CHAPTER 10

MECHANICAL PROPERTIES OF FLUIDS

10.1 WHAT IS A FLUID ?

1. *What are fluids ? Give their important characteristics.*

Fluid. *A fluid is a substance that can flow.* It ultimately assumes the shape of the containing vessel because it cannot withstand shearing stress. Thus, both liquids and gases are fluids.

Important characteristics of fluids :

(*i*) The atoms or molecules in a fluid are arranged in a random manner.

(*ii*) A fluid cannot withstand tangential or shearing stress for an indefinite period. It begins to flow when a shearing stress is applied.

(*iii*) A fluid has no definite shape of its own. It ultimately assumes the shape of the containing vessel. So a fluid has no modulus of rigidity.

(*iv*) A fluid can exert/withstand a force in a direction perpendicular to its surface. So a fluid does have a bulk modulus of rigidity.

2. *Both liquids and gases are fluids. What is the main difference between them ?*

Difference between liquid and gas. A liquid is incompressible and has a definite volume and a free surface of its own. A gas is compressible and it expands to occupy all the space available to it.

3. *Distinguish between the terms fluid statics and fluid dynamics.*

Fluid statics. *The branch of physics that deals with the study of fluids at rest is called fluid statics or hydrostatics.* Its study includes hydrostatic pressure, Pascal's law, Archimedes' principle, floatation of bodies and surface tension.

Fluid dynamics. *The branch of physics that deals with the study of fluids in motion is called fluid dynamics or hydrodynamics.* Its study includes equation of continuity, Bernoulli's theorem, Toricelli's theorem, viscosity, etc.

10.2 THRUST OF A LIQUID

4. *Define the term thrust. Give its SI unit.*

Thrust. A liquid in equilibrium has a fundamental property that it exerts a force on any surface in contact with it and this force acts perpendicular to the surface. *The total force exerted by a liquid on any surface in contact with it is called thrust.* It is because of this thrust that a liquid flows out through the holes of the containing vessel.

As thrust is a force, so its SI unit is newton (N).

5. *Show that a liquid at rest exerts force perpendicular to the surface of the container at every point.*

Liquid in equilibrium.

Consider a liquid contained in a vessel in the equilibrium state of rest. As shown in Fig. 10.1, suppose the liquid exerts a force F on the bottom surface in an inclined direction OA. The surface exerts an equal reaction R to water along OB.

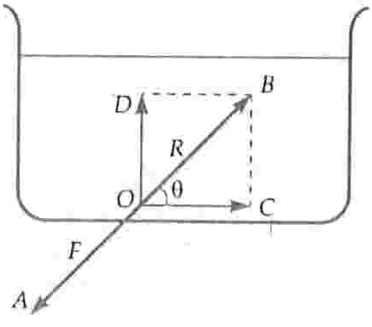


Fig. 10.1 Liquid in equilibrium.

The reaction R along OB has two rectangular components :

(*i*) Tangential component, $OC = R\cos\theta$

(*ii*) Normal component, $OD = R\sin\theta$

Since a liquid cannot resist any tangential force, so the liquid near O should begin to flow along OC. But the liquid is at rest, the force along OC must be zero.

$\therefore \quad R\cos\theta = 0$

As $R \neq 0$, so $\cos\theta = 0$ or $\theta = 90°$

Hence a *liquid always exerts force perpendicular to the surface of the container at every point.*

10.3 PRESSURE

6. *Define the term pressure. Is it a scalar or a vector ? Give its units and dimensions.*

Pressure. *The pressure at a point on a surface is the thrust acting normally per unit area around that point.* If a total force F acts normally over a flat area A, then the pressure is

$$P = \frac{F}{A}$$

If the force is not distributed uniformly over the given surface, then pressure will be different at different points. If a force ΔF acts normally on a small area ΔA surrounding a given point, then pressure at that point will be

$$P = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A} = \frac{dF}{dA}$$

Pressure is a scalar quantity, because fluid pressure at a particular point in fluid has same magnitude in all directions. This shows that a definite direction is not associated with fluid pressure. Though force is a vector quantity, only the magnitude of the normal component of the force appears in the above equations for pressure.

Units and dimensions of pressure :

SI unit of pressure = Nm^{-2} or Pascal (Pa)

CGS unit of pressure = dyne cm^{-2}

Dimensional formula of pressure is $[ML^{-1}T^{-2}]$.

7. *Briefly explain a method for measuring fluid pressure at any point inside the fluid.*

Measurement of fluid pressure. Fig. 10.2 shows a small pressure measuring device placed in a fluid-filled vessel. This pressure sensor consists of a small piston of area ΔA moving in a close fitting cylinder and resting against a spring. The cylinder is evacuated and the spring is calibrated to measure the force acting on the cylinder. The inward force exerted by the fluid on the piston is balanced by the outward spring force and is thereby measured. If ΔF is the magnitude of this normal force acting on the piston of area ΔA, the fluid pressure is

$$p = \frac{\Delta F}{\Delta A}$$

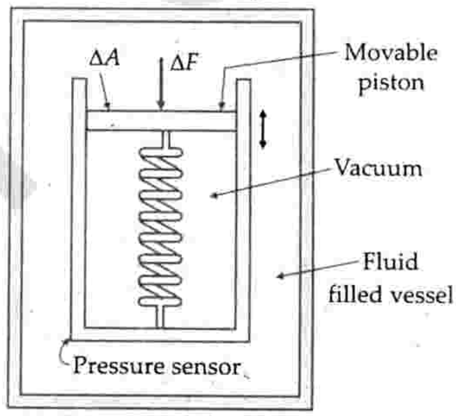


Fig. 10.2 Measurement of fluid pressure.

10.4 PRACTICAL APPLICATIONS OF PRESSURE

8. *Describe some practical applications from daily life which make use of the concept of pressure.*

Practical applications based on the concept of pressure :

(*i*) **A sharp knife cuts better than a blunt one.** The area of a sharp edge is much less than the area of a blunt edge. For the same total force, the effective force per unit area (or pressure) is more for the sharp edge than the blunt edge. Hence a sharp knife cuts better.

(*ii*) **Railway tracks are laid on wooden sleepers.** This spreads force due to the weight of the train on a larger area and hence reduces the pressure considerably. This, in turn, prevents the yielding of the ground under the weight of the train.

(*iii*) **It is difficult for a man to walk on sand while a camel walks easily on sand** inspite of the fact that a camel is much heavier than a man. This is because camel's feet have a larger area than the feet of man. Due to larger area, pressure is less.

(*iv*) **Pins and nails are made to have pointed ends.** Their pointed ends have very small area. When a force is applied over head of a pin or a nail, it transmits a large pressure (= force/area) on the surface and hence easily penetrate the surface.

Examples based on Thrust and Pressure

FORMULAE USED

1. Thrust = Total force exerted by a liquid on the surface in contact
2. Pressure $= \dfrac{\text{Thrust}}{\text{Area}}$ or $P = \dfrac{F}{A}$

UNITS USED

Thrust is in newton and pressure in Nm^{-2} or pascal (Pa).

EXAMPLE 1. *The two thigh bones (femurs), each of cross-sectional area 10 cm^2 support the upper part of a human body of mass 40 kg. Estimate the average pressure sustained by the femurs. Take $g = 10\ ms^{-2}$.* **[NCERT]**

Solution. Total cross-sectional area of the femurs,

$$A = 2 \times 10\ cm^2 = 20 \times 10^{-4}\ m^2$$

Force acting vertically downwards and hence normally on femurs,

$$F = mg = 40 \times 10 = 400\ N$$

$$\therefore \quad P_{av} = \frac{F}{A} = \frac{400}{20 \times 10^{-4}} = \mathbf{2 \times 10^5\ Nm^{-2}}.$$

EXAMPLE 2. *How much pressure will a man of weight 80 kgf exert on the ground when (i) he is lying and (ii) he is standing on his feet ? Given that the area of the body of the man is 0.6 m^2 and that of a foot is 80 cm^2.*

Solution. Force, $F = 80\ kgf = 80 \times 9.8\ N$

(*i*) When the man is lying on the ground,

$$A = 0.6\ m^2$$

$$\therefore \quad P = \frac{F}{A} = \frac{80 \times 9.8}{0.6} = \mathbf{1.307 \times 10^3\ Nm^{-2}}.$$

(*ii*) When the man is standing on his feet,

$$A = 2 \times 80\ cm^2 = 160 \times 10^{-4}\ m^2$$

$$P = \frac{80 \times 9.8}{160 \times 10^{-4}} = \mathbf{4.9 \times 10^4\ Nm^{-2}}.$$

PROBLEMS FOR PRACTICE

1. Find the pressure exerted at the tip of a drawing pin if it is pushed against a board with a force of 20 N. Assume the area of the tip to be 0.1 mm^2. **(Ans.** 2×10^8 Pa)
2. Atmospheric pressure is nearly 100 kPa. How large the force does the air in a room exert on the inside of a window pan that is 40 cm × 80 cm ? **(Ans.** 32 kN)
3. The force on a phonograph needle is 1.2 N. The point has a circular cross-section whose radius is 0.1 mm. Find the pressure (in atm) it exerts on the records. Given 1 atm = 1.013×10^5 Pa. **(Ans.** 377 atm)
4. A cylindrical vessel containing liquid is closed by a smooth piston of mass m. The area of cross-section of the piston is A. If the atmospheric pressure is P_0, find the pressure of the liquid just below the piston. **(Ans.** $P_0 + mg / A$)

HINTS

1. $P = \dfrac{F}{A} = \dfrac{20\ N}{0.1\ mm^2} = \dfrac{20\ N}{0.1 \times 10^{-6}\ m^2} = 2 \times 10^8$ **Pa.**

2. $P = 100\ kPa = 10^5\ Pa,$

$A = 40\ cm \times 80\ cm = 40 \times 80 \times 10^{-4}\ m^2$

$F = P \times A = 10^5 \times 40 \times 80 \times 10^{-4}$

$= 32 \times 10^3\ N =$ **32 kN.**

3. $P = \dfrac{F}{A} = \dfrac{F}{\pi r^2} = \dfrac{1.2}{3.14 \times (10^{-4})^2}$ Pa

$= \dfrac{1.2}{3.14 \times 10^{-8} \times 1.013 \times 10^5}$ atm. $= 377$ **atm.**

10.5 DENSITY

9. *Define the term density. Give its units and dimensions.*

Density. *The density of any material is defined as its mass per unit volume.* If a body of mass M occupies volume V, then its density is

$$\rho = \frac{M}{V} \quad i.e., \quad \text{Density} = \frac{\text{Mass}}{\text{Volume}}.$$

Density is a positive scalar quantity. As liquids are incompressible, their density remains constant at all pressures. Density of a gas varies largely with pressure.

Units and dimensions of density :

SI unit of density = kg m^{-3}

CGS unit of density = g cm^{-3}

Dimensional formula of density is $[ML^{-3}]$.

