equilibrium.

OBJECT IN EQUILIBRIUM

An object on which forces are being exerted **is in equilibrium when the vector sum of all forces (i.e., the net force) being exerted on it is zero. Its acceleration is zero.**

EXAMPLE 6.2: BALANCING LOADS ON HEAD

A common sight around us is people carrying loads on their heads, for example, women in villages carrying water in vessels, porters on railway stations carrying luggage, construction workers carrying materials on their heads, etc. Consider a situation in which a construction worker is standing with a total mass (of the vessel and cement) of 15 kg on her head (Fig. 6.8a). All the weight above our shoulders is primarily supported by the seventh cervical vertebra in our spine (Fig. 6.8b). Let the weight of the woman's neck and head be 50 N. What is the normal force exerted by the vertebra on the head and neck of the woman

a) before she puts the load on her head, and

b) after she puts it on her head? Take $g = 10\,\text{ms}^{-2}$.

SOLUTION ■ The **KEY IDEA** here is that the woman is standing still and therefore, her neck and head are at rest. Thus, the net force acting on them is zero.

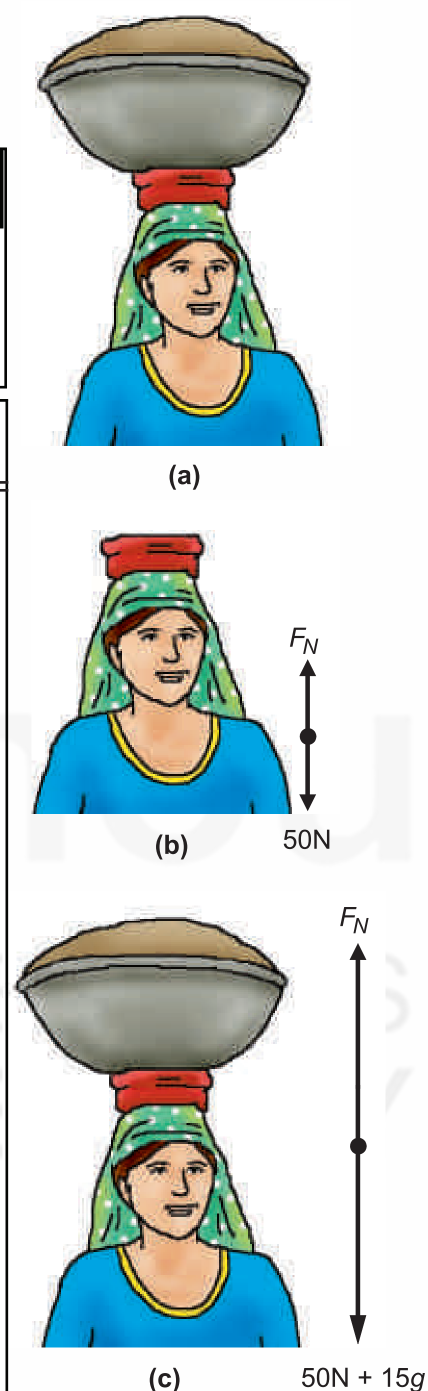


Fig. 6.8: Object in equilibrium.

Figs. 6.8b and c show the forces stated in the problem. Let us identify the **forces on the neck and head of the woman** for both situations. These are:

a) the weight of neck and head of 50 N directed downwards and the normal force directed upwards;

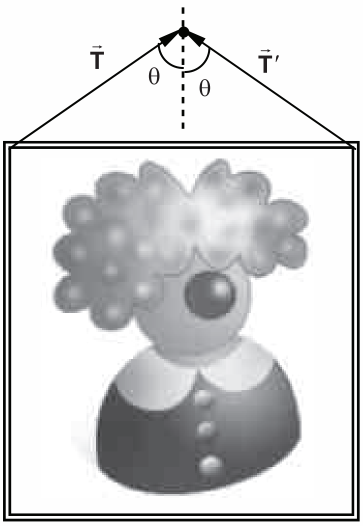
b) the weight of the vessel and cement given by 15g directed downwards plus the weight of 50 N (of the neck and head) directed downwards, and the normal force directed upwards.

Figs. 6.8b and c also show the free-body diagrams for all forces **on** the neck and head of the woman. Since the net force on the neck and head of the woman is zero, the normal force must be equal and opposite to the net weight in each case. Hence, we have

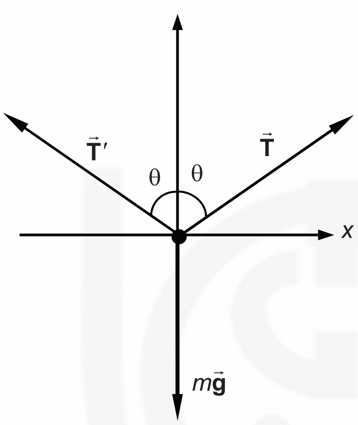
a) $F_N = 50\,\text{N}$ and

b) $F_N = (50\,\text{N} + 15g\,\text{N}) = (50\,\text{N} + 150\,\text{N}) = 200\,\text{N}$

Let us now consider another example of an object in equilibrium.



(a)



(b)

Fig. 6.9: a) Picture in equilibrium; b) free-body diagram for the picture.

EXAMPLE 6.3: PICTURE IN EQUILIBRIUM

A framed picture of mass m is hung on the wall by a mass-less string as shown in Fig. 6.9a. The two parts of the string make an angle θ with the vertical. Determine the tension in the two parts of the string in terms of m and θ.

SOLUTION ■ The **KEY IDEA** here is that the picture is at rest. Thus, the net force acting on it is zero.

Fig. 6.9b shows the forces stated in the problem. Since the string is mass-less, the tension in it is exerted on the picture. Let us identify the **forces on the picture**. These are: its weight $m\vec{\mathbf{g}}$ directed downwards and the tension in the two parts of the string: $\vec{\mathbf{T}}$ and $\vec{\mathbf{T}}'$.

We now draw the free-body diagram with labels and choose the coordinate axes x and y as shown in Fig. 6.9b. Since the net force on the picture is zero, the x and y-components of the net force must be zero. Thus, the sum of the x-components of all forces acting on the picture must be zero and also the sum of the y-components of those forces must be zero:

$$F_{net,x} = \sum F_x = 0 \quad \text{and} \quad F_{net,y} = \sum F_y = 0$$

Since $m\vec{\mathbf{g}}$ is directed vertically downwards along the negative y-axis, on resolving $\vec{\mathbf{T}}$ and $\vec{\mathbf{T}}'$ along the x and y-axes, we get:

$$T\cos(90^\circ - \theta) + T'\cos(90^\circ + \theta) = 0 \quad \Rightarrow \quad T = T' \text{ and}$$

$$T\sin(90^\circ - \theta) + T'\sin(90^\circ + \theta) - mg = 0$$

or $$T\cos\theta + T'\cos\theta - mg = 0 \quad \Rightarrow \quad T = \frac{mg}{2\cos\theta} \quad (\because T = T')$$

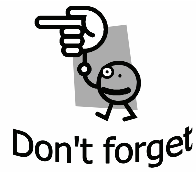
This result tells us that if θ is increased, cos θ will decrease and the tension will increase. This may cause the string to break.

Example 6.3 also explains why the cables supporting a suspension bridge are so curved instead of being horizontal. This is to keep the angle θ that the cable makes with the vertical as small as possible (Fig. 6.10).

Fig. 6.10: **Suspension bridge**.

Always remember the following:

> **An object can be moving and still be in equilibrium provided its velocity is constant, that is, it moves in a straight line with constant speed.**



We now consider the equilibrium of a moving object.

EXAMPLE 6.4: EQUILIBRIUM AT CONSTANT VELOCITY

A plane is flying with constant speed along a straight line at an angle of 30° with the horizontal (Fig. 6.11). The weight $\vec{\mathbf{W}}$ of the plane is 80,000 N and its engine provides a thrust $\vec{\mathbf{T}}$ of 100,000 N in the direction of flight. Two additional forces are exerted on the plane:

- the lift force $\vec{\mathbf{F}}_1$ perpendicular to the plane's wings, and
- the force $\vec{\mathbf{F}}_2$ due to air resistance opposite to the direction of motion.

Determine $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$.

SOLUTION ■ The **KEY IDEA** here is that the plane is moving at a constant velocity. Thus, the net force exerted on it is zero.

Fig. 6.11a shows the forces stated in the problem. Let us draw the free-body diagram for the plane along with the coordinate axes (Fig. 6.11b). Let us choose the *x*-axis to be along the direction of motion. Since the net force on the plane is zero, we can put the sum of the *x* and *y* components of the forces to be equal to zero:

$$\sum F_x = -W\sin 30^\circ + T - F_2 = 0 \quad \text{and} \quad \sum F_y = -W\cos 30^\circ + F_1 = 0$$

Hence, $F_2 = T - W\sin 30^\circ = 100{,}000\,\text{N} - 80{,}000\,\text{N}.(\frac{1}{2}) = 60{,}000\,\text{N}$

and $F_1 = W\cos 30^\circ = 80{,}000\,\text{N}.(\frac{\sqrt{3}}{2}) = 69{,}282\,\text{N}$

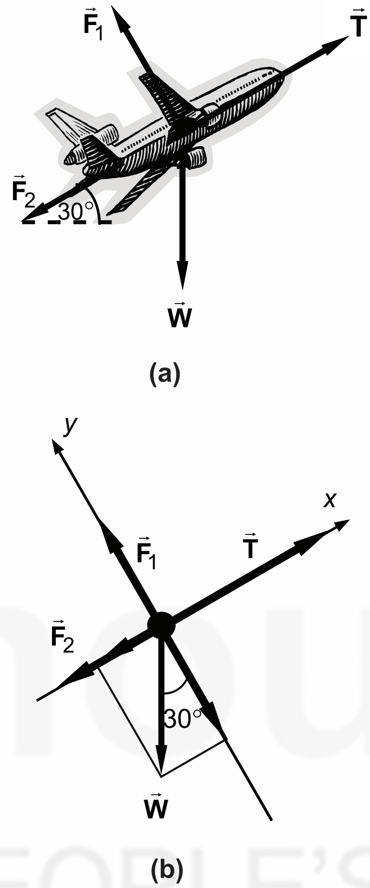


Fig. 6.11: Equilibrium of a moving object.

EXAMPLE 6.5: FRICTION, TENSION AND EQUILIBRIUM

A box of mass 20 kg is pulled up by a rope at a constant velocity on a rough inclined plane, which makes an angle of 30° with the horizontal (Fig. 6.12a). Determine the coefficient of kinetic friction between the box and the plane's surface given that the box moves at a constant velocity and the tension in the rope is 200 N. Assume the rope to be mass-less. Take $g = 10\,\text{ms}^{-2}$.

SOLUTION ■ The **KEY IDEA** here is that the box is moving at a constant velocity. Thus, the net force being exerted on it is zero.

Fig. 6.12b shows the free-body diagram for the box along with the coordinate axes. Let us choose the *x*-axis to be along the direction of motion. Since the net force on the box is zero, we can put the respective sums of the *x* and *y*-components of the forces to be equal to zero.

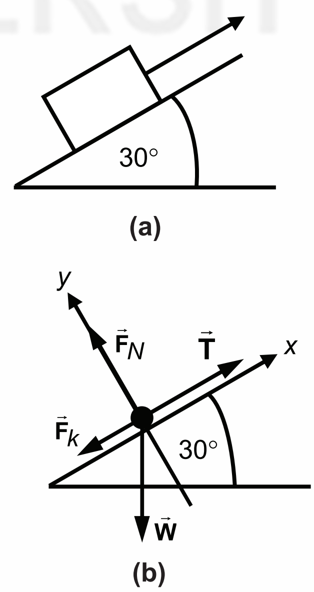


Fig. 6.12: Motion on an inclined plane.

$$\sum F_x = T + (-F_k - W_x) = 0 \quad \text{(i)}$$

and $$\sum F_y = F_N - W_y = 0 \quad \text{(ii)}$$

where $$F_k = \mu_k F_N,\ W_x = mg\sin 30^\circ \text{ and } W_y = mg\cos 30^\circ \quad \text{(iii)}$$

We now have to solve Eqs. (i) and (ii) to obtain μ_k.

For this, we substitute F_N from Eq. (ii) in Eq. (iii) to get

$$F_k = \mu_k F_N = \mu_k W_y = \mu_k mg\cos 30^\circ \quad \text{(iv)}$$

Then substituting F_k from Eq. (iv) in Eq. (i), we get

$$T - \mu_k mg\cos 30^\circ - mg\sin 30^\circ = 0 \quad \text{(v)}$$

From Eq. (v)

$$\mu_k = \frac{T - mg\sin 30^\circ}{mg\cos 30^\circ} = \frac{200 - 20\times 10\times \frac{1}{2}}{20\times 10\times \frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} = 0.58$$

You may now like to solve problems to check whether you have understood these concepts.

SAQ 3 – Applying Newton's laws

a) A table of mass 45.0 kg rests on the floor. A child of mass 30.0 kg stands on the table to reach the top of an almirah. Determine the magnitudes of the normal force that (i) the floor exerts on the table and (ii) the table exerts on the child. Take $g = 9.80\,\text{ms}^{-2}$.

b) A child is sliding down with constant velocity, on an icy hill having slope θ (Fig. 6.13). Determine the coefficient of kinetic friction given that the combined mass of the child and the sled is *m*.

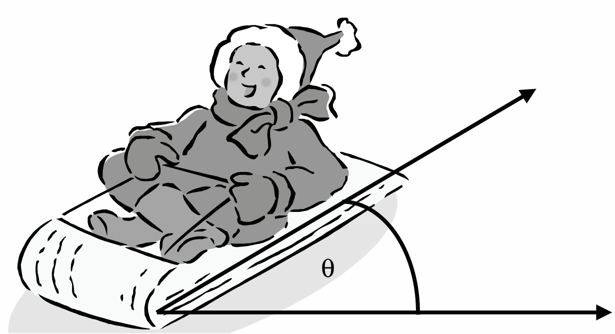


Fig. 6.13

Let us now consider the case when the objects in motion are accelerating, that is, the net force on them is non-zero and they are not in equilibrium.

6.3.3 Objects not in Equilibrium

From the definition of an object in equilibrium, it follows that **an object will not be in equilibrium if the net force on it is not zero**, that is, **if it is accelerating**. Let us illustrate this situation with the help of a simple example.

EXAMPLE 6.6: ACCELERATED MOTION UNDER GRAVITY

A crate of mass M slides down a rough ramp, which has a slope of θ (Fig. 6.14a). Obtain an expression for its acceleration.

SOLUTION ■ The **KEY IDEA** here is to identify all forces being exerted **on** the crate and then apply Newton's laws of motion.

Fig. 6.14b shows the forces on the crate. These are the force of **gravity** (or the **weight** of the crate, $\vec{\mathbf{W}}$), the **normal force** $\vec{\mathbf{F}}_N$ and the **force of kinetic friction** $\vec{\mathbf{F}}_k$ (since the ramp has a rough surface and the crate is moving).

Let us draw the free-body diagram for the crate along with the coordinate axes. We *choose* the positive x-axis to be opposite to the direction of motion (Fig. 6.14b).

Let us now resolve the forces along the x and y-axes and apply Newton's second law. Since the crate is not moving along the y-axis, the net force in that direction is zero. Since the crate is sliding in the opposite direction, we equate the net force in the positive x-direction to ($-Ma$). Thus, we get:

$$\sum F_x = -Ma = F_k - Mg\sin\theta \qquad \text{(i)}$$

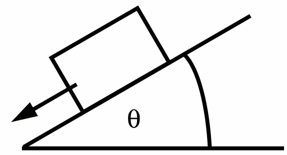
and $$\sum F_y = F_N - Mg\cos\theta = 0 \qquad \text{(ii)}$$

From Eq. (ii), $F_N = Mg\cos\theta$

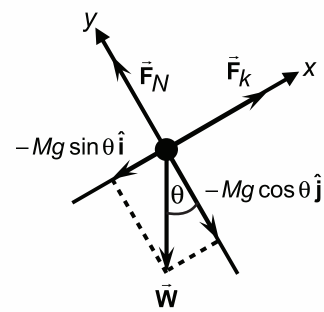
$$\therefore \quad F_k = \mu_k F_N = \mu_k Mg\cos\theta$$

From Eq. (i), $-Ma = \mu_k Mg\cos\theta - Mg\sin\theta$

$$\therefore \quad a = g(\sin\theta - \mu_k\cos\theta)$$



(a)



(b)

Fig. 6.14: a) A crate moving down on a rough ramp; b) free-body diagram for the crate.

Some more situations of the kind we have described so far are shown in Fig. 6.15.

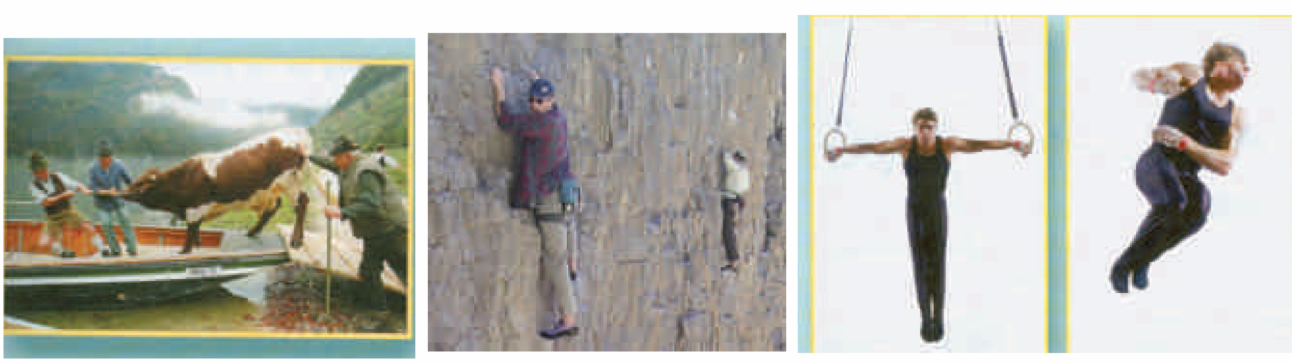


Fig. 6.15: Some more situations in which Newton's laws apply.

In the box below, we summarise what you have studied so far. You have learnt about four external contact forces. You have learnt how to solve problems for motion of objects in equilibrium and accelerated motion under these contact forces except for the spring force. You will learn about the spring force in Block 4 of this course.

Recap

External Contact Forces	Newton's Laws
1. **Normal Force**	$\sum \vec{\mathbf{F}} = m\vec{\mathbf{a}}$
2. **Friction**	
3. **Tension**	**Equilibrium:** $\vec{\mathbf{a}} = \vec{\mathbf{0}}\ \text{ms}^{-2}$
4. **Spring Force**	**Non-equilibrium:** $\vec{\mathbf{a}} \neq \vec{\mathbf{0}}\ \text{ms}^{-2}$

SAQ 4 – **Applying Newton's laws**

A box of mass 15 kg is being pulled on the floor by a mass-less rope with a force of 80 N at an angle of 60° to the horizontal. What is the acceleration of the box if the coefficient of kinetic friction between the floor and the box is $\mu_k = 0.20$? Take $g = 10\,\text{ms}^{-2}$.

So far you have learnt how to apply the equations of motion to objects moving along a straight line. However, in school physics, you have studied about the motion of a projectile. It is a particle launched with an initial velocity, which then falls under gravity. Its path is a parabola. This is an example of motion in a plane along a curved path. **Circular motion** is another important example of motion in a plane. In the following section, you will learn about objects in **uniform circular motion** and analyse it using Newton's laws.

6.4 UNIFORM CIRCULAR MOTION

Motion in a circle is quite common around us. You can yourself experience it by doing the following activity.

Activity

Tie a stone to a string and whirl it around in a horizontal circle. How would you describe its motion? Is the stone undergoing acceleration?

Observe vehicles moving around turns in the road or go to a park and observe children moving on a merry-go-round. What kind of motion is this?

What path does the Moon follow as it revolves around the Earth? What is the path of the Earth and the other planets as they revolve around the Sun?

Suppose you fix a bright arrow on the edge of a fan blade. What path does that arrow follow when you turn on the fan?

Open or shut the door and observe the path followed by its handle. What is the shape of the path?

Did you notice while doing the activity that all these objects move in a circle or in the arc of a circle? Other such examples are: Artificial satellites moving in circular orbits around the Earth, electrons moving near the centre of an electromagnet. In the shot-put throw in any athletic competition, the thrower swings the shot-put in a circle many times to give it increased speed. These are all examples of circular motion (which always takes place in a plane). In this section, we focus on **uniform** circular motion (Fig. 6.16). Let us define it.

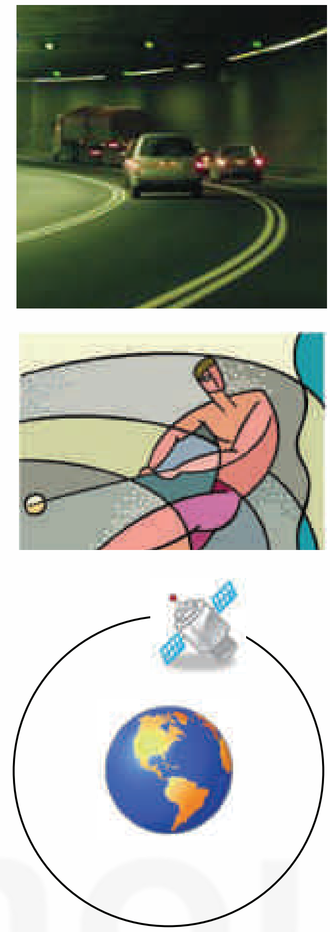


Fig. 6.16: **Examples of circular motion.**

UNIFORM CIRCULAR MOTION

A particle that moves around a circle or a circular arc at a **constant** (uniform) **speed** is said to undergo **UNIFORM CIRCULAR MOTION**.

Let us first determine the velocity and acceleration of a particle in uniform circular motion.

Consider an object moving in a circle with uniform speed (Fig. 6.17). Note that even though its speed is constant, the **direction of its velocity** changes continuously. This means that its **velocity changes continuously** and the object has **non-zero acceleration**. We shall now model the object as a particle and determine the acceleration of a particle undergoing uniform circular motion.

Note that the particle is moving in a circle. Therefore, we use the two-dimensional Cartesian coordinate system to describe its motion.

Let the centre of the circle be at the origin *O* of the coordinate system. In Fig. 6.17a, we show the position vector of the particle with respect to *O* at two different instants of time. Note that the position vector's magnitude will be the same at all instants of time and equal to the radius *r* of the circle. Do you notice that its direction changes with time?

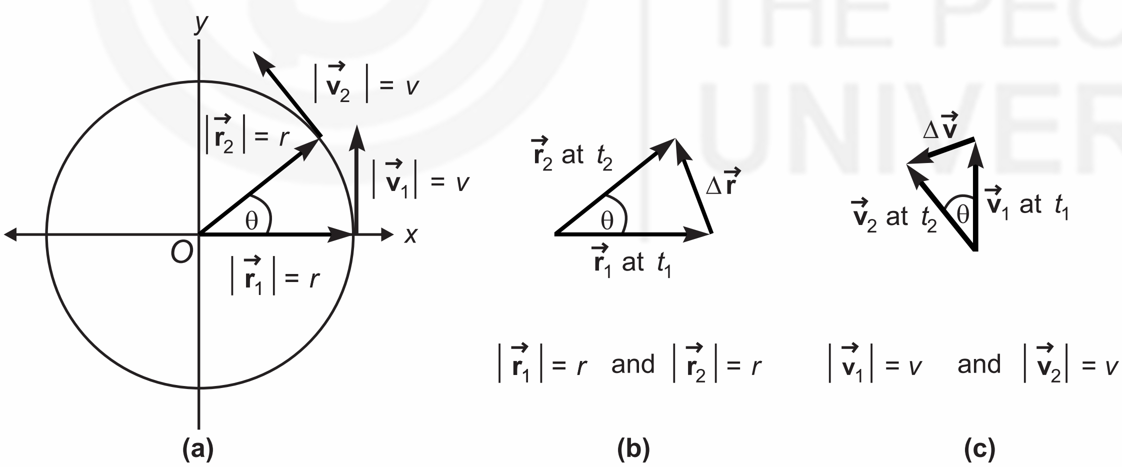


Fig. 6.17: **a) The magnitude of the position vector of a particle in uniform circular motion is constant and equal to the radius of the circle. Its velocity vector at any point is directed along the tangent to the circle at that point and is always perpendicular to the position vector; b) position vectors of the particle at two instants of time; c) velocity vectors of the particle at those instants of time. Note that the magnitude of its velocity (that is, its speed) is constant.**

Let $\vec{\mathbf{r}}$ be the position vector of the particle at any instant of time *t* (see Fig. 6.18a). In component form, it is given by

$$\vec{\mathbf{r}} = r(\cos\theta\,\hat{\mathbf{i}} + \sin\theta\,\hat{\mathbf{j}}) \qquad (6.6a)$$

where θ is the angle by which the particle moves in the circle in time *t*. It is also the angle its position vector makes with the positive *x*-axis at the instant *t*.

From the definition of the unit vector you have learnt in Unit 1, you know that

$$\vec{\mathbf{r}} = r\,\hat{\mathbf{r}} \tag{6.6b}$$

where the unit vector $\hat{\mathbf{r}}$ is given by

$$\hat{\mathbf{r}} = (\cos\theta\,\hat{\mathbf{i}} + \sin\theta\,\hat{\mathbf{j}}) \tag{6.6c}$$

We define the angular speed of the particle as

$$\omega = \frac{\theta}{t} \tag{6.6d}$$

Thus, we can write

$$\vec{\mathbf{r}} = r\,(\cos\omega t\,\hat{\mathbf{i}} + \sin\omega t\,\hat{\mathbf{j}}) \tag{6.6e}$$

Recall from Example 2.4 of Unit 2 that the **velocity** of the particle is given by

$$\vec{\mathbf{v}} = \frac{d\vec{\mathbf{r}}}{dt} \tag{6.7a}$$

or

$$\vec{\mathbf{v}} = \frac{d}{dt} r\,(\cos\omega t\,\hat{\mathbf{i}} + \sin\omega t\,\hat{\mathbf{j}}) = \omega r\,(-\sin\omega t\,\hat{\mathbf{i}} + \cos\omega t\,\hat{\mathbf{j}}) \tag{6.7b}$$

and its magnitude is given by

$$|\vec{\mathbf{v}}| = v = \omega r \tag{6.7c}$$

$\vec{\mathbf{r}}\,.\,\vec{\mathbf{v}} =$
$[r\,(\cos\omega t\,\hat{\mathbf{i}} + \sin\omega t\,\hat{\mathbf{j}})\,.$
$\omega r\,(-\sin\omega t\,\hat{\mathbf{i}} + \cos\omega t\,\hat{\mathbf{j}})]$
$= \omega r^2\,(-\sin\omega t\cos\omega t$
$+ \sin\omega t\cos\omega t) = 0$

Recall from Sec. 2.4.2 of Unit 2 that **the direction of the velocity at any point on the circle is along the tangent at that point** (Fig. 6.18b). You also know that for a circle, the *tangent at any point is perpendicular to the radius*. Therefore, **the velocity vector of a particle at any given point on the circle is perpendicular to its position vector at that point** (see Fig. 6.18a). You can indeed verify (read the margin remark) that for circular motion

$$\vec{\mathbf{r}}\,.\,\vec{\mathbf{v}} = 0 \tag{6.8}$$

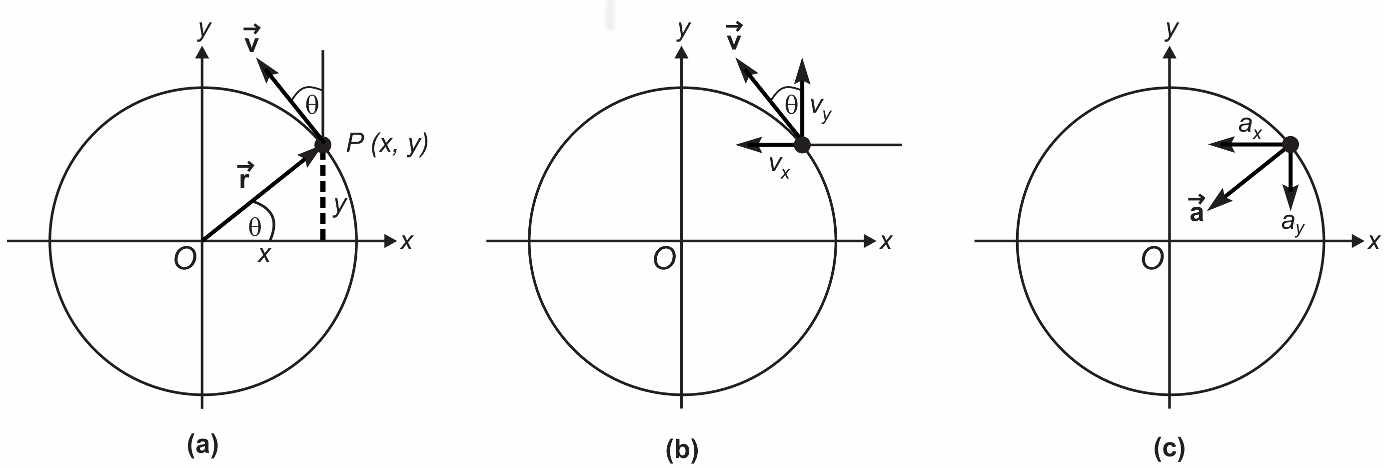


Fig. 6.18: **a) The position and velocity vectors of a particle in uniform circular motion in the anticlockwise direction having coordinates (*x*, *y*) at an instant *t*; b) the *x* and *y* components of the particle's velocity vector at *t*; c) the acceleration vector of the particle at *t* along with its *x* and *y* components.**

The **acceleration** of the particle in uniform circular motion is given by

$$\vec{\mathbf{a}} = \frac{d\vec{\mathbf{v}}}{dt} = \frac{d}{dt}\left[\omega r\,(-\sin\omega t\,\hat{\mathbf{i}} + \cos\omega t\,\hat{\mathbf{j}})\right] = -\omega^2 r\,(\cos\omega t\,\hat{\mathbf{i}} + \sin\omega t\,\hat{\mathbf{j}}) \qquad (6.9a)$$

or

$$\vec{\mathbf{a}} = -\,\omega^2 r\,\hat{\mathbf{r}} \qquad (6.9b)$$

Since $v = \omega r$ [see Eq. (6.7c)], we can write

$$\vec{\mathbf{a}} = -\frac{v^2}{r}\hat{\mathbf{r}} \qquad (6.9c)$$

Centripetal acceleration

The magnitude of $\vec{\mathbf{a}}$ is $a = \dfrac{v^2}{r}$ (6.9d)

When we express $\vec{\mathbf{a}}$ in the form of Eqs. (6.9b or c), the direction of the vector $\vec{\mathbf{a}}$ becomes clear immediately. It is opposite to $\hat{\mathbf{r}}$ as shown by the negative sign in Eq. (6.9b or c). Thus, the **acceleration vector for uniform circular motion is directed opposite to the position vector of the object and towards the centre of the circle** (Fig. 6.18c). That is why it is called **centripetal acceleration**. (The word *centripetal* means *centre-seeking*.)

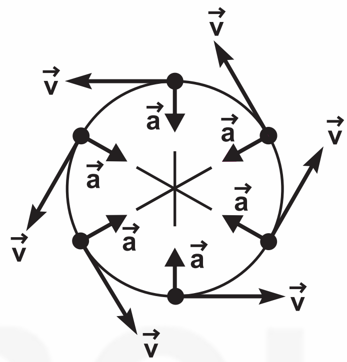
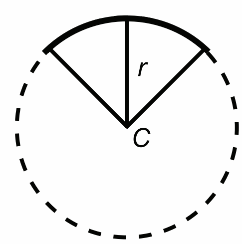


Fig. 6.19: **Velocity and acceleration vectors in uniform circular motion.**

Now study Fig. 6.19. It shows that $\vec{\mathbf{a}}$ is perpendicular to $\vec{\mathbf{v}}$ for uniform circular motion. Note also that the **directions of** $\vec{\mathbf{r}}$, $\vec{\mathbf{v}}$ **and** $\vec{\mathbf{a}}$ **change continuously as the particle moves in the circle**. Eqs. (6.6a to 6.9d) for the position vector, velocity and acceleration derived for uniform circular motion also apply to **particles in uniform motion along a circular arc or a curved path**. When a particle moves in a circular arc or a curved path, then r is the radius of the circle which can be drawn through the arc or the curved path. It is called the **radius of curvature** of the arc or of the curved path (read the note in the margin).

NOTE

The **radius of curvature** (r) of the arc or the curved path is the radius of the circle which can be drawn through that arc or the curved path. The centre of this circle (C) is called the **centre of curvature**.



You also know that a finite acceleration in an object means that a **net force** is being exerted on it. This force is called the **centripetal force**. It is defined as follows:

CENTRIPETAL FORCE

The net force required to keep a particle of mass *m* moving at a constant speed *v* in a circular path of radius *r* with centripetal acceleration (v^2/r) **is called the CENTRIPETAL force.** Its **magnitude** is:

$$F_c = \frac{mv^2}{r} \qquad (6.10a)$$

The centripetal force is always directed towards the centre of the circle and its direction changes continuously as the particle moves in a circle. In the unit vector notation, we can express the vector $\vec{\mathbf{F}}_c$ as follows:

$$\vec{\mathbf{F}}_c = -\frac{mv^2}{r}\hat{\mathbf{r}} = -\,m\omega^2 r\,\hat{\mathbf{r}} \qquad (6.10b)$$

Centripetal force

Let us now revise the concepts of uniform circular motion, centripetal acceleration and centripetal force.

Recap

UNIFORM CIRCULAR MOTION, CENTRIPETAL ACCELERATION AND CENTRIPETAL FORCE

A particle moving with a constant speed in a circle of radius r or a circular arc at a **constant** (uniform) **speed** is said to undergo **UNIFORM CIRCULAR MOTION**.

The particle experiences **centripetal acceleration**. It is given by

$$\vec{\mathbf{a}} = -\,\omega^2 r\,\hat{\mathbf{r}} = -\frac{v^2}{r}\hat{\mathbf{r}}$$

The **magnitude** and **direction** of the centripetal acceleration are given as follows:

Magnitude: $a = \omega^2 r = \dfrac{v^2}{r}$

Direction: Centripetal acceleration vector always points towards the centre of the circle or the centre of curvature of the path of motion. It changes continuously as the particle moves.

The **net force** required to keep the particle (of mass m) **executing uniform circular motion** in a circular path of radius r with **centripetal acceleration** (v^2/r) is called the **centripetal force** and is given by

$$\vec{\mathbf{F}}_c = -\frac{mv^2}{r}\hat{\mathbf{r}} = -\,m\omega^2 r\,\hat{\mathbf{r}}$$

The **magnitude** and **direction** of the centripetal force are given as follows:

Magnitude: $F_c = m\omega^2 r = \dfrac{mv^2}{r}$

Direction: It is in the direction of the centripetal acceleration, directed always towards the centre of the circle or the centre of curvature of the path of motion.

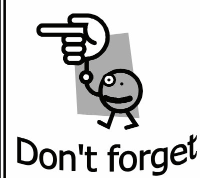
An important parameter used for describing uniform circular motion is the time period of motion. The **time period** T is **the time taken by the particle to travel once around the circle, that is, to make one complete revolution**. Since the distance travelled by the particle in one complete revolution is just the circumference $(2\pi r)$ of the circle, we can write

Time period

$$v = \frac{2\pi r}{T} \quad \text{or} \quad T = \frac{2\pi r}{v} = \frac{2\pi}{\omega} \tag{6.11}$$

You should study the concept of centripetal force carefully. It says that *a net force is required to keep a particle in uniform circular motion and that force is* **called** *a centripetal force*. **Always remember the following about the centripetal force**:

- **The phrase "centripetal force" does not denote a new and separate force existing in nature.**
- **The word centripetal is an adjective that describes any force directed towards the centre of curvature of the path of motion.**
- **The centripetal force is not another force that must be added to the free-body diagram for any object.**
- **Just as we have explained in Sec. 5.3 for the force in the equation of motion $\vec{F} = m\vec{a}$, Eqs. (6.10a and b) are just the equations of motion for circular motion.**
- **Just like $\vec{F}$, the centripetal force $\vec{F}_c$ must also be given by some force law representing the net force on the particle.**
- **Eqs. (6.10a and b) have to be used along with the force law applicable in a given problem.**

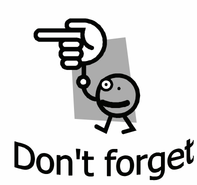


The net force, which keeps a particle moving in uniform circular motion, could be any one of many forces such as the following:

- the tension in a string, such as the one attached to a ball moving in a horizontal circle;
- friction between two surfaces, such as the friction between the tyres of a car and the road when the car moves around a curve in the road;
- some component of the normal force such as in the motion of vehicles on banked curves as discussed in Example 6.9;
- the gravitational force, such as the force between the Earth and the Sun which keeps it in a nearly circular orbit, as we will discuss in the next unit.

A centre-seeking force such as the gravitational force, the radial component of the tension force or the normal force, etc. is required to maintain the motion of an object in a circular path. The force is directed towards the centre of the circle.

Without this force, we cannot maintain circular motion.



In the following example we apply Newton's laws to uniform circular motion.

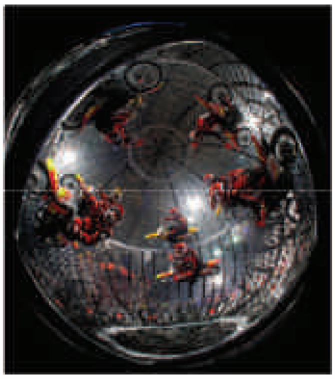


Fig. 6.20: **Force on a motorcyclist in uniform circular motion**.

EXAMPLE 6.7: UNIFORM CIRCULAR MOTION

A motorcyclist in a circus stunt rides a motorcycle inside a 10.0 m wide steel sphere (Fig. 6.20) at a constant speed of $64.0\ \text{kmh}^{-1}$. What is the force being exerted on the motorcyclist given that his mass is 50.0 kg?

SOLUTION ■ The **KEY IDEA** here is to obtain the centripetal acceleration since the motorcyclist is moving in a circle of radius 5.0 m and then the force.

The centripetal acceleration and the force on the motorcyclist are given by

$$a = \frac{v^2}{r} = \frac{(17.8\,\text{ms}^{-1})^2}{5.0\,\text{m}} = 63.4\ \text{ms}^{-2}$$

$$F = ma = 50.0\,\text{kg} \times 63.4\ \text{ms}^{-2} = 3.17 \times 10^3\ \text{N}$$

directed along the acceleration, that is, towards the centre of the sphere.

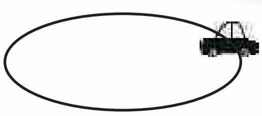
Let us consider another example of circular motion.

EXAMPLE 6.8: UNIFORM CIRCULAR MOTION IN A PLANE

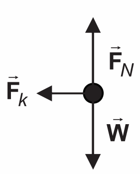
The maximum speed with which a 900 kg car can make a turn in a circular path is $10.0\ \text{ms}^{-1}$ (Fig. 6.21a). The radius of the circle in which the car is turning is 30.0 m. Determine the force of friction being exerted upon the car and the coefficient of friction between the car and the road. Take $g = 10.0\,\text{ms}^{-1}$.

SOLUTION ■ The **KEY IDEA** here is to determine the net force on the car that provides the centripetal force to keep it moving in a circle.

The forces being exerted on the car are: Weight of the car $\vec{\mathbf{W}}$ acting downwards, the normal force $\vec{\mathbf{F}}_N$ acting upwards and the force of friction acting towards the centre of the circle. The free-body diagram is given in Fig. 6.21b. Since the normal force and the weight are equal and opposite, only the force of friction provides the centripetal force required for the car to move in the circle. The force of friction must be equal to the centripetal force.



(a)



(b)

Fig. 6.21: **a) Car on a circular path; b) free-body diagram for the car.**

From Eq. (6.10a), $F_k = \dfrac{mv^2}{r}$ (i)

Substituting $v = 10.0\,\text{ms}^{-1}$, $m = 900\,\text{kg}$, $r = 30.0\,\text{m}$ in Eq. (i), we get

$$F_k = \frac{900\,\text{kg} \times (10.0\,\text{ms}^{-1})^2}{30.0} = 3000\,\text{N}$$

From Eq. (6.3), $F_k = \mu_k mg \quad \Rightarrow \quad \mu_k = F_k / mg$ (ii)

Substituting $F_k = 3000\,\text{N}$ and $m = 900\,\text{kg}$ in Eq. (ii), we get

$$\mu_k = 0.33$$

We now take up an important application of uniform circular motion from real life.

EXAMPLE 6.9: BANKING OF ROADS

Curves on roads are banked, that is, the road is designed so that it makes a finite angle (called the **banking angle**) with the horizontal at the turns (Fig. 6.22a). This is done because friction alone cannot help vehicles go around bends especially on slippery roads (due to rains or snow). Suppose a vehicle of mass M has to move around a turn in the road at a constant speed v. The radius of curvature of the turn is r. What should the banking angle of the road be so that friction is not required to negotiate the turn?

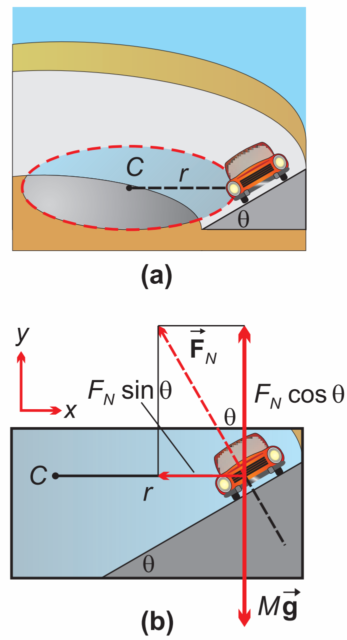


Fig. 6.22: a) Banking of roads; b) free-body diagram.

SOLUTION ■ The **KEY IDEA** here is that the vehicle undergoes centripetal acceleration $\left(= \dfrac{v^2}{r}\right)$ as it moves around the turn in the road.

The road is banked so that the normal force on the vehicle has a component pointing towards the centre of the turn. We now identify all forces being exerted on the vehicle and then apply Newton's laws of motion. The forces on the vehicle are: **gravity** (or its **weight**), and the contact forces: **normal force** and **kinetic friction**, **which should be zero as per the problem**. Let us draw the free-body diagram for the vehicle along with the coordinate axes. We choose the positive x-axis to be along the horizontal (Fig. 6.22b).

You can see from the free-body diagram that the angle that the normal force makes with the vertical is equal to θ. The x-component of the normal force points towards the centre and, therefore, it provides the centripetal force. Let us now resolve the forces along the x and y-axes and apply Newton's second law. Since the vehicle is not moving along the y-axis, the net force in that direction is zero. Equating the centripetal force to $F_N \sin\theta$, we get:

$$M\frac{v^2}{r} = F_N \sin\theta \quad \text{and}$$

$$\sum F_y = F_N \cos\theta - Mg = 0 \quad \text{or} \quad F_N \cos\theta = Mg$$

Dividing the first equation by the second, we get

$$\tan\theta = \frac{v^2}{rg} \qquad \text{or} \qquad \theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$$

Thus, if you want to go around a turn of radius 100 m travelling at a speed of $15\,\text{ms}^{-1}$, the banking angle should be

$$\theta = \tan^{-1}\left(\frac{(15\,\text{ms}^{-1})^2}{(100\,\text{m})(9.8\,\text{ms}^{-2})}\right) = 12.9^\circ \approx 13^\circ$$

SAQ 5 – Uniform circular motion

A fighter pilot flies her aircraft at the speed of 2520 kmh^{-1} in a circular arc having the radius of curvature 6000 m. What is the force experienced by her given that her mass is 60 kg?

Let us now summarise what you have learnt in this unit.

6.5 SUMMARY

Concept | **Description**

Contact forces around us

- **CONTACT FORCES:**

  Normal force: **Direction** perpendicular to the surfaces of contact and opposite to deformation. **Magnitude** non-zero only if surfaces in contact, otherwise zero.

  Friction: **Direction** along the surfaces of contact and opposite to direction of motion. **Magnitude** non-zero only if surfaces in contact, otherwise zero.

  $F_s^{max} = \mu_s F_N$ for **static friction** and

  $F_k = \mu_k F_N$ for **kinetic friction**

  Tension force: **Direction** along the string, rope or cable away from the object. **Magnitude** non-zero if string, rope or cable taut; zero otherwise.

  Spring force: **Direction** opposite to the displacement of the spring from equilibrium. From Hooke's law: $F = -kx$

Objects in equilibrium

- An object is in **equilibrium when the net force on it is zero.** Hence, its **acceleration is zero** and it is **at rest** or **moves with constant velocity.**

  For motion in a plane:

  $$F_{net,x} = \sum F_x = 0 \quad \text{and} \quad F_{net,y} = \sum F_y = 0$$

  For motion in space:

  $$F_{net,x} = \sum F_x = 0, \quad F_{net,y} = \sum F_y = 0 \text{ and } F_{net,z} = \sum F_z = 0$$

- We have discussed a variety of applications of Newton's laws. Some of these are shown in Fig. 6.23 below:

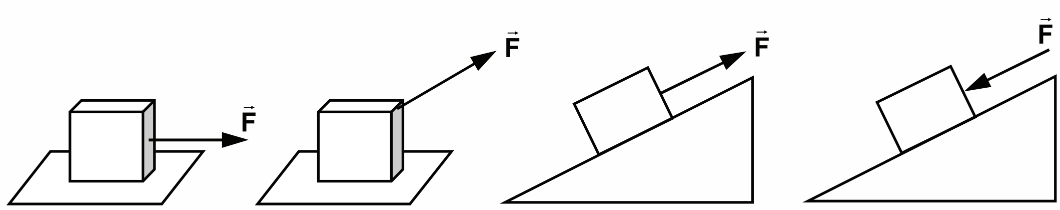


Fig. 6.23: **Some applications of Newton's laws.**

6.6 TERMINAL QUESTIONS

1. Use the statement given ahead and Fig. 6.24 to answer parts (a) and (b). Fig. 6.24 shows a frictionless channel in the shape of a part of a circle with its centre at *O*. The channel is fixed to the top of a frictionless horizontal table. Suppose that you are looking down at the table. Neglect the force exerted by the air. A ball is shot at high speed into the channel at *P* and exits at *Q*. Consider the following distinct forces:

   A. A downward force of gravity.
   B. A force exerted by the channel pointing from *Q* to *O*.
   C. A force in the direction of motion.
   D. A force pointing from *O* to *Q*.

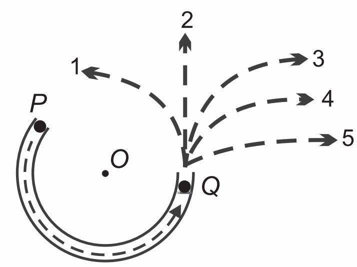


Fig. 6.24

   a) Which of the above forces is (are) acting on the ball when it is within the frictionless channel at position *Q*?

      i) A only
      ii) A and B
      iii) A and C
      iv) A, B and C
      v) A, C and D

   b) Which of the paths 1 to 5 shown in Fig. 6.24 would the ball most closely follow after it exits the channel at *Q* and moves across the frictionless table top?

2. Fig. 6.25 shows a boy swinging, starting at a point higher than *P*. Consider the following distinct forces:

Fig. 6.25

   A. A downward force of gravity.
   B. A force exerted by the rope pointing from *P* to *O*.
   C. A force in the direction of the boy's motion.
   D. A force pointing from *O* to *P*.

   Which of the above forces is (are) acting on the boy when he is at position *P*?

      i) A only
      ii) A and B
      iii) A and C
      iv) A, B, and C
      v) A, C, and D

3. A book is kept at rest on a table. Identify all the forces being exerted on the book and the table. Draw the free-body diagram for both objects.

4. A lamp hangs from two cables. One cable has a tension of 12.6 N and is at an angle of 15° with respect to the ceiling. What is the tension in the other cable if it makes an angle of 65° with respect to the ceiling?

5. A box of mass 0.70 kg sits on a ramp. If the normal force on the box due to the ramp is 5.4 N, what is the angle the ramp makes with the (horizontal) ground?

6. A train of mass 8.00×10^6 kg is moving in a straight line at a constant speed of $80.0\ \text{kmh}^{-1}$. The brakes, which produce a net backward force of 2.40×10^6 N, are applied for 25.0 s. What is the new speed of the train? How far has the train travelled in this time?

7. Determine the mass of a box which requires a minimum pushing force of 64.4 N to start moving across a rough floor. The coefficient of static friction between the box and the floor is 0.350. Take $g = 9.80\ \text{ms}^{-2}$.

8. A ship of mass 2.00×10^8 kg is moving at a constant velocity. Its engines generate a forward thrust of 5.00×10^5 N. Determine (i) the upward bouyant force on the ship due to water and (ii) the resistive force exerted by water on the ship. Take $g = 9.80\ \text{ms}^{-2}$.

9. A rock of mass 0.20 kg is tied to a string that moves in a circular path over a horizontal frictionless surface. If the speed of the rock is $8.0\ \text{ms}^{-1}$ and the radius of the path is 2.0 m, calculate the tension in the string.

10. A circular curve on a highway is designed for traffic moving at $20.0\ \text{m s}^{-1}$. The radius of the curve is 120 m. What is the minimum coefficient of kinetic friction between the tyres and the highway necessary to keep the cars from sliding off the curve. Take $g = 10.0\ \text{ms}^{-2}$.

6.7 SOLUTIONS AND ANSWERS

Self-Assessment Questions

1. See Fig. 6.26.

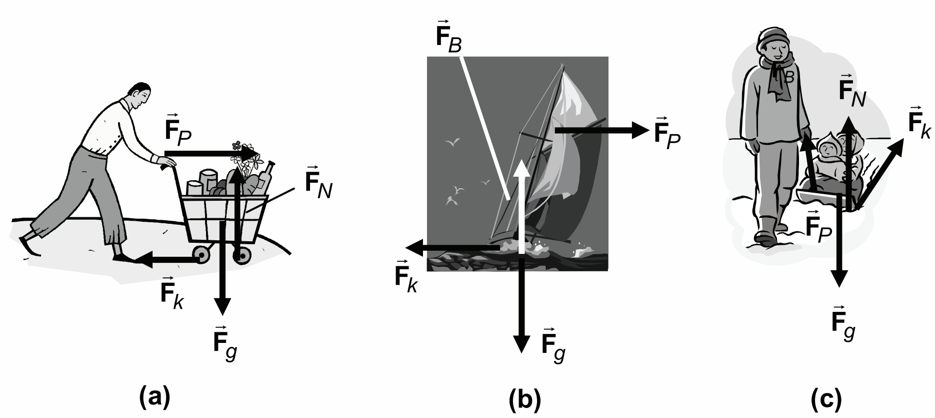


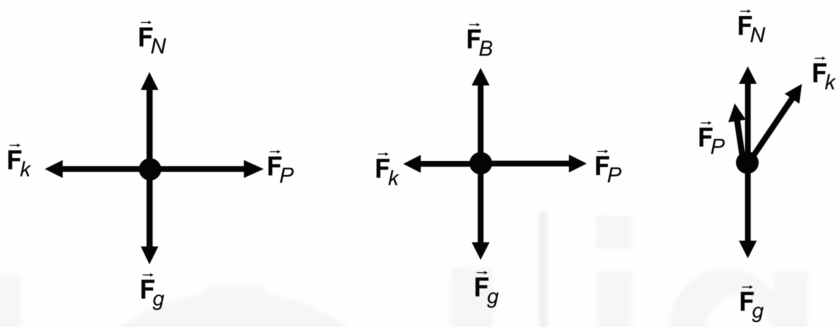
Fig. 6.26

a) $\vec{\mathbf{F}}_g = m\vec{\mathbf{g}}$ is the weight of the cart, $\vec{\mathbf{F}}_N$ is the normal force, $\vec{\mathbf{F}}_k$ is the force of kinetic friction and $\vec{\mathbf{F}}_P$ is the force with which the man is pushing the cart.

b) $\vec{\mathbf{F}}_g = m\vec{\mathbf{g}}$ is the weight of the boat, $\vec{\mathbf{F}}_B$ is the upward buoyant force due to water, $\vec{\mathbf{F}}_k$ is the resistive force exerted by water on the boat and $\vec{\mathbf{F}}_P$ is the force with which the wind pushes on the sails.

c) $\vec{\mathbf{F}}_g = m\vec{\mathbf{g}}$ is the weight of the sled, $\vec{\mathbf{F}}_N$ is the normal force, $\vec{\mathbf{F}}_k$ is the force of kinetic friction and $\vec{\mathbf{F}}_P$ is the force with which the sled is being pulled.

2. The free-body diagrams in each case are given in Fig. 6.27. Note that each body has been modeled as a particle.



(a) **(b)** **(c)**

Fig. 6.27

3. a) i) Normal force of the floor on the table is equal and opposite to the weight of the table and the child. Its magnitude is

$$F_N = 75.0\,\text{kg} \times 9.8\,\text{ms}^{-2} = 735\,\text{N}$$

ii) The magnitude of the normal force exerted by the table on the child is

$$F'_N = 30.0\ \text{kg} \times 9.8\,\text{ms}^{-2} = 294\,\text{N}$$

b) Since the velocity of the child is constant, the net force on her is zero. From the free-body diagram (Fig. 6.28), we can write

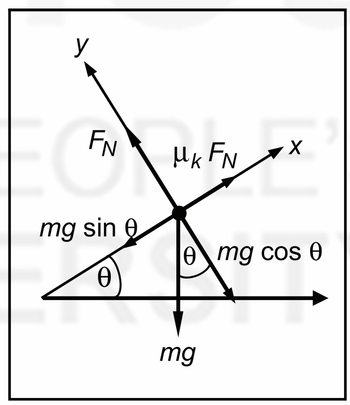


Fig. 6.28

$$mg\cos\theta = F_N \qquad \text{(i)}$$

and

$$mg\sin\theta = \mu_k F_N \qquad \text{(ii)}$$

Dividing Eq. (ii) by Eq. (i), we get the coefficient of kinetic friction:

$$\mu_k = \tan\theta$$

4. The free-body diagram with the choice of axes is shown in Fig. 6.29. $\vec{\mathbf{F}}$ is the applied force which has a magnitude of 80 N, $\vec{\mathbf{F}}_N$ is the normal force, $\vec{\mathbf{W}} = m\vec{\mathbf{g}}$ is the weight of the box and $\vec{\mathbf{F}}_k = \mu_k \vec{\mathbf{F}}_N$ is the force of kinetic friction. The box is moving along the x-axis with acceleration a_x. On resolving the forces along the x-axis, we get

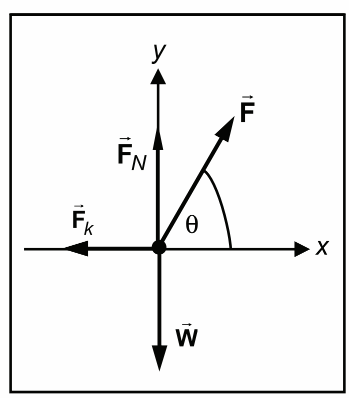


Fig. 6.29

$$F\cos 60^\circ - \mu_k F_N = F_{net,x} = ma_x$$

or $a_x = (F\cos 60° - \mu_k F_N)/m$ (i)

On resolving the forces along the *y*-axis, we get

$$F\sin 60° + F_N - W = F_{net,y} = 0$$

or $F_N = W - F\sin 60°$ (ii)

Substituting $W = mg = 150\,\text{N}$, $F = 80\,\text{N}$, $\sin\theta = \dfrac{\sqrt{3}}{2}$ in Eq. (ii), we get

$$F_N = \left(150 - \frac{80\sqrt{3}}{2}\right)\text{N} = 81\,\text{N} \quad \text{(iii)}$$

Substituting the value of F_N from Eq. (iii) and $\mu_k = 0.20$ in Eq. (i), we get

$$a_x = \frac{80\,\text{N} \times \cos 60° - 0.20 \times 81\text{N}}{15\,\text{kg}} = 1.6\ \text{ms}^{-2}$$

5. The centripetal force on the pilot is

$$F_C = \frac{mv^2}{r} = 60\ \text{kg} \times \frac{(700\ \text{ms}^{-1})^2}{6000\,\text{m}} = 4900\,\text{N} = 4.9 \times 10^3\ \text{N}$$

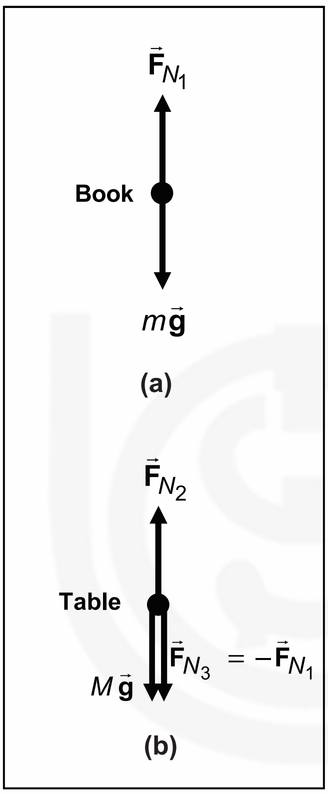


Fig. 6.30

Terminal Questions

1. a) ii.

   b) Path 2, which is tangent to the channel at point *Q* since now there will be no force on the ball.

2. ii) Both the force of gravity and the force exerted by the rope are exerted on the boy.

3. Since the book and the table are at rest, the forces on the book are:

   $m\vec{g}$ = weight of the book

   $\vec{F}_{N_1}$ = normal force on the book due to the table

   The forces on the table are:

   $M\vec{g}$ = weight of the table

   $\vec{F}_{N_2}$ = normal force on the table due to the ground

   $\vec{F}_{N_3} = -\vec{F}_{N_1}$ = normal force on the table due to the book

   The free-body diagrams for both objects are shown in Fig. 6.30.

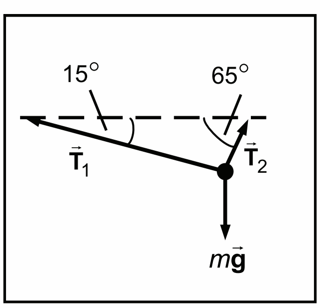


Fig. 6.31

4. See Fig. 6.31. Since the lamp is at rest we must have

$$T_2 \sin 65° + T_1 \sin 15° = mg$$

and $T_1 \cos 15° = T_2 \cos 65°$

$$\therefore\ T_2 = (12.6\,\text{N}) \times \frac{\cos 15°}{\cos 65°} = 28.8\,\text{N}$$

5. See Fig. 6.32. Since the box is at rest on the ramp, we must have

$$mg\cos\theta = F_N$$

where F_N is the normal force. Therefore,

$$\theta = \cos^{-1}\left(\frac{F_N}{mg}\right)$$

$$= \cos^{-1}\left(\frac{5.4\text{ N}}{0.70\text{ kg}\times 9.8\text{ ms}^{-2}}\right) = \cos^{-1}(0.79) = 38^\circ$$

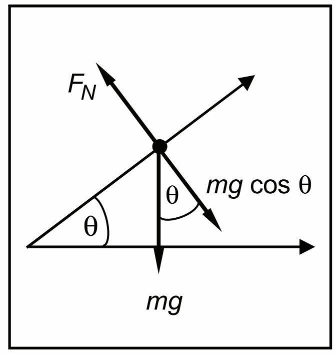


Fig. 6.32

6. Let us take the train to be moving in the positive *x*-direction. The acceleration of the train (a_x) after the brakes are applied is

$$a_x = -\frac{2.40\times10^6\text{ N}}{8.00\times10^6\text{ kg}} = -0.300\text{ ms}^{-2}$$

The speed *u* of train before the brakes are applied is

$$u = 80.0\text{ kmh}^{-1} = 22.2\text{ ms}^{-1}$$

The speed of the train after 25.0 s and the distance travelled are calculated by using the equations of kinematics. The speed *v* of the train after $t = 25.0\text{ s}$ is

$$v = 22.2\text{ ms}^{-1} - 0.300\text{ ms}^{-2}\times 25.0\text{ s} = 14.7\text{ ms}^{-1}$$

In this time the train would have travelled,

$$s = 22.2\text{ ms}^{-1}\times 25.0\text{ s} - \frac{1}{2}\times 0.300\text{ ms}^{-2}\times(25.0\text{ s})^2 = 461\text{m}$$

7. The box will start moving when the pushing force exceeds the static frictional force opposing the motion. Therefore,

$$F_s^{max} = 64.4\text{N} \quad\Rightarrow\quad \mu_s mg = 64.4\text{N}$$

where *m* is the mass of the box and $\mu_s = 0.35$. Thus,

$$m = \frac{64.4\text{N}}{0.350\times(9.80\text{ ms}^{-2})} = 18.8\text{kg}$$

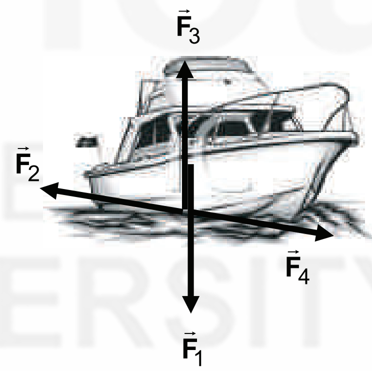


Fig. 6.33

8. Since the ship is moving with constant velocity, the net force on it is zero. The forces on the ship (Fig. 6.33) are

1) $\vec{\mathbf{F}}_1$ = weight of the ship,

2) $\vec{\mathbf{F}}_2$ = resistive force exerted by the water,

3) $\vec{\mathbf{F}}_3$ = upward buoyant force, and

4) $\vec{\mathbf{F}}_4$ = forward thrust of the engines.

Fig. 6.34 shows the free-body diagram for this problem. Since the net force on the ship is zero, we have

$$\vec{\mathbf{F}}_1 = \vec{\mathbf{F}}_3 \qquad\text{and}\qquad \vec{\mathbf{F}}_2 = \vec{\mathbf{F}}_4$$

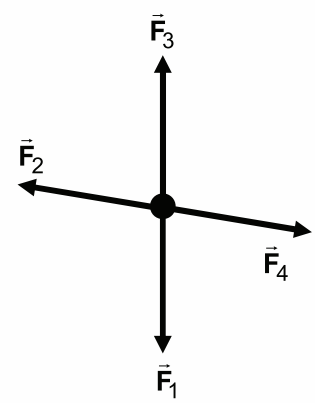


Fig. 6.34

i) The upward buoyant force exerted by water is equal to the weight of the ship:

$$\therefore \quad \vec{\mathbf{F}}_3 = (2.00 \times 10^8 \text{ kg}) \times 9.80 \text{ ms}^{-2} = 19.6 \times 10^8 \text{ N}$$

ii) The resistive force exerted by water is equal to the forward thrust of the engines which is 5.00×10^5 N.

9. The centripetal force is provided by the tension $\vec{\mathbf{T}}$ in the string. So using Eq. (6.10a) with $m = 0.20$ kg, $v = 8.0$ ms^{-1} and $r = 2.0$ m, we can write

$$T = \frac{mv^2}{r} = \frac{0.20 \text{ kg} \times (8.0 \text{ ms}^{-1})^2}{2.0 \text{ m}} = 6.4 \text{ N}$$

10. The car will remain in the curve only if the force of friction provides the necessary centripetal force. Thus using Eqs. (6.4 and 6.10a), we can write,

$$\frac{mv^2}{r} = \mu_k mg \quad \Rightarrow \quad \mu_k = \frac{v^2}{gr}$$

Substituting $v = 20.0 \text{ ms}^{-1}$, $r = 120$ m and $g = 10.0 \text{ ms}^{-2}$, we get

$$\mu_k = \frac{(20.0 \text{ ms}^{-1})^2}{(120 \text{ m}) \times (10 \text{ ms}^{-2})} = 0.33$$

Why do astronauts float in a space station? You can find the answer in this unit!

UNIT 7

GRAVITATION

Structure

7.1 Introduction
 Expected Learning Outcomes
7.2 The Force of Gravitation
 Principle of Superposition
 Gravitational Field
7.3 Gravity
 Variation of *g* with Altitude, Depth and Latitude
 Vertical Circular Motion under Gravity
7.4 Weight
 Weightlessness
7.5 Summary
7.6 Terminal Questions
7.7 Solutions and Answers

STUDY GUIDE

In Unit 6, you have learnt how to apply Newton's laws of motion to a variety of situations around us in which contact forces are exerted on particles. In this unit, you will learn about the long-range force of gravitation and its applications. You will also learn the concepts of force of gravity and weight and apply Newton's laws of motion to situations in which these forces are present.

In order to study this unit well, **you should know very well the concepts of vector algebra from Units 1 and 2 as well as the concepts explained in Units 5 and 6**. You should revise all these concepts before studying this unit.

You should know the methods of solving problems discussed in Unit 6. Try to work out all solved examples, SAQs and Terminal Questions by yourself. This will help you understand the concepts better.

"When you make the finding yourself – even if you're the last person on Earth to see the light – you never forget it."

Carl Sagan

7.1 INTRODUCTION

In Unit 6, you have learnt about **different types of contact forces and force laws for which you can apply Newton's laws to predict motion of particles**. You have applied these to study the motion of particles on which these forces are exerted. In this unit, you will learn about one of the long-range forces, namely, the **force of gravitation**.

Motion of objects is also explained using the work-energy theorem which you will study in Unit 9.

As you have learnt in your school physics, the force of gravitation is a ***universal*** force. It is exerted on each and every pair of macroscopic objects in the universe. This is the force which is responsible for keeping the Moon in its orbit around the Earth. This is also the force which is exerted between the planets and the Sun and all other celestial bodies, and helps us explain many astronomical phenomena.

Therefore, in Sec. 7.2, you will study the law of gravitation, and understand how it applies to extended bodies. We also explain the principle of superposition, which helps us determine the net force of gravitation on a particle when it is subjected to gravitational force by many particles around it. We also explain the concept of gravitational field.

NOTE

In your written work, always use an arrow above the letter you use to denote a vector, e.g., $\vec{r}$. Use a cap above the letter you use to denote a unit vector, e.g. $\hat{r}$.

The force of gravity is used to analyse motion of objects on the Earth, e.g., the trajectory of projectiles and the weight of objects. In Sec. 7.3, you will learn about the force of gravity on Earth, and the factors which cause a variation in gravity at different points on and around the Earth.

In Unit 6, you have learnt about uniform circular motion in a horizontal plane. In Sec. 7.3, you will also study about uniform circular motion in a vertical plane where the centripetal force is provided by the force of gravity, and some of its applications.

While travelling in a lift you may have felt lighter or heavier than you normally do. In Sec. 7.4, you will learn why that is so when you study the concepts of weight and weightlessness.

In the next unit, we discuss the concepts of linear momentum and impulse. We also introduce the law of conservation of linear momentum.

Expected Learning Outcomes

After studying this unit, you should be able to:

- apply Newton's laws of motion to particles moving under the force of gravitation;
- use the principle of superposition and determine the net force of gravitation on a particle;
- solve problems related to motion of particles under the force of gravity; and
- explain the concepts of weight, weightlessness and solve related problems.

7.2 THE FORCE OF GRAVITATION

In your school physics, you have learnt about the force of gravitation. You know that it is exerted between all macroscopic objects in the universe, whatever the distance between them may be. We say that it is a universal force. Let us relate it to our day to day experiences. You know that objects fall towards the Earth with an acceleration due to gravity given by $g = 9.8 \text{ ms}^{-2}$.

Would you like to know: **Why is** $g = 9.8 \text{ ms}^{-2}$? From Newton's second law, you know that there must be some force due to which an object falls towards the Earth with this acceleration. What is that force? What is the force law? The answers were given by Newton who put forth the **law of universal gravitation** and arrived at its mathematical form. Together with his three laws of motion, this law can be used to explain the motion of all macroscopic objects in nature. So let us learn Newton's law of universal gravitation, and write down the mathematical form of the force of gravitation.

NEWTON'S LAW OF UNIVERSAL GRAVITATION

Every particle in the universe exerts an attractive force on every other particle. Consider two particles having masses m_1 and m_2, respectively and separated by a distance r (Fig. 7.1).

The force of gravitation exerted by particle m_1 on particle m_2 acts along the line joining the particles and is directed from m_2 **to** m_1. It is given by

Force of gravitation

$$\vec{\mathbf{F}}_{12} = -G\,\frac{m_1 m_2}{|\vec{\mathbf{r}}_{12}|^2}\,\hat{\mathbf{r}}_{12} \qquad (7.1a)$$

where $\hat{\mathbf{r}}_{12}$ is the unit vector along the line joining the particles and is directed from m_1 to m_2. Note that $|\vec{\mathbf{r}}_{12}| = r$. The negative sign in Eq. (7.1a) tells us that the gravitational force on m_2 due to m_1 is attractive: It is directed opposite to $\hat{\mathbf{r}}_{12}$. G is a constant called the **universal gravitational constant**. Its value has been found experimentally to be

$$G = 6.673 \times 10^{-11} \text{ N.m}^2.\text{ kg}^{-2}$$

In the same way, we can write the force of gravitation exerted **on** m_1 **by** m_2 as

$$\vec{\mathbf{F}}_{21} = -G\,\frac{m_1 m_2}{|\vec{\mathbf{r}}_{21}|^2}\,\hat{\mathbf{r}}_{21} \qquad (7.1b)$$

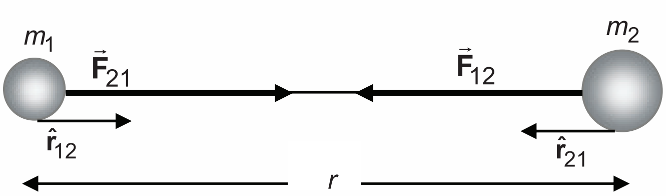


Fig. 7.1: The gravitational force between two particles.

NOTE

Forces which act along the line joining the particles or a particle and the origin (which remains fixed in space) are called **central forces.** They are of the form:

$$\vec{\mathbf{F}} = F\,\hat{\mathbf{r}}$$

where F is the magnitude of the force and $\hat{\mathbf{r}}$ is the unit vector along the line. You will learn about them in Unit 13.

Note from Fig. 7.1 that $|\vec{\mathbf{r}}_{21}| = r$ and $\hat{\mathbf{r}}_{21}$ is the unit vector along the line joining the particles, directed from m_2 to m_1. Also note that $\hat{\mathbf{r}}_{21} = -\hat{\mathbf{r}}_{12}$ and hence

$$\vec{\mathbf{F}}_{21} = -\vec{\mathbf{F}}_{12} \qquad (7.1c)$$

NOTE

In the notation we are using here:

$\vec{\mathbf{F}}_{21}$: Force **on** 1 **by** 2.

and

$\vec{\mathbf{F}}_{12}$: Force **on** 2 **by** 1.

Note that Eqs. (7.1a, b and c) hold for point masses. The mass appearing in Eqs. (7.1a and b) is called the **gravitational mass**. **It is the property of matter, which causes objects to exert the force of gravitation on each other**. You may now like to work out the following SAQ to calculate the force of gravitation between two particles.

NOTE

The inertial and gravitational masses of an object are the same in the Newtonian world.

SAQ 1 – Force of gravitation

Calculate the force of gravitation between two particles of mass 50 kg each when the distance between them is 1.5 m.

We now consider a very important application of the force of gravitation in the dynamics of uniform circular motion, namely, in satellite technology. We see the impact of this technology all around us, whether in TV and telephone communications, internet or mapping the Earth (for example, through the search engines Google Earth and *Bhuvan* launched by our own ISRO). Many of these satellites (for example, the satellites used in communication) are **geosynchronous**.

A geosynchronous satellite takes the same amount of time to orbit the Earth once, as the Earth takes to rotate once about its axis. This means that it moves with the same angular speed as a point directly below it on the surface of the Earth. And so to an observer at that point on the Earth, the satellite appears to be stationary in the sky. Since such a satellite remains in the same place in the sky relative to an observer on the Earth, it is called a **geosynchronous satellite**. Geosynchronous satellites are useful in communication technology because they can provide continuous coverage of the same area. A geosynchronous satellite orbiting the Earth is shown in Fig. 7.2. The force of gravitation between the satellite and the Earth provides the necessary centripetal acceleration to keep it moving in its orbit. Let us use this concept to determine the height of a geosynchronous satellite above the Earth's surface.

Fig. 7.2: **A geosynchronous satellite orbiting the Earth.**

EXAMPLE 7.1: GEOSYNCHRONOUS SATELLITES

What is the height of a geosynchronous satellite above the surface of the Earth?

SOLUTION ■ The **KEY IDEA** is that the force of gravitation between the satellite and the Earth (that points towards the centre of the Earth) provides the necessary centripetal force to keep it moving in its orbit. Thus, we shall apply Eqs. (6.10a and b) and equate the expression for the force of gravitation on the satellite due to the Earth to that of the centripetal force.

Let the masses of the satellite and the Earth be m and M_e, respectively. Then if the satellite is orbiting the Earth in a circle of radius r, we have from Eqs. (7.1a, 6.10a and b) that

$$\frac{GmM_e}{r^2} = \frac{mv^2}{r} = m\omega^2 r \qquad \text{(i)}$$

or
$$r = \left(\frac{GM_e}{\omega^2}\right)^{\frac{1}{3}} = \left(\frac{GM_eT^2}{4\pi^2}\right)^{\frac{1}{3}} \qquad \left(\because\ \omega = \frac{2\pi}{T}\right) \qquad \text{(ii)}$$

Substituting $T = 24$ hours $= (24 \times 60 \times 60)$ s $= 86400$s,

$G = 6.67 \times 10^{-11}\ \text{Nm}^2\,\text{kg}^{-2}$ and $M_e = 5.97 \times 10^{24}$ kg in Eq. (ii), we get:

$$r = 4.22 \times 10^7\ \text{m}$$

Since the radius of the Earth is $R_e = 6.37 \times 10^6\ \text{m},$ the height of the satellite above the Earth's surface is

$$h = (42.2 - 6.37) \times 10^6\ \text{m} = 3.58 \times 10^7\ \text{m} \approx 35{,}800\,\text{km}$$

Global Positioning System

The global positioning system (GPS) is a system of satellites, and computers and receivers on ground stations, which can help us determine the latitude and longitude of a receiver on Earth. This is done by calculating the time difference for signals from different satellites to reach the receiver.

From Eq. (i) in Example 7.1, we get the expression of the speed of the satellite in its orbit, called the **orbital speed**, as

$$v = \sqrt{\frac{GM_e}{r}} \qquad (7.2)$$

Eq. (7.2) tells us that

- For a given value of r, the orbital speed of a satellite does not depend on its mass, that is, for a given orbit, ***a satellite with large mass has the same orbital speed as a satellite with small mass***.
- The smaller the radius r of the orbit, that is, the closer the satellite is to the Earth, the greater is its orbital speed.

Eq. (7.2) applies to all satellites (artificial and natural like the Moon) orbiting the Earth. It also applies to any object orbiting another object of mass M, for example, satellites of Jupiter or other planets as well as to planets orbiting stars. In that case, M_e should be replaced by M in Eq. (7.2).

You may now like to solve a problem based on the concepts discussed so far.

SAQ 2 – Satellites

Eq. (7.2) can be used to determine the mass of any object located in the universe if we know the speed of a satellite orbiting that object. An object located at a distance of 7.50×10^{17} m from the centre of the galaxy orbits it in a nearly circular orbit with a speed of $5.70 \times 10^5\ \text{ms}^{-1}$. Determine the mass of the object located at the centre of the galaxy (see Fig. 7.3).

Fig. 7.3: A black hole is thought to exist at the centre of this galaxy shown by the white circle.

Newton's law of gravitation describes the interaction between two point particles. **REMEMBER**: **Objects that are very small in comparison with the distance between them may be regarded as point particles**. Now suppose we have a system of many particles such as the solar system. How do we calculate the force of gravitation on one particular planet (say, the Earth) due to the Sun and all other objects in the solar system?

We use the **principle of superposition** that is obeyed by the gravitational force. Let us now discuss this principle, which helps us determine the force of gravitation on each particle due to all other particles in a system of particles.

7.2.1 Principle of Superposition

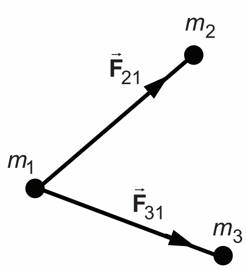


Fig. 7.4: Principle of superposition.

Recall the principle of superposition of forces from Sec. 5.3.1 of Unit 5. It tells us that the force on a particle in a system of particles is just the resultant (vector sum) of the forces exerted by all other individual particles in the system on that particle. To understand this principle for the force of gravitation, let us consider a system of three particles, with masses m_1, m_2 and m_3 (see Fig. 7.4). What is the force of gravitation on one of them, say m_1? If only m_1, m_2 were present, the force **on** m_1 due to m_2 would be

$$\vec{\mathbf{F}}_{21} = -G\frac{m_1 m_2}{r_{21}^2}\hat{\mathbf{r}}_{21} \tag{7.3a}$$

Similarly, if only m_1 and m_3 were present, the force on m_1 due to m_3 would be

$$\vec{\mathbf{F}}_{31} = -G\frac{m_1 m_3}{r_{31}^2}\hat{\mathbf{r}}_{31} \tag{7.3b}$$

Now, what is the force on m_1 due to both m_2 and m_3? According to the principle of superposition, in the three-particle system of Fig. 7.4, the total force $\vec{\mathbf{F}}_1$ on m_1 is the vector sum of $\vec{\mathbf{F}}_{21}$ and $\vec{\mathbf{F}}_{31}$, i.e.,

$$\vec{\mathbf{F}}_1 = \vec{\mathbf{F}}_{21} + \vec{\mathbf{F}}_{31} \tag{7.4a}$$

or

$$\vec{\mathbf{F}}_1 = -G\frac{m_1 m_2}{r_{21}^2}\hat{\mathbf{r}}_{21} - G\frac{m_1 m_3}{r_{31}^2}\hat{\mathbf{r}}_{31} \tag{7.4b}$$

For an N-particle system of masses $m_1, m_2, \ldots, m_N$, the gravitational force on particle 1 of mass m_1 due to all other particles in the system is given by

$$\vec{\mathbf{F}}_1 = -G\sum_{i=2}^{N}\frac{m_1 m_i}{r_{i1}^2}\hat{\mathbf{r}}_{i1} \tag{7.4c}$$

Recap

PRINCIPLE OF SUPERPOSITION

According to the principle of superposition, in a system of *N* particles, the resultant force of gravitation on any particle is the vector sum of the forces of gravitation exerted by all other particles on it [as given by Eq. (7.4c)].