10. *What do you mean by specific gravity or relative density of a substance ?*

Specific gravity or relative density. *The specific gravity or relative density of a substance is defined as the ratio of the density of the substance to the density of water at* 4°C. The density of water at 4°C is 1.0×10^3 kg m^{-3}.

$$\text{Specific gravity} = \frac{\text{Density of substance}}{\text{Density of water at } 4°C}.$$

Specific gravity is a dimensionless positive scalar quantity. Clearly,

Density of a substance

= Specific gravity × Density of water at 4°C

Table 10.1 *Densities of some common fluids at STP*

Fluid	Density (kg m^{-3})
Water	1.00×10^3
Sea water	1.03×10^3
Mercury	13.6×10^3
Ethyl alcohol	0.806×10^3
Whole blood	1.06×10^3
Air	1.29
Oxygen	1.43
Hydrogen	9.0×10^{-2}
Intersteller space	$\approx 10^{-20}$

10.6 PASCAL'S LAW

11. *State and prove Pascal's law of transmission of fluid pressure.*

Pascal's law. This law tells as how pressure can be transmitted in a fluid. It can be stated in a number of equivalent ways as follows :

(*i*) The *pressure exerted at any point on an enclosed liquid is transmitted equally in all directions.*

(*ii*) *A change in pressure applied to an enclosed incompressible fluid is transmitted undiminished to every point of the fluid and the walls of the containing vessel.*

(*iii*) The *pressure in a fluid at rest is same at all points if we ignore gravity.*

Proof of Pascal's law. Pascal law can be proved by using two principles :

(*i*) *The force on any layer of a fluid at rest is normal to the layer* and

(*ii*) *Newton's first law of motion.*

As shown in Fig. 10.3, consider a small element *ABC – DEF* in the form of a right angled prism in the interior of a fluid at rest. The element is so small that all its parts can be assumed to be at same depth from the liquid surface and, therefore, the effect of gravity is same for all of its points. Suppose the fluid exerts pressure P_a, P_b and P_c on the faces *BEFC, ADFC* and *ADEB* respectively of the this element and the corresponding normal forces on these faces are F_a, F_b and F_c. Let A_a, A_b and A_c be the respective areas of the three faces. In right ΔABC, let $\angle ACB = \theta$.

As the prismatic element is in equilibrium with remaining fluid, by Newton's law, the fluid force should balance in various directions.

Along horizontal direction, $F_b \sin\theta = F_c$

Along vertical direction, $F_b \cos\theta = F_a$

From the geometry of the figure, we get

$$A_b \sin\theta = A_c$$

and $$A_b \cos\theta = A_a$$

From the above equations, we get

$$\frac{F_b \sin\theta}{A_b \sin\theta} = \frac{F_c}{A_c}$$

and $$\frac{F_b \cos\theta}{A_b \cos\theta} = \frac{F_a}{A_a}$$

$$\therefore \quad \frac{F_a}{A_a} = \frac{F_b}{A_b} = \frac{F_c}{A_c}$$

or $$P_a = P_b = P_c$$

Hence, pressure exerted is same in all directions in a fluid at rest. This proves Pascal's law of transmission of fluid pressure.

The above discussion again shows that pressure is not a vector quantity. No direction can be assigned to it.

12. *How will you experimentally verify the Pascal's law of transmission of fluid pressure ?*

Experimental verification of Pascal's law. As shown in Fig. 10.4, take a vessel having three openings

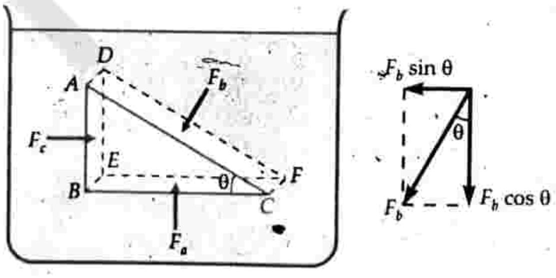


Fig. 10.3 Proof of Pascal's law.

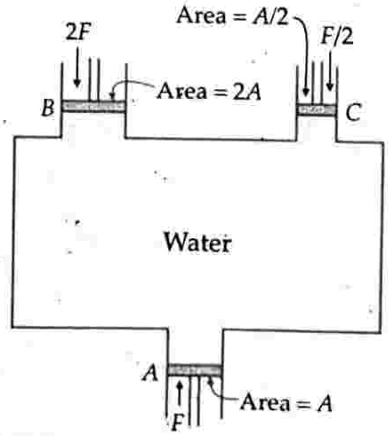


Fig. 10.4 Experimental verification of Pascal's law.

A, B and C and provided with frictionless and water tight pistons. Let their cross-sectional areas be $A, 2A$ and $\frac{A}{2}$ respectively. Fill the vessel with water and apply an additional force F on piston A. To keep the pistons B and C in their positions, forces equal to $2F$ and $\frac{F}{2}$ respectively have to be applied on them. This shows that the pressure P is transmitted equally in all directions because

$$P = \frac{F}{A} = \frac{2F}{2A} = \frac{F/2}{A/2}.$$

10.7 APPLICATIONS OF PASCAL'S LAW

13. *Explain how is Pascal's law applied in a hydraulic lift.*

Hydraulic lift. Hydraulic lift is an application of Pascal's law. It is used to lift heavy objects. It is a force multiplier.

It consists of two cylinders C_1 and C_2 connected to each other by a pipe. The cylinders are fitted with water-tight frictionless pistons of different cross-sectional areas. The cylinders and the pipe contain a liquid. Suppose a force f is applied on the smaller piston of cross-sectional area a. Then

Pressure exerted on the liquid,

$$P = \frac{f}{a}$$

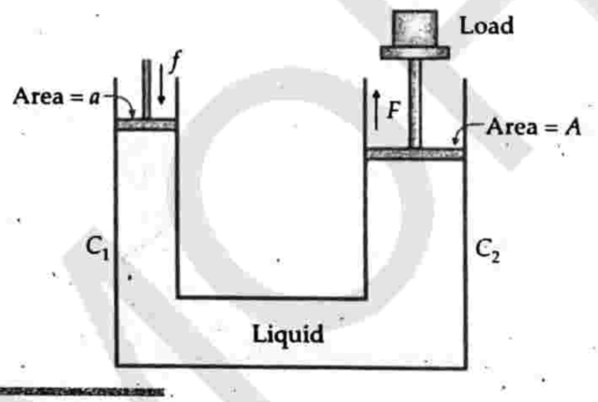


Fig. 10.5 Hydraulic lift.

According to Pascal's law, the same pressure P is also transmitted to the larger piston of cross-sectional area A.

$\therefore$ Force on larger piston is

$$F = P \times A = \frac{f}{a} \times A = \frac{A}{a} \times f$$

As $A > a$, therefore, $F > f$.

Hence by making the ratio A/a large, very heavy loads (like cars and trucks) can be lifted by the application of a small force. However, there is no gain of work. The work done by force f is equal to the work done by F. The piston P_1 has to be moved down by a larger distance compared to the distance moved up by piston P_2.

14. *With the help of a labelled diagram, explain the working of hydraulic brakes.*

Hydraulic brakes. The hydraulic brakes used in automobiles are based on Pascal's law of transmission of pressure in a liquid.

Construction. As shown in Fig. 10.6, a hydraulic brake consists of a tube T containing brake oil. One end of this tube is connected to a master cylinder fitted with piston P. The piston P is attached to the brake pedal through a lever system. The other end of the tube is connected to the wheel cylinder having two pistons P_1 and P_2. The pistons P_1 and P_2 are connected to the brake shoes S_1 and S_2 respectively. The area of cross-section of the wheel cylinder is larger than that of master cylinder.

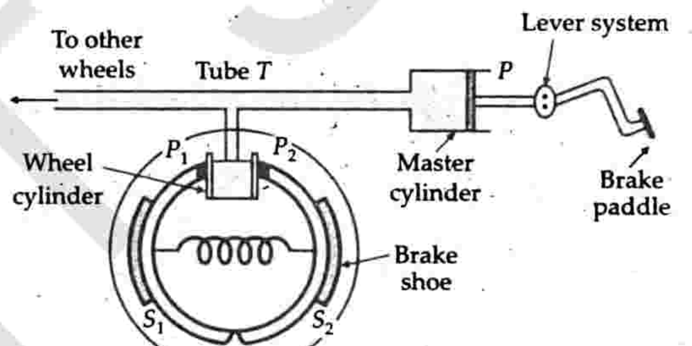


Fig. 10.6 Hydraulic brakes.

Working. When the pedal is pressed, its lever system pushes the piston P into the master cylinder. The pressure is transmitted through the oil to the pistons P_1 and P_2 in the wheel cylinder, in accordance with Pascal's law. The pistons P_1 and P_2 are pushed outwards. The brake shoes get pressed against the inner rim of the wheel, retarding the motion of the wheel. As the cross-sectional area of wheel cylinder is larger than that of master cylinder, a small force applied to the pedal produces a large retarding force.

When the paddle is released, a spring pulls the brake shoes away from the rim. The pistons in both cylinders move towards their normal positions and the oil is forced back into the master cylinder.

Advantages of hydraulic brakes :

(*i*) The master cylinder transmits equal retarding force on each wheel. So a hydraulic brake operates uniformly and hence prevents skidding.

(*ii*) A small force applied to the pedal exerts a much larger force on the wheel drums. It enables the driver to keep the vehicle under control.

Examples based on Pascal's Law and Hydraulic Lift

FORMULAE USED

1. According to Pascal's law, pressure applied at any point of an enclosed mass of fluid is transmitted equally in all directions.
2. For a hydraulic lift, $P = \frac{f}{a} = \frac{F}{A}$.

UNITS USED

Forces f and F are in newton, area of cross-sections a and A in m^2.

EXAMPLE 3. *In a car lift compressed air exerts a force F_1 on a small piston having a radius of 5 cm. This pressure is transmitted to a second piston of radius 15 cm. If the mass of the car to be lifted is 1350 kg, what is F_1 ? What is the pressure necessary to accomplish this task ? Take $g = 9.81\ ms^{-2}$.* **[NCERT ; Central Schools 08]**

Solution. Here $r_1 = 5$ cm, $r_2 = 15$ cm,

$$F_2 = mg = 1350 \times 9.81 \text{ N}$$

As the pressure through air is transmitted equally in all directions, so

$$\frac{F_1}{A_1} = \frac{F}{A_2} \quad \text{or} \quad \frac{F_1}{\pi r_1^2} = \frac{F_2}{\pi r_2^2}$$

or
$$F_1 = F_2 \times \frac{r_1^2}{r_2^2} = 1350 \times 9.81 \times \frac{5 \times 5}{15 \times 15}$$

$$= \mathbf{1.5 \times 10^3\ N.}$$

Required air pressure,

$$P = \frac{F_1}{A_1} = \frac{F_1}{\pi r_1^2} = \frac{1.5 \times 10^3 \text{ N}}{3.14 \times (5 \times 10^{-2} \text{ m})^2} = \mathbf{1.9 \times 10^5\ Pa.}$$

EXAMPLE 4. *The neck and bottom of a bottle are 2 cm and 10 cm in diameter respectively. If the cork is pressed with a force of 1.2 kg f in the neck of the bottle, calculate the force exerted on the bottom of the bottle.*