You may like to work through an example to apply the principle of superposition.

EXAMPLE 7.2: PRINCIPLE OF SUPERPOSITION

Determine the location of the point on the line joining two fixed particles of masses m_1 and m_2, at which a particle of mass m does not feel any resultant gravitational force due to them. Show that it is independent of the mass m. Take the point to be situated between the particles.

SOLUTION ■ The **KEY IDEA** here is to use the principle of superposition to determine the net force on the particle of mass m due to the other two particles and equate it to the null vector.

Refer to Fig. 7.5. Let the distance between the masses m_1 and m_2 be a. Let m be at a distance x from m_1 when the resultant gravitational force on m due to m_1 and m_2 is zero. Now from the principle of superposition, the net force on the particle of mass m due to the particles of masses m_1 and m_2 is given by

$$\vec{\mathbf{F}} = -\frac{Gm_1m}{x^2}\hat{\mathbf{x}} - \frac{Gm_2m}{(a-x)^2}(-\hat{\mathbf{x}}) \qquad \text{(i)}$$

Putting $\vec{\mathbf{F}} = \vec{\mathbf{0}}$ in Eq. (i), we get

$$\frac{Gm_1m}{x^2} = \frac{Gm_2m}{(a-x)^2} \qquad \text{(ii)}$$

Solving Eq. (ii) as shown in the margin remark, we get

$$x = \frac{a}{1+b} \quad \text{where } b = \sqrt{\frac{m_2}{m_1}}$$

Read the margin remark. Note that x is a constant independent of the mass m. Note also that the location of the particle of mass m does depend on the ratio of the two fixed masses.

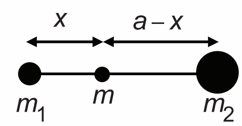


Fig. 7.5: **Applying the principle of superposition.**

We can also solve Eq. (ii) by treating it as a quadratic equation as shown below. You can see that the roots of the quadratic equation $m_1(a-x)^2 = m_2x^2$ i.e.

$$(m_1 - m_2)x^2 - 2am_1x + a^2m_1 = 0$$

are $\frac{a}{1+b}$ and $\frac{a}{1-b}$ of which only $\frac{a}{1+b}$ is possible. This is because x has to be positive and less than a. This does not hold for the solution $\frac{a}{1-b}$ because for $m_2 < m_1$, $x > a$, which is not possible as per the given problem and for $m_2 > m_1$, $x < 0$, which is again not possible as per the given problem.

You should now work out an SAQ on the principle of superposition.

SAQ 3 – Principle of superposition

Two particles of mass $2m$ and $4m$, respectively, are separated by a distance $2r$. An object of mass m is kept at the midpoint of the line joining the two particles. Determine the net force of gravitation on the particle of mass m. At what point on the line should the mass m be kept so that the net gravitational force on it is zero?

Strictly speaking, we do not refer to non-contact forces in nature as action-at-a-distance forces any more. These are all **force fields**, a concept that is beyond the scope of this course. Thus, the gravitational force is described as a gravitational force field. We now introduce the concept of **gravitational field** associated with the gravitational force, which might be new for you.

7.2.2 Gravitational Field

Let us consider two particles of masses m_1 and m_2 placed at distance r from each other. As you know, each particle experiences gravitational force of attraction due to the other, which depends on the inverse square of the distance between them. However large the value of r is, the particles will experience gravitational force. We say that m_1 sets up a field, called the **gravitational field**, in the space around it. Any other particle situated in the gravitational field of m_1 experiences the gravitational force given by Eq. (7.1a).

The strength of a gravitational field is given by its **intensity**.

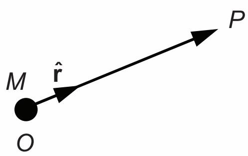


Fig. 7.6: **Intensity of the gravitational field at *P* due to *M* at *O*.**

By definition, the **intensity of the gravitational field** due to a mass M at a point at distance r from it is given by **the force experienced by a unit mass placed at that point**. Mathematically, the gravitational field intensity at point P due to mass M situated at point O (see Fig. 7.6) is given by

$$\vec{\mathbf{E}} = -\frac{GM}{r^2}\hat{\mathbf{r}} \tag{7.5a}$$

where $\hat{\mathbf{r}}$ is the unit vector along OP, and $OP = r$. The force $\vec{\mathbf{F}}$ experienced by a mass m kept at P due to the mass M at point O is given by

$$\vec{\mathbf{F}} = -\frac{GMm}{r^2}\hat{\mathbf{r}} \tag{7.5b}$$

From Eq. (7.5b), we can write Eq. (7.5a) as

$$\vec{\mathbf{F}} = m\vec{\mathbf{E}} \tag{7.5c}$$

7.3 GRAVITY

In your school physics, you have learnt about the force of gravity. As you know, the **gravitational force of attraction between the Earth and any other object on or near it is called gravity**. Let us discuss it in detail here and try to understand why the acceleration due to gravity g has the value $9.8\,\text{ms}^{-2}$ on Earth. Since g is the acceleration due to gravity, from Newton's second law, the force of gravity exerted by the Earth on any object of mass m situated on its surface is given by

$$F = mg \tag{7.6a}$$

It is directed towards the centre of the Earth. Now what is the gravitational force exerted by the Earth on the object, if the radius and mass of the Earth are R_e and M_e, respectively? Its magnitude is given by

$$F = G\frac{mM_e}{R_e^2} \tag{7.6b}$$

and it is also directed towards the centre of the Earth. This should be equal to F given by Eq. (7.6a).

Thus, we can write

$$F = G\frac{mM_e}{R_e^2} = mg \qquad (7.6c)$$

or

$$g = \frac{GM_e}{R_e^2} \qquad (7.6d)$$

Eq. (7.6d) gives the value of g at the Earth's surface. If we substitute the values of the universal constant G, and R_e and M_e, it equals $9.81\,\text{ms}^{-2}$. This, however, is the value of g at the equator. It changes as we move away from the Earth's surface and also as we go deep inside the Earth. It also changes with the latitude of the object since Earth is not a perfect sphere. Let us see how g varies with altitude, depth and latitude.

7.3.1 Variation of *g* with Altitude, Depth and Latitude

Refer to Fig. 7.7. We consider the positions of a particle of mass m at A and B, respectively, where

$SA = h =$ the altitude of A and

$SB = d =$ the depth at which B is situated

Here S is a point on the surface of the Earth and

$OS = R_e =$ the radius of Earth

We denote the forces of attraction experienced by m at A and S by $\vec{\mathbf{F}}_A$ and $\vec{\mathbf{F}}_S$, respectively. Then from Eq. (7.6b), we can write their magnitudes as:

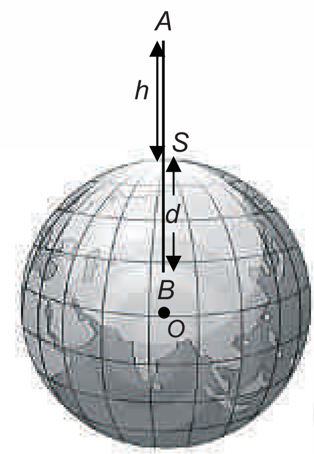


Fig. 7.7: **Acceleration due to gravity at different points from the surface of the Earth.**

$$F_S = \frac{GM_e m}{R_e^2}, \qquad (7.7a)$$

and

$$F_A = \frac{GM_e m}{(R_e + h)^2} \qquad (7.7b)$$

where M_e is the mass of Earth.

Let us denote the magnitudes of acceleration due to gravity on the surface of the Earth, and at point A by g_S and g_A. Then

$$g_S = \frac{F_S}{m} = \frac{GM_e}{R_e^2} = g_0 \qquad (7.8a)$$

and

$$g_A = \frac{F_A}{m} = \frac{GM_e}{(R_e + h)^2} \qquad (7.8b)$$

From Eqs. (7.8a and b), we get

$$g_A = \frac{g_0 R_e^2}{(R_e + h)^2} \qquad (7.9a)$$

g* at altitude *h

We can expand Eq. (7.9a) in a binomial expansion by rewriting it as follows:

$$g_A = g_0 \frac{R_e^2}{R_e^2\left(1 + \frac{h}{R_e}\right)^2} = g_0\left(1 + \frac{h}{R_e}\right)^{-2}$$

or $$g_A = g_0\left(1 - \frac{2h}{R_e}\right) \quad \text{for} \quad h << R_e \qquad (7.9b)$$

Eq. (7.9a) gives the value of g for an altitude h (above the Earth's surface) and Eq. (7.9b) holds for altitudes much smaller than the Earth's radius.

Since the Earth is a solid body, the acceleration due to gravity also varies with depth. We are stating the formulas for the variation of g at a point B situated at depth d under the surface of the Earth and $g(\lambda)$ at latitude λ:

g at depth d

$$g_B = \frac{g_0}{R_e}(R_e - d), \qquad (7.10)$$

g at latitude λ

and $$g(\lambda) = g_0 + \omega^2 R_e \sin^2\lambda, \qquad (7.11)$$

where $g(\lambda)$ = Value of g on the surface of Earth at a place having latitude λ,

g_0 = Value of g on equator = 9.81 ms^{-2} and

ω = Angular speed of rotation of Earth.

Let us now calculate the value of g at a point in the Earth's interior.

EXAMPLE 7.3: VARIATION OF g WITH DEPTH

Calculate the percentage decrease in the value of g from its value on the Earth's surface, if it is measured at the end of a tunnel 30 km below the surface of the Earth. Assume the radius of the Earth to be 6400 km.

SOLUTION ■ The **KEY IDEA** here is to use Eq. (7.10) to calculate the value of g at the depth of 30 km.

Substituting $R_e = 6400$ km and $d = 30$ km in Eq. (7.10), we get.

$$g = \frac{g_0}{R_e}(R_e - 30\,\text{km}) = g_0\left[1 - \frac{30}{6400}\right] \qquad \text{(i)}$$

The percentage decrease from the value at the surface of the Earth is:

$$\Delta g = \left(\frac{g_0 - g}{g_0}\right)\times 100 = \left(1 - \frac{g}{g_0}\right)\times 100 \qquad \text{(ii)}$$

Replacing $\frac{g}{g_0}$ from Eq. (i) in Eq. (ii) we get,

$$\Delta g = \left[1 - \left(1 - \frac{30}{6400}\right)\right]\times 100 = 0.47\%$$

SAQ 4 – Gravitational acceleration

Determine the gravitational acceleration of an object on the planet Jupiter given that its mass is 1.90×10^{27} kg and radius is 7.14×10^{7} m.

7.3.2 Vertical Circular Motion under Gravity

In Sec. 6.4 of Unit 6, you have studied uniform circular motion in a horizontal plane. You have studied examples in which the force required for keeping an object in uniform circular motion was provided by tension in a string or friction. In all those situations, the force of gravity did not play a role as it was balanced by the normal force or its component. We now take up circular motion in a vertical plane in which the force of gravity plays an important role.

You may have seen vertical circular motion in giant wheels in fairs, ancient water wheels, toy Ferris wheels, amusement park rides and motorcycle stunts in circuses (Fig. 7.8).

(a) (b) (c)

Fig. 7.8: **Examples of vertical circular motion around us**: **(a) the Delhi eye is a giant wheel in Kalindi Kunj, Delhi**; **(b) a vertical loop in a roller coaster**; **(c) a water wheel used in ancient times for irrigating fields**. **Source:** Images are courtesy upload. Wikimedia.org

We now work out a few examples to understand the dynamics of vertical circular motion in which the force of gravity also has to be accounted for.

EXAMPLE 7.4: VERTICAL CIRCULAR MOTION

A bucket of water of mass 2.0 kg is tied by a mass-less rope and whirled in a vertical circle with a radius of 1.0 m. At the top of the circular loop, the speed of the bucket is $4.0\,\text{ms}^{-1}$. Determine the tension in the string when the bucket is at the top of the circular loop. Take $g = 10\,\text{ms}^{-2}$.

SOLUTION ■ The **KEY IDEA** here is to identify all forces acting on the bucket at the top of the circle, determine the net force on it and equate it to the centripetal force.

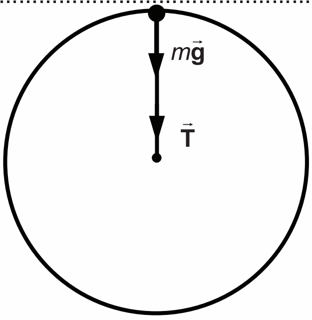


Fig. 7.9: **Free-body diagram for a bucket of water moving in a vertical circle.**

The forces exerted on the bucket at the top of the circle are the force of gravity on the bucket $m\vec{\mathbf{g}}$ and the tension in the string $\vec{\mathbf{T}}$. Both forces are directed vertically downwards as shown in the free-body diagram (see Fig. 7.9). The net force in the downward direction is

$$\vec{\mathbf{F}}_{net} = m\vec{\mathbf{g}} + \vec{\mathbf{T}} \quad \text{(i)}$$

This force, as you have learnt in Sec. 6.4, is equal to the centripetal force $\vec{\mathbf{F}}_c$ required for circular motion. Therefore, we can write

$$\vec{\mathbf{F}}_{net} = \vec{\mathbf{F}}_c \quad \text{(ii)}$$

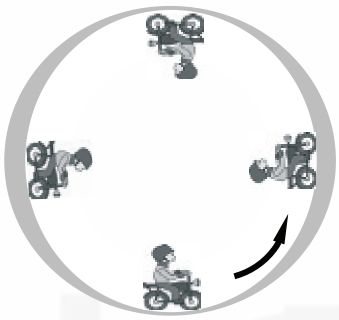
Since all the forces are directed vertically downwards, we can write:

$$\frac{mv^2}{r} = mg + T \quad \text{(iii)}$$

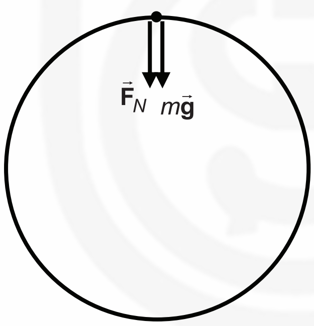
Hence, the tension in the string is given by

$$T = \frac{mv^2}{r} - mg = \frac{(2.0\,\text{kg}) \times (4.0\,\text{ms}^{-1})^2}{1.0\,\text{m}} - 2.0\,\text{kg} \times 10\,\text{ms}^{-2} = 12\,\text{N}$$

Let us take up another example of vertical circular motion.



(a)



(b)

Fig. 7.10: **a) Motorcyclist in a vertical circle**; **b) free-body diagram**.

Example 7.5 : VERTICAL CIRCULAR MOTION

Motorcycle drivers perform a stunt in which they drive their motorcycles around a vertical circular track (Fig. 7.10a). Assume them to be in uniform circular motion. What should the minimum speed of the motorcycle be so that it does not fall even at the top of the vertical circle of radius *r*?

SOLUTION ■ The **KEY IDEA** here is to identify all forces acting on the motorcycle at the top of the circle, determine the net force on it and equate it to the centripetal force needed to keep it moving in the circle.

The net force, which provides the centripetal force at the topmost point of the track (Fig. 7.10b), is equal to the sum of all the force components directed towards the centre of the circle in the radial direction. *We do not consider the engine thrust of the motorcycle and the braking forces because they do not act in the radial direction*. Thus, there are two forces being exerted at each point as you can see in Fig. 7.10b: the **force of gravity** ($m\vec{\mathbf{g}}$) of the motorcycle plus rider and the **normal force** $\vec{\mathbf{F}}_N$ on the motorcycle. Thus, the equation of motion is $m\vec{\mathbf{g}} + \vec{\mathbf{F}}_N = \vec{\mathbf{F}}_c$ where $\vec{\mathbf{F}}_c$ is the centripetal force. At the topmost point, since all the forces are pointing vertically downwards, we get

$$mg + F_N = \frac{mv^2}{r}$$

The minimum value of the speed is obtained when $F_N = 0$. Hence,

$$v_{min} = \sqrt{gr}$$

For a vertical circle of radius 6.0 m, the minimum speed of the motorcycle should be $v_{min} = \sqrt{9.8\,\text{ms}^{-2} \times 6.0\,\text{m}} = 7.7\,\text{ms}^{-1} = 28\,\text{kmh}^{-1}$

SAQ 5 – Vertical circular motion

Suppose the speed of the bucket of Example 7.4 at the bottom of the circular loop is $4.0\,\text{ms}^{-1}$. Calculate the tension in the string.

A force related to the force of gravity is that of **weight**. We now give the formal definition of weight which follows from the law of gravitation.

7.4 WEIGHT

Let us begin by defining weight.

WEIGHT

The **weight** of an object of mass *m* on or above the Earth at a distance *r* from its centre is the **gravitational force exerted on it by the Earth**. It is directed towards the centre of the Earth:

$$\vec{\mathbf{W}} = -\frac{GM_e m}{r^2}\hat{\mathbf{r}} \qquad (7.12a)$$

where $\hat{\mathbf{r}}$ denotes the unit vector directed away from the centre of the Earth towards the point where the object is situated. The magnitude of weight is given by

$$W = \frac{GM_e m}{r^2} = mg \qquad (7.12b)$$

The unit of weight is newton (N).

The **weight** of an object of mass *m* on or above any other astronomical body of mass M_b at a distance *r* from its centre is the gravitational force exerted on it by that body. It is directed towards the centre of that body with magnitude

$$F = G\frac{mM_b}{r^2} \qquad (7.12c)$$

Weight

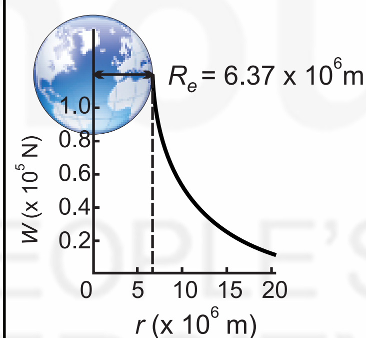


Fig. 7.11: Weight of an object decreases with its distance from the centre of the Earth.

From Eqs. (7.12a and b), you can see that the weight of an object due to Earth's gravitational force decreases as the distance of that object increases from the centre of the Earth. The weight of any spaceship decreases as it moves away from the Earth.

NOTE

Weight is also defined as the contact force that is exerted on an object by whatever is **supporting** it.

Therefore, **always <u>remember</u> that the mass of an object is not the same as its weight**.

Mass and weight are <u>NOT</u> the same quantity. Weight is the gravitational force on the body, which changes with the body's location. Mass is an intrinsic property of the body, which does not change from place to place.

We end this discussion on the concept of weight with an example.

EXAMPLE 7.6: WEIGHT

The weight of an object on the surface of a planet of mass 4.8×10^{24} kg is 60 N. Determine the radius of the planet given that the mass of the object is 30 kg.

SOLUTION ■ The **KEY IDEA** here is to use Eq. (7.12c) for the weight of the object on the planet.

Then the radius of the planet is: $r = \left[\frac{GmM_b}{F}\right]^{1/2}$

Using the values $F = 60$ N, $M_b = 4.8 \times 10^{24}$ kg and $m = 30$ kg, we get

$$r = \left[\frac{(6.67 \times 10^{-11}\,\text{Nm}^2\text{kg}^{-2}) \times (30\,\text{kg}) \times 4.8 \times 10^{24}\,\text{kg}}{(60\,\text{N})}\right]^{1/2}$$

$$\therefore \qquad r = 1.3 \times 10^7\ \text{m}$$

You may like to check your understanding of the concept of weight by solving the following SAQ.

SAQ 6 – Weight

A spaceship lifts off vertically from the Moon, where the free-fall acceleration is $1.60\,\text{ms}^{-2}$. What is the astronaut's weight on the Moon if her weight on the Earth is 500 N? As it lifts off, the spaceship has an upward acceleration of $0.900\,\text{ms}^{-2}$. Determine the magnitude of the force exerted by the spaceship on the astronaut. Take the free-fall acceleration on the Earth to be $10.0\,\text{ms}^{-2}$.

We now discuss the phenomenon of weightlessness.

7.4.1 Weightlessness

You may have seen images of astronauts floating around in the space station and space shuttles. Astronauts orbiting the Earth are known to experience sensations of weightlessness. You may yourself have experienced the sensation of being lighter in a lift just as it starts accelerating downwards. Or if you have enjoyed a ride in a huge giant wheel or a roller coaster, you must have felt a sensation of being weightless as it came down from top. What causes this feeling of weightlessness? To understand it, let us first study what actually causes the sensation of weight as we ***feel*** it.

Suppose that you are sitting at rest on a chair. There are two forces acting upon you – one is the weight due to the Earth's gravitational force (or the force of gravity due to Earth) pulling you downwards, towards the centre of the Earth and the other is the normal force pointing upward, exerted on you by the chair. As you know from Unit 6, the normal force is a **contact** force. Note that the force of gravity acting upon your body is **not** a contact force; it is an *action-at-a-distance* force (strictly speaking a force field), which exists even if you are not in contact with the Earth. (You ***cannot ever*** feel the force of gravity acting on your body – because it is a **not** a contact force). When you sit on the chair, what you feel is the normal force exerted by the chair on you and **it is this force that provides you with a sensation of weight**. When you are at rest, the upward normal force is equal to your weight and this gives a measure of the gravitational force on you or your weight.

Contact forces can only result from the actual touching of the two interacting objects – in this case, the chair and you. The forces that result from contact can actually be felt. For example, when you are pushed by a friend, you feel the applied force (a contact force). When you ride on the swings, you feel the tension force (a contact force).

This means that if the upward normal force being exerted upon you were zero, you would not have any sensation of your weight.

Thus, **it follows that weightlessness is a sensation experienced by an object when the net contact force on it is zero. Such a sensation is observed in situations in which objects are in a state of free fall**. In free fall, the only force acting upon an object is the force of gravity and the net contact force is zero. Therefore, you will have no sensation of weight in a state of free fall. You would of course continue to have the same mass as you always do!

In the following example, we calculate the apparent weight of a person travelling in an accelerating lift using Newton's laws. When you go through it, you will also learn what happens in the state of free fall!

EXAMPLE 7.7: APPARENT WEIGHT

We normally use a spring balance to determine our weight. Let us see what reading the scale gives when it is accelerating. You may have been inside a lift in a multi-storied building. How do you feel when the lift just starts accelerating upwards? You feel heavier. What happens when the lift just starts accelerating downwards? You feel lighter. How do we explain these sensations?

To find the answer, consider a spring balance kept in a lift with a person standing on it (Fig. 7.12a). Suppose that the lift is at rest initially with respect to an inertial observer standing outside the lift. What is the magnitude of the weight of the person in the lift as measured by the inertial observer? You know from Eq. (7.6a) that it is $F = mg$, where m is the mass of the person. This is the **"true" weight** of the person.

Now suppose that the lift (and hence, the balance and the person) **moves upwards** with **acceleration** $\vec{a}$ with respect to the inertial observer. The free-body diagram of the person is shown in Fig. 7.12b. Note the choice of y-axis. Two forces are exerted on the person: The "true" weight of the person given by mg and the normal force exerted by the balance on the person. Applying Newton's second law to the person, we get

$$\sum F_y = + F_N - mg = + ma \qquad \text{(i)}$$

where a is the acceleration of the lift, the balance and the person.

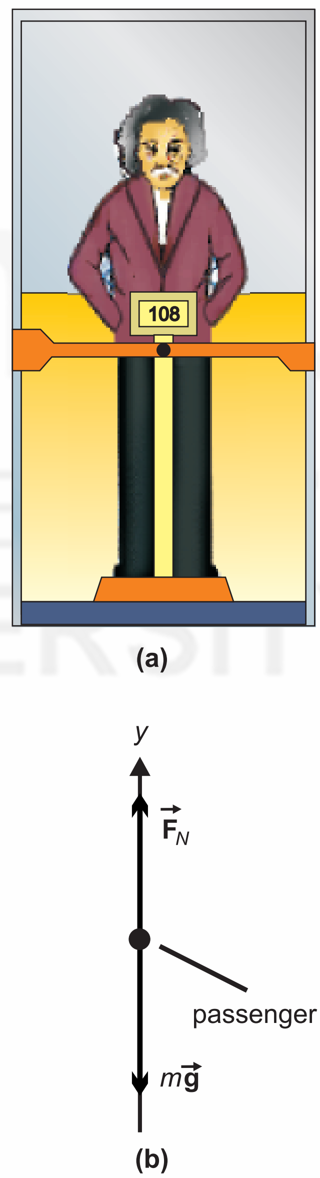


Fig. 7.12: **Apparent and "true" weight of a person.**

Hence, $F_N = mg + ma$ (ii)

Now F_N is the normal force **on** the person **by** the balance. But according to Newton's third law, this is also equal to the magnitude of the downward force exerted **by** the person **on** the balance: This is the weight of the person as measured by the balance in a lift accelerating upwards. It is called the **apparent weight** of the person. It is *greater* than the person's true weight. That is why you feel heavier when the lift accelerates vertically upwards.

If the lift is not accelerating, the apparent weight equals the true weight. Thus, if the lift is at rest or moving with a constant velocity, the balance measures the true weight of the person.

If the lift accelerates **downwards**, the acceleration $\vec{\mathbf{a}}$ is directed opposite and we have

$$\sum F_y = +F_N - mg = -ma \quad \text{(iii)}$$

Hence, $F_N = mg - ma$ (iv)

Thus, the **apparent weight is less than the true weight of the person in a lift accelerating vertically downwards**. That is why you feel lighter when the lift just starts moving downwards. Now, what would happen if the lift were to fall freely with acceleration g? In such a situation, $a = g$ in Eq. (iv), which gives

$$F_N = 0 \quad \text{(v)}$$

Thus, the apparent weight is zero and you would feel **weightless** in a freely-falling lift!

This explains why astronauts in spaceships feel weightless – for the same reason that a person in a freely falling lift feels weightless. Note that both the astronaut and the spaceship are in a state of uniform circular motion around the Earth. The centripetal force for the circular motion is provided by the force of gravity and is directed towards the centre of the Earth. Thus, gravity is the only force acting upon the astronaut and their surroundings(spacecraft). So, their acceleration is the same as the acceleration due to gravity i.e. these are freely falling bodies. Therefore, as you have learnt in Example 7.7, their apparent weight is zero and they feel a sensation of weightlessness.

In this unit, we have discussed the long-range force of gravitation, which is exerted by every object in this universe upon every other object in the macroscopic world. You have learnt the concept of gravitational field. You have also learnt the concepts of the force of gravity and the variation of the acceleration due to gravity with altitude, depth and latitude. We have discussed the concepts of weight and weightlessness. We hope that you have found the applications of these concepts in satellites, vertical motion of giant wheels and spaceships interesting.

Let us now summarise what you have learnt in this unit.

7.5 SUMMARY

Concept	Description
Force of Gravitation	■ The **force of gravitation** is a long-range force of attraction exerted by every object in this universe upon every other object. The gravitational force exerted **by** a particle of mass m_1 **on** a particle of mass m_2 separated by a distance r from it is **directed along the line joining the particles from m_2 to m_1**. It is given by $$\vec{\mathbf{F}}_{12} = -G\frac{m_1 m_2}{\lvert\vec{\mathbf{r}}_{12}\rvert^2}\hat{\mathbf{r}}_{12}$$ where G is the universal constant of gravitation, $\hat{\mathbf{r}}_{12}$ is the unit vector along the line joining the particles and is directed from m_1 to m_2. Its magnitude is given by $$F = G\frac{m_1 m_2}{r^2} \quad \text{where} \quad \lvert\vec{\mathbf{r}}_{12}\rvert = r$$
The Principle of Superposition	■ According to the **principle of superposition,** in a system of N particles, the resultant force of gravitation on any particle is the vector sum of the forces of gravitation exerted by all other particles on it. For an N-particle system of masses $m_1, m_{2,}, \ldots, m_N$, the gravitational force on particle 1 of mass m_1 due to all other particles in the system is given by $$\vec{\mathbf{F}}_1 = -G\sum_{i=2}^{N}\frac{m_1 m_i}{r_{i1}^2}\hat{\mathbf{r}}_{i1}$$
Gravity	■ The gravitational force between the Earth and any other object on or near it is called **gravity**. It is given by $$\vec{\mathbf{F}}_g = m\vec{\mathbf{g}},$$ where g is the acceleration due to gravity directed towards the centre of the Earth. Its values are given by $$g_0 = \frac{GM_e}{R_e^2} \text{ at the equator at the surface of the Earth}$$ $$g_A = \frac{g_0 R_e^2}{(R_e + h)^2} \text{ at height } h \text{ from the surface of the Earth}$$ $$g_B = \frac{g_0}{R_e}(R_e - d), \text{ at depth } d \text{ from the surface of the Earth}$$ $$g(\lambda) = g_0 + \omega^2 R_e \sin^2\lambda, \text{ at latitude } \lambda$$

Weight

- The **WEIGHT** of an object of mass m on or above the Earth at a distance r from its centre is the **gravitational force exerted on it by the Earth** directed towards the centre of the Earth. Its magnitude is given by

$$W = \frac{GM_e m}{r^2} = mg$$

A freely-falling object feels the sensation of weightlessness.

7.6 TERMINAL QUESTIONS

1. Determine the force of gravitation exerted by the Earth on an object of mass 70 kg situated at an altitude of 6.4×10^6 m above the surface of the Earth. Take the mass and radius of the Earth to be $M_e = 6.0 \times 10^{24}$ kg and $R_e = 6.4 \times 10^6$ m, respectively.

2. A small satellite is in circular orbit around a planet at a distance of 3.0×10^8 m from the centre of the planet. The orbital speed of the satellite is $200\ \text{ms}^{-1}$. What is the mass of the planet?

3. Three objects $A\ (m_A = 10\ \text{kg}), B\ (m_B = 10\ \text{kg})$ and $C\ (m_C = 15\ \text{kg})$ are placed 1.0 m apart in a straight line. Calculate the net gravitational force on object B due to objects A and C.

4. The mass of the Earth is about 80 times that of the Moon. Determine the ratio of the radius of the Earth to that of the Moon, if the value of the acceleration due to gravity on the surface of the Moon is $1.67\ \text{ms}^{-2}$. Use $g = 9.80\ \text{ms}^{-2}$.

5. Calculate the value of the acceleration due to the Earth's gravity at an altitude of 2.5×10^3 km and at the depth of 3.0 km given that $g_0 = 9.80\ \text{ms}^{-2}$.

6. A stone of mass 0.50 kg is swinging in a vertical circle of radius 1.0 m. The speed of the stone is constant and equals 6.0 ms $^{-1}$. Determine the tension in the string at the bottom of the circle and at the top of the circle. Take g = 10 ms^{-2}.

7. An object of mass 20.0 kg is orbiting the Mars at an altitude of 200 km. What is the weight of the object, if Mars has a radius of 3430 km and a mass of 6.34×10^{23} kg?

8. One of the vertical circular rides in an amusement park has a radius of 35.0 m. You are sitting in a car that is just at the top of the ride. How fast must the car be moving in order that you momentarily lift off your seat and feel weightless? Take $g = 10.0\ \text{ms}^{-2}$.

9. It is possible to simulate "weightless" conditions by flying a plane in an arc such that the centripetal acceleration exactly cancels the acceleration due

to gravity. Such a plane was used by NASA when training astronauts. What would its required speed be at the top of an arc of radius 1000 m?

10. A girl of mass 40 kg stands in an elevator. Obtain the force which the floor of the lift exerts on the girl

    a) when the lift has an upward acceleration of $2.0\,\text{ms}^{-2}$;

    b) when the lift is rising at constant speed and

    c) when the lift has a downward acceleration of $2.0\,\text{ms}^{-2}$.

    Take $g = 10\,\text{ms}^{-2}$.

7.7 SOLUTIONS AND ANSWERS

Self-Assessment Questions

1. Using $m_1 = m_2 = 50\,\text{kg}$ and $r = 1.5\,\text{m}$ in Eq. (7.1a), we get the magnitude of the force of gravitation as

$$F = \frac{(6.67\times10^{-11}\,\text{Nm}^2\text{kg}^{-2})\times(50\,\text{kg})\times(50\,\text{kg})}{(1.5\,\text{m})^2} = 7.4\times10^{-8}\,\text{N}$$

The force is directed along the line joining the two particles, as in Fig. 7.1.

2. We use Eq. (7.2) with $v = 5.70\times10^5\,\text{ms}^{-2}$ and $R = 7.50\times10^{17}\,\text{m}$:

$$M = \frac{v^2 r}{G} = \frac{(5.70\times10^5\,\text{ms}^{-1})^2\times(7.50\times10^{17}\,\text{m})}{(6.67\times10^{-11}\,\text{Nm}^2\,\text{kg}^{-2})} = 3.65\times10^{39}\,\text{kg}$$

3. Since the mass m is at the midpoint of the line joining the two masses $2m$ and $4m$, the gravitational force on m due to the mass $4m$ is

$$\vec{\mathbf{F}}_1 = G\frac{4m^2}{r^2}\hat{\mathbf{r}}$$

where $\hat{\mathbf{r}}$ is the unit vector along $\vec{\mathbf{F}}_1$. The gravitational force on m due to the mass $2m$ is $\vec{\mathbf{F}}_2 = -G\dfrac{2m^2}{r^2}\hat{\mathbf{r}}$

The net gravitational force on m is $\vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 = G\dfrac{2m^2}{r^2}\hat{\mathbf{r}}$

It is directed towards the greater mass $4m$. Let x be the distance of the mass m from the mass $4m$. Then the distance of m from the mass $2m$ is $(2r - x)$. For the net force on m to be zero we must have, $|\vec{\mathbf{F}}_1'| = |\vec{\mathbf{F}}_2'|$, where

$$|\vec{\mathbf{F}}_1'| = \frac{4Gm^2}{x^2} \quad \text{and} \quad |\vec{\mathbf{F}}_2'| = \frac{2Gm^2}{(2r-x)^2}$$

$$\therefore \quad \frac{4Gm^2}{x^2} = \frac{2Gm^2}{(2r-x)^2} \quad \Rightarrow \quad 2\,(2r-x)^2 = x^2$$

We can also solve SAQ 3 using Eq. (ii) of Example 7.2 with $m_1 = 4m$, $m_2 = 2m$ and $a = 2r$. Hence, we get

$$x = \frac{2r}{\sqrt{\frac{2m}{4m}} + 1} = \frac{2\sqrt{2}r}{\sqrt{2}+1} = 1.17r$$

Solving for x we get 2 values for x: $x_1 = \frac{2\sqrt{2}\ r}{(\sqrt{2}-1)} = 6.83r$ and

$$x_2 = \frac{2\sqrt{2}\ r}{(\sqrt{2}+1)} = 1.17r$$

Of these, only the second is acceptable because m lies between the two masses for only that value of x. Hence, the mass m must be placed at a distance $1.17r$ from the mass $4m$ so that the net force on it is zero. Read the margin remark as well.

4. The gravitational acceleration g_J on Jupiter is given by Eq. (7.6d), on replacing M_e and R_e by the mass and radius of Jupiter, respectively:

$$g_J = \frac{(6.67\times10^{-11}\,\text{Nm}^2\,\text{kg}^{-2})\times(1.90\times10^{27}\,\text{kg})}{(7.14\times10^7\,\text{m})^2} = 24.9\ \text{ms}^{-2}$$

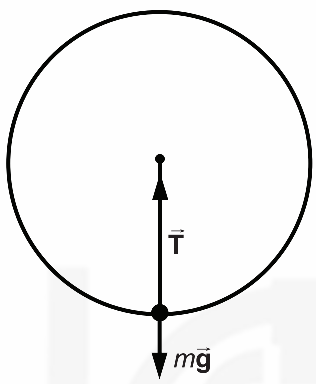


Fig. 7.13: **Free-body diagram for solution of SAQ 5.**

5. Refer to Fig. 7.13. At the bottom of the circle, the tension in the string $\vec{\mathbf{T}}$ and the force of gravity are exerted in the opposite directions. Therefore,

$$\sum F_y = T - mg$$

For uniform circular motion: $F_C = \sum F_y = T - mg$

or $$\frac{mv^2}{r} = T - mg$$

$$\therefore\ T = \frac{mv^2}{r} + mg = \frac{2.0\,\text{kg}\ \times(4.0\,\text{ms}^{-1})^2}{1.0\,\text{m}} + 2.0\,\text{kg}\ \times 10\,\text{ms}^{-2}$$

$$= 32\,\text{N} + 20\,\text{N} = 52\,\text{N}$$

6. The mass of the astronaut $= \frac{500\,\text{N}}{10.0\,\text{ms}^{-2}} = 50.0\ \text{kg}$

The astronaut's weight on the Moon $= 50.0\ \text{kg}\times1.60\,\text{ms}^{-2} = 80.0\ \text{N}$

The force exerted by the spaceship on the astronaut is
$80.0\ \text{N} + (50.0\ \text{kg}\times 0.900\ \text{ms}^{-2}) = 125\ \text{N}$

Terminal Questions

1. Using Eq. (7.1a), we can write the magnitude of the force of gravitation as:

$$F = G\frac{mM_e}{(R_e + r)^2}$$

where $r = 6.4\times10^6\,\text{m}$. Using $R_e = 6.4\times10^6\,\text{m}$, $M_e = 6.0\times10^{24}\,\text{kg}$ and $m = 70\,\text{kg}$ we get:

$$F = \frac{(6.67\times10^{-11}\text{Nm}^2\text{kg}^{-2})\times(70\,\text{kg})\times(6.0\times10^{24}\text{kg})}{(6.4\times10^6\text{m}+6.4\times10^6\text{m})^2} = 1.7\times10^2\,\text{N}$$

2. For circular motion, the centripetal force is provided by the gravitational force on the satellite and, therefore, we have:

$$\frac{mv^2}{r} = G\frac{mM_P}{r^2}$$

where M_P is the mass of the planet, $r = 3.0 \times 10^8$ m and $v = 200\ \text{ms}^{-1}$.

$$M_P = \frac{rv^2}{G} = \frac{(3.0\times10^8\ \text{m})\times(200\ \text{ms}^{-1})^2}{6.67\times10^{-11}\ \text{Nm}^2\text{kg}^{-2}} = 1.8\times10^{23}\ \text{kg}$$

3. Let B be located at the origin of the coordinate system. Let the gravitational forces exerted by A and C on B be $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$, respectively. From Eqs. (7.4a and b), the resultant gravitational force on B is

$$\vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 = -G\frac{m_A m_B}{r_{AB}^2}\hat{\mathbf{i}} + G\frac{m_B m_C}{r_{BC}^2}\hat{\mathbf{i}}$$

where $m_A =$ mass of $A = 10$ kg, $m_B =$ mass of $B = 10$ kg and $m_C =$ mass of $C = 15$ kg and $r_{AB} = r_{BC} = 1.0$ m. Thus,

$$\vec{\mathbf{F}} = 6.67\times10^{-11}\ \text{Nm}^2\ \text{kg}^{-2}\times\left[-\frac{10\ \text{kg}\times10\ \text{kg}}{(1.0\ \text{m})^2} + \frac{10\ \text{kg}\times15\ \text{kg}}{(1.0\ \text{m})^2}\right]\hat{\mathbf{i}}$$

$$= 3.3\times10^{-9}\ \text{N}\ \hat{\mathbf{i}}$$

4. We use Eq. (7.6d) for the value of g on Earth. For the gravitational acceleration on the Moon (g_m), we replace M_e by M_m (the mass of the Moon) and R_e by R_m (the radius of the Moon) in Eq. (7.6d). It is given that $M_e = 80\,M_m$. Let α be the ratio of the radii of the Earth and the Moon. So,

$$g_m = G\frac{M_m}{R_m^2} = G\frac{M_e/80}{\frac{R_e^2}{\alpha^2}} = \frac{\alpha^2}{80}\left[\frac{GM_e}{R_e^2}\right]$$

$$\Rightarrow \qquad 1.67\ \text{ms}^{-2} = \frac{\alpha^2}{80}.g = \frac{\alpha^2}{80}(9.80\ \text{ms}^{-2})$$

$$\therefore \qquad \alpha = \sqrt{\frac{80\times1.67}{9.80}} = \sqrt{13.63} \approx 3.69$$

5. We calculate the acceleration due to the Earth's gravity at an altitude of 2.5×10^3 km (g_1) using Eq. (7.9a). Note that we use Eq. (7.9a) instead of Eq. (7.9b) because h (2500 km) is comparable to R_e (6371 km). Thus,

$$g_1 = g_0\left(1+\frac{2500}{6371}\right)^{-2}\text{ms}^{-2} = 5.07\ \text{ms}^{-2}$$

We calculate the acceleration at a depth of 3.0 km (g_2) using Eq. (7.10):

$$g_2 = 9.80\ \text{ms}^{-2}\times\left(1-\frac{3.0}{6371.0}\right) = 9.795\ \text{ms}^{-2} \approx 9.8\ \text{ms}^{-2},$$

up to 2 significant digits.

6. As in Example 7.4, we can write the tension at the top of the circle as

$$T = \frac{mv^2}{r} - mg = \frac{0.50\,\text{kg} \times (6.0\,\text{ms}^{-1})^2}{(1.0\,\text{m})} - 0.50\,\text{kg} \times 10\,\text{ms}^{-2} = 13\,\text{N}$$

For the tension at the bottom of the circle we write (as in SAQ 5),

$$T = \frac{mv^2}{r} + mg = 18\,\text{N} + 5\,\text{N} = 23\,\text{N}$$

7. We use Eq. (7.12c) with M_b replaced by the mass of Mars and $r = (3430 + 200)\,\text{km}$. Then the weight of the object is

$$F = \frac{(6.67 \times 10^{-11}\text{Nm}^2\text{kg}^{-2}) \times (20.0\,\text{kg}) \times (6.34 \times 10^{23}\text{kg})}{(3.63 \times 10^6\,\text{m})^2} = 64.2\,\text{N}$$

8. To feel weightless at the top of the ride, we must have $\vec{\mathbf{F}}_N = 0$ (see Fig. 7.10). Therefore, $\frac{mv^2}{r} = F_N + mg$ and

for $F_N = 0, \quad \frac{mv^2}{r} = mg$

$$\therefore \quad v^2 = gr \;\Rightarrow\; v = \sqrt{gr} \text{ or } v = \sqrt{(10.0\,\text{ms}^{-2}) \times 35.0\,\text{m}} = 18.7\,\text{ms}^{-1}$$

9. In this case, the centripetal force is provided by the force of gravity when the astronaut feels weightless, i.e., $F_N = 0$. Thus, we have

$$\frac{mv^2}{r} = mg \;\Rightarrow\; v = \sqrt{gR} = \sqrt{(10\,\text{ms}^{-2}) \times 1000\,\text{m}} = 100\,\text{ms}^{-1}$$

10. The free-body diagram for the problem is shown in Fig. 7.14. Note that the following two forces are exerted on the girl:

$\vec{\mathbf{F}}_g$ = The force of gravity exerted by the Earth on the girl $= m\vec{\mathbf{g}}$

$\vec{\mathbf{F}}_f$ = The force exerted by the floor on the girl

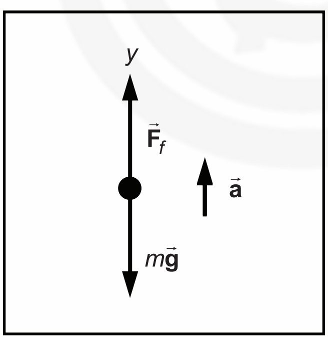


Fig. 7.14: Diagram for the solution of TQ 10.

Since motion is only in the *y* direction, according to Newton's second law, the equation of motion to be solved in the problem is:

$$\vec{\mathbf{F}}_{net} = m\vec{\mathbf{a}} \quad \text{or} \quad F_{net,y} = m a_y$$

The solution for each situation is as follows:

a) When $a_y > 0$, $\sum F_y = F_f - mg = ma_y \Rightarrow F_f = m(g + a_y)\,\text{N}$ or $F_f = 40(10 + 2) = 480\,\text{N}$

b) When $a_y = 0$, (equilibrium), $\sum F_y = F_f - mg = 0 \Rightarrow F_f = mg\,\text{N}$ or $F_f = 40(10) = 400\,\text{N}$

c) When $a_y < 0$, $\sum F_y = F_f - mg = -ma_y \Rightarrow F_f = m(g - a_y)\text{N}$ or $F_f = 40(10 - 2) = 320\,\text{N}$

How is the speed of a rocket increased by adding many stages in it? This unit will help you answer this question!

UNIT 8

LINEAR MOMENTUM AND IMPULSE

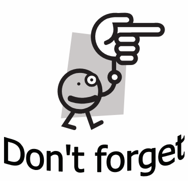
Structure

8.1 Introduction
    Expected Learning Outcomes
8.2 Linear Momentum
    Conservation of Linear Momentum
    Linear Momentum and the Flow of Mass
    Rocket Motion
8.3 Impulse
8.4 Summary
8.5 Terminal Questions
8.6 Solutions and Answers

STUDY GUIDE

In this unit, you will study the **law of conservation of linear momentum for two-particle systems** and the concept of **impulse**. In order to learn these concepts well, **you should know the concepts of vector algebra and integral calculus**. You may like to review these concepts before studying this unit.

Try to work out all solved examples, SAQs and Terminal Questions by yourself. This will help you understand the concepts better.



IN YOUR WRITTEN WORK, ALWAYS USE AN ARROW ABOVE THE LETTER YOU USE TO DENOTE A VECTOR, E.G., $\vec{r}$. USE A CAP ABOVE THE LETTER YOU USE TO DENOTE A UNIT VECTOR, E.G., $\hat{r}$.

"The entire preoccupation of the physicist is with things that contain within themselves a principle of movement and rest."

Aristotle

8.1 INTRODUCTION

In Units 5, 6 and 7, you have studied Newton's laws of motion and their applications. You have solved a variety of problems involving objects in motion. While introducing Newton's second law, we had defined the **linear momentum** of an object and derived the equation of motion $(\vec{\mathbf{F}} = m\vec{\mathbf{a}})$ for objects of constant mass. In this unit, we discuss this concept in some detail. More importantly, we introduce the **law of conservation of linear momentum** in Sec. 8.2.1 for two-particle systems. This is a **fundamental law of nature**, which allows us to determine the velocities of particles without going into the detailed description of their motion. We also use the concept of linear momentum to study the motion of systems in which there is a flow of mass, for example, the motion of rockets.

So far we have dealt with forces that do not depend on time. There are many situations in which the forces exerted on an object vary with time, for example, the ball hit by a bat or a tennis racket. These are exerted for a very small time (the time of contact) but are quite large. In Sec. 8.3, we introduce the concept of **impulse** to study the motion of an object under time-varying forces. We also discuss the **impulse-momentum theorem**, which is useful for solving problems of motion of such objects.

In the next unit, we discuss the concepts of work and kinetic energy, which came from further refinement of Newton's mechanics.

Expected Learning Outcomes

After studying this unit, you should be able to:

- determine the linear momentum and impulse of a particle;
- apply the law of conservation of linear momentum to simple two-particle systems;
- calculate the linear momentum for two-particle systems with constant mass and variable mass; and
- apply the impulse-momentum theorem to solve simple problems.

8.2 LINEAR MOMENTUM

You have learnt in Sec. 5.3 of Unit 5 that the linear momentum of an object is the product of its mass *m* and velocity $\vec{\mathbf{v}}$. You have also learnt that the **net external force** on an object is equal to the rate of change of its **linear momentum** with time:

$$\vec{\mathbf{F}}_{net} = \frac{d\vec{\mathbf{p}}}{dt} \quad \text{where} \quad \vec{\mathbf{p}} = m\vec{\mathbf{v}} \tag{8.1a}$$

This equation tells us that **the net external force exerted on the object changes its linear momentum**. It also implies that the linear momentum of an object can be changed only by a net external force. If there is no net external force on a system, $\vec{\mathbf{p}}$ remains constant. This is actually a fundamental law of nature and gives us an extremely powerful tool for solving problems on motion. So we now explain this law for a two-particle system.

8.2.1 Conservation of Linear Momentum

If the net external force on a particle or a system of particles is zero, Eq. (8.1a) gives us

$$\frac{d\vec{\mathbf{p}}}{dt} = \vec{\mathbf{0}} \qquad \Rightarrow \qquad \vec{\mathbf{p}} = \text{constant} \qquad (8.1b)$$

Conservation of linear momentum

This means that the linear momentum of the object, on which the net external force is zero, does not change. This result is called the **law of conservation of linear momentum**. It is extremely useful when we are considering a system of more than one particle/object.

Let us consider two-particle systems on which the net external force is zero. We call such systems as **isolated systems**. We have shown a few examples of such systems in Fig. 8.1: A book at rest on a table, passenger standing in a boat in still water, a billiards ball moving at a constant velocity. In all these systems, the net external force is zero as these are either at rest or moving with a constant velocity. Moreover, these systems are **closed**, that is, no particles leave or enter the system. The total linear momentum $\vec{\mathbf{p}}$ of such a system of two particles having linear momentum $\vec{\mathbf{p}}_1$ and $\vec{\mathbf{p}}_2$, respectively, is

$$\vec{\mathbf{p}} = \vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2 \qquad (8.1c)$$

The law of conservation of linear momentum is then stated as follows:

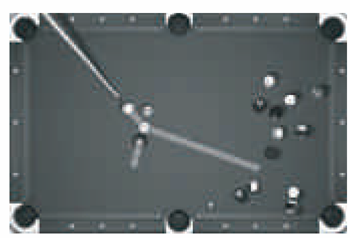


Fig. 8.1: **Some examples of isolated systems.**

LAW OF CONSERVATION OF LINEAR MOMENTUM

If no net external force is exerted on an isolated and closed system of particles, **the total linear momentum of the system cannot change.** We say that **the total linear momentum of the system is conserved:**

$$\vec{\mathbf{p}} = \text{constant} \qquad \text{(for a closed, isolated system)} \qquad (8.2a)$$

We also say that for a **closed and isolated system of particles**

$$\begin{pmatrix}\text{The total linear momentum} \\ \text{at some initial instant } t_i\end{pmatrix} = \begin{pmatrix}\text{The total linear momentum} \\ \text{at some final instant } t_f\end{pmatrix}$$

We can also write this result as: $\vec{\mathbf{p}}_i = \vec{\mathbf{p}}_f$ (8.2b)

This law is valid for a system of many particles and you will learn about it in Block 3. Here we will take up systems of two particles and apply it to some simple cases.

EXAMPLE 8.1: CONSERVATION OF LINEAR MOMENTUM

A car (1) is standing at a red traffic light on a slippery straight road. The driver of another car (2) is unable to apply brakes in time and hits the car at rest (see Fig. 8.2). With what speeds do the two cars move forward together if the speed of the moving car at the time of hitting the stationary car is constant and equals $2.0\ \text{ms}^{-1}$? The mass of car 1 and its driver is 1000 kg and the mass of car 2 and its passengers is 1500 kg.

SOLUTION ■ The **KEY IDEA** here is that the net external force on the system of the two cars is zero when they collide. This is because the speed of car 2 is constant and it is moving in a straight line, and car 1 is at rest. Hence, we can treat the system of the two cars as a closed, isolated system and apply the law of conservation of linear momentum to it.

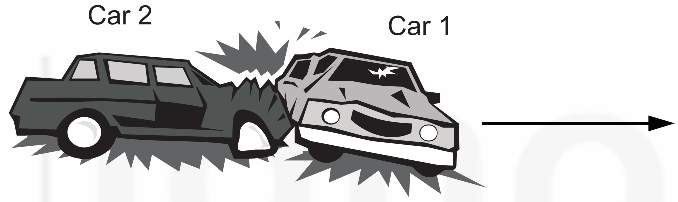


Fig. 8.2

We use Eq. (8.2b) and write the initial momentum of the system as

$$\vec{\mathbf{p}}_i = m_{car1}\vec{\mathbf{v}}_{car1} + m_{car2}\vec{\mathbf{v}}_{car2} = \vec{\mathbf{0}} + (1500\ \text{kg} \times 2.0\,\text{ms}^{-1})\hat{\mathbf{i}}$$

$$= 3.0 \times 10^3\ \text{kg.ms}^{-1}\,\hat{\mathbf{i}}$$

where the direction of motion of the moving car (2) is along the positive x- axis (see Fig. 8.2). After the moving car hits the stationary car (1) , the two travel together in the same direction. Let their velocity be $\vec{\mathbf{v}}$. Then we have:

$$\vec{\mathbf{p}}_f = (m_{car1} + m_{car2})\vec{\mathbf{v}} = \vec{\mathbf{p}}_i$$

or $$\vec{\mathbf{p}}_f = (2.5 \times 10^3\,\text{kg})\vec{\mathbf{v}} = 3.0 \times 10^3\,\text{kg.ms}^{-1}\,\hat{\mathbf{i}}$$

$$\therefore \quad \vec{\mathbf{v}} = 1.2\,\text{ms}^{-1}\,\hat{\mathbf{i}}$$

Now you should apply this law to a simple system by solving SAQ 1.

SAQ 1 – Conservation of linear momentum

Two identical cars have the same speed; one travels due east and another due west. Do the cars have the same linear momentum? What is the total linear momentum of the system of the cars? Explain your answers.

So far in this section, you have learnt that when no net external force is exerted on a system of particles, we can apply the law of conservation of linear momentum to study its motion. The advantage is that we need not know the details of the forces acting on each part of the system. You will study more examples of this law in Block 3.

So far, we have taken up examples of simple systems in which the mass remains constant. There are many examples around us where there is a flow of mass in the system and the mass of the object in the system changes. Rocket motion, ketchup coming out of a bottle, water sprayed from a pipe, coal falling onto rail wagons for transport are some such examples (Fig. 8.3).

Fig. 8.3: **Examples of systems having variable mass**.

For studying the motion of such systems, we determine $\Delta\vec{\mathbf{p}}$ and then find the net force from $\left(\vec{\mathbf{F}}_{net} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\mathbf{p}}}{\Delta t}\right)$.

8.2.2 Linear Momentum and the Flow of Mass

The concept of linear momentum is very useful when we study a system in which there is flow of mass with time. For such systems, we write Newton's second law as follows:

$$\vec{\mathbf{F}} = \frac{d\vec{\mathbf{p}}}{dt} = \frac{d(m\vec{\mathbf{v}})}{dt} = m\frac{d\vec{\mathbf{v}}}{dt} + \vec{\mathbf{v}}\frac{dm}{dt} \qquad (8.3)$$

We shall solve Eq. (8.3) for rocket motion, which is a very important example of mass flow and linear momentum problem. But you may like to attempt an SAQ based on Eq. (8.3).

SAQ 2 – Variable mass system

Sand falls on a conveyor belt at a constant rate of $50\,\text{kgs}^{-1}$ from a hopper (Fig. 8.4). Determine the force required to maintain a constant velocity of $2.0\ \text{ms}^{-1}$ of the belt.

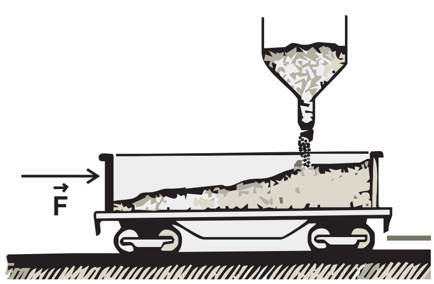


Fig. 8.4: **Sand falling on a conveyor belt.**

Fig. 8.5: **Lift-off of the PSLV rocket that launched Chandrayaan-1 on October 22, 2008.**

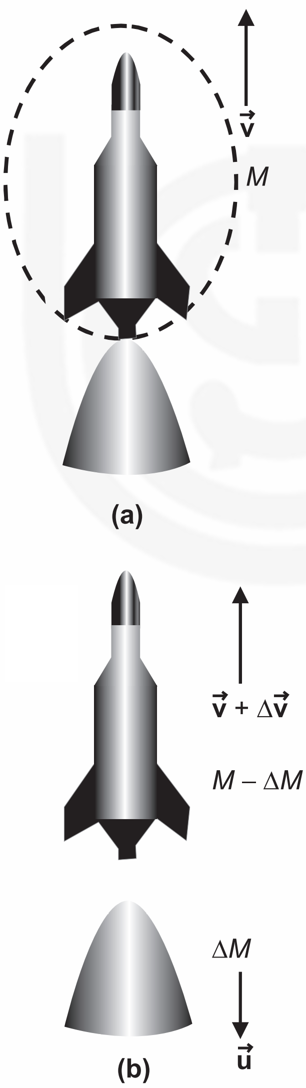


Fig. 8.6: **Analysing rocket motion.**

8.2.3 Rocket Motion

Underlying principle: In a rocket, a stream of gas produced at very high temperature and pressure is ejected at very high velocity through an exhaust nozzle (Fig. 8.5). The rocket moves with an increased velocity in a direction opposite to that of the ejected gas.

We now solve Eq. (8.3) to answer the question: **What is the velocity of a rocket of mass M which ejects gas of mass ΔM in time interval Δt?**

Let us first apply Newton's second law of motion for a variable mass system [(Eq. (8.3)] to the rocket alone. So we just analyse the motion of the rocket and do not worry about the motion of ejected gas. So the system boundary is as shown by the dotted line in Fig. 8.6a.

We observe the rocket's motion from an **inertial frame attached to the Earth**.

Let the mass of the rocket be M at some instant of time t. Let the mass of the gas ejected by the rocket during the time interval Δt be ΔM. During this time interval, the rocket's mass decreases by ΔM and at the instant $(t + \Delta t)$ it is $(M - \Delta M)$.

Let $\vec{\mathbf{u}}$ be the velocity of the ejected gas of mass ΔM and $(\vec{\mathbf{v}} + \Delta\vec{\mathbf{v}})$ be the rocket's velocity relative to the inertial frame at the end of the time interval Δt (see Fig. 8.6b).

Newton's second law tells us that $\vec{\mathbf{F}}_{net} = \dfrac{d\vec{\mathbf{p}}}{dt}$

Now $\vec{\mathbf{p}}_i = M\vec{\mathbf{v}}$

and $\vec{\mathbf{p}}_f = (M - \Delta M)(\vec{\mathbf{v}} + \Delta\vec{\mathbf{v}}) + (\Delta M)\vec{\mathbf{u}}$

The change in linear momentum of the rocket is:

$$\Delta\vec{\mathbf{p}} = \vec{\mathbf{p}}_f - \vec{\mathbf{p}}_i = \left[(M - \Delta M)(\vec{\mathbf{v}} + \Delta\vec{\mathbf{v}}) + (\Delta M)\vec{\mathbf{u}}\right] - M\vec{\mathbf{v}}$$

or

$$\Delta\vec{\mathbf{p}} = M\vec{\mathbf{v}} + M\Delta\vec{\mathbf{v}} - \Delta M\vec{\mathbf{v}} - \Delta M\Delta\vec{\mathbf{v}} + (\Delta M)\vec{\mathbf{u}} - M\vec{\mathbf{v}}$$

$$= M\Delta\vec{\mathbf{v}} + \Delta M(\vec{\mathbf{u}} - \vec{\mathbf{v}} - \Delta\vec{\mathbf{v}})$$

Note that $\left[\vec{\mathbf{u}} - (\vec{\mathbf{v}} + \Delta\vec{\mathbf{v}})\right]$ is the **velocity of the gas stream relative to the rocket**. We denote it by $\vec{\mathbf{v}}_{rel}$ so that we have

$$\Delta\vec{\mathbf{p}} = M\Delta\vec{\mathbf{v}} + \Delta M\,\vec{\mathbf{v}}_{rel} \tag{8.4a}$$

We now divide Eq. (8.4a) by Δt and take the limit as $\Delta t \to 0$ to obtain $\dfrac{d\vec{\mathbf{p}}}{dt}$. In this process, we assume that $\vec{\mathbf{v}}_{rel}$ is constant. Thus,

$$\frac{d\vec{\mathbf{p}}}{dt} = \lim_{\Delta t \to 0}\left(\frac{\Delta M}{\Delta t}\right)\vec{\mathbf{v}}_{rel} + M\lim_{\Delta t \to 0}\left(\frac{\Delta\vec{\mathbf{v}}}{\Delta t}\right)$$

Since M decreases with time, $\lim_{\Delta t \to 0}\left(\frac{\Delta M}{\Delta t}\right) = -\frac{dM}{dt}$ and we have

$$\frac{d\vec{\mathbf{p}}}{dt} = -\frac{dM}{dt}\vec{\mathbf{v}}_{rel} + M\frac{d\vec{\mathbf{v}}}{dt} \qquad (8.4b)$$

Substituting Eq. (8.4b) in Newton's second law, we get

$$\vec{\mathbf{F}}_{net} = M\frac{d\vec{\mathbf{v}}}{dt} - \frac{dM}{dt}\vec{\mathbf{v}}_{rel} \qquad (8.4c)$$

We now consider two cases:

Case 1: Rocket in Earth's gravity, and

Case 2: Rocket in free space when the net external force on it is zero.

Case 1: Rocket in Earth's gravity

When the rocket moves in the Earth's gravity, $\vec{\mathbf{F}}_{net} = -M\vec{\mathbf{g}}$ and we have

$$-M\vec{\mathbf{g}} = M\frac{d\vec{\mathbf{v}}}{dt} - \frac{dM}{dt}\vec{\mathbf{v}}_{rel}$$

$$\text{or} \quad -\vec{\mathbf{g}} = \frac{d\vec{\mathbf{v}}}{dt} - \frac{1}{M}\frac{dM}{dt}\vec{\mathbf{v}}_{rel}$$

For determining the velocity of the rocket, we integrate this equation. Suppose the mass of the rocket is M_0 and its velocity $\vec{\mathbf{v}}_0$ at the instant $t = 0$. Then we have,

NOTE

Note that we have changed the limits for the integral over variable M.

$$\int_0^t \frac{d\vec{\mathbf{v}}}{dt}\,dt = \vec{\mathbf{v}}_{rel}\int_{M_0}^{M}\frac{dM}{M} - \vec{\mathbf{g}}t \qquad \left[\because \left(\frac{dM}{dt}\right)dt = dM\right]$$

$$\text{or} \quad \vec{\mathbf{v}}(t)\Big|_{\vec{\mathbf{v}}_0}^{\vec{\mathbf{v}}_t} = \vec{\mathbf{v}}_{rel}\ln M\Big|_{M_0}^{M} - \vec{\mathbf{g}}t$$ (where $\vec{\mathbf{v}}_t$ is the velocity at time t)

Thus, the rocket velocity at any instant t is given by

$$\vec{\mathbf{v}}_t - \vec{\mathbf{v}}_0 = \vec{\mathbf{v}}_{rel}\ln\frac{M}{M_0} - \vec{\mathbf{g}}t \qquad (8.5a)$$

Rocket motion under gravity

The symbol "ln" in this equation means natural logarithm.

Case 2: Rocket in free space when the net external force on it is zero

In free space, when no net external force acts on the system, the equation for rocket motion simply becomes:

$$\vec{\mathbf{v}}_t - \vec{\mathbf{v}}_0 = \vec{\mathbf{v}}_{rel}\ln\frac{M}{M_0} \qquad (8.5b)$$

Rocket motion in free space

Let us put some numbers in these equations so that you may appreciate the results. Study Example 8.2.

EXAMPLE 8.2: ROCKET MOTION

The mass of a two-stage rocket launched in free space is 1000 kg at some instant of time. When 800 kg of fuel of the first stage burns in it, the rocket ejects a stream of gas at a relative velocity of $1500\,\text{ms}^{-1}$. What is the rocket's velocity after the first stage is ejected? Then another 180 kg of fuel is burnt in the second stage ejecting gas at the same relative speed of $1500\,\text{ms}^{-1}$. What is the rocket's velocity after the second stage is ejected?

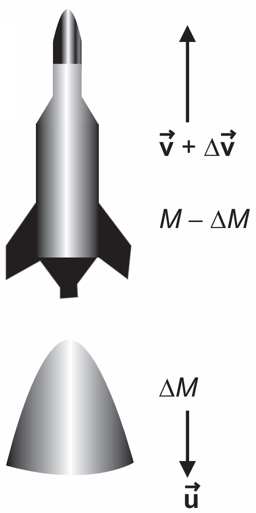


Fig. 8.7: **Analysing rocket motion**.

SOLUTION ■ The **KEY IDEA** here is to apply Eq. (8.5b) in both instances since the rocket is travelling in free space.

Let us choose the coordinate system as shown in Fig. 8.7 (Fig. 8.6b repeated here) so that $\vec{\mathbf{v}}_t$ is along the positive *y*-axis. Then, $\vec{\mathbf{v}}_{rel}$ is opposite to the positive *y*-axis. Hence, with $\vec{\mathbf{v}}_0 = \vec{\mathbf{0}}$, we have

$$v_t = -v_{rel} \ln \frac{M}{M_0} = -(1500\text{ ms}^{-1}) \ln \left(\frac{200}{1000}\right)$$

$$= -(1500\text{ ms}^{-1})(\ln 2 - \ln 10)$$

$$= -(1500\text{ ms}^{-1}) \times (-1.6) = 2400\text{ ms}^{-1}$$

When another 180 kg of fuel is burnt, then we have $M_0 = 200\,\text{kg}$, $M = 20\,\text{kg}$ and $v_0 = 2400\text{ ms}^{-1}$. Thus,

$$v_t = v_0 - v_{rel} \ln \frac{M}{M_0} = 2400\text{ ms}^{-1} - (1500\text{ ms}^{-1}) \ln \left(\frac{20}{200}\right)$$

$$= 2400\text{ ms}^{-1} - (1500\text{ ms}^{-1}) \times (-2.3) = 5850\text{ ms}^{-1}$$

Thus, using this principle, rocket speed can be increased manifold by attaching many stages in the rocket that drop off along with the fuel.

NOTE that when we apply Eqs. (8.5a and b) in numerical problems on rocket motion, we just replace $\frac{dM}{dt}$ by its magnitude because we have accounted for the decrease of M with t in the derivation itself.

So far, the forces that you have learnt about do not change with time. But there are many situations around us where the force varies with time: for example, when a cricket ball is hit by a bat, a tennis ball is hit by a racquet or a child falls on a hard floor, etc. In all such cases, the force just before the impact and just after the impact is zero. But it is very large during the brief time interval of impact. The motion in such cases is analysed in terms of the **impulse**. But before studying about impulse, you may like to try an SAQ on rocket motion.

***SAQ 3* – Rocket motion**

A rocket having initial mass of 1000 kg is fired from rest in free space with a payload of 100 kg. What is the velocity of the rocket when 900 kg of its fuel is burnt, if the speed of the exhaust gas relative to the rocket is 1000 ms^{-1}?

8.3 IMPULSE

Fig. 8.8 shows some situations in which a large force is exerted on an object during a short time interval and the force itself changes with time. The change in the force with time for a typical situation is also shown in the figure. If the net force on the object is $\vec{\mathbf{F}}_{net}(t)$, the change in its linear momentum during the short time of impact is given by

$$d\vec{\mathbf{p}} = \vec{\mathbf{F}}_{net}\ (t)\ dt \tag{8.6}$$

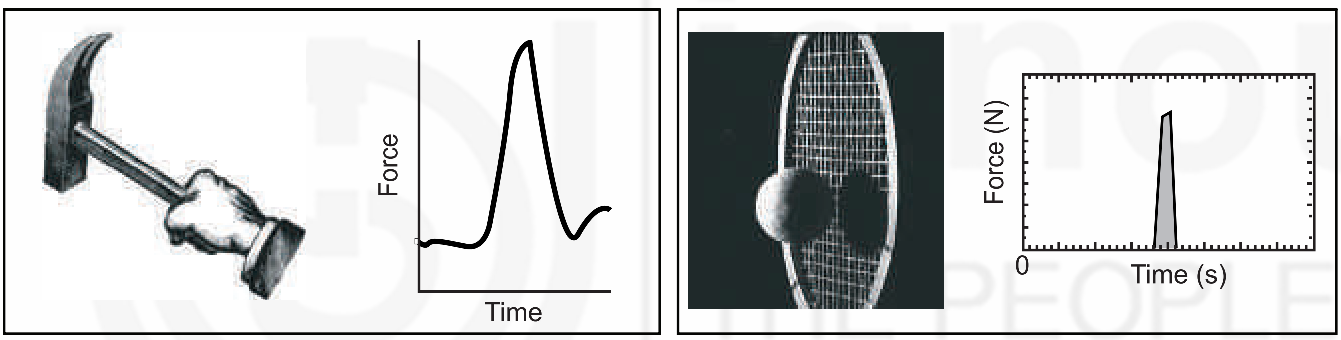


Fig. 8.8: **Situations in which a large time-varying force acts for a short time interval**.

Suppose the force is exerted between the instants t_1 and t_2. Let

$\vec{\mathbf{p}}_1$ = linear momentum of the object at $t = t_1$, that is, just before the force is exerted, and

$\vec{\mathbf{p}}_2$ = linear momentum of the object at $t = t_2$, that is, immediately after the impact.

Since the force changes continuously with time during that time inteval, we need to integrate it from t_1 to t_2 to determine the change in linear momentum in that time interval. Thus, we have

$$\vec{\mathbf{p}}_2 - \vec{\mathbf{p}}_1 = \int_{t_1}^{t_2} \vec{\mathbf{F}}_{net}(t)\ dt \tag{8.7a}$$

The right hand side of this equation is a measure of both the strength and duration of the force of impact exerted on the object. It is called **impulse** and denoted by $\vec{\mathbf{J}}$. Note that it is a vector quantity and has the dimensions of linear momentum. Thus, we define

Impulse

$$\vec{\mathbf{J}} = \int_{t_1}^{t_2} \vec{\mathbf{F}}_{net}(t)\, dt \qquad (8.7b)$$

Comparing this expression with Eq. (8.7a), we can write

Impulse

$$\vec{\mathbf{J}} = \vec{\mathbf{p}}_2 - \vec{\mathbf{p}}_1 = \Delta\vec{\mathbf{p}} \qquad (8.8)$$

The result in Eq. (8.8) is called the **impulse-momentum theorem**.

The impulse-momentum theorem tells us that *the impulse experienced by an object on which a time-varying force is exerted for a short duration is given by the change in its momentum in that duration*. Let us put these concepts together and revise them.

Recap

IMPULSE AND THE IMPULSE-MOMENTUM THEOREM

When a time-varying net external force is exerted on an object for a short time interval, the **impulse** experienced by the object during this time interval due to the force is defined as

$$\vec{\mathbf{J}} = \int_{t_1}^{t_2} \vec{\mathbf{F}}_{net}(t)\, dt \qquad (8.7b)$$

Comparing this expression with Eq. (8.7a), we can write

$$\vec{\mathbf{J}} = \vec{\mathbf{p}}_2 - \vec{\mathbf{p}}_1 = \Delta\vec{\mathbf{p}} \qquad (8.8)$$

The result in Eq. (8.8) is called the **IMPULSE-MOMENTUM THEOREM**: It tells us that **the impulse experienced by an object on which a time-varying force is exerted for a short duration is given by the change in its momentum in that duration**.

NOTE

The **impulse-momentum theorem** is **not a new law**. It is an alternate form of Newton's second law of motion.

In order to determine the impulse experienced by an object during impact, we need to know how the net force changes with time. However, in many cases, we do not know the change in force with time but we do know the **magnitude of the average force** and **the duration of time** for which it is exerted. Now, by definition, the average force over a time interval Δt is given as:

Average force

$$\vec{\mathbf{F}}_{av} = \frac{1}{\Delta t}\int_{t_1}^{t_2} \vec{\mathbf{F}}_{net}(t)\, dt, \qquad (8.9)$$

where $\Delta t = t_2 - t_1$. Comparing Eq. (8.9) with Eq. (8.7b), we can write

Average force and impulse

$$\vec{\mathbf{J}} = \vec{\mathbf{F}}_{av}\,\Delta t \qquad (8.10)$$

The results given in Eqs. (8.8) and (8.10) are very useful since it is far easier to measure the change in linear momentum and the average force. Let us now consider some common applications of these results.

EXAMPLE 8.3: IMPULSE

Why do you get hurt more when you fall on a hard floor compared to when you fall on a soft mattress?

SOLUTION ■ The **KEY IDEA** here is to use Eq. (8.10) to determine the average force being exerted on you in both cases.

In both cases, you come to rest finally and your velocity can be assumed to be the same at the instant when you hit the floor or the mattress. Therefore, the **change in momentum** or the **impulse experienced by you due to the floor or the mattress is the same** in both cases.

But the time taken to come to a stop is different in the two situations: **since the mattress is soft**, **you take longer time to come to a stop on it compared to the hard floor**.

Note that $\vec{\mathbf{J}}$ is the same in both cases and $\vec{\mathbf{F}}_{av} = \dfrac{\vec{\mathbf{J}}}{\Delta t}$

Since **the duration of impact is longer for the mattress compared to the floor**, we have

$$(\Delta t)_{mattress} \gg (\Delta t)_{hard\ floor}$$

$$\therefore \quad (\vec{\mathbf{F}}_{av})_{mattress} \ll (\vec{\mathbf{F}}_{av})_{hard\ floor}$$

Thus, the average force exerted by you on the floor and hence the equal and opposite reaction **force exerted by the floor on you is <u>much larger</u>** compared to the force exerted by the mattress on you. That is why you get hurt more when you fall on a hard floor compared to when you fall on a mattress.

EXAMPLE 8.4: IMPULSE-MOMENTUM THEOREM

A ball of mass 0.25 kg coming straight at the bat with speed $20.0\,\text{ms}^{-1}$ is hit so that it travels with the same speed but in the opposite direction after leaving the bat. What impulse is experienced by the ball while it is in contact with the bat? What average force is exerted on the ball if the duration of contact is 1 ms?

SOLUTION ■ The **KEY IDEA** here is to use Eqs. (8.8) and (8.10) keeping in mind that momentum is a vector quantity and the change in its direction has to be taken into account while using the equations.

Let us assume that the ball moves along the positive *x*-direction after being hit by the bat. Then, the ball initially moves in the negative *x*-direction. Therefore, from Eq. (8.8), we have

$$J_x = p_{2x} - p_{1x} = mv_{2x} - m(-v_{1x})$$
$$= 0.25\,\text{kg}\,(20.0\,\text{ms}^{-1} + 20.0\,\text{ms}^{-1}) = 10\ \text{kgms}^{-1}$$

The impulse is directed along the positive *x*-axis, i.e., the direction in which the bat hits the ball. The average force on the ball in the same direction during the time interval $1\,\text{ms}\ (=10^{-3}\ \text{s})$ is

$$F_{av} = \frac{J_x}{\Delta t} = \frac{10\ \text{kgms}^{-1}}{10^{-3}\ \text{s}} = 10^4\ \text{N}$$

Note that this is the **average net force** on the ball. The maximum net force is larger than the average force.

We end this section with an SAQ for you.

SAQ 4 – Linear momentum and impulse

An average force has magnitude two times as large as another average force. Both forces produce the same impulse. The larger average force is exerted for a time interval of 2.0 ms. For what time interval does the smaller average force act?

In this unit, we have discussed the concepts of linear momentum and impulse. You have learnt how these concepts help us solve many problems on motion of particles without having to apply Newton's laws of motion. You have studied the **law of conservation of linear momentum** and the **impulse-momentum theorem** and their simple applications. You have also learnt about time-varying forces exerted for a brief interval of time and how the average force in such situations can be calculated if we know the impulse. We now sum up what you have learnt in this unit.

8.4 SUMMARY

Concept	Description
Law of conservation of linear momentum	The **TOTAL LINEAR MOMENTUM of an isolated and closed system is CONSERVED if no net external force is exerted on it**: $\vec{\mathbf{p}} = \text{constant}$ for an isolated and closed system
Impulse	When a time-varying net external force is exerted on an object for a short time interval, the **IMPULSE** experienced by the object during this time interval due to the force is defined as $\vec{\mathbf{J}} = \int_{t_1}^{t_2} \vec{\mathbf{F}}_{net}(t)\, dt$ It is related to the average force as follows: $\vec{\mathbf{J}} = \vec{\mathbf{F}}_{av}\, \Delta t$

where $$\vec{\mathbf{F}}_{av} = \frac{1}{\Delta t}\int_{t_1}^{t_2} \vec{\mathbf{F}}_{net}(t)\, dt$$

Impulse-momentum theorem

- The **IMPULSE-MOMENTUM THEOREM** states that the impulse experienced by an object on which a time-varying force is exerted for a short duration is given by the change in its linear momentum in that duration:

$$\vec{\mathbf{J}} = \vec{\mathbf{p}}_2 - \vec{\mathbf{p}}_1 = \Delta\vec{\mathbf{p}}$$

8.5 TERMINAL QUESTIONS

1. A child of mass 32.0 kg jumps on to a stationary skateboard while running at a speed of 4.0 ms^{-1}. If the mass of the skateboard is 8.0 kg, the speed of the child and skateboard is

   a) 16.0 ms^{-1}

   b) 5.0 ms^{-1}

   c) 8.1 ms^{-1}

   d) 3.2 ms^{-1}

2. How long must a force of 200 N be exerted to produce a change in linear momentum of 500 kgms^{-1}?

   a) 0.4 s
   b) 2.5 s
   c) 10.0 s
   d) 5.0 s

3. In trying to catch a ball, a cricketer extends the hand forward before the impact with the ball. After the impact, he moves his hand backward in the direction of the ball's direction of motion. This is because

   a) the force of impact on the hand is reduced.
   b) the relative velocity is reduced.
   c) the time of impact increases.
   d) the time of impact decreases.

4. A trailer truck and a car coming down a hill at the same speed are forced to stop in the same amount of time. Compared to the force required to stop the car, the force needed to stop the truck is

   a) greater.
   b) smaller.
   c) the same.

5. A car of mass 1500 kg is travelling due west at a speed of $55\ \text{kmh}^{-1}$. It comes to a stop in 10 s when the driver applies brakes. Using the impulse-momentum theorem, determine the average force exerted on the car.

6. Using the concept of impulse, explain why a safety net is used in circuses when circus artists perform on the trapeze.

7. A rocket of mass 6000 kg moving at speed $5.0 \times 10^3\ \text{ms}^{-1}$ fires its engine ejecting 1000 kg of exhaust gas at a speed of $2.0 \times 10^3\ \text{ms}^{-1}$ relative to the rocket. What is the final velocity of the rocket?