Solution. Here $2r = 2$ cm or $r = 1$ cm

and $2R = 10$ cm or $R = 5$ cm

$$a = \pi r^2 \text{ and } A = \pi R^2$$

$$f = 1.2 \text{ kg f} = 1.2 \times 9.8 \text{ N}$$

$$F = \frac{f}{a} \times A = \frac{1.2 \times 9.8}{\pi r^2} \times \pi R^2$$

$$= 1.2 \times 9.8 \times \frac{R^2}{r^2}$$

$$= 1.2 \times 9.8 \times \frac{25}{1} \text{ N} = 1.2 \times 25 \text{ kgf} = \mathbf{30\ kg\ f.}$$

EXAMPLE 5. *Two syringes of different cross-sections (without needles) filled with water are connected with a tightly fitted rubber tube filled with water. Diameters of the smaller piston and larger piston are 1.0 cm and 3.0 cm respectively. (a) Find the force exerted on the larger piston when a force of 10 N is applied to the smaller piston. (b) If the smaller piston is pushed in through 6.0 cm, how much does the larger piston move out ?* **[NCERT ; Chandigarh 07]**

Solution. (*a*) As the pressure is transmitted undiminished through water, so

$$\frac{F_1}{A_1} = \frac{F_2}{A_2} \quad \text{or} \quad \frac{F_1}{\pi r_1^2} = \frac{F_2}{\pi r_2^2}$$

$$\therefore \quad F_2 = \frac{r_2^2}{r_1^2} F_1 = \frac{\left(\frac{3}{2} \times 10^{-2}\right)^2}{\left(\frac{1}{2} \times 10^{-2}\right)^2} \times 10 = \mathbf{90\ N.}$$

(*b*) As water is incompressible, so

Volume covered by inward movement of smaller piston
= Volume covered by outward movement of larger piston

or
$$L_1 A_1 = L_2 A_2$$

or
$$L_1 \times \pi r_1^2 = L_2 \times \pi r_2^2$$

or
$$L_2 = \frac{r_1^2}{r_2^2} L_1 = \frac{\left(\frac{1}{2} \times 10^{-2}\right)^2}{\left(\frac{3}{2} \times 10^{-2}\right)^2} \times 6.0 \times 10^{-2}$$

$$= 0.67 \times 10^{-2} \text{ m} = \mathbf{0.67\ cm.}$$

EXAMPLE 6. *Two pistons of hydraulic press have diameters of 30.0 cm and 2.5 cm. What is force exerted by larger piston, when 50.0 kg wt. is placed on the smaller piston ? If the stroke of the smaller piston is 4.0 cm, through what distance will the larger piston move after 10 strokes ?*

Solution. Here $2r = 2.5$ cm, $r = 1.25$ cm

and $2R = 30.0$ cm, $R = 15.0$ cm

$$f = 50.0 \text{ kg wt}, \quad F = ?$$

As
$$\frac{f}{a} = \frac{F}{A} \quad \text{or} \quad \frac{f}{\pi r^2} = \frac{F}{\pi R^2}$$

$$\therefore \quad F = f \times \frac{R^2}{r^2} = 50 \times \left(\frac{15.0}{1.25}\right)^2$$

$$= 50 \times 144 = 7200 \text{ kg wt}$$

Also $f \times l = F \times L$

$$\therefore \quad L = \frac{f \times l}{F} = \frac{50 \times 4.0}{7200} = 0.028 \text{ cm}$$

Distance through which larger piston moves in 10 strokes

$$= 10 \times 0.028 = \mathbf{0.28\ cm.}$$

✖ PROBLEMS FOR PRACTICE

1. The area of the smaller piston of a hydraulic press is 1 cm and that of larger piston is 22 cm^2. How much weight can be raised on the larger piston by a 200 kg f exerted on the smaller piston ?

   (**Ans.** 4400 kg)

2. The average mass that must be lifted by a hydraulic press is 80 kg. If the radius of the larger piston is five times that of the smaller piston, what is the minimum force that must be applied ? (**Ans.** 31.4 N)

3. In a hydraulic press used for compressing cotton, the area of the piston is 0.1 m^2 and the force exerted along the piston rod is 200 N. If the area of the larger cylinder is 0.8 m^2, find the pressure produced in the cylinder and the total crushing force exerted on the bale of cotton. (**Ans.** 2000 Nm^{-2}, 1600 N)

4. An automobile back is lifted by a hydraulic jack that consists of two pistons. The large piston is 1 m in diameter and the small piston is 10 cm in diameter. If W be the weight of the car, how much smaller a force is needed on the small piston to lift the car ? (**Ans.** 1% of the weight of the car)

✖ HINTS

1. Here $a = 1\,cm^2 = 10^{-4}\,m^2$,

$$A = 22\,cm^2 = 22 \times 10^{-4}\,m^2,$$
$$f = 200\text{ kg f} = 200 \times 9.8\text{ N}$$

Suppose m mass can be raised on the larger piston. Then

$$F = m \times 9.8\text{ newton}$$

As $\dfrac{f}{a} = \dfrac{F}{A}$ $\therefore$ $\dfrac{200 \times 9.8}{10^{-4}} = \dfrac{m \times 9.8}{22 \times 10^{-4}}$

or $m = 200 \times 22 =$ **4400 kg.**

4. As $\dfrac{f}{\pi r^2} = \dfrac{F}{\pi R^2}$

$$\therefore \quad f = F \times \left(\frac{r}{R}\right)^2 = W \times \left(\frac{0.05}{0.5}\right)^2$$
$$= 0.01\,W = 1\% \text{ of the weight of the car.}$$

10.8 PRESSURE EXERTED BY A LIQUID COLUMN

15. *Derive an expression for the pressure exerted by a liquid column of height h.*

Pressure exerted by a liquid column. Consider a vessel of height h and cross-sectional area A filled with a liquid of density ρ. The weight of the liquid column exerts a downward thrust on the bottom of the vessel and the liquid exerts pressure.

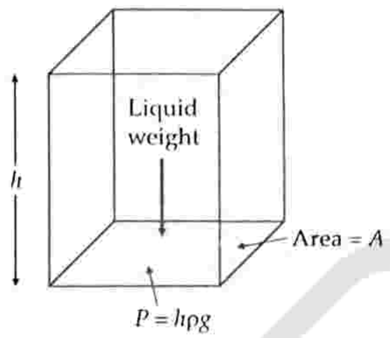


Fig. 10.7 Pressure exerted by a liquid column.

Weight of liquid column,

$$W = \text{Mass of liquid} \times g$$
$$= \text{Volume} \times \text{density} \times g$$
$$= Ah \times \rho \times g = Ah\rho g$$

Pressure exerted by the liquid column on the bottom of the vessel is

$$P = \frac{\text{Thrust}}{\text{Area}} = \frac{W}{A} = \frac{Ah\rho g}{A}$$

or $$P = h\rho g$$

Thus the pressure exerted by a liquid column at rest is proportional to (*i*) height of the liquid column and (*ii*) density of the liquid.

10.9 EFFECT OF GRAVITY ON FLUID PRESSURE

16. *Discuss the variation of fluid pressure with depth. Hence explain how is Pascal's law affected in the presence of gravity.*

Variation of liquid pressure with depth. As shown in Fig. 10.8, consider a liquid at rest in a container. The liquid pressure must be same at all points which are at the same depth, as otherwise liquid will not be in equilibrium. Imagine a cylindrical element of the liquid of cross-sectional area A and height h. Let P_1 and P_2 be the liquid pressures at its top point 1 and bottom point 2 respectively.

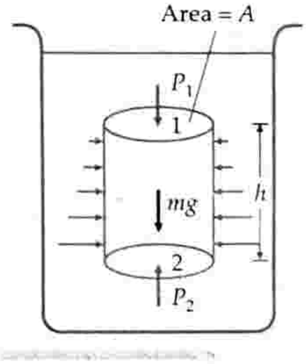


Fig. 10.8 Variation of liquid pressure with depth.

As the liquid cylinder is at rest, the resultant horizontal force should be zero. Various force acting on it in the vertical direction are :

1. Force due to the liquid pressure at the top,

$$F_1 = P_1 A, \text{ acting downwards}$$

2. Force due to the liquid pressure at the bottom,

$$F_2 = P_2 A, \text{ acting upwards}$$

3. Weight of the liquid cylinder acting downwards,

$$W = \text{Mass} \times g = \text{Volume} \times \text{density} \times g$$
$$= Ah\rho g$$

where ρ is the density of the liquid.

As the liquid cylinder is in equilibrium,

Net downward force = Net upward force

or $$F_1 + W = F_2$$

or $$F_2 - F_1 = W$$

or $$P_2A - P_1A = Ah\rho g$$

or $$P_2 - P_1 = h\rho g$$

If we shift point 1 to the liquid surface, which is open to the atmosphere, then we can replace P_1 by atmospheric pressure P_a and P_2 by P in the above equation and we get

$$P - P_a = h\rho g$$

or $$P = P_a + h\rho g$$

We can note the following points :

(*i*) The liquid pressure is the same at all points at the same horizontal level or at same depth.

(*ii*) Pressure at any point inside the fluid depends on the depth h.

(*iii*) The absolute (actual) pressure P, at a depth h below the liquid surface open to the atmosphere is greater than the atmospheric pressure by an amount $h\rho g$. The excess pressure $P - P_a$, at depth h is called a ***gauge pressure*** at that point.

(*iv*) Pressure does not depend on the cross-section or base-area or the shape of the vessel.

Effect of gravity on Pascal's law. If we neglect the effect of gravity, then

$$P_2 - P_1 = h\rho g = 0$$

or $$P_2 = P_1$$

That is, pressure at all points inside the liquid is same in the absence of gravity. This is Pascal's law. However, in the presence of gravity, Pascal's law gets modified as $P_2 - P_1 = h\rho g$.

10.10 PASCAL'S VASES : HYDROSTATIC PARADOX

17. *Explain hydrostatic paradox with suitable example.*

Hydrostatic paradox. Pascal demonstrated experimentally that *the pressure exerted by a liquid column depends only on the height of the liquid column and not on the shape of the containing vessel.* As shown in Fig. 10.9, the experimental arrangement consists of three glass vessels A, B and C of different shapes. The area of the lower open end of all the vessels is same. The lower end of each vessel is closed by supporting a disc against it. Each disc is connected to a pressure-meter. When the three vessels are filled with the same liquid upto the same height, all the three meters record the same pressure. This appears anomalous because the three vessels have different shapes and contain different amounts of liquid. This apparently unexpected result is known as **hydrostatic paradox.**

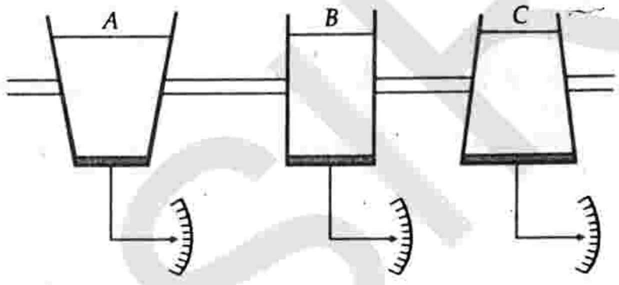


Fig. 10.9 Pascal's Vases.

Explanation. First consider the vessel A. Its one wall is shown in Fig. 10.10.(*a*). The liquid presses normally on the surface of the container. In the case of vessel A, the reaction R of the wall is inclined upwards. The vertical component V reduces the downward thrust of liquid. In the case of vessel B, the pressure is normal to the walls. The pressure acts horizontally on the walls. The reaction R of the walls is also horizontal [Fig. 10.10.(*b*)]. In the case of vessel C, the reaction R is inclined in the downward direction as shown in Fig. 10.10(*c*). The vertical component V increases the downward thrust of the liquid.

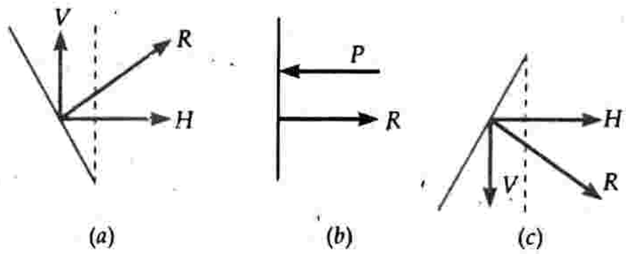


Fig. 10.10 Explanation of hydrostatic paradox.

10.11 ATMOSPHERIC PRESSURE AND GAUGE PRESSURE

18. *What is atmospheric pressure ?*

Atmospheric pressure. The gaseous envelope surrounding the earth is called the atmosphere. *The pressure exerted by the atmosphere is called atmospheric pressure.* The force exerted by air column of air on a unit area of the earth's surface is equal to the atmospheric pressure. The atmospheric pressure at sea level is 1.013×10^5 Nm^{-2} or Pa.

19. *Describe mercury barometer for measuring atmospheric pressure.*

Or

Describe Torricelli's experiment of measuring atmospheric pressure.

Mercury barometer. An Italian scientist *E. Torricelli* was first to device a method for measuring atmospheric pressure accurately. It is called a simple barometer. A 1 m long glass tube closed at one end is filled with clean and dry mercury. After closing the end of the tube with the thumb, the tube is inverted into a dish of mercury. As the thumb is removed, the mercury level in the tube falls down a little and comes to rest at a vertical height of 76 cm above the mercury level in the dish.

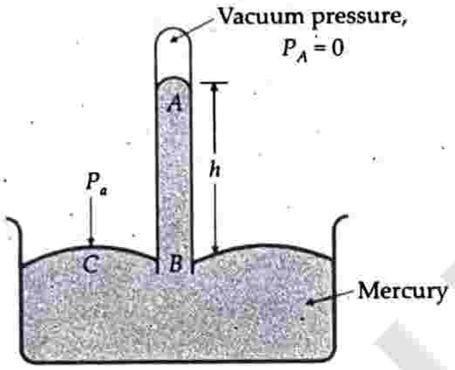


Fig. 10.11 Mercury barometer.

The space above mercury in the tube is almost a perfect vacuum and is called Torricellian vacuum. Therefore, pressure $P_A = 0$. Consider a point C on the mercury surface in the dish and point B in the tube at the same horizontal level. Then

$$P_B = P_C = \text{Atmospheric pressure, } P_a$$

If h is the height of mercury column and ρ is the density of mercury, then

$$P_B = P_A + h\rho g$$

or $$P_a = 0 + h\rho g$$

or $$P_a = h\rho g$$

For a mercury barometer, $h = 76 \text{ cm} = 0.76 \text{ m}$, $\rho = 13.6 \times 10^3 \text{ kg m}^{-3}$, $g = 9.8 \text{ m s}^{-2}$, therefore

Atmospheric pressure,

$$P_a = 0.76 \times 13.6 \times 10^3 \times 9.8 = 1.013 \times 10^5 \text{ Pa.}$$

20. *Describe how an open tube manometer can be used to measure the pressure of a gas. Distinguish between absolute pressure and gauge pressure.*

Open-tube manometer. It is a simple device used to measure the pressure of a gas enclosed in a vessel. It consists of a *U*-tube containing some liquid. One end of the tube is open to the atmosphere and the other end is connected to the vessel.

The total pressure P of the gas is equal to the pressure at A. Thus

$$P = P_A = P_C + h\rho g \quad \text{or} \quad P = P_a + h\rho g$$

where P_a is the atmospheric pressure, $h = BC$ = difference in the levels of the liquid in the two arms and ρ is the density of the liquid.

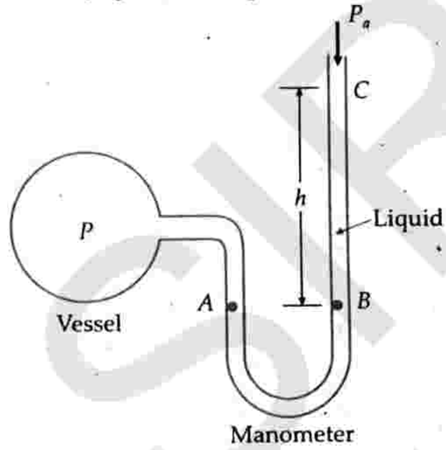


Fig. 10.12 Open tube manometer.

Absolute pressure and gauge pressure. *The total or actual pressure P at a point is called absolute pressure. Gauge pressure is the difference between the actual pressure (or absolute pressure) at a point and the atmospheric pressure, i.e.,* $P_g = P - P_a = h\rho g$

The gauge pressure is proportional to h. Many pressure measuring devices directly measure the gauge pressure. These include the tyre pressure gauge and the blood pressure gauge (sphygmomanometer).

10.12 HEIGHT OF ATMOSPHERE

21. *Calculate the height of the atmosphere above the earth's surface. Also state the assumptions used.*

Height of atmosphere. For calculating the height of atmosphere, we make use of the following assumptions :

(*i*) The value of g does not change appreciably upto a certain height.

(*ii*) Temperature remains uniform throughout.

(*iii*) Although density of air decreases with height, we assume it to be uniform and take $\rho = 1.3 \text{ kg m}^{-3}$.

Pressure exerted by h height of air column

= Pressure exerted by 0.76 m of Hg

or $$h\rho g = 1.013 \times 10^5 \text{ Pa}$$

or $$h = \frac{1.013 \times 10^5}{\rho g} = \frac{1.013 \times 10^5}{1.3 \times 9.8}$$

$$= 7951 \text{ m} \simeq 8 \text{ km.}$$

In actual practice, both the density of air and the value of g decrease with height, so the atmospheric cover extends with decreasing pressure even beyond 100 km.

10.13 DIFFERENT UNITS OF PRESSURE

22. *Name the various units used for measuring pressure.*

Various units for pressure :

(*i*) SI unit of pressure = $\mathbf{Nm^{-2}}$ or **Pascal (Pa).**

(*ii*) CGS unit of pressure = $\mathbf{dyne\ cm^{-2}}$.

(*iii*) **Atmosphere** (atm). It is the pressure exerted by 76 cm of Hg column (at 0°C, 95° latitude and mean sea level).

$$1 \text{ atm} = 1.013 \times 10^5 \text{ Pa} = 1.013 \times 10^6 \text{ dyne cm}^{-2}.$$

(*iv*) In meteorology, the atmospheric pressure is measured in bar and millibar.

$$1 \text{ bar} = 10^5 \text{ Pa} = 10^6 \text{ dyne cm}^{-2}$$
$$1 \text{ millibar} = 10^{-3} \text{ bar} = 100 \text{ Pa}$$

(*v*) Atmospheric pressure is also measured in **torr**, a unit named after Torricelli.

$$1 \text{ torr} = 1 \text{ mm of Hg}$$
$$1 \text{ atm} = 1.013 \text{ bar} = 760 \text{ torr}$$

23. *In what units is the blood pressure measured ?*

Units for blood pressure. The blood pressure is measured in mm of Hg. When the heart is contracted to its smallest size, the pumping is hardest and the pressure of blood flowing in major arteries is nearly 120 mm of Hg . This is known as ***systolic pressure.*** When the heart is expanded to its largest size, the blood pressure is nearly 80 mm of Hg. This is known as ***diastolic pressure.***

For Your Knowledge

- While describing a fluid, we are concerned with properties that vary from point to point and not with properties associated with a specific piece of matter. So the role of force in a solid is replaced in a fluid by pressure and that of mass by density.
- A fluid exerts pressure not only on a solid piece immersed in fluid or on the walls of container, fluid pressure exists at all points in a fluid. A volume element (of fluid) inside a fluid is in a equilibrium because the pressures exerted on its various faces get balanced.
- Pressure at a point in a liquid acts equally in all directions.
- Pressure in a liquid is the same for all points at the same horizontal level.
- Pressure in a liquid increases with depth h according to the relation $P = P_a + h\rho g$

  This expression is valid only for incompressible fluids *i.e.*, liquids.
- Liquid pressure is independent of the area and the shape of the containing vessel.
- The mean pressure on the walls of a vessel containing liquid upto height h is $h\rho g/2$.
- Most of the pressure-measuring devices measure the pressure difference between the true pressure and the atmospheric pressure. This difference is called *gauge pressure* and the pressure is called *absolute pressure.*

  Absolute pressure

  = Gauge pressure + Atmospheric pressure

  i.e., $P = P_g + P_a$
- The gauge pressure may be positive or negative depending on $P > P_a$ or $P < P_a$. In inflated tyres or the human circulatory system, the absolute pressure is greater than atmospheric pressure, so gauge pressure is positive, called the *overpressure*. However, when we suck a fluid through a straw, the absolute pressure in our lungs is less than atmospheric pressure and so the gauge pressure is negative.
- A diver in water at a depth of 10 m is under twice the atmospheric pressure.
- At a depth of 100 km in a sea, the increase in pressure is 100 atm. Submarines are designed to withstand such high pressures.
- The pressure at the centre of the earth is estimated to be 3 million atmospheres.
- The atmospheric pressure is nearly 100 kPa. The tyres of a car are usually inflated to a pressure of about 200 kPa.
- It is because of the blood pressure from inside that we do not feel such a high atmospheric pressure.
- A drop in the atmospheric pressure by 10 mm of Hg or more is a sign of an approaching storm.

Examples based on Pressure Exerted by a Liquid Column and Gauge Pressure

Formulae Used

1. Pressure exerted by a liquid column of height h and density ρ is $P = h\rho g$
2. Absolute pressure

   = Atmospheric pressure + Gauge pressure

   $P = P_a + P_g$

Units Used

Height h is in metre, density ρ in kg m^{-3} and pressure P in Nm^{-2} or Pa.

Conversions Used

$$1 \text{ atm} = 1.013 \times 10^6 \text{ dyne cm}^{-2} = 1.013 \times 10^5 \text{ Nm}^{-2} \text{ (or Pa)}$$
$$1 \text{ bar} = 10^6 \text{ dyne cm}^{-2} = 10^5 \text{ Nm}^{-2}$$
$$1 \text{ millibar (m bar)} = 10^{-3} \text{ bar} = 10^3 \text{ dyne cm}^{-2} = 10^2 \text{ Nm}^{-2}$$
$$1 \text{ torr} = 1 \text{ mm Hg}$$
$$1 \text{ atm} = 101.3 \text{ kPa} = 1.013 \text{ bar} = 760 \text{ torr}.$$

EXAMPLE 7. *Express standard atmospheric pressure in (i) Nm^{-2} (ii) bars and (iii) torr.*

Solution. (*i*) Standard atmospheric pressure

$= 76$ cm of Hg

We know that,

$$P = h\rho g$$

Here $h = 76 \text{ cm} = 0.76 \text{ m}$,

$\rho(\text{Hg}) = 13.6 \times 10^3 \text{ kg m}^{-3}$, $g = 9.8 \text{ ms}^{-2}$

$\therefore$ $1 \text{ atm} = 0.76 \times 13.6 \times 10^3 \times 9.8$

$= \mathbf{1.013 \times 10^5 \ Nm^{-2}}$.