8. In a movie, the Superman is shown hanging still in midair without any support when a villain approaches him. In the next scene he is shown to grab the villain and throw him away while he himself remains stationary. Explain what is wrong with this scene.

9. Determine the impulse for a ball of mass 50 g, which strikes a wall straight with a velocity of magnitude $5.0\,\text{ms}^{-1}$ and rebounds at the same velocity.

10. A woman of mass 60 kg and her car are suddenly accelerated from rest to a speed of $5.0\ \text{ms}^{-1}$ as a result of a rear-end collision. Assuming that the duration of the collision is 0.50 s, obtain the

    a) impulse on the woman, and

    b) the average force exerted by her on the seat of her car.

8.6 SOLUTIONS AND ANSWERS

Self-Assessment Questions

1. Linear momentum is a vector quantity. It depends on the velocity of the cars and not just the speed. Since the velocities of the two cars are directed opposite to each other, **the cars will not have the same linear momentum**. However, since the cars are identical, their masses are the same. Since their velocities are equal and opposite, the total linear momentum of the system would be zero.

2. We use Eq. (8.3). The velocity of the conveyor belt is constant.

$$\therefore \qquad \frac{d\vec{\mathbf{v}}}{dt} = \vec{\mathbf{0}}$$

Let $\vec{\mathbf{v}} = 2.0\ \text{ms}^{-1}\ \hat{\mathbf{i}}$, where $\hat{\mathbf{i}}$ is the direction of motion of the conveyor belt. It is given that $\frac{dm}{dt} = 50.0\,\text{kg}\,\text{s}^{-1}$. Then from Eq. (8.3):

$$\vec{\mathbf{F}} = 50.0\,\text{kgs}^{-1} \times 2.0\ \text{ms}^{-1}\ \hat{\mathbf{i}} = 1.0 \times 10^2\ \text{N}\ \hat{\mathbf{i}}$$

3. We use Eq. (8.5b) with $M_0 = 1000$ kg, $M = 100$ kg, $v_0 = 0$ and $v_{rel} = 1000\ \text{ms}^{-1}$. So the final velocity of the rocket is

$$v_t = -1000\,\text{ms}^{-1} \times \ln\frac{100}{1000} = 2.30 \times 10^3\ \text{ms}^{-1}$$

The negative sign in the above equation appears because the final velocity is directed opposite to the exhaust velocity.

4. We use Eq. (8.10). If the smaller average force is F_{AV}, the larger average force is $2F_{AV}$. The impulse of both these forces is the same, so we can write

$$F_{AV} \times \Delta t = 2F_{AV} \times (2.0 \times 10^{-3}\ \text{s})$$

where Δt is the time for which the smaller average force is exerted. Then

$$\Delta t = 4.0 \times 10^{-3}\ \text{s} = 4.0\ \text{ms}$$

Terminal Questions

1. The correct option is (d). We use Eq. (8.2b). The initial linear momentum of the system is

$$p_i = 32.0\,\text{kg} \times 4.0\,\text{ms}^{-1} = 128.0\ \text{kgms}^{-1}$$

The final linear momentum is $p_f = (32.0\,\text{kg} + 8.0\,\text{kg}) \times v$

where v is the speed of the child and the skateboard. From conservation of linear momentum, we get

$$v = \frac{128.0\,\text{kgms}^{-1}}{40.0\,\text{kg}} = 3.2\,\text{ms}^{-1}$$

2. The correct option is (b). We use Eqs. (8.8) and (8.10) with $F_{av} = 200\,\text{N}$ and $p_2 - p_1 = 500\ \text{kgms}^{-1}$ so that

$$\Delta t = \frac{500\ \text{kgms}^{-1}}{200\,\text{N}} = 2.5\,\text{s}$$

3. The correct option is (c). The time of impact increases and so the average force on the hand reduces.

4. The correct option is (a). The change in linear momentum is greater for the truck because it has a larger mass. Hence the force required to stop the truck will be greater [Eqs. (8.8) and (8.10)].

5. Let us assume that the car is moving in the negative x-direction. The average force as given by Eq. (8.10) is $\vec{\mathbf{F}}_{AV} = \dfrac{\vec{\mathbf{J}}}{\Delta t}$

The initial velocity of the car is $\vec{\mathbf{v}} = -55\,\text{kmh}^{-1}\,\hat{\mathbf{i}} = -15.3\,\text{ms}^{-1}\,\hat{\mathbf{i}}$

The impulse $\vec{\mathbf{J}} = \vec{\mathbf{0}} - \left[1500\,\text{kg} \times (-15.3\,\text{ms}^{-1}\hat{\mathbf{i}})\right] = 2.3 \times 10^4\ \text{kgms}^{-1}\ \hat{\mathbf{i}}$

The average force $\vec{\mathbf{F}}_{av} = \dfrac{2.3 \times 10^4\ \text{kgms}^{-1}}{10\,\text{s}}\ \hat{\mathbf{i}} = 2.3 \times 10^3\,\text{N}\,\hat{\mathbf{i}}$

6. Refer to Example 8.3. The presence of the safety net increases the duration of impact and the average force exerted by the circus artist on the net is less. Hence, the force of reaction on the circus artist is also

less. If the safety net were not there, they would fall on the hard floor and hurt themselves since the duration of impact would be much less.

7. We use Eq. (8.5b) with

$$v_0 = 5.0 \times 10^3 \,\text{ms}^{-1}, \quad M_0 = 6000 \text{ kg},$$

$$M = (6000 \text{ kg} - 1000 \text{ kg}) = 5000 \text{ kg}, \; v_{rel} = 2.0 \times 10^3 \,\text{ms}^{-1}$$

So the final velocity of the rocket is

$$v_t = 5.0 \times 10^3 \,\text{ms}^{-1} - 2.0 \times 10^3 \,\text{ms}^{-1} \ln\left(\frac{5000}{6000}\right)$$

$$= 5.0 \times 10^3 \,\text{ms}^{-1} - 2.0 \times 10^3 \,\text{ms}^{-1}(-0.18) = 5.4 \times 10^3 \,\text{ms}^{-1}$$

8. The scene does not agree with the principle of conservation of linear momentum. Since there is no net external force on the system, the Superman should move backwards after throwing the villain so that the linear momentum of the system is conserved.

9. Let us assume that the ball moves along the positive *x*-axis.

The linear momentum of the ball when it hits the wall is

$$\vec{\mathbf{p}}_1 = (50 \times 10^{-3} \text{ kg}) \times 5.0 \,\text{ms}^{-1} \,\hat{\mathbf{i}} = 0.25 \,\text{kgms}^{-1} \,\hat{\mathbf{i}}$$

The linear momentum of the ball after it rebounds is

$$\vec{\mathbf{p}}_2 = -(50 \times 10^{-3} \text{ kg}) \times 5.0 \,\text{ms}^{-1} \,\hat{\mathbf{i}} = -0.25 \,\text{kgms}^{-1} \,\hat{\mathbf{i}}$$

$$\therefore \quad \vec{\mathbf{J}} = \vec{\mathbf{p}}_2 - \vec{\mathbf{p}}_1 = -0.25 \,\text{kgms}^{-1} \,\hat{\mathbf{i}} - 0.25 \,\text{kgms}^{-1} \,\hat{\mathbf{i}} = -0.50 \,\text{kgms}^{-1} \,\hat{\mathbf{i}}$$

10. We use Eq. (8.8) with the initial linear momentum to be zero, since the car starts from rest. The final linear momentum is

$$\vec{\mathbf{p}}_2 = 60 \,\text{kg} \times 5.0 \,\text{ms}^{-1} \,\hat{\mathbf{i}} = 300 \,\text{kgms}^{-1} \,\hat{\mathbf{i}}$$

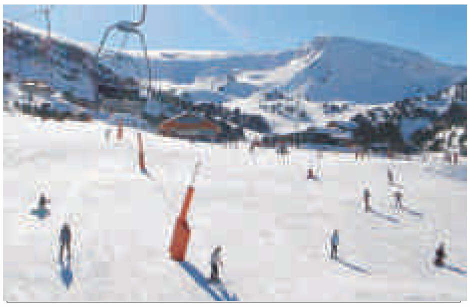
where $\hat{\mathbf{i}}$ is in the direction of motion of the car.

a) The impulse $\vec{\mathbf{J}} = \vec{\mathbf{p}}_2 - \vec{\mathbf{p}}_1 = 3.0 \times 10^2 \,\text{kgms}^{-1} \,\hat{\mathbf{i}}$

b) To find the average force $\vec{\mathbf{F}}_{av}$, we use Eq. (8.10) with

$\vec{\mathbf{J}} = 3.0 \times 10^2 \,\text{kgms}^{-1} \,\hat{\mathbf{i}}$ and $\Delta t = 0.50 \,\text{s}$. Hence,

$$\vec{\mathbf{F}}_{av} = \frac{3.0 \times 10^2 \,\text{kgms}^{-1}}{0.50 \,\text{s}} \,\hat{\mathbf{i}} = 6.0 \times 10^2 \,\text{N} \,\hat{\mathbf{i}}$$



How do we calculate the speed of a skier as she skis downhill on a curved path with a changing slope? This unit will help you answer this question!

UNIT 9

WORK AND KINETIC ENERGY

Structure

9.1 Introduction
Expected Learning Outcomes
9.2 Work
No-work Force
Positive and Negative Work
9.3 The Work-energy Theorem and Kinetic Energy
9.4 Work done by a Variable Force
9.5 Power
9.6 Summary
9.7 Terminal Questions
9.8 Solutions and Answers

STUDY GUIDE

In this unit, you will learn about some more fundamental concepts of mechanics, namely, **work**, **kinetic energy** and the **work-energy theorem**. These concepts help us analyse motion without having to solve the equations of motion. You have studied these concepts in your school physics. We also use the concept of the scalar product, which you have studied in Units 1 and 2 of Block 1, to define work done by a force. We also use the concepts of **integral calculus** in this unit, which you have studied in school mathematics particularly that of integral of a function of a single variable as area under the curve.

Do revise the concepts of vectors and vector components before studying this unit. The work-energy theorem discussed in Sec. 9.3 may be altogether new for you. In Sec. 9.4, we derive the equations for work done by a variable force. You may take more time to understand these sections. Finally, we advise you to solve all examples, SAQs and Terminal Questions on your own to understand the unit well.

"Energy is eternal delight."

William Blake

9.1 INTRODUCTION

Fig. 9.1: The force on the bus moving on a hilly road changes with time. Can we determine the velocity and position of the bus without applying Newton's laws?

In Unit 5, you have studied Newton's laws of motion and in Unit 6, you have learnt how to apply these laws to determine the velocity and acceleration of an object at any instant of time, if you know the net force being exerted on it. But in many situations applying Newton's laws to determine the object's position and velocity is not very easy.

For example, consider the motion of a bus on a hilly road with a continuously changing slope (Fig. 9.1). Suppose you want to find its speed at any given time by applying Newton's laws. You will need to know the force on it at every instant of time. Since the slope of the road is changing continuously with time, the force on the bus ($mg \sin \theta$ in this case) also keeps changing with it and so does the acceleration of the bus.

Unless we know exactly how the angle θ and hence the force on the bus changes with time, we cannot apply Newton's laws to the bus and determine its velocity and position at any given instant of time. And even if we know the force as a function of time, it is a complex problem to solve.

In such cases we would like to know: **Is there any simple way of analysing motions like that of this bus? How can we determine the changes in a particle's velocity and position without applying Newton's laws**?

NOTE

The concepts we discuss in this unit apply to objects that can be treated as **point particles**. However, the **law of conservation of energy is universal and applies to all particles and extended bodies.** We shall need added information when we apply these concepts to extended objects and systems of particles. You will study about this in the next block.

The answer is that we can do so by applying the **work-energy theorem**, which you will study in Sec. 9.3 and Sec. 9.4 of this unit. However, in order to study the theorem, you will need to master certain related concepts, which we discuss in Sec. 9.2 and Sec. 9.4. These concepts are **work done by a constant force** (Sec. 9.2), **kinetic energy** (Sec. 9.3) and **work done by variable forces** (Sec. 9.4). We also introduce the concept of **power** in Sec. 9.5.

In Unit 8, you have studied the law of conservation of linear momentum, which is a fundamental law of nature. In the next unit, we develop the concept of energy further and introduce yet another fundamental law of nature, namely, the **law of conservation of energy**. We also discuss the related concepts, namely, **conservative forces**, **potential energy** and **conservation of mechanical energy**.

Expected Learning Outcomes

After studying this unit, you should be able to:

- define and calculate work done by constant and variable forces;
- calculate the kinetic energy of a particle;
- apply the work-energy theorem for constant and variable forces; and
- calculate power in mechanical systems.

9.2 WORK

You have studied the concept of work in your school physics courses and so you should know it very well. For example, let a **net force** $\vec{\mathbf{F}}$ be exerted on a box resulting in its displacement by $\vec{\mathbf{d}}$ (Fig. 9.2). What is the work done on the box by the force? You may recall the answer from school physics: For a **net constant force** $\vec{\mathbf{F}}$ **exerted on an object**, which undergoes a displacement of magnitude d in the direction of the force, the work done by the force on the object is defined as:

$$W = F d \tag{9.1}$$

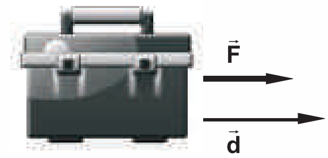


Fig. 9.2: **What is the work done on the box by the force $\vec{\mathbf{F}}$ exerted on it if the box undergoes displacement $\vec{\mathbf{d}}$ in the direction of the force?**

Notice that in Eq. (9.1), F and d are the magnitudes of the force and displacement, respectively. Thus, as long as the distance travelled by the box is the same and the displacement is **along** the direction of the force applied, the ***work done on the box is the same. This is true whether the box*** (of Fig. 9.2) ***moves from north to south or from east to west or along any other straight line***. **Work done is a scalar quantity** since it does not depend upon direction. The **unit of work done** is **joule** (J) in SI units, named so in honour of the English physicist James Joule (1818 – 1889).

Fig. 9.3: **The work done on an object is zero if its displacement is zero.**

Eq. (9.1) also tells us that the **work done is zero if the displacement of an object is zero** even if a force is applied on the object. So even if you apply a large force to move any object such as a heavy bookshelf and it does not move, you will not be doing any work as per this definition (Fig. 9.3).

Let us now consider a situation in which the constant force and displacement are not in the same direction. In Fig. 9.4, a box is being pulled by a string. The force being exerted on the box makes an angle θ with its displacement. In such cases, the **component of force along the displacement** is used in defining the work done. You can see from Fig. 9.4 that it is given by $F\cos\theta$. Note that in the situations we have taken up so far, the force is constant. Thus, we can define **work done by constant force** as follows:

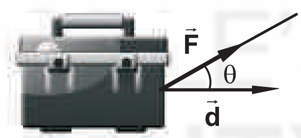


Fig. 9.4: **The work done on an object by a force applied at a finite angle to the displacement.**

WORK DONE BY CONSTANT FORCE

The **work done by a constant force** $\vec{\mathbf{F}}$ **on an object that undergoes displacement** $\vec{\mathbf{d}}$ is defined as

$$W = F d \quad \text{when } \vec{\mathbf{F}} \text{ is along } \vec{\mathbf{d}} \tag{9.2a}$$

$$W = (F\cos\theta) d \quad \text{when } \vec{\mathbf{F}} \text{ makes an angle } \theta \text{ with } \vec{\mathbf{d}} \tag{9.2b}$$

Using the definition of the scalar product from Unit 1, we can also express the work done as the scalar product of force and displacement:

$$W = \vec{\mathbf{F}}.\vec{\mathbf{d}} \tag{9.3}$$

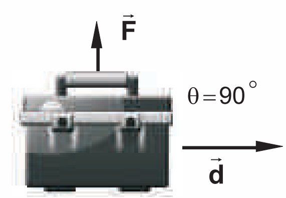


Fig. 9.5: **The work done on an object by a force applied perpendicular to the displacement is zero.**

When the force and displacement are in the **same direction**, the angle between $\vec{\mathbf{F}}$ and $\vec{\mathbf{d}}$ is zero $(\theta = 0°)$. Then the work done is maximum (Fig. 9.2). When $\vec{\mathbf{F}}$ and $\vec{\mathbf{d}}$ are not in the same direction and the angle between them increases, the work done for the same force and displacement decreases (Fig. 9.4). When $\vec{\mathbf{F}}$ is perpendicular to $\vec{\mathbf{d}}$, the work done is zero (Fig. 9.5). Also **REMEMBER**: **Work done is a scalar quantity and does not**

depend on the observer's frame of reference. Let us now take up an example for determining the work done on an object by a constant force.

> **NOTE**
>
> Note that in Eqs. (9.2a and b), *F* is the magnitude of the **net force**. Thus, if many forces act on the object, the work done is due to the resultant force. Since work is a scalar quantity, it can be obtained by simply adding the work done by individual forces. Of course, we have to take into account the angle for each force.

Example 9.1: WORK DONE BY CONSTANT FORCE

Suppose the box in Fig. 9.4 is being pulled by a force of magnitude 20 N and the angle made by the string with the horizontal is $\theta = 45^\circ$. Determine the work done on the box if it is displaced by 10 m in the horizontal direction.

SOLUTION ■ The **KEY IDEA** here is to obtain the work done by the component of the force **along** the displacement since it makes an angle of 45° with the displacement. We, therefore, use Eq. (9.2b).

The work done by the force is:

$$W = (F\cos\theta) \times d = 20\,\text{N} \times \cos 45^\circ \times 10\,\text{m} = 1.4 \times 10^2\ \text{J}$$

Fig. 9.6: **Parachutist falling under the force of gravity.**

You may now like to determine the work done yourself. Try SAQ 1.

SAQ 1 – Work Done

A parachutist of mass 60 kg falls vertically downwards from the plane by a distance of 500 m (Fig. 9.6). What is the work done by the force of gravity during the fall? Take $g = 10\ \text{ms}^{-2}$.

You have seen that work done by a force perpendicular to the displacement is zero. This brings us to the concept of **no-work force**.

9.2.1 No-work Force

Eq. (9.2b) tells us that work is done only by the component of the force **along** the direction of displacement.

The component of force perpendicular to the direction of displacement does no work. And if the force itself is applied perpendicular (that is, at an angle of 90°) to the direction of displacement, it does no work. Such a force is called **no-work force**.

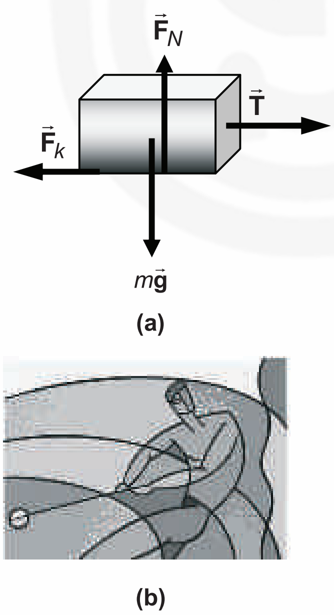


Fig. 9.7: **Examples of no-work force: a) $\vec{F}_N$ and $m\vec{g}$ are no-work forces**; **b) the work done by tension in the string on the ball is zero for circular motion**.

Fig. 9.7 shows some examples of no-work forces. In Fig. 9.7a, a box is being pulled horizontally on the floor by a rope. The work done by the force of gravity $(= m\vec{g})$ and the normal force $(\vec{F}_N)$ is zero as the displacement of the box is perpendicular to both these forces. Work is done on the box by only the force exerted by the tension $(\vec{T})$ in the rope and the force of kinetic friction $(\vec{F}_k)$ opposing the box's motion.

When a ball on a string moves in uniform circular motion (as in Fig. 9.7b), the work done by the tension in the string on the ball is zero because the **force has no component in the direction of motion**.

You may like to check whether you have learnt these concepts well enough.

SAQ 2 – Work Done

State giving reason whether work is being done in each of the following situations:

a) A bus moves with constant velocity on the road.

b) A student carries a pile of books from one place to another. Is work being done on the books by the student?

c) A box is pulled on the floor with a finite acceleration. Is work being done on the box? Is work being done on the floor?

d) A child pushes a heavy cupboard but cannot shift it by even an inch. Is work being done on the cupboard?

e) A desk is pulled a distance of 1.0 m by a rope with a force of 100 N. What is the work done on the desk if the force makes an angle of $0°$, $90°$ and $60°$ with the displacement?

Solving SAQ 2 should have made it clear to you that

> **Work is done on an object** only when
>
> (i) the force exerted on it **is non-zero,**
>
> (ii) it undergoes **finite displacement,** and
>
> (iii) the **force and displacement are not perpendicular to each other.**

Now, in some situations, work done may be positive and in others, it may be negative. Let us find out about negative and positive work.

9.2.2 Positive and Negative Work

When the component of force points in the same direction as the displacement, **work done is <u>positive</u>** (Fig. 9.8). You can see that this result follows from Eq. (9.2b). In Example 9.1 based on Fig. 9.4, the force applied is **not along** the direction of the displacement and θ is an acute angle, i.e., $0^\circ \leq \theta < 90^\circ$. From Eq. (9.2b), the work done by $\vec{\mathbf{F}}$ is positive. We say that **work is done <u>by</u>** the force **<u>on</u> the object**.

Now suppose the force has a finite component in a direction **<u>opposite</u>** to the displacement of the particle. This is the case when you throw an object up against the force of gravity or in the case of projectile motion until the moment it starts falling down. In all such cases, **the work done by the force is <u>negative</u>**. In such cases the angle θ is an obtuse angle, i.e., $90^\circ < \theta \leq 180^\circ$.

NOTE

When one body does negative work on a second body, the second body does an equal amount of *positive* work on the first body.

From Eq. (9.2b), the work done by the force on the object is negative. We say that **work is done by the object**.

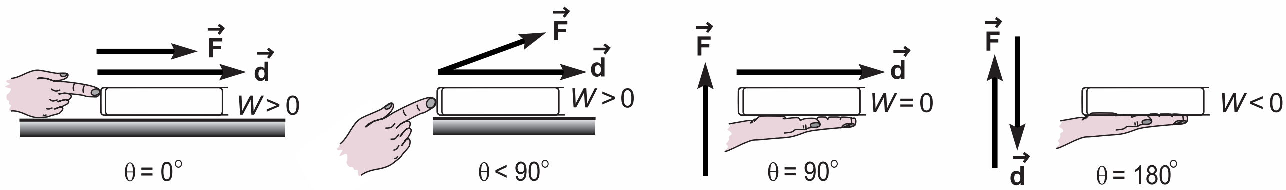


Fig. 9.8: **Positive, zero and negative work.**

We summarise this information as follows:

- ***W* > 0 or POSITIVE if the force (or a component of the force) points <u>along</u> the direction of displacement (θ is acute: $0° \leq \theta < 90°$). *W* = 0 when $\theta = 0°$.**
- ***W* < 0, i.e., NEGATIVE if the force (or a component of the force) points <u>opposite</u> to the direction of displacement (θ is obtuse: $90° < \theta \leq 180°$).**
- ***W* < 0 also represents work done <u>BY</u> the object.**

The following example will help you understand this concept.

EXAMPLE 9.2 : POSITIVE AND NEGATIVE WORK

Fig. 9.9: **Work done by a weight lifter in lifting the weights is positive. In lowering the weights, the work done by the weight lifter is negative.**

A weight lifter lifts weights of mass 70 kg by a distance of 0.50 m above his shoulders (Fig. 9.9) and lowers them by the same distance with a constant velocity. Calculate the work done by the weight lifter while raising them and lowering them. Take $g = 10\,\text{ms}^{-2}$.

SOLUTION ■ The **KEY IDEA** here is to determine the force exerted on the weights by the weight lifter and then use Eq. (9.2b). The directions of the force and the displacement must be taken into account.

Since the weights move with a constant velocity, they are in equilibrium. Therefore, the force exerted by the weight lifter is equal and opposite to the weight of the weights. Therefore, $F = mg = 700\text{ N}$. When the weights are being raised, the force is exerted in the same direction as the displacement. Therefore, the work done is positive:

$$W = (F\cos\theta)d = 700\text{N} \times (\cos 0°) \times 0.50\,\text{m} = 3.5 \times 10^2\text{ J}$$

When the weights are lowered, force is still being exerted upwards by the weight lifter but now it is directed opposite to the displacement of the weights. Therefore, the work done is:

$$W = (F\cos\theta)d = 700\,\text{N} \times (\cos 180°) \times 0.50\,\text{m} = -\ 3.5 \times 10^2\text{ J}$$

since $\cos 180° = -1$. Thus, the work done by the weight lifter is negative, that is the weights do positive work on the weight lifter.

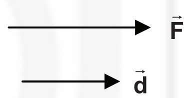
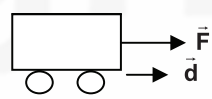
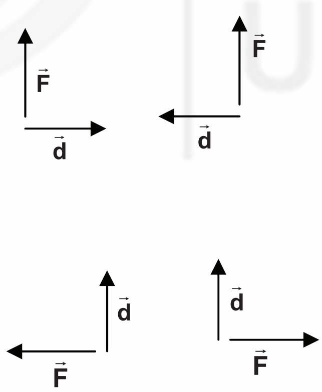
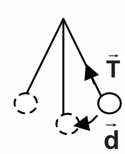
Please note that in all numerical problems, we use the rules for significant digits explained in the course BPHCL-132 entitled 'Mechanics : Laboratory'.

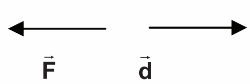
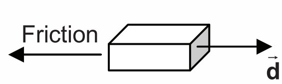
In general, in terms of the components of force $\vec{\mathbf{F}}$ and displacement $\vec{\mathbf{r}}$, the work done is given as follows for one-dimensional, two-dimensional and three-dimensional motion:

WORK DONE IN COMPONENT FORM		
Force	**Displacement**	**Work done**
$\vec{\mathbf{F}} = F_x\,\hat{\mathbf{i}}$	$\vec{\mathbf{r}} = x\,\hat{\mathbf{i}}$	$W = F_x\,x$ (9.4a)
$\vec{\mathbf{F}} = F_x\,\hat{\mathbf{i}} + F_y\,\hat{\mathbf{j}}$	$\vec{\mathbf{r}} = x\,\hat{\mathbf{i}} + y\,\hat{\mathbf{j}}$	$W = F_x\,x + F_y\,y$ (9.4b)
$\vec{\mathbf{F}} = F_x\,\hat{\mathbf{i}} + F_y\,\hat{\mathbf{j}} + F_z\,\hat{\mathbf{k}}$	$\vec{\mathbf{r}} = x\,\hat{\mathbf{i}} + y\,\hat{\mathbf{j}} + z\,\hat{\mathbf{k}}$	$W = F_x\,x + F_y\,y + F_z\,z$ (9.4c)

All other cases (e.g., 1-d displacement and 2-d force or 2-d displacement and 3-d force, etc.) can be worked out using Eqs. (9.4a to c) for work done. We would like to present some special cases of work done in Table 9.1.

Table 9.1: Special Cases of the General Definition of Work Done.

<table>
<tr><th>Description</th><th>Equation</th><th>Examples</th></tr>
<tr><td>• If the displacement is along the direction of force, then $\theta = 0^\circ$ and $\cos\theta = 1$. Note that in this case the work done is maximum, as the cosine of the angle is 1.</td><td>$W = F\,d\cos 0^\circ = F\,d$

</td><td>Pulling a cart horizontally ($\vec{\mathbf{F}}$ along $\vec{\mathbf{d}}$)

</td></tr>
<tr><td>• If the displacement is perpendicular (normal) to the direction of force, then $\theta = 90^\circ$ and $\cos\theta = 0$. In this case, the work done is zero because the component of the force along the direction of motion is zero. We also refer to such a force as 'no-work force'. Thus, no work is done when
i) the force is zero,
ii) the displacement is zero,
iii) the displacement and force are perpendicular to each other.</td><td>$W = F\,d\cos 90^\circ = 0$

</td><td>When we walk, the force of reaction exerted by the ground on our feet is perpendicular to our displacement. Hence, the force of reaction is a no-work force. Tension in the string of a pendulum is a no-work force.


Vertical forces do not produce horizontal displacement. Horizontal forces do not produce vertical displacement.
A force does not produce displacement in a perpendicular direction.</td></tr>
<tr><td>• If the displacement is in a direction opposite to the direction of force, then $\theta = 180^\circ$ and $\cos\theta = -1$. Thus, the work done is negative.</td><td>$W = F\,d\cos 180^\circ = -Fd$

</td><td>Work done against the force of friction or viscous force.

</td></tr>
</table>

You may quickly like to check your understanding before studying further.

SAQ 3 – Work done by constant force

The box of mass 2.0 kg shown in Fig. 9.10 is displaced by 5.0 m towards right. Calculate the work done on the box by each individual force. What is the total work done on the box?

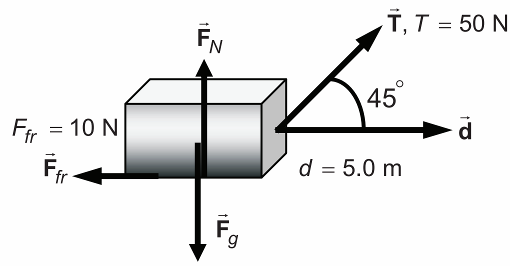


Fig. 9.10

Identify which one of these forces does negative work and which one, positive work. Which amongst these is/are no-work forces? Explain, giving reason.

Let us now go back to our aim stated in the introduction: To look for an alternative way to find the positions and velocities of an object without having to apply Newton's laws. In fact, we can arrive at a relation between the position and velocity of a particle starting from Newton's second law itself. This relationship is known as the **work-energy theorem**. In the next section, we derive this theorem for constant forces. However, to do so, we need to define the **kinetic energy** of a particle.

9.3 THE WORK-ENERGY THEOREM AND KINETIC ENERGY

All around us we see examples of work being done on objects. A football player (for example, Ronaldo) kicks the ball from rest and lands it in the goal. A car starts from rest and *moves* under the action of force. The switch is turned on and the fan blades start *moving*. You can think of many more such examples of force being exerted on objects to make them ***move*** and ***change their speeds***. To begin with, we consider motion in a straight line (e.g., a bus on a straight road) to keep the mathematics simple.

Suppose a body of *constant mass m* moves a distance *d* when a constant net force F_{net} is exerted on it. You know that the work done is $W = F_{net}\, d$. If the acceleration of the body is *a*, then from Newton's second law, we have

$$F_{net} = ma \qquad (9.5)$$

Let the body be at rest initially $(u = 0)$. What is its speed for constant acceleration *a*? It is $v^2 = 2ad$ or $ad = \frac{v^2}{2}$.

Therefore, work done $= (ma)\,d = \dfrac{1}{2}mv^2$

This is a discovery! What is this new quantity $\frac{1}{2}mv^2$ on the right hand side?

We call it the **kinetic energy** of the body:

$$\text{K.E.} = \frac{1}{2}mv^2 \qquad (9.6a)$$

The word **energy**, coming from the Greek *energos* or active (from *en*, at + *ergos*, work), is best translated as *the ability to do work*. We define energy as follows:

ENERGY

Energy is the capacity of a body to do work and is always measured by the work the body is capable of doing.

REMEMBER: Energy is **not** work. It is the **capacity of a body to do work**. You have learnt from the above derivation that *when work is done, there is a change in the energy of a body. The converse is also true*. When there is a change in energy, work is done. "**Kinetic**" ***identifies the unique condition of motion***. We define kinetic energy as follows:

KINETIC ENERGY

The kinetic energy of a particle of mass *m* travelling with speed *v* is given by

$$\text{Kinetic Energy (K.E.)} = \frac{1}{2}mv^2 \qquad (9.6a)$$

Kinetic energy

The kinetic energy of a particle represents its capacity to do work by virtue of its velocity. The SI unit of kinetic energy is joule (J).

If the body is not at rest initially but moves at a speed *u*, then $v^2 = u^2 + 2ad$ and hence $ad = \frac{1}{2}(v^2 - u^2)$. Therefore, the work done is

$$W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2 = \Delta\text{K.E.} \qquad (9.6b)$$

Work-energy Theorem

where ΔK.E. is the **change** in the kinetic energy of a body. By doing some simple mathematics, we have discovered that the work done by a force brings about a change in the speed of the object from *u* to *v*. Moreover, it depends only on its initial and final speeds.

Thus, we have arrived at a very important result, which is called the **work-energy theorem**. The work done on the left hand side of Eq. (9.6b) is the difference between two terms: both these terms are of the form $\frac{1}{2}(\text{mass}).(\text{speed})^2$.

NOTE

If the force $\vec{\mathbf{F}}$ acts along the direction of motion, the work done is positive and **kinetic energy of the object increases**; if force acts opposite to motion, work done is negative and **kinetic energy decreases**. And, if the force acts perpendicular to motion (as in uniform circular motion), work done is zero and there is **no change in kinetic energy**. Conversely, **if there is no change in the kinetic energy of an object, the work done on the object by the net external force is zero**.

As you have just learnt, the quantity $\frac{1}{2}$(mass).(speed)^2 is called the **kinetic energy** of the particle of mass *m* moving with speed *v*. The form of this quantity remains the same for all types of motion, whether in a plane or in space. Did you note that the unit of kinetic energy is the same as that of work? It is a scalar quantity like work. This is not surprising as these two are related through the work-energy theorem. Let us state this theorem now.

THE WORK-ENERGY THEOREM

The work done by the net external force on a particle of mass *m* is equal to the change in the kinetic energy of that object:

$$W = (\text{K.E.})_f - (\text{K.E.})_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \qquad (9.7)$$

Here *W* is the work done by the net external force on a particle of mass *m,*

$(\text{K.E.})_f$ is the final kinetic energy of the particle,

$(\text{K.E.})_i$ is the initial kinetic energy of the particle,

v_f is the final speed of the particle, and

v_i is the initial speed of the particle.

What does the work-energy theorem tell us? It tells us that when a net external force acts on a particle that *moves* from one position to another, its kinetic energy changes from some initial value to some final value. **The work done on the particle by the force is always equal to the change in its kinetic energy.**

Here, you should also understand **what the work-energy theorem is NOT**. **Eq. (9.7) follows from Newton's second law and hence is not a new law in itself**. **That is why**, **it is known as a theorem**.

The work-energy theorem simply gives us a relation between work and kinetic energy derived from Newton's second law of motion. You must also understand the **limitations of the work-energy theorem**. It involves work and energy that are SCALAR quantities, not vectors. The work-energy theorem will never explain **why** a physical system moves the way it does when a force is exerted on it. That explanation lies in the analysis of *force applied* and *resultant motion* that involve vector quantities.

The work-energy theorem is only an accounting concept. Can an accountant explain why a business makes or loses money? No, s/he can only tell how much money was spent and on what; only the business owner can explain why. Similarly, work-energy theorem only tells us how much work is done when an object's speed changes or how much change in speed takes place when work is done.

The work-energy theorem provides an extremely powerful method that connects a particle's speed with its position no matter how complicated its motion is. Thus, it is useful in solving many problems on motion.

As you shall learn in the next few sections, the work-energy theorem given by Eq. (9.7) remains the same for all types of motion – one, two or three-dimensional. It also holds for all types of forces, whether constant or varying from point to point. It is also true for constant mass as well as variable mass systems such as rockets. You may now like to study some applications of the work-energy theorem and learn how to determine the positions and velocities of particles in motion without having to apply Newton's second law.

EXAMPLE 9.3: WORK-ENERGY THEOREM

A spacecraft of mass 200 kg is travelling in deep space at a speed of 300 ms^{-1}. The only force that is exerted on it is due to a weak thrust of the engine of magnitude 0.60 N, which displaces the craft along a straight line in its direction by the distance of 3000 km (see Fig. 9.11). What is the final speed of the probe, assuming that its mass remains constant?

SOLUTION ■ The **KEY IDEA** here is to determine the final speed using the work-energy theorem.

We know	$m = 200$ kg, $v_i = 300\,\text{ms}^{-1}$, $F = 0.60$ N, $s = 3000$ km
We have to find	$v_f = ?$

It is given that the direction of the force and the displacement are the same. Therefore, $W = F\,d$ and applying the work-energy theorem (Eq. 9.7) we get

$$\frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 + (F)\,(s)$$

$$\therefore\ \frac{1}{2}mv_f^2 = \frac{1}{2}(200\,\text{kg})\times(300\,\text{ms}^{-1})^2 + (0.60\,\text{N})\times(3.0\times10^6\,\text{m}) = 10.8\times10^6\,\text{J}$$

$$\therefore\ v_f = 328.6\ \text{ms}^{-1} \approx 3.3\times10^2\ \text{ms}^{-1}$$ up to 2 significant digits.

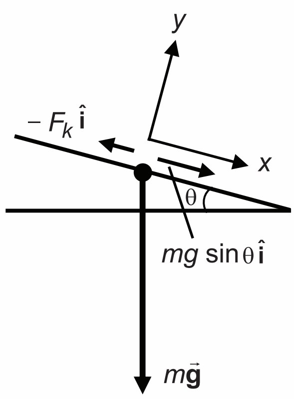
Fig. 9.11: **Work-energy theorem can be applied to determine the speed of a space probe travelling in deep space.**

In Example 9.3, we knew the object's position, and the force was directed along the displacement. We now take up an example in which force is not along the displacement.

EXAMPLE 9.4: WORK-ENERGY THEOREM

A child of mass 30 kg skates down from the top of a ramp having a constant slope of 15° (Fig. 9.12a). The child's speed increases from 1.5 ms^{-1} to 3.0 ms^{-1} as she reaches the bottom of the ramp. Assuming that a force of kinetic friction of magnitude 50 N opposes her motion, determine the length of the ramp. Take $g = 10\,\text{ms}^{-2}$.

(a)



(b)

Fig. 9.12: **a) Application of work-energy theorem to determine distance if speed is given; b) the free-body diagram for the problem.**

SOLUTION ■ The **KEY IDEA** here is to determine the distance travelled by the child (which is the same as the length of the ramp) using the work-energy theorem.

We know	$m = 30$ kg, $v_i = 1.5\,\text{ms}^{-1}$, $v_f = 3.0\,\text{ms}^{-1}$, $F_k = 50$ N, $\theta = 15°$
We have to find	$d = ?$

In this case $W = F_{net}\,d$, where F_{net} is the component of the net external force along the displacement of the child. F_{net} is given by (Fig. 9.12b):

$$F_{net} = mg\,\sin 15° - F_k = (300 \times 0.259 - 50)\,\text{N} = 27.7\,\text{N}$$

From the work-energy theorem:

$$W = F_{net}\,d = \frac{m}{2}(v_f^2 - v_i^2)$$

$$\therefore\; d = \frac{m}{2F_{net}}(v_f^2 - v_i^2) = 3.67\,\text{m} \approx 3.7\,\text{m} \text{ up to 2 significant digits.}$$

We now use the work-energy theorem to answer a very interesting question from our daily lives. As per traffic rules, a motor driver is supposed to maintain a speed limit and a minimum distance from the vehicle right in front. Why is it so? You can intuitively give the reason that this is because the driver should not bump into the vehicle in front if that vehicle stops suddenly. But is there a way to determine how much minimum distance should be maintained for a given speed limit? The work-energy theorem gives us the answer in a very simple way as you can find from the following example!