(*ii*) As $1 \text{ Nm}^{-2} = 10^{-5}$ bar

$\therefore$ 1 atm = **1.013 bar** = **1013 millibar.**

(*iii*) As 1 torr = 1 mm of Hg

$\therefore$ 1 atm = 760 mm of Hg = **760 torr.**

EXAMPLE 8. *What will be the length of mercury column in a barometer tube, when the atmospheric pressure is 75 cm of mercury and the tube is inclined at an angle of 60° with the horizontal direction ?*

Solution. Here $h = 75$ cm, $\theta = 60$

If l is the length of mercury coloumn in the barometer tube, then

$$\frac{h}{l} = \sin 60 \quad \text{or} \quad \frac{75}{l} = \frac{\sqrt{3}}{2}$$

$$\therefore \quad l = \frac{75 \times 2}{\sqrt{3}} = \mathbf{86.6\ cm}.$$

EXAMPLE 9. *What is the pressure on a swimmer 10 m below the surface of a lake ?* **[NCERT]**

Solution. Here $h = 10$ m,

$\rho(\text{water}) = 1000 \text{ kg m}^{-3}$, $g = 9.8 \text{ ms}^{-2}$

Pressure on a swimmer 10 m below the surface of the lake,

$$P = P_a + h\rho g$$
$$= 1.0 \times 10^5 + 10 \times 1000 \times 9.8$$
$$= 1.98 \times 10^5 \text{ Pa} \simeq \mathbf{2\ atm}.$$

EXAMPLE 10. *The density of the atmosphere at sea level is 1.29 kg m^{-3}. Assume that it does not change with altitude. Then how high would the atmosphere extend ? Take $g = 9.81 \ ms^{-2}$.* **[NCERT]**

Solution. Here $\rho = 1.29 \text{ kg m}^{-3}$, $g = 9.81 \text{ ms}^{-2}$, $P_a = 1.01 \times 10^5$ Pa

As $P_a = h\rho g$

$$\therefore \quad h = \frac{P_a}{\rho g} = \frac{1.01 \times 10^5}{1.29 \times 9.81} = 7981 \text{ m} \simeq \mathbf{8\ km}.$$

EXAMPLE 11. *A rectangular tank is 10 m long, 10 m broad and 3 m high. It is filled to the rim with water of density 10^3 kg m^{-3}. Calculate the thrust at the bottom and walls of the tank due to hydrostatic pressure. Take $g = 9.8 \text{ ms}^{-2}$.*

Solution. Pressure on the bottom of the tank

$= h\rho g = 3 \times 10^3 \times 9.8 = 2.94 \times 10^3 \text{ Nm}^{-2}$

Area of bottom = Length × Breadth

$= 10 \times 5 = 50 \text{ m}^2$

$\therefore$ Thrust on the bottom

= Pressure × Area

$= 2.94 \times 10^3 \times 50 = 1.47 \times 10^6$ N

The hydrostatic pressure on the walls of the tank increases uniformly from zero at the free surface of water to $h\rho g$ at the bottom of the tank.

$\therefore$ Average hydrostatic pressure on the walls

$$= \frac{0 + h\rho g}{2} = \frac{1}{2} h\rho g = \frac{1}{2} \times 3 \times 10^3 \times 9.8$$
$$= 1.47 \times 10^4 \text{ Nm}^{-2}$$

Now, area of broad walls

= 2 × Length × Height

$= 2 \times 10 \times 3 = 60 \text{ m}^2$

Area of narrow walls

= 2 × Breadth × Height

$= 2 \times 5 \times 3 = 30 \text{ m}^2$

Total area of walls $= 90 \text{ m}^2$

$\therefore$ Thrust on the walls

= Average pressure × Area

$= 1.47 \times 10^4 \times 90 = 1.323 \times 10^6$ N

Total thrust on walls and bottom

$= 1.47 \times 10^6 + 1.323 \times 10^6$

$= \mathbf{2.793 \times 10^6\ N}.$

EXAMPLE 12. *The manual of a car instructs the owner to inflate the tyres to a pressure of 200 kPa. (a) What is the recommended gauge pressure ? (b) What is the recommended absolute pressure ? (c) If, after the required inflation of the tyres, the car is driven to a mountain peak where the atmospheric pressure is 10% below that at sea level, what will the tyre gauge read ?*

Solution. (*a*) The pressure instructed by a manual is the gauge pressure.

$\therefore$ $P_g = \mathbf{200\ kPa}.$

(*b*) Absolute pressure

= Atmospheric pressure + Gauge pressure

or $P = P_a + P_g = 101 \text{ kPa} + 200 \text{ kPa} = \mathbf{301\ kPa}.$

(*c*) At the mountain peak, the atmospheric pressure P_a' is 10% less.

$\therefore$ $P_a' = 90$ kPa.

If we assume that the absolute pressure in the tyres does not change during the driving, then

$$P_g = P - P'_a = 301 - 90 = \mathbf{211\ kPa}.$$

As the tyre gauge reads the gauge pressure, so it will read 211 kPa.

EXAMPLE 13. *At a depth of 1000 m in an ocean (a) What is the absolute pressure ? (b) What is the gauge pressure ? (c) Find the force acting on the window of area 20 cm × 20 cm of a submarine at this depth, the interior of which is maintained at sea-level atmospheric pressure. (The density of sea water is* $1.03 \times 10^3\ kg\ m^{-3}$*,* $g = 10\ ms^{-2}$*)* [NCERT]

Solution. Here $h = 1000$ m, $\rho = 1.03 \times 10^3$ kg m^{-3}

Atmospheric pressure, $P_a = 1$ atm $= 1.01 \times 10^5$ Pa

The absolute pressure,

$$P = P_a + h\rho g = 1.01 \times 10^5 + 1000 \times 1.03 \times 10^3 \times 10$$

$$= 104.01 \times 10^5\ \text{Pa} = \frac{104.01 \times 10^5}{1.01 \times 10^5}\ \text{atm} = \mathbf{104\ atm.}$$

(*b*) Gauge pressure,

$$P_g = P - P_a = h\rho g = 100 \times 1.03 \times 10^3 \times 10\ \text{Pa}$$

$$= 103 \times 10^5\ \text{Pa} \approx \mathbf{103\ atm.}$$

(*c*) Pressure outside the submarine,

$$P = P_a + h\rho g$$

Pressure inside the submarine $= P_a$

Net pressure on the window

= Gauge pressure

$$P_g = h\rho q$$

Area of window,

$$A = 20\ \text{cm} \times 20\ \text{cm} = 0.04\ \text{m}^2$$

Force acting on the window,

$$F = P_g A = 103 \times 10^5\ \text{Pa} \times 0.04\ \text{m}^2 = \mathbf{4.12 \times 10^5\ N.}$$

EXAMPLE 14. *What is the absolute and gauge pressure of the gas above the liquid surface in the tank shown in Fig. 10.13 ? Density of oil* $= 820\ kg\ m^{-3}$*, density of mercury* $= 13.6 \times 10^3\ kg\ m^{-3}$*. Given 1 atmospheric pressure* $= 1.01 \times 10^5$ *Pa.* [NCERT]

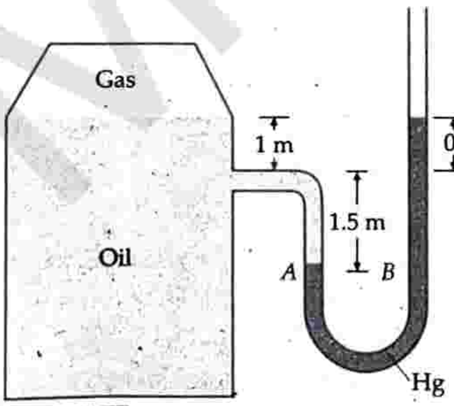


Fig. 10.13

Solution. As the points A and B are at the same level in the mercury coloumn, so

$$P_A = P_B$$

Now $\quad P_A = P + (1.50 + 1.00) \times 820 \times 9.8$

where P is the pressure of the gas in the tank.

And $\quad P_B = P' + (1.50 + 0.75) \times 13.6 \times 10^3 \times 9.8$

where P' is the atmospheric pressure.

As $\quad P_A = P_B$

$$P - P' = 2.25 \times 13.6 \times 10^3 \times 9.8 - 2.50 \times 820 \times 9.8$$

$$= 3 \times 10^5 - 0.2 \times 10^5 = 2.8 \times 10^5\ \text{Pa}$$

∴ Gauge pressure

= Absolute pressure – Atmospheric pressure.

$$P_g = P - P' = \mathbf{2.8 \times 10^5\ Pa.}$$

Absolute pressure

= Gauge pressure + Atmospheric pressure

$$P = 2.8 \times 10^5 + 1.01 \times 10^5 = \mathbf{3.81 \times 10^5\ Pa.}$$

EXAMPLE 15. *A liquid stands at the same level in the U-tube when at rest. If A is the area of cross-section and g the acceleration due to gravity, what will be the difference in height h of the liquid in the two limbs of U-tube, when the system is given an acceleration 'a' towards right, as shown in Fig. 10.14 ?*

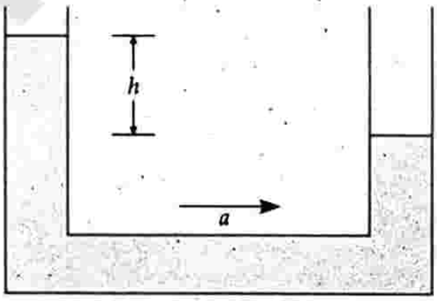


Fig. 10.14

Solution. If L is the length of the horizontal portion of the tube, then mass of liquid in this portion

$$= AL\rho.$$

Force exerted on the above mass towards left

$$= AL\rho \times a$$

Due to the difference h in the height of the liquid in the two limbs, the downward force exerted on the liquid in the horizontal portion

$$= h\rho g \times A$$

$$\therefore \quad h\rho g A = AL\rho a \quad \text{or} \quad h = \frac{La}{g}.$$

✘ PROBLEMS FOR PRACTICE

1. What is the minimum pressure required to force blood from the heart to the top of the head (a

vertical distance of 50 cm) ? Assume that the density of blood is 1.04 g cm^{-3} and neglect friction.
(**Ans.** 50960 dyne cm^{-2} or 38 mm of Hg)

2. A column of water 40 cm high supports a 30 cm column of an unknown liquid. What is the density of the liquid ? (**Ans.** 1.33×10^3 kgm^{-3})
3. If the water pressure gauge shows the pressure at ground floor to be 270 k Pa, how high would water rise in the pipes of a building ? (**Ans.** 27.6 m)
4. A cylindrical jar of cross-sectional area of 50 cm^2 is filled with water to a height of 20 cm. It carries a tight fitting piston of negligible mass. Calculate the pressure at the bottom of the jar when a mass of 1 kg is placed on the piston. Ignore atmospheric pressure. (**Ans.** 3920 Nm^{-2})
5. Water is filled in a flask upto a height of 20 cm. The bottom of the flask is circular with radius 10 cm. If the atmospheric pressure is 1.013×10^5 Pa, find the force exerted by the water on the bottom. Take $g = 10$ ms^{-2} and density of water = 1000 kgm^{-3}.
(**Ans.** 3246 N)
6. A vertical U-tube of uniform inner cross-section contains mercury in both of its arms. A glycerine (density 1.3 g cm^{-3}) column of length 10 cm is introduced into one of the arms. Oil of density 0.8 g cm^{-3} is poured in the other arm until the upper surfaces of the oil and glycerine are in the same horizontal level. Find the length of the oil column. **[IIT]**
(**Ans.** 9.6 cm)
7. The area of cross-section of the wider tube shown in Fig. 10.15 is 800 cm^2. If a mass of 12 kg is placed on the massless piston, what is the difference h in the level of water in the two tubes ? (**Ans.** 15.0 cm)

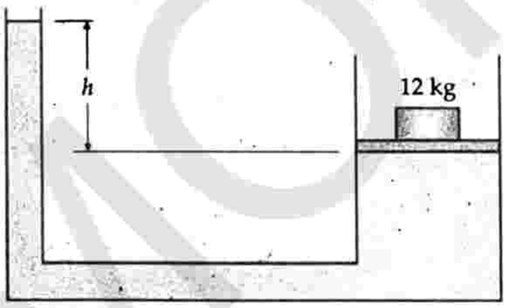


Fig. 10.15

8. A barometer kept in an elevator accelerating upwards reads 76 cm of Hg. If the elevator is accelerating upwards at 4.9 ms^{-2}, what will be the air pressure in the elevator ? (**Ans.** 114 cm of Hg)

✖ HINTS

1. $P = h\rho g = 50 \times 1.04 \times 980 = \mathbf{50960\ dyne\ cm^{-2}}$
$= \mathbf{38\ mm\ of\ Hg.}$
$[\because 1\ \text{mm of Hg} = 1333\ \text{dyne cm}^{-2}]$

2. As $h_1 \rho_1 g = h_2 \rho_2 g$
$$\therefore \quad \rho_2 = \frac{h_1}{h_2} \times \rho_1 = \frac{0.40}{0.30} \times 10^3 = \mathbf{1.33 \times 10^3\ kg\ m^{-3}}.$$

3. $P = h\rho g = 270$ kPa $= 270 \times 10^3$ Pa
$$\text{or} \quad h = \frac{270 \times 10^3}{\rho g} = \frac{270 \times 10^3}{10^3 \times 9.8} = \mathbf{27.6\ m.}$$

4. Here $A = 50\ cm^2 = 50 \times 10^{-4}\ m^2$,
$h = 20$ cm $= 0.20$ m,
$m = 1$ kg, ρ (water) $= 10^3$ kg m^{-3}
Total force on the bottom $= mg + h\rho g \times A$
$= 1 \times 9.8 + 0.20 \times 10^3 \times 9.8 \times 50 \times 10^{-4}$
$= 9.8 + 9.8 = 19.6$ N
Pressure at the bottom
$$= \frac{\text{Force}}{\text{Area}} = \frac{19.6}{50 \times 10^{-4}} = \mathbf{3920\ Nm^{-2}}.$$

5. Pressure at the bottom,
$P = P_0 + h\rho g = 1.013 \times 10^5 + 0.20 \times 1000 \times 10$
$= 1.033 \times 10^5$ Pa
Area of the bottom,
$A = \pi r^2 = 3.142 \times (0.1)^2 = 0.03142\ m^2$
$F = PA = 1.033 \times 10^5 \times 0.03142 = \mathbf{3246\ N.}$

6. The situation is shown in Fig. 10.16. Let h be the length of oil column. As B and E are the two points in mercury at the same level, so
$$P_E = P_B$$
$$h \times 0.8 \times g + (10 - h) \times 13.6 \times g = 10 \times 1.3 \times g$$
or $\quad h = \mathbf{9.6\ cm.}$

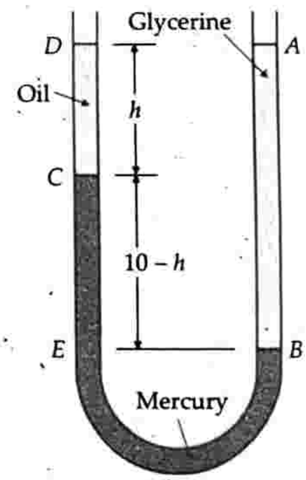


Fig. 10.16

7. Area of cross-section of the piston, $A = 800\ cm^2$
$F = mg$
$= 12 \times 1000 \times 980$ dyne
Pressure on the liquid,
$$P = \frac{F}{A} = \frac{12 \times 1000 \times 980}{800}\ \text{dyne cm}^{-2}$$
But $\quad P = h\rho g = h \times 1 \times 980$
$$\therefore \quad h \times 1 \times 980 = \frac{12 \times 1000 \times 980}{800}$$
or $\quad h = \mathbf{15.0\ cm.}$

8. When the elevator moves upward with acceleration a, net acceleration $= g + a$
$\therefore$ Pressure $= h\rho\,(g + a)$ dyne cm^{-2}
$$= \frac{76 \times 13.6 \times (9.8 + 4.9)}{13.6 \times 9.8} = \mathbf{114\ cm\ of\ Hg.}$$

10.14 BUOYANCY

24. *What do you understand by buoyancy and centre of buoyancy ?*

Buoyancy. When body is immersed in a fluid, the fluid exerts pressure on all faces of the body. But the fluid pressure increases with depth. The upward thrust at the bottom is more than the downward thrust on the top because the bottom is at the greater depth than the top. Hence a resultant upward force acts on the body. *The upward force acting on a body immersed in a fluid is called upthrust or buoyant force and the phenomenon is called buoyancy.* For example, a cork taken inside water experiences an upward thrust and comes to the surface. Similarly, while drawing water from a well, a bucket is found too much lighter when it is inside water than when it comes out of it.

The force of buoyancy acts through the centre of gravity of the displaced fluid which is called centre of buoyancy.

10.15 ARCHEMEDES' PRINCIPLE*

25. *State Archemedes' principle and prove it mathematically. Deduce an expression for the apparent weight of the immersed body.*

Archemedes' principle. This principle was discovered by the great Greek scientist, *Archimedes* around 225 B.C. and it gives the magnitude of buoyant force on a body.

Archimedes' principle *states that when a body is partially or wholly immersed in a fluid, it experiences an upward thrust equal to the weight of the fluid displaced by it and its upthrust acts through the centre of gravity of the displaced fluid.*

Proof. As shown in Fig. 10.17, consider a body of height h lying inside a liquid of density ρ, at a depth x below the free surface of the liquid. Area of cross-section of the body is a. The forces on the sides of the body cancel out.

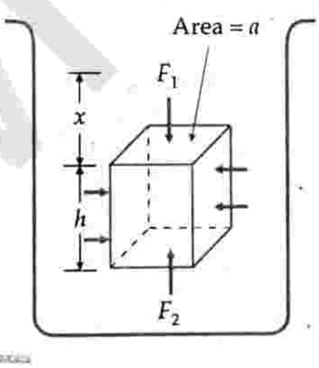


Fig. 10.17 Buoyant force on a body.

Pressure at the upper face of the body,

$$P_1 = x\rho g$$

Pressure at the lower face of the body,

$$P_2 = (x+h)\rho g$$

Thrust acting on the upper face of the body is

$$F_1 = P_1\, a = x\rho g a,$$

acting vertically downwards.

Thrust acting on the lower face of the body is

$$F_2 = P_2\, a = (x+h)\rho g a,$$

acting vertically upwards.

The resultant force $(F_2 - F_1)$ is acting on the body in the upward direction and is called upthrust (U).

$$\therefore \quad U = F_2 - F_1 = (x+h)\rho g a - x\rho g a = ah\rho g$$

But $ah = V$, the volume of the body = Volume of liquid displaced

$$\therefore \quad U = V\rho g = Mg$$

$$[\because M = V\rho = \text{mass of liquid displaced}]$$

i.e., Upthrust or buoyant force

= Weight of liquid displaced

This proves the Archimedes' principle.

Apparent weight of immersed body. The actual weight W of the immersed body acts downwards and the upthrust U acts upwards.

$\therefore$ Apparent weight

= Actual weight – Buoyant force

$$W_{app} = W - U = V\sigma g - V\rho g = V\sigma g\left(1 - \frac{\rho}{\sigma}\right)$$

or $$W_{app} = W\left(1 - \frac{\rho}{\sigma}\right)$$

Here $W = V\sigma g$ is the true weight of the body and σ is its density.

10.16 LAW OF FLOATATION*

26. *State and explain the law of floatation. Deduce an expression for the fraction of volume of the floating body submerged in the liquid.*

Law of floatation. *The law of floatation states that a body will float in a liquid if the weight of the liquid displaced by the immersed part of the body is equal to or greater than the weight of the body.*

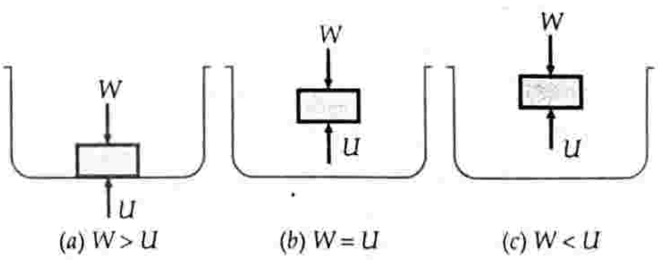


Fig. 10.18 Law of floatation.

Explanation. When a body is immersed fully or partly in a liquid, the following two vertical forces act on it :

(*i*) Its true weight W which acts vertically downward through its centre of gravity.

(*ii*) Force of buoyancy or upthrust U which acts vertically upwards through the centre of buoyancy.

As shown in Fig. 10.18, three cases are possible :

(*a*) **When $W > U$.** The downward pull of the weight of the body is higher than the upthrust. The net force $(W - U)$ acts in the downward direction and hence the body sinks.

$$W > U \quad \Rightarrow \quad V\sigma g > V\rho g \quad \text{or} \quad \sigma > \rho$$

Thus *a body sinks in a liquid if its density greater than the density of the liquid.* That is why an iron piece or a stone sinks in water.

(*b*) **When $W = U$.** The weight of the body is just balanced by the upthrust. No net force acts on the body. The body floats fully immersed.

$$W = U \quad \Rightarrow \quad V\sigma g = V\rho g \quad \text{or} \quad \sigma = g$$

Thus a drop of olive oil stands at rest anywhere in a mixture of equal quantities of water and alcohol because the density of olive oil is equal to that of the mixture.