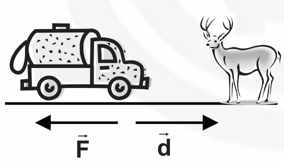


Fig. 9.13: **Applying work-energy theorem to determine the stopping distance.**

EXAMPLE 9.5: WORK-ENERGY THEOREM

A truck of mass *m* is travelling with speed *v* and the driver spots a deer in front of him. He applies a braking force *F* and the truck stops after travelling a distance *d* (Fig. 9.13). What is this distance, called the ***stopping distance***?

SOLUTION ■ The **KEY IDEA** here is to determine the **stopping distance** using the work-energy theorem.

We know	m, v and F
We have to find	$d = ?$

Note that the direction of the force and the displacement are opposite in this case and the final speed is zero. Therefore, applying the work-energy theorem we get

$$-F\,d = -\frac{1}{2}\,m v^2 \quad \text{whence} \quad d = \frac{1}{2}\left(\frac{m}{F}\right)v^2$$

Notice that for equal braking force (generated by applying equal force on the brakes) and mass, d is proportional to v^2. Thus, if the speed is doubled (say, the driver is travelling at 80 kmh^{-1} instead of the mandated 40 kmh^{-1} on any road), the stopping distance becomes 4 times as much. If the speed is tripled, the stopping distance is 9 times as much, i.e., the truck stops only after travelling 9 times the original distance d. That is why while driving, we have **speed limits** and it is emphasised that a certain minimum distance must always be kept between two vehicles on road. Do you follow this rule while driving on the roads? **REMEMBER:** We could prevent many accidents from happening if all of us understood just this little bit of physics.

Even a small difference in vehicle speed can make a large difference to the probability of serious injury. Put simply, if a car hits a pedestrian at 50 kmh^{-1}, the car driver is more likely to kill the pedestrian than if the car hits a pedestrian at 40 kmh^{-1}.

You should now apply the work-energy theorem to a simple situation.

SAQ 4 – Applying the work-energy theorem

A block of mass 1.0 kg slides along a rough floor with an initial velocity of 12 ms^{-1}. The coefficient of kinetic friction between the box and the floor is 0.50. Determine the work done by the force of kinetic friction when the box travels a distance of 10 m from its initial position. Obtain the initial and final kinetic energies of the box. Take $g = 9.8\,ms^{-2}$.

You may now like to pause and think over what you have learnt so far. So far in this unit, you have learnt how to determine work done by constant force. You have also learnt how to apply the work-energy theorem to solve problems on motion under constant forces. But you know that there are many **forces** around us **which change** as the position of the object changes. The most common examples of such forces are the force due to a spring, the gravitational force and the electrostatic force. How do we determine the work done by such variable forces (that is, forces which are not constant)? Can we apply the work-energy theorem to variable forces? This is what you will learn in the next section.

9.4 WORK DONE BY A VARIABLE FORCE

Let us first take up the simplest case of one-dimensional motion: The force exerted on an object depends on its position along the x-direction. Consider a very simple situation as shown in Fig. 9.14. A constant force F_1 displaces the object from $x = 0$ to $x = d_1$ by a distance d_1: The work done by F_1 is $W_1 = F_1 d_1$. Then another constant force F_2 displaces it from $x = d_1$ to $x = d_2$ by the distance $(d_2 - d_1)$. The work done is $W_2 = F_2 (d_2 - d_1)$.

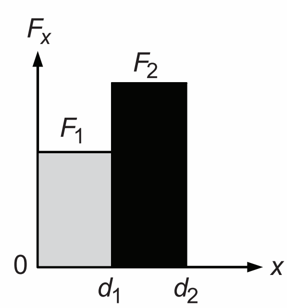


Fig. 9.14: **Work done by two constant forces acting one after the other.**

The total work done is

$$W = W_1 + W_2 = F_1 d_1 + F_2 (d_2 - d_1)$$

Note that this is just the **sum of areas of the grey and black rectangles** in the force vs. displacement graph. Let us now extend this to a more general case, but we still let the force depend only on the variable x (the particle's position along the x-direction). In school mathematics, you have studied the concept of integral of a function as an area under the curve.

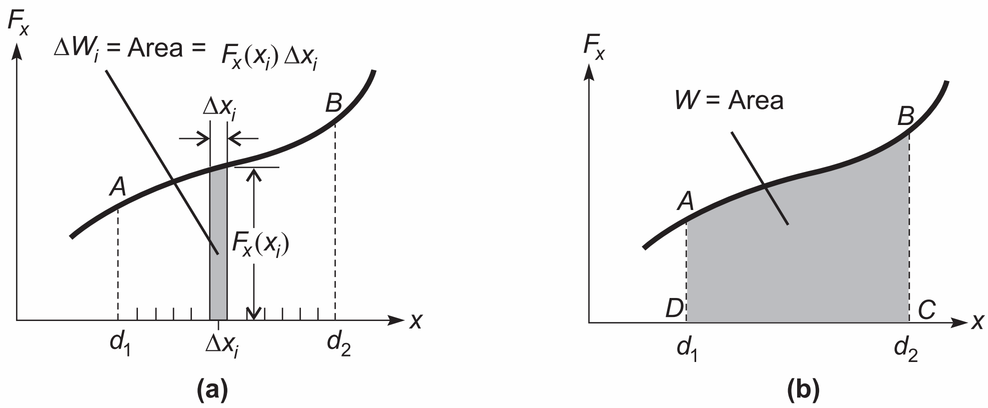


Fig. 9.15: **Work done by a variable force.**

Let us take a force F_x that is changing continuously with x, the displacement of an object (see Fig. 9.15a). We now ask: What is the work done in moving the object from position A $(x = d_1)$ to position $B(x = d_2)$? To answer this question, we use the basic concept of integral as area under a curve.

We divide the region under the curve from point A to point B in Fig. 9.15a into a very large number of **narrow rectangular strips** such that the width of the i^{th} strip is Δx_i. Let us take the width of each rectangle (Δx_i) to be very small so that the force exerted on the object over the displacement Δx_i is constant. Let us label the constant force for the i^{th} such rectangular strip of width Δx_i as $F_x(x_i)$. Then the work done by $F_x(x_i)$ for the displacement Δx_i is just the area of the i^{th} strip, that is, the area of the rectangle of height $F_x(x_i)$ and width Δx_i:

$$\Delta W_i = F_x(x_i)\Delta x_i \qquad (9.8a)$$

The area under the curve is divided into many such strips, say n in number. Therefore, the total work done from $x = d_1$ to $x = d_2$ is the sum of the areas of all the n rectangles.

$$W = W_1 + W_2 + W_3 + \cdots + W_n = \sum_{i=1}^{n} W_i$$

$$= F_x(x_1)\Delta x_1 + F_x(x_2)\Delta x_2 + F_x(x_3)\Delta x_3 + \cdots + F_x(x_n)\Delta x_n$$

or $$W = \sum_{i=1}^{n} F_x(x_i)\Delta x_i \qquad (9.8b)$$

This gives us only the ***approximate*** area under the curve. We obtain the exact value of the area **by making the width Δx_i infinitesimally small (tending to zero) or making the number of strips infinitely large (tending to infinity)**. In this limiting case, we have

$$\Delta x_i \to 0 \quad \text{and} \quad n \to \infty$$

and the sum given in Eq. (9.8b) becomes equal to the **exact area under the curve**:

$$W = \lim_{\substack{\Delta x_i \to 0 \\ n \to \infty}} \sum_{i=1}^{n} F_x(x_i)\Delta x_i$$

This sum is represented by the following definite integral:

$$W = \int_{d_1}^{d_2} F_x \, dx \qquad (9.9)$$

Work done by a variable force

Thus, the **work done** by a variable force F_x in displacing an object along the x-axis from position A ($x = d_1$) to position B ($x = d_2$) is given by (Eq. 9.9), which **is a definite integral of force with respect to displacement**. It is the area of the shaded region $ABCD$ under the curve $F_x(x)$ (Fig. 9.15b). Let us apply Eq. (9.9) to determine work done by a variable force in one-dimension.

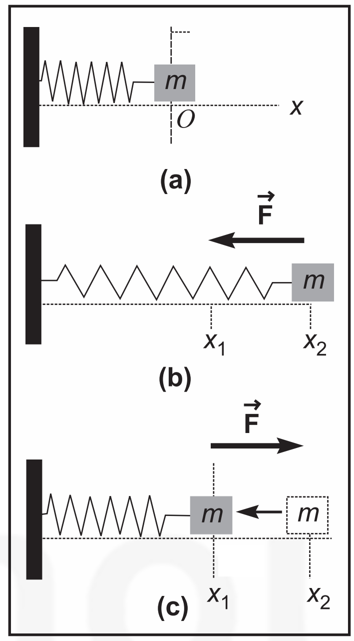


Fig. 9.16: **The work done by spring force on a) a spring-mass system; b) the block is stretched from** $x = x_1$ **to** $x = x_2$; **c) the block is compressed from** $x = x_2$ **to** $x = x_1$

EXAMPLE 9.6: WORK DONE BY SPRING FORCE

One end of a spring is attached to a fixed wall and the other end to a block that is free to slide on a horizontal surface (see Fig. 9.16). Determine the work done by the spring force as the block is moved from some initial position $x = x_1$ to a final position $x = x_2$ along the x-axis. What is the work done by the spring force on the block as it moves back from $x = x_2$ to $x = x_1$?

SOLUTION ■ The **KEY IDEA** here is that the force is one-dimensional. It is a variable force as it depends on the position of the spring along the x-axis and we can apply Eq. (9.9) to determine the work done by the spring force in moving the spring from $x = x_1$ to $x = x_2$. However, in applying Eq. (9.9), we shall make two simplifying assumptions: (1) the spring is mass-less and (2) the spring is ideal, that is, it obeys Hooke's law.

We substitute the expression for spring force (Eq. 6.5b) in Eq. (9.9) and integrate the resulting expression for work (read the margin remark):

$$W = \int_{x_1}^{x_2} F_x \, dx = -\int_{x_1}^{x_2} k\,x\,dx = -\frac{1}{2}k\,x^2\Big|_{x_1}^{x_2} = -\frac{k}{2}(x_2^2 - x_1^2)$$

In determining the work done, we have used this result :

$$\int x\,dx = \frac{x^2}{2}$$

Note that since $x_2 > x_1$, **the block is farther away from its relaxed position than it was initially**. In this case, $W < 0$ and **work is done <u>on</u> the spring <u>by</u> the block**. The work done when the spring is compressed so that the block moves back from $x = x_2$ to $x = x_1$ is

$$W = -\int_{x_2}^{x_1} k\,x\,dx = -\frac{k}{2}(x_1^2 - x_2^2) = \frac{k}{2}(x_2^2 - x_1^2)$$

In this case, the **block is closer to its relaxed position than it was initially**. Since $x_2 > x_1$, $W > 0$, and **work is done <u>by</u> the spring <u>on</u> the block**.

NOTE

The **work done** by the spring force on the block **is positive** if the block moves closer to the relaxed position ($x = 0$). The **work done** by the spring force on the block **is negative** if it moves farther away from the relaxed position ($x = 0$). It is **zero** if the block ends up at the same distance from ($x = 0$).

Let us now extend Eq. (9.9) to motion in a plane and motion in space. We consider the general case of work done by a three-dimensional variable force given by

$$\vec{\mathbf{F}} = F_x\,\hat{\mathbf{i}} + F_y\,\hat{\mathbf{j}} + F_z\,\hat{\mathbf{k}} \tag{9.10a}$$

The components of the force depend on the position of the particle but again to simplify the calculations, we assume that

- F_x depends on x but not on y and z,
- F_y depends on y but not on x and z, and
- F_z depends on z but not on x and y.

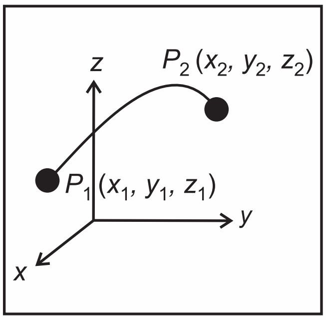


Fig. 9.17: **Work done by a variable force on an object moving in space.**

For motion in space (Fig. 9.17), for a small displacement

$$d\vec{\mathbf{r}} = dx\,\hat{\mathbf{i}} + dy\,\hat{\mathbf{j}} + dz\,\hat{\mathbf{k}} \tag{9.10b}$$

the work done by the variable force of Eq. (9.10a) is given by

$$dW = \vec{\mathbf{F}}\,.\,d\vec{\mathbf{r}} = F_x\,dx + F_y\,dy + F_z\,dz \tag{9.10c}$$

The work done by $\vec{\mathbf{F}}$ when the particle moves in space, from the point $P_1\,(x_1, y_1, z_1)$ to the point $P_2\,(x_2, y_2, z_2)$ is given by

Work done by a variable force

$$W = \int_{P_1}^{P_2} \vec{\mathbf{F}}\,.\,d\vec{\mathbf{r}} = \int_{x_1}^{x_2} F_x\,dx + \int_{y_1}^{y_2} F_y\,dy + \int_{z_1}^{z_2} F_z\,dz \tag{9.11}$$

If the force is two-dimensional with $\vec{\mathbf{F}} = F_x\,\hat{\mathbf{i}} + F_y\,\hat{\mathbf{j}}$ and the displacement $d\vec{\mathbf{r}} = dx\,\hat{\mathbf{i}} + dy\,\hat{\mathbf{j}}$, is in the xy plane, the work done by $\vec{\mathbf{F}}$ when the particle moves in the x-y plane, from the point $P_1\,(x_1, y_1)$ to $P_2\,(x_2, y_2)$ is given by

Work done by a variable force

$$W = \int_{P_1}^{P_2} dW = \int_{x_1}^{x_2} F_x\,dx + \int_{y_1}^{y_2} F_y\,dy \tag{9.12}$$

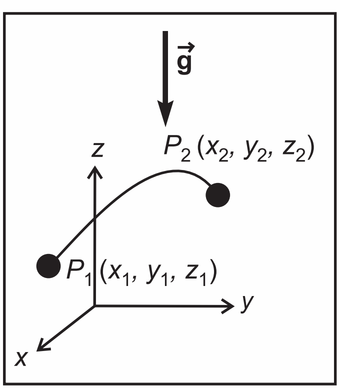


Fig. 9.18: **Work done by force of gravity on an object moving in space.**

If $\vec{\mathbf{F}}$ has only one component, say, the x-component, then the y term in Eq. (9.12) becomes zero and it reduces to Eq. (9.9). We now take up an example to show how to apply Eq. (9.11).

EXAMPLE 9.7 : WORK DONE BY FORCE OF GRAVITY

A particle of mass m undergoes displacement from a point $P_1\,(x_1, y_1, z_1)$ to another point $P_2\,(x_2, y_2, z_2)$ close to Earth under the influence of the force of gravity given by $\vec{\mathbf{F}} = m\vec{\mathbf{g}}$ (see Fig. 9.18). What is the work done by this force on the particle?

SOLUTION ■ The **KEY IDEA** here is that displacement is taking place in space but the force is one-dimensional and constant near the surface of the Earth.

Let us apply Eq. (9.11) to determine the work done by the force of gravity on the particle. Notice that the choice of the coordinate system is such that the force of gravity is directed along the negative z-axis. So we have $F_x = 0, \quad F_y = 0 \quad$ and $\quad F_z = -mg$. Substituting this in Eq. (9.11), the work done is given by

$$W = = \int_{x_1}^{x_2} F_x\,dx + \int_{y_1}^{y_2} F_y\,dy + \int_{z_1}^{z_2} F_z\,dz = 0 + 0 + \left(-\int_{z_1}^{z_2} m\,g\,dz\right)$$

or $W = -m\,g\,(z_2 - z_1) = -m\,g\,\Delta z$, where Δz is the change in height.

NOTE

Work done by the force of gravity depends only on the vertical separation of the initial and final positions of the particle. The path followed by the particle does not matter. Thus, the details of motion do not count.

Let us calculate the work done by the force of gravitation as a last example. You have learnt in Unit 7 that the force of gravitation between two objects depends inversely on the distance between them. So this is also a variable force, which changes with the position of objects.

EXAMPLE 9.8: WORK DONE BY GRAVITATIONAL FORCE

A satellite of mass m is displaced from a point A near the Earth's surface to a point B at a distance of $3R_e$ from the Earth's surface (Fig. 9.19). What is the work done by the force of gravitation on the satellite?

Remember that we take the force of gravitation to be equal to $m\vec{\mathbf{g}}$ near the surface of the earth, where it is constant at a given place. We then term it the **force of gravity**.

SOLUTION ■ The **KEY IDEA** here is that the force of gravitation is a variable force and we use Eq. (9.11) since motion is in space.

In Unit 7, you have learnt about the gravitational force between any two objects separated by a distance r. In this case, it is given by

$$\vec{\mathbf{F}} = -\frac{GM_e\,m}{r^2}\,\hat{\mathbf{r}}$$

The work done by the force of gravitation in moving the satellite from the point $r = R_e$ to the point $r = 3R_e$ is given by Eq. (9.11). The diagram for the problem is shown in Fig. 9.19 and you can see that $\hat{\mathbf{r}}$ is a unit vector along $d\vec{\mathbf{r}}$, that is, $\hat{\mathbf{r}} \parallel d\vec{\mathbf{r}}$. Therefore,

$$\hat{\mathbf{r}}.d\vec{\mathbf{r}} = 1 \times |d\vec{\mathbf{r}}| = dr$$

and $$\vec{\mathbf{F}}.\,d\vec{\mathbf{r}} = -\frac{GM_e\,m}{r^2}(\hat{\mathbf{r}}.d\vec{\mathbf{r}}) = -\frac{GM_e\,m}{r^2}dr$$

Thus, the work done by the force of gravitation on the satellite is given by

$$W = \int_A^B \vec{\mathbf{F}}.\,d\vec{\mathbf{r}} = -\int_{r_A}^{r_B} \frac{GM_e\,m}{r^2}dr$$

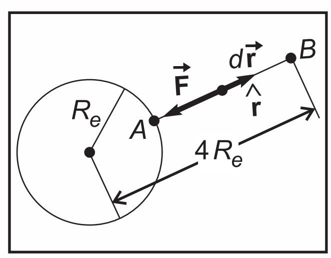


Fig. 9.19: **The work done by gravitational force (not to scale).**

Putting

$G = 6.67 \times 10^{-11}\, Nm^2\, kg^{-2}$,

$M_e = 5.97 \times 10^{24}$ kg,

$R_e = 6.37 \times 10^6$ m,

and $m = 100$ kg, in

$$W = -\frac{3}{4}\frac{GM_e m}{R_e},$$

we get:

$$W = -4.70 \times 10^9 \text{ J}$$

Notice that we have replaced the limits by the actual distance of the points A and B from the centre of the Earth. We now use the following result from integral calculus: $\int \frac{1}{r^2}\, dr = -\frac{1}{r}$ and obtain

$$W = \left.\frac{GM_e m}{r}\right|_{r_A}^{r_B} = GM_e m\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

In this case $r_A = R_e$ and $r_B = 4R_e$. Therefore, the work done on the satellite is given by

$$W = GM_e m\left(\frac{1}{4R_e} - \frac{1}{R_e}\right) = -\frac{3}{4}\frac{GM_e m}{R_e}$$

The work done is negative as the force is directed towards the Earth and displacement is away from the Earth. The work done on a satellite of mass 100 kg by the force of gravitation $= -4.70 \times 10^9$ J (see margin remark).

SAQ 5 – Work done by variable force

A suitcase moves on a straight conveyor belt when a force of magnitude $F = (2.0x - 5.0x^2)$N is exerted on it. Calculate the work done on the suitcase as it moves from the position $x_1 = 1.0\,\text{m}$ to the position $x_2 = 3.0\,\text{m}$.

In this section, so far you have learnt how to determine work done by variable forces in one, two and three dimensions. In the previous section, you have learnt how to apply the **work-energy theorem** for **constant forces**. Remember that **the work-energy theorem (Eq. 9.7) can also be applied to variable forces**. The proof of the work-energy theorem for variable forces, however, is beyond the scope of this syllabus. Before we end this unit, we would like to discuss the concept of power, which is related to work done.

9.5 POWER

Why is this concept important? It is important in situations where we are interested in doing work in the smallest possible time, for example, lifting a load of bricks or other materials in buildings, lifting water to upper floors, pumping petrol, using machines to transport goods or people, e.g., trains, buses, trucks, conveyor belts, etc. (Fig. 9.20). Then we need to talk of the rate at which work is done and the concept of power.

Fig. 9.20: **Some examples in which the concept of power is used: conveyor belt, pumping of petrol and water.**

POWER

Power is defined as the rate at which work is done by force. If an amount of work ΔW is done in the duration of time Δt by a force, we define the **average power** due to the work done by the force in that time interval as

$$\langle P \rangle = \frac{\Delta W}{\Delta t} \qquad (9.13)$$

Power

We define the **instantaneous power** P as the **rate** of doing work:

$$P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt} \qquad (9.14)$$

The SI unit of power is **watt** (W).

We can also find the relationship between power and constant or average force as follows:

Let force $\vec{\mathbf{F}}$ be exerted on a system, which gets displaced by $\Delta\vec{\mathbf{r}}$ **in time** Δt. The work done in this time is $\mathbf{F}.\Delta\vec{\mathbf{r}}$ and the instantaneous power, that is, the rate at which the force does work is

$$P = \frac{dW}{dt} = \lim_{\Delta t \to 0} \frac{\vec{\mathbf{F}}.\Delta\vec{\mathbf{r}}}{\Delta t} = \vec{\mathbf{F}}.\frac{d\vec{\mathbf{r}}}{dt}$$

or

$$P = \vec{\mathbf{F}}.\vec{\mathbf{v}} \qquad (9.15)$$

Let us now take up an example to determine power.

EXAMPLE 9.9: AVERAGE AND INSTANTANEOUS POWER

A suitcase is carried by a conveyor belt and the amount of work done by the belt's force on the suitcase is given by $W(t) = ct^2$. Suppose the average power due to the belt's force during the time interval from $t_1 = 0\,\text{s}$ to $t_2 = 30\,\text{s}$ is 300 W. What is the instantaneous power due to the belt at $t = 10\,\text{s}$? Assume the belt to be frictionless.

SOLUTION ■ The **KEY IDEA** here is to apply Eq. (9.13) to determine the constant c and then apply Eq. (9.14) to determine the instantaneous power.

At $t_1 = 0\,\text{s}$, $W_1 = 0$ and at $t_2 = 30\,\text{s}$, $W_2 = 900c$. From Eq. (9.13),

$$\langle P \rangle = \frac{\Delta W}{\Delta t} = \left(\frac{900\,c}{30}\right)\text{W} = (30\,c)\,\text{W} = 300\,\text{W} \Rightarrow c = 10\ \text{Js}^{-2}$$

From Eq. (9.14), the instantaneous power is

$$P = \frac{dW}{dt} = 2ct = (20t)\,\text{W}$$

At $t = 10\,\text{s}$, $\quad P = 200\,\text{W} = 2.0 \times 10^2\,\text{W}$

SAQ 6 – Power

A lift of mass 4000 kg moves 200 m upwards at a constant speed in 25.0 s. At what average rate does the force due to the cable do work on the lift? Take $g = 9.8\,\text{ms}^{-2}$.

We now present the summary of this unit.

9.6 SUMMARY

Concept	Description
Work done by constant force	■ **Work done by a constant force** on an object that undergoes displacement $\vec{\mathbf{d}}$ is defined as: $$W = (F\cos\theta)d = \vec{\mathbf{F}}\cdot\vec{\mathbf{d}}$$ **Work done does not depend on the observer's frame of reference. The component of force perpendicular to the direction of displacement does no work**. If the force itself is applied perpendicular (that is, at an angle of 90°) to the direction of displacement, it does no work. Such a force is called **no-work force**. When the force or the component of force points in the same direction as the displacement, **work done is positive**. When the force or the component of force points in the opposite direction as the displacement, **work done is negative**, that is, **work is done by the object**. In component form, the work done is $$W = F_x x \quad \text{(One-dimension)}$$ $$W = F_x x + F_y y \quad \text{(Two-dimensions)}$$ $$W = F_x x + F_y y + F_z z \quad \text{(Three-dimensions)}$$
Energy	■ **Energy** is the capacity of a body to do work and is always measured by the work the body is capable of doing.
Kinetic energy	■ The **kinetic energy** of a particle of mass m travelling with speed v is given by $$\text{Kinetic Energy (K.E.)} = \frac{1}{2}mv^2$$
Work done by variable force	■ The **work done by variable force** in displacing an object is given by $$W = \int_{x_1}^{x_2} F_x\,dx \quad \text{(One-dimension)}$$ $$W = \int_{x_1}^{x_2} F_x\,dx + \int_{y_1}^{y_2} F_y\,dy \quad \text{(Two-dimensions)}$$

$$W = \int_{x_1}^{x_2} F_x\,dx + \int_{y_1}^{y_2} F_y\,dy + \int_{z_1}^{z_2} F_z\,dz \qquad \text{(Three-dimensions)}$$

- Work done by the **force of gravity** $\vec{\mathbf{F}} = m\vec{\mathbf{g}}$ on an object that undergoes a finite displacement from the position $\vec{\mathbf{r}}_1\ (= x_1\,\hat{\mathbf{i}} + y_1\,\hat{\mathbf{j}} + z_1\,\hat{\mathbf{k}})$ to the position $\vec{\mathbf{r}}_2\ (= x_2\,\hat{\mathbf{i}} + y_2\,\hat{\mathbf{j}} + z_2\,\hat{\mathbf{k}})$:

$$W = \int_{\vec{\mathbf{r}}_1}^{\vec{\mathbf{r}}_2} \vec{\mathbf{F}}.d\vec{\mathbf{r}} = -\ mg(z_2 - z_1)$$

- Work done by the **spring force** $(\vec{\mathbf{F}} = -\,kx\,\hat{\mathbf{i}})$ in stretching a spring from x_1 to x_2:

$$W = \int_{x_1}^{x_2} F_x\,dx = -\int_{x_1}^{x_2} k\,x\,dx = -\frac{k}{2}(x_2^2 - x_1^2)$$

- Work done by the **force of gravitation** due to an object of mass M on another object of mass m that undergoes displacement from position $\vec{\mathbf{r}}_A$ to $\vec{\mathbf{r}}_B$:

$$W = \int_A^B \vec{\mathbf{F}}\,.\,d\vec{\mathbf{r}} = -\int_{r_A}^{r_B} \frac{GMm}{r^2}\,dr = GMm\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

Work-energy theorem

- Work done by the net force on an object of mass m is equal to the change in the kinetic energy of that object:

$$W = (\text{K.E.})_f - (\text{K.E.})_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

The work-energy theorem follows from Newton's second law and hence is not a new law in itself.

Power

- **Power is defined as the rate at which work is done by a force.**

$$\langle P \rangle = \frac{\Delta W}{\Delta t} \qquad \text{(average power)}$$

$$P = \frac{dW}{dt} \qquad \text{(instantaneous power)}$$

Also $\qquad P = \vec{\mathbf{F}}.\vec{\mathbf{v}}$

9.7 TERMINAL QUESTIONS

1. A person travels in a lift with a bag hanging from his hand. Work is done on the bag by the force exerted by his hand. Which of the following is the correct statement about the work done?

   a) The work done is always positive whether the lift goes up or down.

   b) The work done is always negative whether the lift goes up or down.

   c) The work done is positive when the lift goes up and negative when it comes down.

d) The work done is negative when the lift goes up and positive when it comes down.

2. A particle undergoes displacement $\vec{\mathbf{d}} = (4.0\,\text{m})\hat{\mathbf{i}} - (5.0\,\text{m})\hat{\mathbf{j}}$ when a force $\vec{\mathbf{F}} = (a\text{N})\hat{\mathbf{i}} + (2.0\,\text{N})\hat{\mathbf{j}}$ is exerted on it. Determine the value of the constant a if the work done is (a) zero, (b) 20 J and (c) – 16 J.

3. What is the work done by the force of gravity on an object which takes the five paths shown in Fig. 9.21 to reach from point 1 to point 2 in each case? Explain.

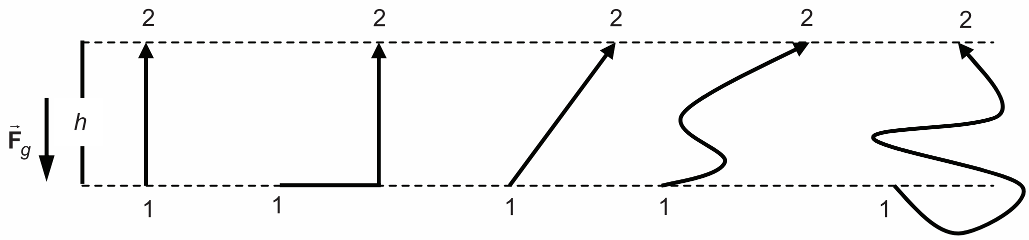


Fig. 9.21

4. Which of the two forces of equal magnitude F acting on the sled shown in Fig. 9.22 does more work on it as it moves a distance d?

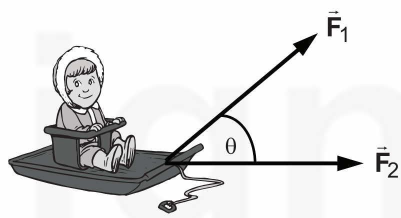


Fig. 9.22

5. The one-dimensional restoring force exerted by an anharmonic spring is given by $F = -ax - bx^2$, where a and b are constants. Calculate the work done by the spring force in moving a mass m from the point $x = x_1$ to $x = x_2$.

6. A spaceship of mass 5000 kg is travelling in deep space at a speed of $10000\ \text{ms}^{-1}$. Its speed is reduced by firing rockets in the opposite direction. The rockets generate a thrust of 45 000 N over a distance of 2000 km. Calculate the final speed of the spaceship.

7. A stationary ball of mass 450 g travels at a speed of $40.0\,\text{ms}^{-1}$ when it is hit by a stick. What is the work done on the ball by the stick? Suppose that the force exerted by the stick is parallel to the displacement of the ball and the stick is in contact with the ball for a distance of 5.00×10^{-3} m. What is the average force exerted by the stick on the ball, if its weight is neglected?

8. From what height would an object need to be dropped from rest so that it acquires kinetic energy equal to that it has when travelling at a speed of $36.0\,\text{ms}^{-1}$? Take $g = 9.80\,\text{ms}^{-2}$.

9. What power is required to pull a 5.0 kg block at a steady speed of $1.5\ \text{ms}^{-1}$? The coefficient of friction is 0.30. Take $g = 9.80\,\text{ms}^{-2}$.

10. A child of mass 30.0 kg rides a bicycle of mass 15.0 kg. If the force of friction in each case is 30.0 N, what power must she supply to maintain a steady speed of 1.50 ms^{-1} on

   i) level ground, and

   ii) while going up an incline of slope 5°? Take $g = 10.0\,ms^{-2}$.

9.8 SOLUTIONS AND ANSWERS

Self-Assessment Questions

1. We use Eq. (9.2a) to obtain work done since force and displacement are in the same direction. The magnitude of the force of gravity is *mg* and the displacement is *d*.

$$\therefore \quad W = mgd$$

Substituting the numerical values of *m* (in kg) and *d* (in m), we get

$$W = 60\,kg \times 10\,ms^{-2} \times 500\,m = 3.0 \times 10^5\,J$$

2. a) Since the bus moves with constant velocity, the net force on it is zero. Therefore, no work is done.

   b) No work is done by the student on the books since the displacement of the books with respect to the student is zero.

   c) Work is being done on the box, since both the force on the box and its displacement are finite. No work is done on the floor (since the displacement of the floor is zero.)

   d) No, because displacement of the cupboard is zero.

   e) We use Eq. (9.2b) to calculate the work done. Here $F = 100\,N$ and $d = 1.0\,m$.

   For $\theta = 0^\circ$, $W = (100\,N) \times \cos 0^\circ \times (1.0\,m) = 100\,J = 1.0 \times 10^2\,J$

   For $\theta = 90^\circ$, $W = 0$

   For $\theta = 60^\circ$, $W = (100\,N) \times \cos 60^\circ \times (1.0\,m) = 50\,J$

3. No work is done by $\vec{\mathbf{F}}_g$ and $\vec{\mathbf{F}}_N$ because these forces are perpendicular to the displacement. These are no-work forces. The total work done on the box is the sum of the work done by $\vec{\mathbf{F}}_{fr}$ and $\vec{\mathbf{T}}$:

$$W = (T\cos 45^\circ)\times 5.0\,m + (F_{fr}\cos 180^\circ)\times 5.0\,m$$

$$= 50\,N \times \frac{1}{\sqrt{2}} \times 5.0\,m + 10\,N \times (-1) \times 5.0\,m = 126.8\,J \approx 1.3 \times 10^2\,J$$

The tension does positive work and the force of friction negative work on the box.

4. Let us assume that the box is sliding along the positive *x*-direction. The

force of kinetic friction is $\vec{\mathbf{F}} = -\,0.50\times1.0\,\text{kg}\times(9.8\,\text{ms}^{-2})\,\hat{\mathbf{i}} = -\,(4.9\,\text{N})\,\hat{\mathbf{i}}$

The displacement $\vec{\mathbf{d}} = (10\,\text{m})\ \hat{\mathbf{i}}$

The work done by the force of friction is negative: $W = -\,49\,\text{J}$

Let the final velocity be v. Using Eq. (9.7) with $v_i = 12\ \text{ms}^{-1},\ m = 1.0\,\text{kg}$ and $W = -49\,\text{J}$, we have

$$\frac{1}{2}\times(1.0\,\text{kg})\times v^2 - \frac{1}{2}\times(1.0\ \text{kg})\times(12\ \text{ms}^{-1})^2 = -\,49\,\text{J}$$

$$\therefore \quad v = 6.8\,\text{ms}^{-1}$$

5. Using Eq. (9.9) with $F = (2.0\,x - 5.0\,x^2)\ \text{N},$ we can write

$$W = \int_{1.0}^{3.0}(2.0x - 5.0x^2)\,dx\ \text{J} = -\ 35.3\,\text{J} \approx -\,35\ \text{J}$$

6. Let us assume that the lift is moving in the positive y-direction. The force exerted by the cable on the lift is equal and opposite to the force of gravity on the lift. Thus, we use Eq. (9.15) with

$$\vec{\mathbf{F}} = (4000\,\text{kg})\times(9.8\,\text{ms}^{-2})\,\hat{\mathbf{j}} = (3.92\times10^4\,\text{N})\,\hat{\mathbf{j}} \quad \text{and}$$

$$\vec{\mathbf{v}} = \left(\frac{200\,\text{m}}{25.0\,\text{s}}\right)\hat{\mathbf{j}} = (8.00\,\text{ms}^{-1})\,\hat{\mathbf{j}}$$

$$\therefore\ P_{av} = (3.92\times10^4\,\text{N})\,\hat{\mathbf{j}}\,.\,(8.00\,\text{ms}^{-1})\,\hat{\mathbf{j}} = 31.36\times10^4\ \text{W} \ \approx 3.1\times10^5\ \text{W}$$

Terminal Questions

1. The correct choice is (c). The force exerted by the hand is equal and opposite to the weight of the bag. On the upward journey of the lift, the work done is positive because the force is directed along the displacement. On the downward journey the work done is negative because the force is opposite to the direction of the displacement.

2. a) Since work done is zero, from Eq. (9.2b), we have

$$[(a\text{N})\,\hat{\mathbf{i}} + (2.0\text{N})\,\hat{\mathbf{j}}]\,.\,[(4.0\,\text{m})\ \hat{\mathbf{i}} - (5.0\,\text{m})\,\hat{\mathbf{j}}] = 0$$

or $\quad (4.0\,a)\text{J} - 10\,\text{J} = 0 \quad \Rightarrow \quad a = 2.5$

b) In this case $(4.0\,a)\text{J} - 10\,\text{J} = 20\,\text{J} \quad \Rightarrow \quad a = 7.5$

c) In this case $(4.0\,a)\text{J} - 10\,\text{J} = -\,16\,\text{J} \quad \Rightarrow \quad a = -\,1.5$

3. Let m be the mass of the object. The work done in each case is the same and equal to $-\,mgh$, since the displacement (opposite to the direction of the force $\vec{\mathbf{F}}_g = m\vec{\mathbf{g}}$) is the same in each case.

4. The work done by $\vec{\mathbf{F}}_1$ is $W_1 = F\,d\ \cos\theta$ and the work done by $\vec{\mathbf{F}}_2$ is $W_2 = F\,d,$ since the magnitudes of the two forces are equal. Since $F\ \cos\,\theta < F$ for non-zero θ, $\vec{\mathbf{F}}_2$ does more work on the sled.

5. We use Eq. (9.9) for the work done by a variable force because here we have a force which depends only on x:

$$W = \int_{x_1}^{x_2} (-ax - bx^2)\,dx = -\frac{a}{2}(x_2^2 - x_1^2) - \frac{b}{3}(x_2^3 - x_1^3)$$

6. We use the work-energy theorem, Eq. (9.7), to solve this problem. The work done by the thrust force is

$$W = (-45000\,\text{N})\hat{\mathbf{i}} \,.\, (2000 \times 10^3\,\text{m})\hat{\mathbf{i}} = -9 \times 10^{10}\,\text{J}$$

where $\hat{\mathbf{i}}$ is in the direction of motion of the spaceship. The work done is negative because the thrust is opposite to the direction of motion of the spaceship. The change in kinetic energy of the spaceship is,

$$(\text{K.E.})_f - (\text{K.E.})_i = \frac{1}{2} \times (5000\,\text{kg}) \times v_f^2 - \frac{1}{2} \times (5000\,\text{kg}) \times (10000\,\text{ms}^{-1})^2$$

$$= (2500\, v_f^2 - 2500 \times 10^8)\,\text{J}$$

where v_f is the final velocity of the spaceship. Using Eq. (9.7) we get,

$(2500\, v_f^2 - 2500 \times 10^8)\,\text{J} = -900 \times 10^8\,\text{J}$ and hence $v_f = 8000\,\text{ms}^{-1}$

7. From Eq. (9.7) the work done is

$$W = \frac{1}{2} \times (450 \times 10^{-3}\,\text{kg}) \times (40.0\,\text{ms}^{-1})^2 - 0 = 360\,\text{J}$$

The average force is obtained from the definition of work done. The work done is $W = Fd$ since $\vec{\mathbf{F}}$ and $\vec{\mathbf{d}}$ are parallel.

$$\therefore \quad F = \frac{W}{d} = \frac{360\,\text{J}}{0.005\,\text{m}} = 72000\,\text{N} = 7.20 \times 10^4\,\text{N}$$

8. The kinetic energy of an object of mass m moving with a speed of $36.0\,\text{ms}^{-1}$ is

$$K = \frac{1}{2} m \times (36.0\,\text{ms}^{-1})^2 = (648\,m)\,\text{J}$$

Suppose the object were to be dropped from a height h to attain this kinetic energy. The work done by the force of gravity on the object is:

$$(-mg\,\hat{\mathbf{j}}) \,.\, (-h\,\hat{\mathbf{j}}) = mgh$$

From the work-energy theorem: $mgh = (648\,m)\,\text{J}$

$$\Rightarrow \qquad h = \frac{648}{9.80}\,\text{m} = 66.1\,\text{m}$$

9. The force of friction on the block is $\vec{\mathbf{F}}_K = \mu_K \vec{\mathbf{F}}_N = -\mu_K mg\,\hat{\mathbf{i}}$. In order to move the block at a constant speed, the net force on the block must be zero. Since the only force on the block is that of friction, an equal and opposite force must be exerted on it, which is given by

$$\vec{\mathbf{F}} = \mu_K mg\,\hat{\mathbf{i}}$$

The power required is $P = \vec{\mathbf{F}}\,.\,\vec{\mathbf{v}} = (\mu_k mg\,\hat{\mathbf{i}}\,.\,v\,\hat{\mathbf{i}})$

$$= (0.30)(5.0\,\text{kg})\,(9.8\,\text{ms}^{-2})\,(1.5\,\text{ms}^{-1}) = 22\,\text{W}$$

10. Let the child ride the bicycle in the positive *x*-direction. The velocity is $\vec{\mathbf{v}} = 1.50\ \text{ms}^{-1}\,\hat{\mathbf{i}}$.

i) The power supplied on level ground is

$$P = \vec{\mathbf{F}}\,.\,\vec{\mathbf{v}} = (30.0\,\text{N}\,\hat{\mathbf{i}})\,.\,(1.50\ \text{ms}^{-1}\,\hat{\mathbf{i}}) = 45.0\,\text{W}$$

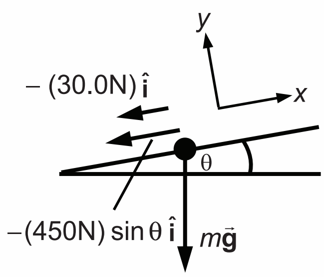


Fig. 9.23: **Angle θ is not to scale in the figure.**

ii) Here we take the *x* and *y*-axes to be along and perpendicular to the inclined plane, respectively (Fig. 9.23). Thus, the velocity is $\vec{\mathbf{v}} = 1.50\ \text{ms}^{-1}\,\hat{\mathbf{i}}$.