(*c*) **When $W < U$.** The gravitational force W is less than the upward force U. The body floats partly immersed. This is because the body sinks only to the extent that $W = U$.

Here $\sigma < \rho$. The density of the floating body is less than that of liquid. That is why a piece of cork floats on water.

Fractional submerged volume of floating body. When the weight of the body is less than the weight of the liquid displaced, the body floats partially submerged. If V is the total volume of the body and V' is the submerged volume, then at equilibrium,

Weight of the body

= Weight of liquid displaced

or $\quad V\sigma g = V'\rho g$

or $\quad \dfrac{V'}{V} = \dfrac{\sigma}{\rho}$

or $\quad \dfrac{\text{Volume of submerged part}}{\text{Total volume of the body}} = \dfrac{\text{Density of body}}{\text{Density of liquid}}$

27. *Give some examples of floating bodies.*

Some examples of floating bodies :

(*i*) The ship is made of steel (8 times denser than water) but its interior is made hollow by giving it a concave shape. It can displace much more water than its own weight. So the ship floats and can carry a lot of cargo.

(*ii*) Ice floats on water because the density of ice is less than that of water.

(*iii*) Human body is slightly more denser than water. An inflated rubber tube has low weight and large volume and increases the upthrust. It helps a person to float.

(*iv*) A person can swim in sea water more easily than in river water. The density of sea water is more than that of river water and so it exerts a greater upthrust.

(*v*) The average density of a fish is slightly greater than water. By means of an anatomical attachment called swim bladder whose size it can adjust, the fish is able to swim with ease.

10.17 EQUILIBRIUM OF FLOATING BODIES*

28. *State the conditions for the equilibrium of floating bodies. Also discuss the stability of a floating body.*

Conditions for the equilibrium of a floating body :

(*i*) Weight of the liquid displaced must be equal to the weight of the body.

(*ii*) The centre of gravity of the body and the centre of buoyancy must lie on the same vertical line.

Stability of a floating body. When the centre of gravity of the body and the centre of buoyancy do not lie on the same vertical line, the two forces ; the weight (W) of the body and the upthrust (U) form a couple which produces rotation. As the floating body is slightly displaced from the equilibrium position, the

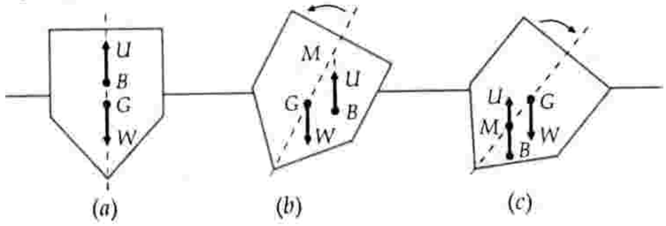


Fig. 10.19 Stability of a floating body.

centre of buoyancy shifts to a new position. *The point at which the vertical line passing through the new centre of buoyancy meets the initial vertical line is called* **metacentre** (M), as shown in Fig. 10.19.

(*i*) If the metacentre M lies above the centre of gravity G, the couple tends to bring the body back to its original position, as shown in Fig. 10.19(*b*). The floating body is in **stable equilibrium**.

(*ii*) If the metacentre M lies below the centre of gravity G, the couple tends to rotate the body away from the original position, as shown in Fig. 10.19(*c*). The floating body is in **unstable equilibrium**. The couple topples the floating body.

Examples based on Archimedes' principle and Law of Floatation

FORMULAE USED

1. According to Archemedes' principle, Loss in weight of a body in a liquid = Weight of liquid displaced = Volume × Density of liquid × g.
2. Apparent weight of solid in a liquid
   = True weight – Weight of liquid displaced,
   $$= mg - V'\rho' g = mg - \frac{m}{\rho}\rho' g = mg\left(1 - \frac{\rho'}{\rho}\right),$$
   where ρ' is the density of the liquid and ρ that of solid.
3. What a body just floats, Weight of the body
   = Weight of liquid displaced
   or $V\rho g = V'\rho' g$ or $\frac{V'}{V} = \frac{\rho}{\rho'}$
   $$\frac{\text{Volume of immerssed part}}{\text{Total volume of the solid}} = \frac{\text{Density of solid}}{\text{Density of liquid}}$$
4. Relative density $= \frac{\text{Density of substance}}{\text{Density of water at } 4°\text{C}}$
5. Relative density of a solid
   $$= \frac{\text{Weight of solid in air}}{\text{Loss in weight in water}}$$
6. Relative density of a liquid
   $$= \frac{\text{Loss in weight in liquid}}{\text{Loss in weight in water}}$$

UNITS USED

Volumes V and V' are in m^3, densities ρ and ρ' are in kgm^{-3} and relative density has no units.

EXAMPLE 16. ***The tip of the iceberg.*** *The density of ice is 917 kg m^{-3}. What fraction of ice lies below water ? The density of sea water is 1024 kg m^{-3}. What fraction of the ice berg do we see assuming that it has the same density as ordinary ice (917 kg m^{-3}) ?* **[NCERT]**

Solution. Density of ice, $\rho = 917$ kg m^{-3}

Density of water = 1000 kg m^{-3}

According to the law of floatation,

Weight of the piece of ice

= Weight of liquid displaced

$$V\rho g = V'\rho' g$$

$$\frac{V'}{V} = \frac{\rho}{\rho'} = \frac{917}{1000} = 0.917$$

So 91.7% of the ice lies below water.

In the case of the iceberg at sea, the fraction visible to us is given by

$$f = 1 - \frac{V'}{V} = 1 - \frac{\rho}{\rho'} = 1 - \frac{917}{1024} = 0.105$$

[ρ' (sea water) = 1024 kg m^{-3}]

So 10.5% of ice berg is visible to us. As most of the ice lies below the surface of the sea, hence the phrase. "The tip of the ice berg".

EXAMPLE 17. *The density of ice is 0.918 g cm^{-3} and that of water is 1.03 g cm^{-3}. An iceberg floats with a portion of 224 m^3 outside the surface of water. Find the total volume of the iceberg.*

Solution. Density of ice, $\rho = 0.918 \times 10^3$ kgm^{-3}

Density of water, $\rho' = 1.03 \times 10^3$ kgm^{-3}

Let volume of the iceberg = V m^3

Then volume of water displaced,

$$V' = (V - 224) \text{ m}^3$$

$\because$ Weight of iceberg = Weight of water displaced

or $\quad V\rho g = V'\rho' g$

or $\quad V \times 0.918 \times 10^3 \times g = (V - 224) \times 1.03 \times 10^3 \times g$

or $\quad V(1.03 - 0.918) = 224 \times 1.03$

or $\quad V = \frac{224 \times 1.03}{0.112} = \mathbf{2060\ m^3}$.

EXAMPLE 18. *A body of mass 6 kg is floating in a liquid with 2/3 of its volume inside the liquid. Find (i) buoyant force acting on the body, and (ii) ratio between the density of body and density of liquid. Take $g = 10$ ms^{-2}.* **[Delhi 04]**

Solution. When a body floats, its apparent weight is zero.

$\therefore$ Buoyant force = Weight of body

= 6 kg × 10 ms^{-2} = **60 N.**

Also, Buoyant force = Weight of liquid displaced

or $\quad V\rho_b g = \frac{2}{3} V \rho_l g$

or $\quad \frac{\rho_b}{\rho_l} = \frac{2}{3}$.

EXAMPLE 19. *A piece of pure gold ($\rho = 19.3$ g cm^{-3}) is suspected to be hollow from inside. It weighs 38.250 g in air and 33.865 g in water. Calculate the volume of the hollow portion in gold, if any.* **[NCERT]**

Solution. Density of pure gold, $\rho = 19.3$ g cm^{-3}

Weight of gold piece, $M = 38.250$ g

$\therefore$ Volume of gold piece,

$$V = \frac{M}{\rho} = \frac{38.250}{19.3} = 1.982 \text{ cm}^3$$

Mass of gold piece in water,

$$M' = 33.865 \text{ g}$$

$\therefore$ Apparent loss in weight of the gold piece in water

$$= 38.250 - 33.865 = 4.385 \text{ g}.$$

Density of water $= 1 \text{ g cm}^{-3}$

$\therefore$ Volume of water displaced

$$= \frac{4.385}{1} = 4.385 \text{ cm}^3$$

Volume of hollow portion of the gold piece

$$= 4.385 - 1.982 = \mathbf{2.403 \text{ cm}^3}.$$

EXAMPLE 20. *A solid body floating in water has 1/6th of the volume above surface. What fraction of its volume will project upward if it floats in a liquid of specific gravity 1.2 ?*

Solution. Let volume of the body $= V \text{ m}^3$

Then volume of body lying above surface

$$= \frac{V}{6} \text{ m}^3$$

Volume of water displaced $= V - \dfrac{V}{6} = \dfrac{5}{6} V \text{ m}^3$

$\therefore$ Weight of body = Weight of water displaced

or $$V\rho g = \frac{5}{6} V \times 10^3 \times g \quad ...(i)$$

Let V' be the volume of the body that lies outside the liquid of specific gravity 1.2. Then

Volume of liquid displaced $= V - V'$

Again, weight of body

= Weight of liquid displaced

$\therefore$ $$V\rho g = (V - V') \times 1.2 \times 10^3 \times g \quad ...(ii)$$

From (*i*) and (*ii*), we get

$$(V - V') \times 1.2 \times 10^3 \times g = \frac{5}{6} V \times 10^3 \times g$$

or $$\frac{V - V'}{V} = \frac{5}{6} \times \frac{10}{12} = \frac{25}{36}$$

or $$1 - \frac{V'}{V} = \frac{25}{36}$$

or $$\frac{V'}{V} = 1 - \frac{25}{36} = \mathbf{\frac{11}{36}}.$$

EXAMPLE 21. *A spring balance reads 10 kg when a bucket of water is suspended from it. What is the reading on the spring balance when*

(*i*) *an ice cube of mass 1.5 kg is put into the bucket*

(*ii*) *an iron piece of mass 7.8 kg suspended by another spring is immersed with half its volume inside the water in the bucket ?*

Relative density of iron = 7.8. **[NCERT ; Delhi 06]**

Solution. (*i*) When the ice is put into the bucket, its total weight $= 10 + 1.5 = 11.5$ kg f

The spring balance shows the reaction of the above force $= \mathbf{11.5 \text{ kg f}}$

(*ii*) Density of iron

$$= \text{R.D.} \times \text{Density of water} = 7.8 \times 10^3 \text{ kgm}^{-3}.$$

Volume of iron piece

$$= \frac{\text{Mass}}{\text{Density}} = \frac{7.8}{7.8 \times 1000} = 0.001 \text{ m}^3$$

As only half iron piece is immersed,

Volume of water displaced $= \dfrac{0.001}{2} \text{ m}^3$

Upthrust = Weight of water displaced

$$= \text{Volume} \times \text{Density} \times g$$

$$= \frac{0.001}{2} \times 1000 \times g \text{ newton}$$

$$= 0.5 g \text{ newton} = 0.5 \text{ kg f}$$

Total upward reaction $= 10 + 0.5 = 10.5$ kg f

$\therefore$ Reading on the spring balance = **10.5 kg f.**

EXAMPLE 22. *A cube of wood floating in water supports a 200 g mass at the centre of its top face. When the mass is removed, the mass rises by 2 cm. Determine the volume of cube.* **[Chandigarh 03]**