The force the child should overcome in this case is

$$\vec{\mathbf{F}} = -(450\,\text{N})\sin 5^\circ\ \hat{\mathbf{i}} - (30.0\,\text{N})\,\hat{\mathbf{i}}$$

Since the child has to apply a force equal to ($-\vec{\mathbf{F}}$), the power to be supplied by her is

$$P = -\vec{\mathbf{F}}\,.\,\vec{\mathbf{v}} = (450\,\text{N}\times\sin 5^\circ\ \hat{\mathbf{i}} + 30.0\,\text{N}\ \hat{\mathbf{i}})\,.\ (1.50\ \text{ms}^{-1}\ \hat{\mathbf{i}})$$

$$= (39.2\,\text{N}\,\hat{\mathbf{i}} + 30.0\,\text{N}\ \hat{\mathbf{i}})\,.\ (1.50\ \text{ms}^{-1}\ \hat{\mathbf{i}}) = 103.8\ \text{W} \approx 104\ \text{W}$$

How much energy is transferred as thermal energy if we take air friction into account? This unit will help you answer such questions! (Image of space shuttle making a landing on the Earth; picture source: www.nasa.gov)

UNIT 10

POTENTIAL ENERGY AND CONSERVATION OF ENERGY

Structure

10.1 Introduction

Expected Learning Outcomes

10.2 Conservative Forces

10.3 Potential Energy

Potential Energy and Stability

10.4 Conservation of Mechanical Energy

10.5 Law of Conservation of Energy

10.6 Summary

10.7 Terminal Questions

10.8 Solutions and Answers

STUDY GUIDE

In this unit, you will learn the **law of conservation of energy**. In order to arrive at this law, you need to know some fundamental concepts of mechanics in addition to what you have learnt in Unit 9, namely, **conservative forces**, **potential energy**, and **non-conservative forces**. The way we explain these concepts may be new for you. Therefore, study them carefully.

Do revise the concepts of vector algebra, scalar product and integral calculus before studying this unit if you have not mastered them so far. **Finally, you should solve all examples, SAQs** and **Terminal Questions on your own to understand it well.**

IN YOUR WRITTEN WORK, ALWAYS USE AN ARROW ABOVE THE LETTER YOU USE TO DENOTE A VECTOR, E.G., $\vec{r}$. USE A CAP ABOVE THE LETTER YOU USE TO DENOTE A UNIT VECTOR, E.G., $\hat{r}$.

"There is inherent in nature a hidden harmony that reflects itself in our minds under the image of simple mathematical laws."

Hermann Weyl

10.1 INTRODUCTION

(a)

(b)

Fig. 10.1: **What form of energy is involved in these cases?**

In Unit 9, you have learnt how to determine **work done by constant and variable forces**. You have also studied the **work-energy theorem**, which relates work done by a force on an object to the change in its **kinetic energy**. In this unit, we shall discuss a fundamental law of nature: the **law of conservation of energy**. However, in order to arrive at the law, we need to introduce the concept of **potential energy**. Potential energy is a quantity associated with a special class of forces known as **conservative forces**. Therefore, we begin our study with a discussion of these two concepts in Sec. 10.2 and Sec. 10.3.

In school physics courses, you may have studied about potential energy as the energy that an object possesses by virtue of its position, shape and configuration. In fact, in real life, we come across many situations where a force acts on an object, which is displaced but its speed does not change; instead, the position, shape or size of the object changes.

For example, suppose a worker lifts a box from the ground and puts it on his shoulder (Fig. 10.1a). The box undergoes displacement along the direction of force, but its initial and final speeds are zero. Work is being done and energy is involved as we have defined it as the capacity to do work. It is certainly not the kinetic energy. What is it? Similarly, when we stretch an elastic band or a spring, we are doing work, though no change in speed is involved (Fig. 10.1b). So, what is the energy associated with this work? In this way, you have been introduced to the concept of **potential energy**.

However, in Sec. 10.3, we introduce the concept of potential energy in a different way that helps us describe mathematically, the law of conservation of mechanical energy (Sec. 10.4). Finally, we introduce non-conservative forces to present the law of conservation of energy in Sec. 10.5. We advise you to study these concepts and the law carefully so that you understand them well.

In the next block, we develop the concepts needed for describing and analysing **angular motion and the motion of many-particle systems**. You will also learn the third conservation law: The law of **conservation of angular momentum**.

Expected Learning Outcomes

After studying this unit, you should be able to:

- define conservative forces and distinguish between conservative and non-conservative forces;
- calculate the potential energy of simple systems;
- apply the law of conservation of mechanical energy to simple problems;
- determine whether an object is in stable, unstable or neutral equilibrium; and
- explain the law of conservation of energy and apply it to simple problems.

10.2 CONSERVATIVE FORCES

Let us go back to Examples 9.6, 9.7 and 9.8 of Unit 9 in which we have determined the work done by the spring force, the force of gravity and the force of gravitation. Let us write down the general expressions for work done for each case:

a) ***Work done by the spring force*** $(\vec{\mathbf{F}} = -k\,\vec{\mathbf{x}})$ ***in stretching a spring*** by displacement $\vec{\mathbf{x}}$ from x_1 to x_2: In Example 9.6, we have determined the work done to be

$$W = \int_{x_1}^{x_2} F_x\,dx = -\int_{x_1}^{x_2} k\,x\,dx = -\frac{k}{2}\left(x_2^2 - x_1^2\right)$$

b) ***Work done by the force of gravity*** $(\vec{\mathbf{F}} = m\vec{\mathbf{g}})$ on an object that undergoes a finite displacement from the position $\vec{\mathbf{r}}_1$ to $\vec{\mathbf{r}}_2$: From Example 9.7, we have found the work done to be

$$W = \int_{\vec{\mathbf{r}}_1}^{\vec{\mathbf{r}}_2} \vec{\mathbf{F}}\,.\,d\vec{\mathbf{r}} = -\,mg\,(z_2 - z_1)$$

c) ***Work done by the force of gravitation on an object that undergoes*** displacement from position $\vec{\mathbf{r}}_A$ to $\vec{\mathbf{r}}_B$: In Example 9.8, we have determined the work done to be

$$W = \int_A^B \vec{\mathbf{F}}\,.\,d\vec{\mathbf{r}} = -\int_{r_A}^{r_B} \frac{GM_e\,m}{r^2}\,dr = GM_e\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

This exercise leads us to the concept of conservative force. Let us see how.

Do you observe anything common between the above three results for work done? In all examples, **the work done depends only on the initial and final positions of the object. It does not depend on the path taken by the object to reach the final position from its initial position**.

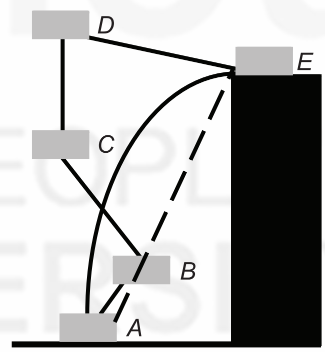


Fig. 10.2: Path independence of work done by a conservative force. Work done by the force of gravity is the same for the same initial and final positions, whatever the path of the particle may be – ***AE*** **or** ***ABCDE***.

For example, the work done by the spring force in Example 9.6 will remain the same whatever be the path of the particle. This also holds true for work done by the force of gravity on the particle in Example 9.7 and the force of gravitation in Example 9.8. You can verify this statement for all examples given above. All these forces are examples of a special class of forces called the **conservative forces**. We define a conservative force as follows (see Fig.10.2).

DEFINITION OF CONSERVATIVE FORCE

A force is **CONSERVATIVE** when the work done by this force on a moving particle is independent of the path of the particle between the particle's initial and final positions: it is the same for any path connecting the same two points.

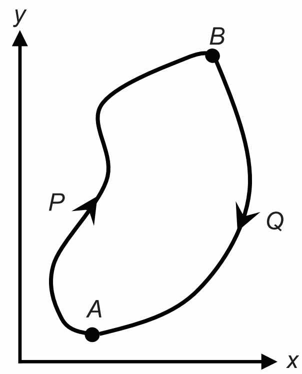


Fig. 10.3: **Work done by a conservative force over a closed path** ***APBQA*** **is zero.**

ANOTHER DEFINITION OF CONSERVATIVE FORCE

A force is **conservative** when the **total work done by this force along any closed path** (which starts and finishes at the same point) **is zero** (Fig. 10.3).

Let us apply these definitions to the force of gravity to test whether it is a conservative force.

EXAMPLE 10.1: THE CONSERVATIVE FORCE TEST

Is the force of gravity a conservative force?

SOLUTION ■ The **KEY IDEA** here is to verify whether either of the definitions of the conservative force holds for this force.

In Example 9.7, you have learnt that the work done by the force of gravity depends only on the vertical separation between the initial and final positions, whatever may be the path followed by the object (see Fig. 10.4). Hence, **the force of gravity is conservative**. You can also test whether the second definition holds for the force of gravity.

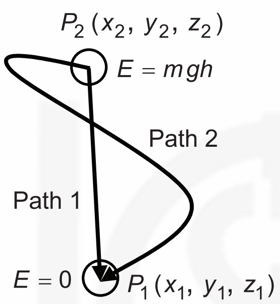


Fig. 10.4: **The work done by the force of gravity around a closed path is zero. Hence, it is a conservative force.**

Suppose the object in Example 9.7 returns via another path from the finishing point $P_2\ (x_2, y_2, z_2)$ to the starting point $P_1\ (x_1, y_1, z_1)$ (see Fig. 10.4). What is the work done by the force of gravity in this case?

Following the method of Example 9.7, we get

$$W_{P_2 \to P_1} = \int_{x_2}^{x_1} F_x\, dx + \int_{y_2}^{y_1} F_y\, dy + \int_{z_2}^{z_1} F_z\, dz = 0 + 0 + \left(-\int_{z_2}^{z_1} mg\, dz\right)$$

$$= -mg\,(z_1 - z_2) = mg\,(z_2 - z_1) = mg\,\Delta z$$

Since work done is a scalar quantity, we simply add up the work done along the individual path segments making up the closed path. Thus, the work done by the force of gravity around the closed path starting from $P_1\ (x_1, y_1, z_1)$ and ending at it is

$$W_{P_1 \to P_2 \to P_1} = W_{P_1 \to P_2} + W_{P_2 \to P_1}$$

$$= -mg\,(z_2 - z_1) + mg\,(z_2 - z_1) = 0$$

Thus, the force of gravity also satisfies the second definition of the conservative force.

You may now like to check for yourself whether the spring force is conservative.

SAQ 1 – Test of conservative force

Using the definition of a conservative force given in Sec. 10.2, show that the spring force is conservative.

The definition of **non-conservative force** follows from these definitions of conservative force. **A force is non-conservative if the work done by it depends on the path of the particle or is non-zero for a closed path**.

So far in this section you have learnt about the definition of conservative force and how to check whether a force is conservative or not. Based on this, you can identify non-conservative forces as well. Let us work out an example to check whether the force of friction is a non-conservative force.

EXAMPLE 10.2: NON-CONSERVATIVE FORCE

Consider the motion of a box being pushed on a rough floor with a constant velocity, which means that the net force on it is zero. Suppose the box is moved from point *A* to point *B* along two different paths: 1 along *AB* and 2 along *ACDB* as shown in Fig. 10.5. Determine the work done by the force of friction along these two paths. Hence, determine whether the force of friction is conservative or not.

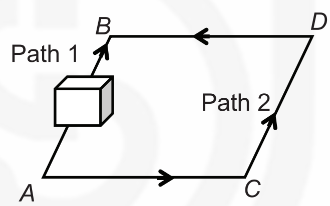


Fig. 10.5: **Calculating work done by the force of friction along two paths.**

SOLUTION ■ The **KEY IDEA** here is to verify whether either of the definitions of the conservative force holds for this force.

The force of friction is always directed opposite to the direction of motion. Therefore, the work done along path 1 is $W_1 = -F_{fr}\, d_{AB}$, where F_{fr} is the force of friction on the box and d_{AB} is the distance from *A* to *B*. The work done along path 2 is

$$W_2 = -F_{fr}\,(d_{AC} + d_{CD} + d_{DB})$$

where d_{AC}, d_{CD} and d_{DB} are the distances from *A* to *C*, from *C* to *D* and from *D* to *B*, respectively. W_1 is not the same as W_2 and so the work done by the force of friction depends on the path followed. Hence, the force of friction is **not** conservative. It is a **non-conservative** force.

The definition of a conservative force allows us to introduce the concept of a very useful function called the **potential energy**.

10.3 POTENTIAL ENERGY

You have learnt that the **work done by a conservative force** on a particle as it moves from one point to another **depends only on the starting and finishing positions, and not on the path between them**. Therefore, for a conservative force, we can write

$$\int_A^B \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} = \int_{\vec{\mathbf{r}}_A}^{\vec{\mathbf{r}}_B} \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} = \text{function of } (\vec{\mathbf{r}}_B) - \text{function of } (\vec{\mathbf{r}}_A)$$

We denote this function of the position of the particle by $U(\vec{\mathbf{r}})$ and write this equation as follows:

$$\int_{\vec{\mathbf{r}}_A}^{\vec{\mathbf{r}}_B} \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} = -U(\vec{\mathbf{r}}_B) + U(\vec{\mathbf{r}}_A) \qquad (10.1a)$$

Potential energy

or

$$U(\vec{\mathbf{r}}_B) - U(\vec{\mathbf{r}}_A) = -\int_{\vec{\mathbf{r}}_A}^{\vec{\mathbf{r}}_B} \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} \qquad (10.1b)$$

We call the function $U(\vec{\mathbf{r}})$ of the position of the particle as the **potential energy function** or simply the **potential energy** of the particle. The reason for the sign convention (the negative sign) in Eqs. (10.1a and b) will become clear to you in a little while. Note that we have not given any proof that this function $U(\vec{\mathbf{r}})$ exists.

However, we have seen at least three cases where the work done by a force does not depend on the path of the object. So we can say that $U(\vec{\mathbf{r}})$ exists for at least these forces. Note also that Eq. (10.1b) defines only the **difference in potential energy** of an object at any two positions $\vec{\mathbf{r}}_A$ and $\vec{\mathbf{r}}_B$. We could add a constant to $U(\vec{\mathbf{r}}_B)$ and the same constant to $U(\vec{\mathbf{r}}_A)$ and Eq. (10.1b) would still be satisfied (read the note in the margin). So because of the way we define potential energy, only the **differences in potential energy are meaningful**.

NOTE

If we redefine the potential energy at A $(U_A = a)$ to be

$U_A = a + c$

and the potential energy at B $(U_B = b)$ to be

$U_B = b + c$

where c is a constant that must be the same for all points, then the work done in going from A to B is

$-\Delta U = -[(b+c)-(a+c)]$

$= a - b$

which is the same as before. This means that you can set the zero of U anywhere you like. You might set it to be zero at the surface of the Earth or you might find it more convenient to set the zero at infinity.

POTENTIAL ENERGY

The **change in potential energy** ΔU of a particle between any two positions $\vec{\mathbf{r}}_B$ and $\vec{\mathbf{r}}_A$ is defined as the **negative of the work done** by a conservative force in moving the particle from $\vec{\mathbf{r}}_A$ to $\vec{\mathbf{r}}_B$:

$$\Delta U = U_B - U_A = U(\vec{\mathbf{r}}_B) - U(\vec{\mathbf{r}}_A) = -\int_{\vec{\mathbf{r}}_A}^{\vec{\mathbf{r}}_B} \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} \qquad (10.1b)$$

Let us now determine the potential energies (strictly speaking the difference between potential energies) associated with some conservative forces.

EXAMPLE 10.3: POTENTIAL ENERGY

Determine the potential energy function associated with the force of gravity and the force of gravitation.

SOLUTION ■ The **KEY IDEA** here is to apply Eq. (10.1b).

a) **Potential energy for the force of gravity**

In Example 9.7, we have calculated the work done by the force of gravity on a particle which moves from a point $P_1\ (x_1, y_1, z_1)$ to another point $P_2\ (x_2, y_2, z_2)$ as $W = \left(\int_{z_1}^{z_2} (-mg)\, dz \right) = -mg\,(z_2 - z_1)$

Using Eq. (10.1b), we can write

$$U_2 - U_1 = -\left(\int_{z_1}^{z_2} (-mg)\, dz \right) = mg(z_2 - z_1)$$

Let us adopt the convention that $U_1 = 0$ at the ground level where $z_1 = 0.$ Let the height of the particle above the ground be h. Then

$$U_2(h) = U = mgh$$

The potential energy $U = mgh$ due to work done by or against the force of gravity, with reference to ground, is called the **gravitational potential energy**.

b) **Potential energy for the force of gravitation**

In Example 9.8, we have determined the work done by the force of gravitation on an object that undergoes displacement from position $\vec{\mathbf{r}}_A$ to $\vec{\mathbf{r}}_B$ as

$$W = G m_1 m_2 \left(\frac{1}{r_B} - \frac{1}{r_A} \right)$$

Using Eq. (10.1b), we can write

$$U_B - U_A = -W = -G m_1 m_2 \left(\frac{1}{r_B} - \frac{1}{r_A} \right)$$

By convention, we choose the reference or starting position for this potential energy to be at infinity, i. e., in the above expression we take $r_A = \infty$ and put $r_B = r.$ Then we have

$$U_B = U(r) = -\frac{G m_1 m_2}{r}$$

The potential energy $U(r) = -\frac{G m_1 m_2}{r}$ due to work done by or against the force of gravitation in bringing an object from infinity to a position r is called the **gravitational potential energy**.

You may like to identify the potential energy function associated with the spring force yourself. Try SAQ 2.

SAQ 2 – Potential energy function for spring force

Identify the potential energy function for the spring force.

Potential energy is defined <u>only for conservative forces</u> and only the <u>DIFFERENCES</u> in potential energy are meaningful.

Let us now establish a connection between what you have learnt in school about potential energy and the mathematical concept you have studied so far. We shall use the example of the force of gravity.

Note that the factors that affect an object's potential energy due to gravity are its mass and its position relative to some reference position. Thus, with respect to ground, a book lying on a table has less gravitational potential energy than the same book lying on a higher table. It also has less gravitational potential energy than a heavier book lying on the same table.

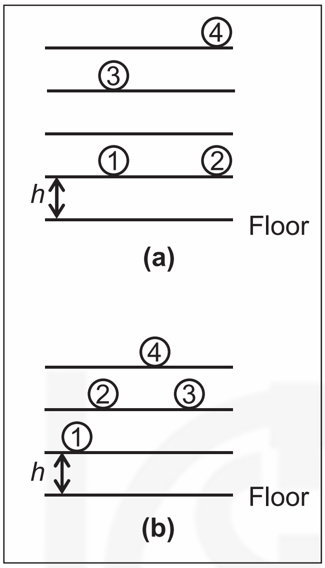


Fig. 10.6: **Potential energy due to the configuration of a system.**

Let us now consider a group of marbles, each of mass *m* at various heights on a rack. What is the total potential energy of the marbles if we pick them up from the floor and place them on the rack as shown in Fig. 10.6a? Let us take the distance of each shelf from the other and the lowest shelf from the floor to be *h*. Then the gravitational potential energy of these marbles with reference to the floor is

$$U = U_1 + U_2 + U_3 + U_4 = mgh + mgh + mg(3h) + mg(4h) = 9mgh$$

Let us now pick up the marbles from the floor and place them on the rack as shown in Fig. 10.6b. What is the total potential energy of the marbles in this case? With reference to the floor, it is

$$U = U_1 + U_2 + U_3 + U_4 = mgh + mg(2h) + mg(2h) + mg(3h) = 8mgh$$

This is different from the earlier result. Why? It is so because the relative positions of the marbles in both cases are different. We say that **the system of marbles has a different <u>CONFIGURATION</u> (different arrangement or distribution) in each case**. You can arrange these marbles differently to get different configurations. For each different configuration, you might find that the gravitational potential energy is different. You may also find that some configurations have the same energy even if they are different! But you will need to calculate the energy for each different configuration.

Thus, the potential energy of a system depends both on the position of the particle(s) in it and on its configuration (that is, the distribution or relative positions of the particles in it). We can thus define the potential energy of a system as follows:

POTENTIAL ENERGY

The **potential energy** of a particle or a system of particles is defined as the **negative of the work done** when it undergoes a **change** in its **position**, **shape** or **configuration**.

Potential energy represents the capacity (of a particle or a system of particles) to do work by virtue of the change in position, shape or its configuration in space.

We can also say that **the energy which a particle or a system possesses solely because of its position or configuration with respect to a reference level is called its potential energy**.

You may wonder: **What exactly is this potential energy**?

You can understand this as follows:

Consider an object such as a brick kept at some height above the ground that has some potential energy by virtue of its height. This is the negative of the work done on the brick while lifting it against the force of gravity. Now, if the brick falls on a pile of crockery, the brick can break it into pieces. That is, the brick ***can do work***. Thus, when we change the position or configuration of a body, it acquires potential energy, i.e., the ***potential to do work***. Hence, this kind of energy is given the name potential energy. So, **the potential energy may be thought of as energy acquired by an object due to a change in its shape, position or configuration, which has the potential of doing work**.

You may also have noted that we have called both $U = mgh$ and $U(r) = -\frac{G m_1 m_2}{r}$ as **gravitational potential energy.** $U(r) = -\frac{G m_1 m_2}{r}$ is the **gravitational potential energy** of a body of mass m_1 at any point at a distance r from the centre of a body of mass m_2. You have learnt in Example 10.2b that it is **equal to *the negative of the work done* in bringing the body** (of mass m_1) **from infinity** (∞) **to that point**. There might be some confusion in your mind about calling both $U = mgh$ and $U(r) = -\frac{G m_1 m_2}{r}$ **gravitational potential energy**. **These two are not different quantities**; it is a misconception if you think so.

You must understand that in calculating $U = mgh$, we assume constant (gravitational) force of attraction due to the Earth. This is a good approximation as long as h << radius of the Earth. If we were to take a body to a height of 5000 km, we would not be able to use the expression $U = mgh$ for the gravitational potential energy. The expression $U(r) = -\frac{G m_1 m_2}{r}$ is for the general case of any distance between any two masses and follows from the inverse square law. $U = mgh$ is a special case of $U(r) = -\frac{G m_1 m_2}{r}$.

NOTE

The gravitational potential energy

$$U(r) = -\frac{G m_1 m_2}{r}$$

is mutual to the masses m_1 and m_2: If we bring m_2 from infinity to a distance r from mass m_1, the gravitational potential energy acquired by m_2 is the same.

NOTE

The **zero of energy in the two situations is different**.

For $U = mgh$, it is taken to be the ground, that is, the surface of the Earth.

But for arriving at the expression

$$U(r) = -\frac{Gm_1 m_2}{r},$$

we take the zero or the reference level to be at infinity.

The **difference** (final potential energy minus initial potential energy) **is independent of the choice of origin**.

So far, we have defined conservative forces and introduced the concept of potential energy associated with such forces. Together these concepts lead us to the concept of **conservation of mechanical energy**. But you may like to first check whether you have understood the concept of potential energy.

SAQ 3 – Potential energy

A child (of mass M) climbs ten steps to the first floor situated at a height of 4.0 m from the ground. What is the gravitational potential energy of the child if we choose the reference point to be (i) the ground, (ii) the second step and (iii) the fifth step? What is the potential energy of the child with reference to the second floor, which is another 4.0 m from the first floor? Take $g = 9.8\,\text{ms}^{-2}$.

We can also use the concept of potential energy and its relationship with force to help us determine the **stability** of an object or a system.

10.3.1 Potential Energy and Stability

You have learnt in Unit 5 that an object is in equilibrium if the net force exerted on it is zero. So let us begin by using Eq. (10.1b) to obtain the force on a particle moving in a straight line from the position x_1 to x_2 (in terms of its potential energy). Its potential energy is given as

$$\Delta U = -\int_{x_1}^{x_2} F_x \, dx \qquad (10.2)$$

From calculus, it follows that F_x is just the derivative of $U(x)$:

$$F_x = -\frac{dU(x)}{dx} \quad \text{(one-dimension)} \qquad (10.3)$$

You can check this result by taking a concrete example. Recall the elastic potential energy of a spring moving along the x-axis given by $U(x) = \frac{1}{2}k\,x^2$.

Then from Eq. (10.3), we get

$$F_x = -\frac{dU(x)}{dx} = -\frac{d}{dx}\left(\frac{1}{2}k\,x^2\right) = -k\,x$$

This is the spring force given by Hooke's law. Eq. (10.3) is useful for computing the force from the potential energy function. It also helps us determine the **stability** of a system. For an object or system moving under conservative force, the condition for equilibrium using Eq. (10.3) becomes

$$F_x = -\frac{dU(x)}{dx} = 0 \quad \text{or} \quad \frac{dU(x)}{dx} = 0 \qquad (10.4)$$

Suppose that Eq. (10.4) is satisfied at some point x_0. We now state the conditions for **stable**, **unstable** and **neutral** equilibrium of an object/system:

- If at $x = x_0$, $\frac{d^2U}{dx^2} > 0$ (the **second derivative of** U **is positive**), the **object/system is in stable equilibrium at that point**;
- If at $x = x_0$, $\frac{d^2U}{dx^2} < 0$ (the **second derivative of** U **is negative**), the **object/system is in unstable equilibrium at that point**.
- If at $x = x_0$, $\frac{d^2U}{dx^2} = 0$, we must look at higher derivatives. If all derivatives vanish so that U is constant in a region about x_0, the **object/system is said to be in a condition of neutral stability or neutral equilibrium. No force results from a displacement; the object/system is effectively free**.

We show all three types of equilibria in Fig. 10.7: Fig. 10.7a shows stable equilibrium. A marble placed at the bottom of a bowl is an example of stable equilibrium. Fig. 10.7b shows unstable equilibrium. A marble balanced on top of a bowl is an example of unstable equilibrium. Fig. 10.7c shows neutral equilibrium. A marble placed on a horizontal tabletop is in the state of neutral equilibrium.

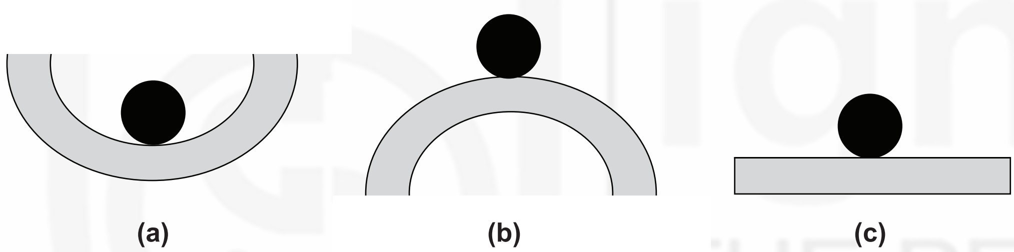


Fig. 10.7: Examples of a) stable; b) unstable; c) neutral equilibria.

So far you have learnt about two kinds of energy: kinetic energy and potential energy associated with conservative forces. We now combine these two concepts with the work-energy theorem discussed in Unit 7 to arrive at a very interesting result: the **conservation of mechanical energy**.

10.4 CONSERVATION OF MECHANICAL ENERGY

Recall the work-energy theorem given by Eq. (9.6b) or Eq. (9.7) as

$$W = \Delta \text{K.E.} = (\text{K.E.})_f - (\text{K.E.})_i = K_f - K_i \quad (10.5a)$$

Using Eq. (10.1b), we can also write

$$-W = \Delta U = U_f - U_i \quad (10.5b)$$

Combining Eqs. (10.5a) and (10.5b), we get

$$K_f - K_i = -(U_f - U_i)$$

Upon rearranging the terms, we can write

$$K_i + U_i = K_f + U_f \quad (10.6)$$

The left hand side of this equation depends on the initial speed of the particle and its potential energy at some initial position $\vec{\mathbf{r}}_i$; it does not depend on $\vec{\mathbf{r}}_f$. Similarly, the right hand side of the equation depends on the final speed and the potential energy at some final position $\vec{\mathbf{r}}_f$; it does not depend on $\vec{\mathbf{r}}_i$. Under such a situation, Eq. (10.6) can be satisfied only if each side of the equation is equal to a constant, since $\vec{\mathbf{r}}_i$ and $\vec{\mathbf{r}}_f$ can be any positions; these are not fixed. We denote this constant by E and write

$$K_i + U_i = K_f + U_f = E \tag{10.7a}$$

E is called the **total mechanical energy** of the particle. **It is the sum of the kinetic energy and potential energy of the particle:**

$$K + U = E \tag{10.7b}$$

Thus, you have learnt that

If the force exerted on a particle is conservative, the total mechanical energy of the particle is independent of the position of the particle: It remains constant.

In the language of physics, we say that

The **total mechanical energy of the particle is conserved**.

What this means is that the kinetic energy and potential energy of the particle (which moves under the action of a conservative force) may change but **their sum remains constant** at all times. Thus, as the kinetic energy of the particle increases, its potential energy will decrease and vice-versa. Its initial mechanical energy and final mechanical energy would always be equal:

$$E_i = E_f \tag{10.8}$$

NOTE

Eq. (10.8) also explains the nomenclature ***conservative*** force: When this kind of force acts on a system, the total mechanical energy of the system is **conserved**, i.e., it does not change with time.

Note that conservation of mechanical energy is a property of conservative forces. In fact the phrase conservative is attached to such forces only because of this reason. While arriving at the law of conservation of mechanical energy the reason for the particular sign convention we chose in defining potential energy would also have become clear to you. We are trying to express a fundamental law of nature in terms of mathematical and physical concepts. Let us now state this law formally.

Conservation of mechanical energy

CONSERVATION OF MECHANICAL ENERGY

If ONLY conservative forces do work on a particle or a system, the total mechanical energy (sum of kinetic energy and potential energy) of the particle or the system is conserved:

$$E_f = E_i \tag{10.8}$$

where $E_f = K_f + U_f$ and $E_i = K_i + U_i$

You can observe this law in action to a reasonably good approximation and the connection between potential and kinetic energy by doing the simple activity described below. This will also help you understand the relationship between kinetic energy and potential energy in mechanical systems. You will need a few balls or marbles of different masses (or any other object that can roll smoothly) and a smooth flat board for this activity.

Activity

KINETIC AND POTENTIAL ENERGY IN MECHANICAL SYSTEMS

Take table tennis balls or marbles of the same kind. Take a smooth (almost frictionless) flat board or a book having smooth hard cover. You could also use the playground slide for this activity.

Hold the board/book at an angle to make an inclined plane and roll a ball (or a marble) down its surface from some height. Note the horizontal distance travelled by the ball after it leaves the plane (see Fig. 10.8). Next hold the inclined plane at a different height but at the same angle with the horizontal as before. Thus, the ball has a different potential energy due to gravity. Again roll it down the plane from the same point and measure the horizontal distance travelled by it. Repeat the measurement for different heights of the plane.

Now answer the following questions based on this activity.

What do you infer when you compare the measurements of distance in each case? What horizontal distance does a ball rolling from a greater height travel? Is it more or less if the ball is rolled from a lesser height? What is the potential energy of the ball in each case?

What happens when you keep the inclined plane at the same height but use balls of different masses? Is the distance travelled by the balls same or different this time?

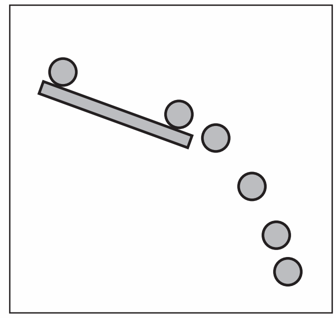


Fig. 10.8: **The connection between kinetic and potential energy.**

Do you now understand that greater initial height of the inclined plane means that the ball has more potential energy and hence greater capacity to do work? This is reflected in the greater change of the speed of the ball (since ΔK.E. is greater from work-energy theorem). Therefore, **a ball with more potential energy travels farther**.

But the **distance travelled is the same** if you keep the inclined plane at the same height. The result is independent of the mass of the object. It depends only on the height from which the mass is moved. Remember that friction should be minimised (by making sure that the plane is as smooth as possible) in this activity.

You can do a similar activity with a toy gun or a bow and arrow. When we insert a dart (small arrow) in the gun, we compress the spring which acquires elastic potential energy. If now we pull the trigger, the spring uncoils. The elastic potential energy of the spring gets converted into the K.E. of the dart. The dart rushes out to hit the target!

The same thing is happening in the case of a bow and arrow: the P.E. of the stretched string gets converted into the K.E. of the arrow.

Thus **for systems acted upon by conservative forces**,

When kinetic energy increases	***potential energy decreases***

and

When potential energy increases	***kinetic energy decreases***

such that the total mechanical energy of the system remains constant.

Let us now take up an example and apply the law of conservation of mechanical energy to solve problems on motion.

NOTE

Suppose, we were to attempt this question using Newton's laws of motion by assuming that only the force of gravity is acting on the object. We would need to integrate the equation of motion

$\vec{\mathbf{F}} = m\dfrac{d\vec{\mathbf{v}}}{dt}$ for $\vec{\mathbf{F}} = m\vec{\mathbf{g}}$

along the path of the object and solve it by applying the initial conditions. Imagine how complicated using Newton's laws would be for complicated paths such as a spiral path or a curved path, etc.

Example 10.4: THE ENERGY APPROACH TO MOTION

A stuntman wants to leap from one cliff to another on his motorcycle (Fig. 10.9). He drives horizontally off the cliff of height 100 m at a speed of $36.0\,\text{ms}^{-1}$. Determine the speed with which he lands on the other cliff of height 65.0 m. Neglect air resistance. Take $g = 10.0\ \text{ms}^{-2}$.

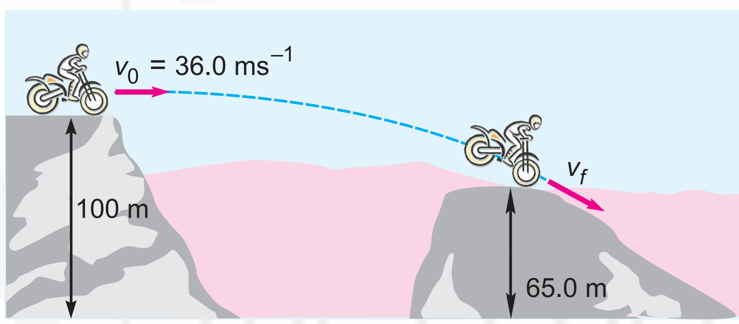


Fig. 10.9: The energy approach to motion.

SOLUTION ■ The **KEY IDEA** here is to apply the law of conservation of mechanical energy to determine the speed.

The potential energy is only due to the force of gravity.

We have $K_i + U_i = K_f + U_f$ or $\frac{1}{2}mv_i^2 + mgh_i = \frac{1}{2}mv_f^2 + mgh_f$

Hence, $v_f = \sqrt{v_i^2 + 2g(h_i - h_f)}$

$$v_f = \sqrt{(36.0\,\text{ms}^{-1})^2 + 2 \times (10.0\,\text{ms}^{-2})\,(100\,\text{m} - 65.0\,\text{m})} = 44.7\,\text{ms}^{-1}$$

SAQ 4 – Conservation of mechanical energy

A diver jumps with an initial speed of $1.5\ \text{ms}^{-1}$ and enters water at the speed of $4.5\ \text{ms}^{-1}$. What height did she jump from? Take $g = 10\,\text{ms}^{-2}$.

You have studied in Unit 7 that at any finite distance from the Earth, g is non-zero. So whatever be the distance of an object from the centre of the Earth, it would still feel the effect of gravity. However, an object can escape from the bounds of the gravitational attraction of the Earth if it is provided with a certain minimum velocity. This is called the **velocity of escape**. We now obtain its expression in Example 10.5.

EXAMPLE 10.5: VELOCITY OF ESCAPE

Determine the velocity of escape of a particle of mass m from a huge body of mass M.

SOLUTION ■ The **KEY IDEA** here is that the total energy of the object must be greater than or equal to zero: $E \geq 0$.

Let the particle be at a distance r from the huge body. The gravitational potential energy of the particle is given by $U = -\frac{GMm}{r}$.

If the particle is to escape from the bounds of the gravitational attraction of the huge body then its total energy $E \geq 0$. If at a distance r, the particle has velocity v, then the total mechanical energy of the particle is

$$E = \frac{1}{2}mv^2 + U = \frac{1}{2}mv^2 - \frac{GMm}{r}$$

So the condition for the particle's escape becomes

$$\frac{1}{2}mv^2 - \frac{GMm}{r} \geq 0 \Rightarrow v^2 \geq \frac{2GM}{r} \quad \text{or} \quad v \geq \sqrt{\frac{2GM}{r}}$$

Hence,

$$v_e = \sqrt{\frac{2GM}{r}} \qquad (10.9)$$

is the required minimum velocity for the particle to escape the gravitational field of the huge object. It is called the **velocity of escape**. Note that it is independent of m, the mass of the particle.

If the particle were originally on the surface of Earth, then $r = R_e$, $M = M_e$ and

$$v_e = \sqrt{\frac{2GM_e}{R_e}} = \sqrt{2gR_e} \qquad (10.10)$$

Now, taking $g = 9.8\,\text{ms}^{-2}$, we get $v_e = 1.1 \times 10^4\,\text{ms}^{-1} = 11\,\text{km s}^{-1}$ – a velocity that will take you from Srinagar to Kanyakumari in about five minutes!

SAQ 5 – Velocity of escape on the Moon

Calculate the velocity of escape on the surface of Moon. The mass and the radius of the Moon are 7.35×10^{22} kg and 1.74×10^6 m, respectively.

With this, we end the discussion on conservative forces, potential energy and conservation of mechanical energy as a tool for describing motion under conservative forces. In the next section, we arrive at the general law of conservation of energy by including the non-conservative forces in the discussion.

10.5 LAW OF CONSERVATION OF ENERGY

In Example 10.2, you have determined the work done by the force of friction along two different paths. What result did you arrive at? The **work done was different for both the paths**. Hence, as per the definition of conservative force, the **force of friction is not conservative** and we **<u>cannot</u>** associate a potential energy with it. **Such forces** (friction, air resistance, resistance due to water, etc.) **for which the work done depends on the path being followed by the particle are called non-conservative forces.**

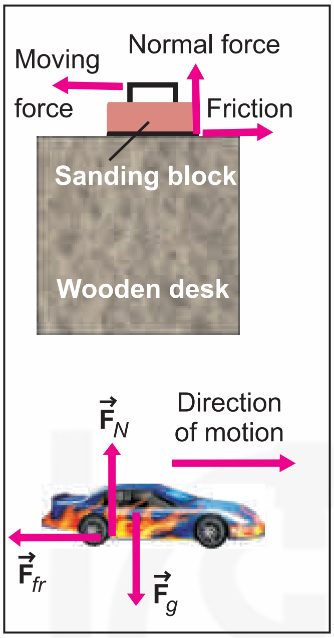


Fig. 10.10: **Forces of dry friction.**

When non-conservative forces are exerted on a particle, its mechanical energy decreases and the energy loss generally gets dissipated as heat. Non-conservative forces are, therefore, also referred to as **dissipative forces**.

We now briefly describe some **important non-conservative forces**.

- **<u>Force of dry friction</u>**

  You have already learnt about the **forces of friction between two solid surfaces in contact** when the surfaces are at rest relative to each other (force of static friction) and when they are in relative motion (force of kinetic friction). These forces always act opposite to the direction of motion of a given body (Fig. 10.10).

- **<u>Force of fluid friction</u>**

  The force of friction which is exerted when a solid object moves in a medium of gas or liquid is called the **force of <u>fluid</u> friction**. It acts between the surface of the solid and fluid in contact and depends on the relative velocity $\vec{\mathbf{v}}$ of the object in the fluid medium. Examples are the **drag force** exerted by the air (also called **air resistance**) when an object moves in air or the viscous force when an object moves in water or any other liquid (Fig. 10.11). For a wide range of values of *v*, the force of fluid friction is given by

$$F_{fr}(v) = C_1 v + C_2 v^2$$

  where C_1 and C_2 are constants for a given object in a given fluid. The force of fluid friction also always acts opposite to $\vec{\mathbf{v}}$.

Fig. 10.11: **Forces of fluid friction.**

- **<u>Forces of inelastic deformations</u>**

  Forces of inelastic deformation exerted on an object deform its surface beyond elastic limits (Fig. 10.12). In such cases, the mechanical energy is transformed into the internal energy of inelastic deformations.

Another well known example of inelastic deformation is that of a spring which is stretched or compressed beyond elastic limit. It does not restore to its original length when the external force of deformation is withdrawn. We say that the spring has undergone inelastic deformation. The forces causing such deformations are non-conservative and are not precisely defined. The work done by these forces can be estimated only indirectly, i.e., by finding the deficit in total mechanical energy.

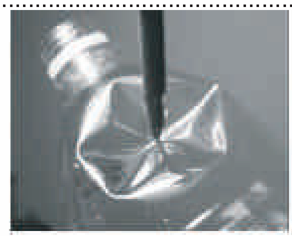


Fig. 10.12: **Force of inelastic deformation.**

In order to arrive at the general law of conservation of energy, we determine how the work-energy theorem gets modified for systems on which both conservative and non-conservative forces are exerted.

We write the total force on the system as the sum of the conservative forces $(\vec{\mathbf{F}}_c)$ and the non-conservative forces $(\vec{\mathbf{F}}_{nc})$:

$$\vec{\mathbf{F}} = \vec{\mathbf{F}}_c + (\vec{\mathbf{F}}_{nc}) \qquad (10.11)$$

From the work-energy theorem, which applies to all kinds of forces, we get the work done in moving a particle from point *A* to point *B* as:

$$W_{total} = \int_A^B \vec{\mathbf{F}}.d\vec{\mathbf{r}} = \int_A^B \vec{\mathbf{F}}_c.d\vec{\mathbf{r}} + \int_A^B \vec{\mathbf{F}}_{nc}.d\vec{\mathbf{r}} = \Delta \text{K.E.} \qquad (10.12)$$

You have learnt that the **work done by a conservative force can also be expressed in terms of the potential energy function** (recall Eq. 10.1b). Hence, we can write

$$W_{total} = -U_B + U_A + W_{nc}, \text{ where } W_{nc} = \int_A^B \vec{\mathbf{F}}_{nc}\,.\,d\vec{\mathbf{r}} \qquad (10.13)$$

Substituting Eq. (10.13) in Eq. (10.12), we get

$$W_{total} = -U_B + U_A + W_{nc} = K_B - K_A \qquad (10.14)$$

Rearranging the terms in Eq. (10.14), we can write

$$K_B + U_B - (K_A + U_A) = W_{nc} \qquad (10.15)$$

or $$E_B - E_A = \Delta E_{mechanical} = W_{nc} \qquad (10.16)$$

Work-energy theorem for non-conservative forces

where E_A and E_B are the total mechanical energies of the particle at point *A* and point *B*, respectively. Eq. (10.12) and Eqs. (10.13) to (10.16) represent the **work-energy theorem for forces which have a non-conservative component**. If the work done by non-conservative forces is zero, these equations give us the law of conservation of mechanical energy: $E_B = E_A$. Thus, we have enlarged the scope of the law of conservation of mechanical energy:

Mechanical energy is conserved for a particle on which non-conservative forces are exerted provided the work done by the non-conservative

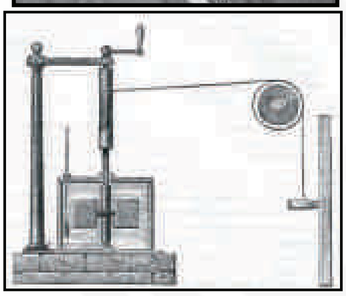
forces is zero. This point is reflected in the word **ONLY** in the statement of the **law of conservation of mechanical energy**.

Let us now use the general work-energy theorem to arrive at the law of conservation of energy.

The **law of conservation of energy** is an extension of Eq. (10.16) and the work-energy theorem given by Eqs. (9.7 and 9.16b). It is a very general law, which applies to all forms of energy in an isolated system. Let us explain what we mean by this statement.

You have studied so far that the **work done by a given force** can be expressed as a **change in some form of energy**. For example,

- Work done by the force of gravity or the force of gravitation is expressed as the **change in the gravitational potential energy**.
- Work done by the spring force in the elastic stretching of a spring is expressed as the **change in its elastic energy**.



James Prescott Joule (4 December 1818 – 11 October 1889) and the apparatus he used to study the nature of heat, and discover its relationship with mechanical work. His work led to the theory of conservation of energy and the development of the first law of thermodynamics. He also worked with Lord Kelvin to develop the absolute scale of temperature and found the relationship between the current through a resistance and the heat dissipated, now called Joule's law.

What about the **work done by** a **non-conservative force** such as **friction**? What form of energy is it associated with? Let us find out.

What do we observe when the force of friction is exerted between two surfaces? For example, what happens when a block slides across a rough table? We observe that the block and the table get warmer. Or when we rub two rough surfaces against each other, both of them get warm.

We say that *mechanical energy gets converted into heat due to the force of friction*. How do we arrive at this conclusion?

We owe this understanding to the British physicist **James Prescott Joule** who first demonstrated this and was the first to appreciate that heat itself represents a form of energy.

Joule conducted a series of carefully done experiments on the heating of water by a paddle wheel driven by a falling weight. He showed that the **loss of mechanical energy by friction is accompanied by the appearance of an equivalent amount of heat**. Thus, he concluded that **heat must be a form of energy** and that the **sum of the mechanical energy and the heat energy of a system is conserved**.

We can say that work done by the **force of friction** is expressed as **energy lost** (or *dissipated*) **as heat** into environment.

If both spring force and friction are exerted on an object/system, the net work done by the spring force and friction is expressed as the change in elastic energy as well as heat.

It has been known for more than one hundred years that there are many **different forms of energy**: mechanical energy, heat, gravitational energy, elastic energy, electrical energy, sound energy, chemical energy, radiant energy (due to light and other electromagnetic radiations), the energy of nuclear forces, and the energy associated with mass. And we have a formula or an expression for each one of them. It is clear now that the concept of energy is much wider than the simple idea of kinetic and potential energy of a mechanical system. And therefore, **the scope of the law of conservation of energy extends to all forms of energy in the universe and applies to an isolated system**.

You may ask: What do we mean by an **isolated system**?

In an isolated system, neither energy nor matter can be transferred to or from the system. This is the type of system for which the law of conservation of energy holds.

We define **the total energy** of an object to be the **sum of its mechanical energy** and **all other forms of energy** including thermal energy, chemical energy, radiant energy (due to light and other forms of electromagnetic waves), sound energy, etc. Then the **change in the total energy** is given by

$$\Delta E_{total} = \Delta E_{mechanical} + \Delta E_{other} \qquad (10.17)$$

LAW OF CONSERVATION OF ENERGY

1. **The CHANGE in the total energy of an ISOLATED SYSTEM is zero if all forms of energy are taken into account**:

$$\Delta E_{total} = \Delta E_{mechanical} + \Delta E_{other} = 0 \qquad (10.18)$$

2. This means that

$$E = \text{constant} \qquad (10.19)$$

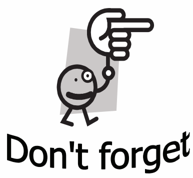
The TOTAL ENERGY of an ISOLATED SYSTEM is CONSERVED if all forms of energy are taken into account.

Conservation of energy

The law of conservation of energy applies to isolated systems, which means that we are neither taking out any energy or matter nor putting in any energy in any form in the system. So when we calculate the energy of an isolated system (to which we do not add any energy or matter nor do we take away any energy or matter from it), the total energy of the system always remains constant.

In isolated systems, the loss in one form of energy will show up as a gain in another form of energy. For example, a loss in kinetic energy due to the decrease in speed because of friction will show up as an increase in heat energy. Thus, **the total change in energy** (which is just the total of the change in various forms of energy) **of such a system will always be zero**. This is also expressed by saying that

Energy can neither be created nor destroyed. It changes or gets transformed from one form to another.



The law of conservation of energy is an extremely general law that applies to all possible situations and processes: those involving motion as well as thermal, chemical, electrical, nuclear, radiant energy, etc. *Motion is just one of the processes.* The law of conservation of energy is more meaningful in the context of a system of particles rather than a single particle and applies to an "isolated" system.

Let us now apply the law of conservation of energy to a physical situation.

EXAMPLE 10.6: LAW OF CONSERVATION OF ENERGY

A block attached to a spring slides horizontally on a surface, which has some friction. The block is stretched by the spring from rest by the distance $d = 0.02\,\text{m}$ and then released. What is the speed of the block at the point of equilibrium? The mass of the block is 1.0 kg and the spring constant is $500\ \text{Nm}^{-1}$. The coefficient of kinetic friction $\mu_k = 0.12$ and $g = 10\ \text{ms}^{-2}$.

SOLUTION ■ The **KEY IDEA** here is to choose the boundary of the system so that it is isolated and then apply the law of conservation of energy. We have to include the mechanical energy, elastic energy and thermal energy due to the non-conservative force of friction while determining the change in energy.

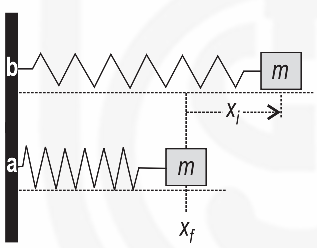


Fig. 10.13: **Diagram for Example 10.6.**

Refer to Fig. 10.13, which shows the spring-mass system. Applying Eq. (10.18), we have $\Delta E_{total} = \Delta E_{mechanical} + \Delta E_{other} = 0.$

Since the block is attached to the spring, its total mechanical energy is the sum of its kinetic energy, the gravitational potential energy and the elastic potential energy. Also since the block is moving horizontally, it remains at the same height and its gravitational potential energy does not change.

Therefore, $(\Delta\text{P.E.})_{grav} = 0$ and hence, the change in its mechanical energy from its initial to final position is given by

$$\Delta E_{mechanical} = \Delta\text{K.E.} + (\Delta\text{P.E.})_{elastic} = \frac{m}{2}\left(v_f^2 - v_i^2\right) + \frac{k}{2}\left(x_f^2 - x_i^2\right)$$

Since the block is released from rest, $v_i = 0$ and the block's position at equilibrium is $x_f = 0.$ Therefore,

$$\Delta E_{mechanical} = \frac{1}{2}\left(m v_f^2\right) - \frac{1}{2}\left(k\,x_i^2\right)$$

Now since the force of friction is the only non-conservative force, we have

$$\Delta E_{thermal} = -W_{fr}, \text{ where } W_{fr} = -(\mu_k\,mg)d \qquad (\because F_{fr} = \mu_k F_N = \mu_k mg)$$

$$\therefore \quad \Delta E_{total} = \frac{1}{2}\left(m v_f^2\right) - \frac{1}{2}\left(k\,x_i^2\right) + \mu_k\ mg\,d = 0$$

$$\therefore \quad v_f^2 = \left(\frac{k\,x_i^2}{m}\right) - 2\mu_k\,g\,d$$

Substituting the values given in the problem, we get $v_f = 0.39\ \text{ms}^{-1}$

We end this unit by highlighting the problem solving strategy using the energy approach. Fig. 10.14 tells you how to select proper energy equations.

Problem solving strategy using the energy approach

Conservation of mechanical energy:

REMEMBER: The law applies to particles or systems on which **ONLY** conservative forces do work.

1. Identify the **initial** and **final states** of the system characterized by positions and velocities. Use the subscript *i* or 1 for the initial state and *f* or 2 for the final state.
2. Choose the coordinate system for the problem and the reference level for the potential energy depending on the type of conservative force (gravity, spring force or gravitation, etc.).
3. List the initial and final kinetic and potential energies and identify which ones are known and can be calculated and which ones are unknown.
4. Substitute the known and unknown kinetic energies and potential energies (using Eq. 10.1b) in the law of conservation of mechanical energies (Eq. 10.8) and solve for the unknown quantities.

Conservation of energy:

REMEMBER: The law applies **ONLY to isolated systems**, that is, systems on which the net external force is zero and no energy and matter is transferred to or from the system.

1. Identify the system that you wish to consider, and determine what is *internal* to the system and what is *external*.
2. Identify the **initial** and **final states** of the system characterized by positions and velocities. Use the subscript *i* or 1 for the initial state and *f* or 2 for the final state.
3. Identify the non-conservative forces being exerted on the system and determine the energy associated with them.
4. Identify the known and unknown variables and use Eq. (10.18) to solve for the unknown variable.

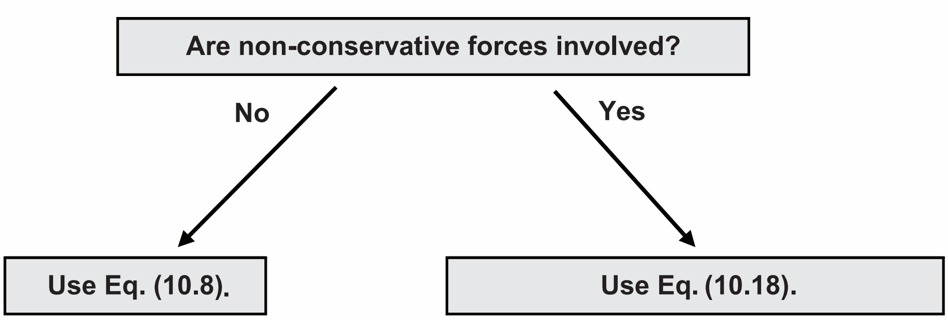


Fig. 10.14: How to select the energy equation suitable for a given problem?

We now summarise the contents of this unit.

10.6 SUMMARY

Concept	Description
Conservative force	■ A force is **conservative** when the work done by this force on a moving particle is independent of the path of the particle between the particle's initial and final positions: it **depends only on the initial and final positions** and is the same whatever be the path connecting the two points. $$\int_A^B \vec{\mathbf{F}} \,.\, d\vec{\mathbf{r}} = \int_{\vec{\mathbf{r}}_A}^{\vec{\mathbf{r}}_B} \vec{\mathbf{F}} \,.\, d\vec{\mathbf{r}} = \text{function of } (\vec{\mathbf{r}}_B) - \text{function of } (\vec{\mathbf{r}}_A)$$ A force is **conservative** when the **total work done by this force along any closed path** (which starts and finishes at the same point) **is zero**.
Potential energy	■ The **change in potential energy** ΔU of a particle between any two positions $\vec{\mathbf{r}}_B$ and $\vec{\mathbf{r}}_A$ is defined as the **negative of the work done** by a conservative force in moving the particle from $\vec{\mathbf{r}}_A$ to $\vec{\mathbf{r}}_B$: $$\Delta U = U_B - U_A = U(\vec{\mathbf{r}}_B) - U(\vec{\mathbf{r}}_A) = -\int_{\vec{\mathbf{r}}_A}^{\vec{\mathbf{r}}_B} \vec{\mathbf{F}} \,.\, d\vec{\mathbf{r}}$$ **Potential energy is defined only for conservative forces** and only the **difference** in potential energy is important. **The potential energy** of a particle or a system of particles is also **defined as the negative of the work done when it undergoes a change in its position, shape or configuration. It represents the capacity (of a particle or a system of particles) to do work by virtue of the position, shape or the configuration in space.**
Conservation of mechanical energy	■ **If work on a particle or a system is done only by conservative forces, the total mechanical energy (sum of kinetic energy and potential energy) of the particle or the system is conserved**: $$E_f = E_i, \qquad \text{where } E_f = K_f + U_f \text{ and } E_i = K_i + U_i$$
Force from potential energy	■ The **force** corresponding to the potential energy function $U(x)$ is given by $$F_x = -\frac{dU(x)}{dx}$$
Non-conservative force	■ A force that is **not conservative** is called a **non-conservative** force. For such forces (friction, air resistance, resistance due to water, etc.), **the work done depends on the path being followed by the particle**.

Work-energy theorem for forces which have a non-conservative component

- If the force being exerted on a particle has **a non-conservative component**, that is,

$$\vec{\mathbf{F}} = \vec{\mathbf{F}}_c + \vec{\mathbf{F}}_{nc}$$

then the **work-energy theorem** for the force can be expressed as:

$$W_{nc} = \Delta E_{mechanical}$$

where $$W_{nc} = \int_A^B \vec{\mathbf{F}}_{nc} \cdot d\vec{\mathbf{r}}$$

and $$\Delta E_{mechanical} = (K_f + U_f) - (K_i + U_i)$$

Law of conservation of energy

- **The change in the total energy of an isolated system is zero if all forms of energy are taken into account.**

$$\Delta E_{total} = \Delta E_{mechanical} + \Delta E_{thermal} + \Delta E_{other} = 0$$

that is,

$$E = \text{constant}$$

The total energy of an isolated system is conserved if all forms of energy are taken into account.

10.7 TERMINAL QUESTIONS

1. Identify the conservative and non-conservative forces being exerted in each of the following cases:

   a) A block attached to a spring executes vertical oscillations and air resistance is negligible.

   b) A bird dives vertically downwards in a lake to catch fish. Resistance due to water is finite.

   c) A motorcyclist travels uphill on a rough track against the direction of the wind.

   d) A box slides on a smooth slope with constant acceleration.

2. Suppose the total mechanical energy of an object is conserved. (a) If the kinetic energy decreases, what must be true about its potential energy? (b) If the potential energy decreases, what must be true about the kinetic energy? (c) If the kinetic energy does not change, what must be true about the potential energy?

3. Select the correct option from among the following and give reason for your choice. The sum of potential and kinetic energies of a freely falling body when air resistance is neglected

   a) decreases.

   b) increases.

   c) remains constant.

   d) approaches zero.

4. Select the correct option. A diver is diving from rest from the diving board, which is at a height of 15.0 m above the water. When she is 7.50 m above the surface of the water, her

   a) velocity is half of the velocity she will have when she touches water.

   b) kinetic energy and her potential energy with respect to the surface of the water are equal.

   c) kinetic energy and linear momentum are equal.

   d) linear momentum and potential energy are equal.

5. A ball of mass 100 g is thrown vertically downwards from rest from a height of 10 m. Determine its kinetic energy, gravitational potential energy and total mechanical energy at 10 m, 5.0 m and 0 m. Neglect air resistance and take $g = 10\text{ ms}^{-2}$.

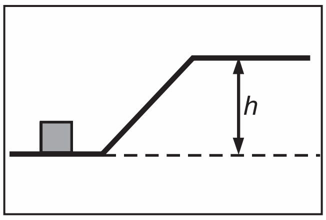


Fig. 10.15

6. A block of mass *m* slides up without friction to the top of a ramp of height *h* (see Fig. 10.15). What must be its initial minimum speed?

7. A labourer throws a brick (of weight 5.0 N) that leaves his hand at a distance of 4.0 m above the ground. (a) Determine the work done by the force of gravity when the brick has risen to a height of 4.5 m above the ground. Include the correct sign for the work. (b) Determine the change in the gravitational potential energy $(\Delta U = U_f - U_i)$ of the brick.

8. A box of mass 100 kg is moved down an inclined plane from rest at a height of 6.0 m and has a speed of 10 ms^{-1} when it reaches the bottom. Calculate the amount of heat energy produced. Use $g = 10\,\text{ms}^{-2}$.

9. A child of mass 25 kg slides from rest down a tree, a distance of 12 m. Her speed is $6.0\,\text{ms}^{-1}$ just before she hits the ground. What is the average force of friction acting on the child? Use $g = 10\,\text{ms}^{-2}$.

10. A wooden cube is pushed across a rough floor at constant speed by a horizontal force of 9.0 N. It moves a distance of 5.0 m. The temperature of the cube is monitored as it moves and it is found that its thermal energy increases by 25 J. What is the increase in the thermal energy of the floor along which the cube slides?

10.8 SOLUTIONS AND ANSWERS

Self-Assessment Questions

1. In Example 9.6, we have determined the work done by the spring force $(\vec{\mathbf{F}} = -k\,\vec{\mathbf{x}})$ in stretching a spring by displacement $\vec{\mathbf{x}}$ from x_1 to x_2 to be

$$W = W_{1 \to 2} = -\frac{k}{2}\left(x_2^2 - x_1^2\right)$$

   Now suppose we stretch the spring by displacement $\vec{\mathbf{x}}$ from x_1 to x_2 along a different path: we first compress the spring from x_1 to some position x_3 and then stretch it to another position x_4 before returning it to the final position x_2. The work done by the spring in this case would be

$$W = W_{1\to 3} + W_{3\to 1} + W_{1\to 2} + W_{2\to 4} + W_{4\to 2}$$

$$= \int_{x_1}^{x_3} F_x\,dx + \int_{x_3}^{x_1} F_x\,dx + \int_{x_1}^{x_2} F_x\,dx + \int_{x_2}^{x_4} F_x\,dx + \int_{x_4}^{x_2} F_x\,dx$$

Following the calculations of Example 9.6, you can see that

$$W_{1\to 3} = -W_{3\to 1} \text{ and } W_{2\to 4} = -W_{4\to 2} \text{ so that}$$

$$W = W_{1\to 2} = -\frac{k}{2}\left(x_2^2 - x_1^2\right)$$

Since the work done by the spring force does not depend on the path, we can say that **the ideal spring force is conservative**. You can also see that the spring force satisfies the alternative definition of the conservative force since

$$W_{1\to 1} = W_{1\to 2} + W_{2\to 1}$$

$$= W_{1\to 2} - W_{1\to 2} = 0$$

2. Since the work done by the spring force $(\vec{\mathbf{F}} = -k\,\vec{\mathbf{x}})$ in stretching a spring by displacement $\vec{\mathbf{x}}$ from x_1 to x_2 is

$$W = -\int_{x_1}^{x_2} k\,x\,dx = -\frac{k}{2}\left(x_2^2 - x_1^2\right)$$

from Eq. (10.1b), we can write $U_2 - U_1 = -W = \dfrac{k}{2}\left(x_2^2 - x_1^2\right)$

Once again if we choose the starting position to be $x_1 = 0$ and the finishing position to be at $x_2 = x$, we can write $U = \frac{1}{2}k\,x^2$. The potential energy $U = \frac{1}{2}k\,x^2$ due to work done by or against the spring force with reference to the equilibrium position at $x = 0$ is called the **elastic potential energy**.

3. Let $\hat{\mathbf{j}}$ be along the upward direction. The height of 10 steps is 4.0 m. So the height of each step is 0.4 m.

i) The height of the child from the ground is 4.0 m. So the child's gravitational potential energy with reference to the ground is

$$U = M\,\text{kg} \times 9.8\,\text{ms}^{-2} \times 4.0\,\text{m} = (39.2\,M)\,\text{J} \approx (39\,M)\,\text{J}$$

ii) The height of the second step from the ground is $(0.4\,\text{m}) \times 2 = 0.8\,\text{m}$. So the child's potential energy with reference to the second step is

$$U = M\,\text{kg} \times 9.8\,\text{ms}^{-2} \times (4.0\,\text{m} - 0.8\,\text{m}) = (31.36\,M)\,\text{J} \approx (31M)\,\text{J}$$

iii) The height of the fifth step from the ground is $(0.4\,\text{m}) \times 5 = 2.0\,\text{m}$. The child's potential energy with reference to the fifth step is

$$U = M\,\text{kg} \times 9.8\,\text{ms}^{-2} \times (4.0\,\text{m} - 2.0\,\text{m}) \;\; = (19.6\,M)\,\text{J} \;\approx (20\,M)\,\text{J}$$

iv) The potential energy of the child with reference to the second floor which is at a height of 4.0 m from the child, when he is on the first floor is,

$$U = M\,\text{kg} \times 9.8\,\text{ms}^{-2} \times (0 - 4.0\,\text{m}) \;=\; -\;(39.2\,M)\,\text{J} \;\approx -\,(39\,M)\,\text{J}$$

4. The total mechanical energy of the diver when she jumps from a height of h m is given by Eq. (10.4b),

$$E \;=\; K_i \;+ U_i \;=\; \frac{1}{2} m \;\times\; (1.5\,\text{ms}^{-1})^2 \;+\; m \;\times\; 10\;\text{ms}^{-2} \;\times\; h \qquad \text{(i)}$$

which is the sum of her kinetic energy and the potential energy. Here m is the mass of the diver. The final energy of the diver is just the kinetic energy, since the potential energy is zero.

$$\therefore \qquad K_f \;+\; U_f \;=\; \frac{1}{2} m \;\times\; (4.5\,\text{ms}^{-1})^2 \qquad \text{(ii)}$$

Since total mechanical energy is conserved, we equate Eqs. (i) and (ii).

$$\frac{1}{2} m \;\times\; (1.5\,\text{ms}^{-1})^2 + \; m \;\times\; 10\,\text{ms}^{-2} \;\times\; h = \frac{1}{2} m \;\times (4.5\,\text{ms}^{-1})^2$$

$$\therefore \;\; h \;=\; \frac{[(4.5\,\text{ms}^{-1})^2 \;-\; (1.5\,\text{ms}^{-1})^2\,]}{2 \;\times\; 10\;\text{ms}^{-2}} \;=\; 0.90\,\text{m}$$

5. The velocity of escape on Moon $v_{em} = \sqrt{\dfrac{2GM_m}{R_m}}$.

where $M_m =$ mass of the Moon and $R_m =$ radius of the Moon.

Putting the values of G, M_m and R_m, we get

$$v_{em} = 2.37 \times 10^3\;\text{ms}^{-1}$$

Terminal Questions

1. a) The forces in this case are gravity and the restoring force of the spring, both of which are conservatives forces.

   b) The force of gravitation on the bird is conservative. The resistive force due to water is non-conservative.

   c) The force of gravitation on the motorcyclist is conservative. The force of friction due to the rough track is non-conservative. Resistive force due to wind is also non-conservative.

   d) Since the slope is smooth, friction is absent. The only forces are the forces of gravitation and the normal force, which are conservative.

2. The total mechanical energy is the sum of the kinetic energy and the potential energy of the object. Therefore, if the total mechanical energy is conserved then,

a) If the kinetic energy decreases, the potential energy must increase.
b) If the potential energy decreases, the kinetic energy must increase.
c) If the kinetic energy does not change then the potential energy must remain constant as well.

3. The correct option is (c) because the only force acting on a freely falling body is the force of gravitation which is a conservative force. Therefore, the total mechanical energy which is a sum of the kinetic and potential energies is a constant.

4. The correct option is (b). Let *m* be the mass of the diver. At a height of 15.0 m above the water, her kinetic energy is zero since her speed is zero and the total energy is just the potential energy: *mgh* = (147*m*J). The potential energy at a height of 7.50 m is (73.5 *m*) J. Since the gravitational force is a conservative force, the total mechanical energy of the diver is constant. So at every stage of the motion of the diver, the total mechanical energy is (147 *m*) J. Since the potential energy at 7.50 m is (73.5 *m*) J, the kinetic energy is [(147 – 73.5) *m*] J = (73.5 *m*) J, which is equal to the potential energy.

5. At a height of 10 m, the gravitational potential energy of the ball with respect to the ground is

$$U = 0.10\text{ kg} \times 10\,\text{ms}^{-2} \times 10\,\text{m} = 10\,\text{J}$$

Since the ball is initially at rest, its initial kinetic energy is zero. From Eq. (10.7b), its total mechanical energy is $E = 10\,\text{J} + 0 = 10\,\text{J}$. At the height of 5.0 m, the gravitational potential energy is

$$U = 0.10\,\text{kg} \times 10\,\text{ms}^{-2} \times 5.0\,\text{m} = 5.0\,\text{J}$$

Since the force of gravitation is a conservative force, the total mechanical energy of the ball, which is 10 J, is conserved. Hence, the kinetic energy at 5.0 m is $K = 10\,\text{J} - 5.0\,\text{J} = 5.0\,\text{J}$. At the height of 0 m, the potential energy is zero, and the kinetic energy is equal to the total mechanical energy. It is 10 J.

6. At the start of its motion the mass has only a kinetic energy: $K_i = \frac{1}{2}mu^2$ where *u* is the initial speed of the mass. The initial potential energy $U_i = 0$. At the maximum height, the kinetic energy is zero and the potential energy is *mgh*. Using Eq. (10.8), we can write:

$$\frac{1}{2}mu^2 + 0 = 0 + mgh \quad \Rightarrow \quad u = \sqrt{2gh}$$

7. a) The work done by the labourer is $W = \int_{4.0\text{m}}^{4.5\text{m}} \vec{\mathbf{F}} \,.\, d\vec{\mathbf{r}}$ where $\vec{\mathbf{F}} = -5.0\text{N}\,\hat{\mathbf{j}}$ is the weight of the brick and $d\vec{\mathbf{r}} = dy\,\hat{\mathbf{j}}$ is the displacement. Here $\hat{\mathbf{j}}$ is the unit vector in the direction of the displacement.

$$W = -\int_{4.0\,\text{m}}^{4.5\,\text{m}} 5.0\,\text{N}\,dy = -\,5.0\,\text{N} \times (4.5\,\text{m} - 4.0\,\text{m}) = -\,2.5\,\text{J}$$

b) The change in the gravitational potential energy is the negative of the work done and is therefore 2.5 J.

8. We calculate the change in the total mechanical energy of the system using Eq. (10.16):

$$\Delta E_{mechanical} = \frac{1}{2} \times (100\,\text{kg}) \times (10\,\text{ms}^{-1})^2 - (100\,\text{kg}) \times (10\,\text{ms}^{-2}) \times 6.0\,\text{m}$$

$$= 5000\text{ J} - 6000\text{ J} = -1000\text{ J} = -1.0 \times 10^3\text{ J}$$

$\therefore$ From Eq. (10.18), the amount of heat energy produced is 1.0×10^3 J.

9. Once again we calculate the change in total mechanical energy

$$\Delta E_{mech} = \frac{1}{2} \times 25 \times (6.0\,\text{ms}^{-1})^2 - 25\,\text{kg} \times 10\,\text{ms}^{-2} \times 12\,\text{m}$$

$$= 450\text{ J} - 3000\text{ J} = -2550\text{ J}$$

The work done against the average force of friction $\vec{\mathbf{F}}_{fr}$ is 2550 J.
So $\vec{\mathbf{F}}_{fr} . \vec{\mathbf{d}} = 2550$ J where $\vec{\mathbf{F}}_{fr} = F_{fr}\,\hat{\mathbf{j}}$ and $\vec{\mathbf{d}} = (12\,\text{m})\,\hat{\mathbf{j}}$

and $$F_{fr} = \frac{2550}{12}\text{N} = 2.1 \times 10^2\text{ N}$$

10. Let the system comprise the cube and the floor and let the force be included in it. Then it is an isolated system. Let $\hat{\mathbf{i}}$ be along the direction of the force with which the cube is pushed. The work done by the force is

$$W = \int_0^{5.0\text{m}} 9.0\,\text{N}\,\hat{\mathbf{i}} \,.\, dx\,\hat{\mathbf{i}} = 45\text{ J}$$

Since the cube moves with constant velocity, the change in its kinetic energy is zero. The change in its mechanical energy equals the change in its potential energy, which is the negative of the work done by the force. Therefore, $\Delta E_{mech} = -45$ J. The change in the thermal energy of the cube is 25 J. Since the total energy of an isolated system is conserved, using Eq. (10.18), we can write,

$$-45\text{ J} + 25\text{ J} + \Delta E_{thermal}^{floor} = 0 \quad \Rightarrow \quad \Delta E_{thermal}^{floor} = 20\text{ J}$$

where $\Delta E_{\text{thermal}}^{\text{floor}}$ is the increase in the thermal energy of the floor.

FURTHER READINGS

1. **Mechanics** (**Berkeley Physics Course**, **Volume I**); C. Kittel, W. D. Knight, M. A. Ruderman, A. C. Helmholz and B. J. Moyer; McGraw Hill International Book Company (2017).

2. **Fundamentals of Physics**; D. Halliday, R. Resnick and J. Walker, Eighth Edition; Wiley India Ltd. (2008).

TABLE OF PHYSICAL CONSTANTS

Symbol	Quantity	Value
c	Speed of light in vacuum	$3.00 \times 10^{8}\ \text{ms}^{-1}$
μ_0	Permeability of free space	$1.26 \times 10^{-6}\ \text{NA}^{-2}$
ε_0	Permittivity of free space	$8.85 \times 10^{-12}\ \text{C}^2\,\text{N}^{-1}\,\text{m}^{-2}$
$1/4\pi\varepsilon_0$		$8.99 \times 10^{9}\ \text{Nm}^2\,\text{C}^{-2}$
e	Charge of the proton	$1.60 \times 10^{-19}\ \text{C}$
$-e$	Charge of the electron	$-1.60 \times 10^{-19}\ \text{C}$
h	Planck's constant	$6.63 \times 10^{-34}\ \text{Js}$
$\hbar$	$h/2\pi$	$1.05 \times 10^{-34}\ \text{Js}$
m_e	Electron rest mass	$9.11 \times 10^{-31}\ \text{kg}$
$-e/m_e$	Electron charge to mass ratio	$-1.76 \times 10^{11}\ \text{C kg}^{-1}$
m_p	Proton rest mass	$1.67 \times 10^{-27}\ \text{kg}$ (1 amu)
m_n	Neutron rest mass	$1.68 \times 10^{-27}\ \text{kg}$
a_0	Bohr radius	$5.29 \times 10^{-11}\ \text{m}$
N_A	Avogadro constant	$6.02 \times 10^{23}\ \text{mol}^{-1}$
R	Universal gas constant	$8.31\ \text{Jmol}^{-1}\text{K}^{-1}$
k_B	Boltzmann constant	$1.38 \times 10^{-23}\ \text{J K}^{-1}$
G	Universal gravitational constant	$6.67 \times 10^{-11}\ \text{Nm}^2\,\text{kg}^{-2}$

Astrophysical Data

Celestial Body	Mass (kg)	Mean radius (m)	Mean distance from the centre of Earth (m)
Sun	1.99×10^{30}	6.96×10^{8}	1.50×10^{11}
Moon	7.35×10^{22}	1.74×10^{6}	3.84×10^{8}
Earth	5.97×10^{24}	6.37×10^{6}	0

LIST OF BLOCKS AND UNITS: BPHCT-131

BLOCK 1:	**MATHEMATICAL PRELIMINARIES**
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Unit 2	Vector Algebra-II
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Unit 17	Superposition of Harmonic Oscillations
Unit 18	Damped Oscillations
Unit 19	Wave Motion

SYLLABUS: MECHANICS (BPHCT-131) 4 Credits

Vector Algebra: Geometrical and algebraic representation of vectors, Vector algebra; Scalar and vector products; Derivatives of a vector with respect to a scalar.

First Order Ordinary Differential Equations: First order homogeneous differential equations (separable and linear first order differential equations).

Second Order Ordinary Differential Equations: 2^{nd} order homogeneous differential equations with constant coefficients.

Laws of Motion: Frames of reference; Newton's Laws of motion; Straight line motion; Motion in a plane; Uniform circular motion; 3-d motion.

Applications of Newton's Laws of Motion: Friction; Tension; Gravitation; Spring-mass system – Hooke's law; Satellite in circular orbit and applications; Geosynchronous orbits; Basic idea of global positioning system (GPS); Weight and Weightlessness.

Linear Momentum and Impulse: Conservation of momentum; Impulse; impulse-momentum Theorem; Motion of rockets.

Work and Energy: Work and energy; Conservation of energy; Head-on and 2-d collisions.

Kinematics of Angular Motion: Kinematics of angular motion: Angular displacement, angular velocity and angular acceleration; General angular motion.

Dynamics of Rotational Motion: Torque; Rotational inertia; Kinetic energy of rotation; Angular momentum; Conservation of angular momentum and its applications.

Motion under Central Force Field: Motion of a particle in a central force field (motion in a plane, conservation of angular momentum; constancy of areal velocity); Kepler's Laws (statement only).

Dynamics of Many Particle Systems: Dynamics of a system of particles; Centre of Mass, determination of the centre of mass of discrete mass distributions, centre of mass of a rigid body (qualitative).

Conservation Laws: Linear momentum, angular momentum and energy conservation for many-particle systems.

Simple Harmonic Motion: Simple Harmonic Motion; Differential equation of SHM and its solutions; Kinetic Energy, Potential Energy, and Total Energy of SHM and their time averages.

Superposition of Harmonic Oscillations: Linearity and Superposition Principle; Superposition of Collinear Oscillations having equal frequencies and having different frequencies (beats); Superposition of Orthogonal Oscillations with equal and unequal frequency; Lissajous Figures and their uses.

Damped Oscillations: Equation of Motion of Damped Oscillations and its solution (without derivation); Qualitative description of the solution for Heavy, Critical and Weak Damping; Characterising Damped Oscillations – Logarithmic Decrement, Relaxation Time and Quality Factor.

Wave Motion: Qualitative Description (Wave formation and Propagation; Describing Wave Motion, Frequency, Wavelength and Velocity of Wave; Mathematical Description of Wave Motion).

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