Solution. Let the side of the cube be l cm. As the mass of 200 g is removed, the cube rises by 2 cm. So by law of floatation,

Upthrust on cube due to displaced volume

$(V = l \times l \times 2 \text{ cm}^3)$ of water = 200 gf

or $$l \times l \times 2 \times 1 \times g = 200 \times g$$

or $$l^2 = 100 \quad \text{or} \quad l = 10 \text{ cm}$$

$\therefore$ Volume of the cube $= (10)^3 = \mathbf{1000 \text{ cm}^3}.$

EXAMPLE 23. *A block weighs 15 N in air. It weighs 12 N when immersed in water. When immersed in another liquid, it weighs 13 N. Calculate the relative density (specific gravity) of (i) the block and (ii) the other liquid.* **[REC 89]**

Solution. (*i*) Relative density of the block

$$= \frac{\text{Weight of the block in air}}{\text{Loss in weight when immersed in water}}$$

$$= \frac{15}{15 - 12} = \mathbf{5}.$$

(*ii*) Relative density of the liquid

$$= \frac{\text{Loss in weight when immersed in liquid}}{\text{Loss in weight when immersed in water}}$$

$$= \frac{15 - 13}{15 - 12} = \mathbf{\frac{2}{3}}.$$

EXAMPLE 24. *A jeweller claims that he sells ornaments made of pure gold that has the relative density of 19.3. He sells a necklace weighing 25.250 g f to a person. The clever* *customer weighs the necklace when immersed in pure water and finds that it weighs 23.075 g f in water. Is the ornament made of pure gold ?*

Solution. Weight of ornament in air = 25.250 g f

Weight of ornament in water = 23.075 g f

Loss of weight in water

= 25.250 − 23.075 = 2.175 g f

Relative density of the ornament

$$= \frac{\text{Weight in air}}{\text{Loss of weight in water}} = \frac{25.250}{2.175} = 11.61$$

As the relative density of the ornament is much less than that of pure gold which is 19.3, therefore, the ornament is not made of pure gold.

EXAMPLE 25. *A tank contains water and mercury as shown in Fig. 10.20. An iron cube of edge 6 cm is in equilibrium as shown. What is the fraction of cube inside the mercury ? Given density of iron $= 7.7 \times 10^3\ kgm^{-3}$ and density of mercury $= 13.6 \times 10^3\ kg\ m^{-3}$.*

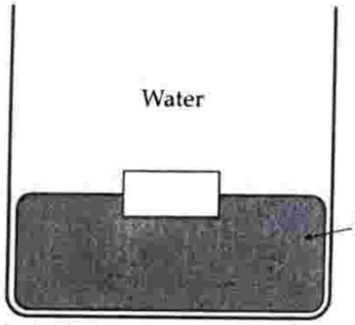


Fig. 10.20

Solution. Let x be the depth of cube in mercury. The depth of cube in water will be $(0.06 - x)$ m. From Archemedes' principle, the buoyant force on cube due to mercury is

$$B_1 = (0.06)^2 \times x \times (13.6 \times 10^3) \times 9.8\ \text{N}$$

Similarly, the buoyant force on cube due to water is

$$B_2 = (0.06)^2 \times (0.06 - x) \times 10^3 \times 9.8\ \text{N}$$

When cube is in equilibrium

$$B_1 + B_2 = \text{Weight of iron cube}$$

or $$(0.06)^2 \times 10^3 \times 9.8\,[13.6\,x + (0.06 - x)] = (0.06)^3 \times 7.7 \times 10^3 \times 9.8$$

On Solving, $x = \frac{0.40}{12.6} = 0.032$ m

Fraction of cube inside mercury $= \frac{0.032}{0.06} = \mathbf{0.533.}$

EXAMPLE 26. *A body of density ρ floats with a volume V_1 of its total volume V immersed in one liquid of density ρ_1 and with the remainder of volume V_2 immersed in another liquid of density ρ_2, where $\rho_1 > \rho_2$. Find the relative volumes immersed in two liquids.*

Solution. Weight of the body $= V\rho g$

Weight of first liquid displaced $= V_1 \rho_1 g$

Weight of second liquid displaced $= V_2 \rho_2 g$

When the body floats,

Weight of the body = Weight of two liquids displaced

$$V\rho g = V_1\rho_1 g + V_2\rho_2 g$$

or $$V\rho = V_1\rho_1 + V_2\rho_2 \quad ...(i)$$

Also $$V = V_1 + V_2 \quad ...(ii)$$

or $$V\rho_1 = V_1\rho_1 + V_2\rho_1 \quad ...(iii)$$

Subtracting (*iii*) from (*ii*), we get

$$V(\rho - \rho_1) = V_2(\rho_2 - \rho_1)$$

or $$V(\rho_1 - \rho) = V_2(\rho_1 - \rho_2)$$

or $$V_2 = \left(\frac{\rho_1 - \rho}{\rho_1 - \rho_2}\right) V \quad ...(iv)$$

Substituting the value of V_2 in (*ii*), we get

$$V = V_1 + \left(\frac{\rho_1 - \rho}{\rho_1 - \rho_2}\right) V$$

or $$V_1 = V\left(1 - \frac{\rho_1 - \rho}{\rho_1 - \rho_2}\right) = \left(\frac{\rho - \rho_2}{\rho_1 - \rho_2}\right) \mathbf{V.} \quad ...(v)$$

EXAMPLE 27. *A sample of milk diluted with water has a density of $1032\ kgm^{-3}$. If pure milk has a density of $1080\ kgm^{-3}$, find the percentage of water by volume in milk.*

Solution. Let volume of diluted sample of milk $= V$

Volume of water in the sample $= v$

Volume of pure milk in the sample $= V - v$

Density of diluted milk = 1032 kgm^{-3}

Density of pure milk = 1080 kgm^{-3}

Density of water = 1000 kgm^{-3}

Density of diluted milk

$$= \frac{\text{Mass of pure milk + Mass of water}}{\text{Volume of diluted milk}}$$

$$1032 = \frac{(V - v) \times 1080 + v \times 1000}{V}$$

or $$1032\,V = 1080\,V - 1080\,v + 1000\,v$$

or $$\frac{v}{V} = \frac{48}{80} = 0.6$$

Percentage of water by volume in milk

$= 0.6 \times 100 = \mathbf{60\%}.$

✖ PROBLEMS FOR PRACTICE

1. How much will a body of 70 N weigh in water if it displaces 200 ml of water ? (**Ans.** 68.04 N)

2. A solid weighs 6 kg in air. If its density is 2000 kg m^{-3}, what will be its apparent weight in water ? (**Ans.** 3 kg)

3. A solid weighs 10 N in air. Its weight decreases by 2 N when weighed in water. What is the density of solid ? (**Ans.** 5000 kgm^{-3})

4. A copper cube of mass 0.50 kg is weighed in water. The mass comes out to be 0.40 kg. Is the cube hollow or solid ? Given density of copper $= 8.96 \times 10^3 \text{ kgm}^{-3}$ and density of water $= 10^3 \text{ kgm}^{-3}$.

(**Ans.** Density of cube $= 5 \times 10^3 \text{ kgm}^{-3}$, so it is hollow)

5. A solid floats in water with 3/4 of its volume below the surface of water. Calculate the density of the solid. (**Ans.** 750 kg m^{-3})

6. A piece of wood of relative density 0.25 floats in a pail containing oil of relative density 0.81. What is the fraction of volume of the wood above the surface of the oil ? (**Ans.** 0.69)

7. A boat having a length of 3 m and breadth 2 m is floating on a lake. The boat sinks by one cm, when a man gets on it. What is the mass of the man ? (**Ans.** 60 kg)

8. When a boulder of mass 240 kg is placed on an iceberg floating in sea water it is found that the iceberg just sinks. What is the mass of the iceberg ? Take the relative density of ice as 0.9 and that of sea water as 1.02. (**Ans.** 1800 kg)

9. A cubical block of wood 10.0 cm on a side floats at the interface between oil and water, with its lower surface horizontal and 4.0 cm below the interface. What is the mass of the block ? The density of the oil is 0.6 g cm^{-3}. (**Ans.** 760 g)

10. A piece of brass (alloy of zinc and copper) weighs 12.9 g in air. When completely immersed in water it weighs 11.3 g. What is the mass of copper contained in the alloy ? Specific gravity of zinc and copper are 7.1 and 8.9 respectively. (**Ans.** 7.61 g)

11. A piece of iron floats in mercury. Given that the density of iron is $7.8 \times 10^3 \text{ kg m}^{-3}$ and that of mercury is $13.6 \times 10^3 \text{ kgm}^{-3}$, calculate the fraction of the volume of iron piece that remains outside the mercury. (**Ans.** 0.43)

12. A metal cube of 5 cm side and relative density 9 is suspended by a thread so as to be completely immersed in a liquid of density $1.2 \times 10^3 \text{ kg m}^{-3}$. Find the tension in the thread. (**Ans.** 9.56 N)

13. A cube of side 4 cm is just completely immersed in liquid A. When it is put in liquid B, it floats with 2 cm outside the liquid. Calculate the ratio of densities of two liquids. (**Ans.** 1 : 2)

14. An iron ball has an air space in it. It weighs 1 kg in air and 0.6 kg in water. Find the volume of air space. Density of iron $= 7200 \text{ kgm}^{-3}$.

(**Ans.** $0.262 \times 10^{-3} \text{ m}^3$)

HINTS

1. Volume of water displaced = 200 ml = 0.2 litre

$\therefore$ Mass of water displaced = 0.2 kg

Loss in weight of body in water

= Weight of water displaced

$= 0.2 \times 9.8 \text{ N} = 1.96 \text{ N}$

Apparent weight of body $= 70 - 1.96 =$ **68.04 N.**

2. Volume of solid $= \dfrac{\text{Mass}}{\text{Density}} = \dfrac{6}{2000} \text{ m}^3$

= Volume of water displaced

Mass of water displaced

$= \text{Volume} \times \text{Density} = \dfrac{6}{2000} \times 1000 = 3 \text{ kg}$

Apparent weight in water $= 6 - 3 =$ **3 kg.**

3. Relative density of solid

$$= \frac{\text{Weight of solid in air}}{\text{Loss in weight in water}} = \frac{10}{2} = 5$$

Density of solid = R.D. × Density of water

$= \mathbf{5 \times 10^3 \text{ kg m}^{-3}}$.

4. Loss of weight in water

= Weight of water displaced

$(0.50 - 0.40) \times g = V \times 10^3 \times g$

Volume of cube, $V = 10^{-4} \text{ m}^3$

Density of cube $= \dfrac{0.50}{10^{-4}} = 5 \times 10^3 \text{ kg m}^{-3}$

which is less than the density of copper.

5. Let the density and volume of the solid be ρ and V. Then

Weight of solid $= V\rho g$

Volume of block in water $= \dfrac{3}{4} V$

= Volume of water displaced

Weight of water displaced

$= V'\rho' g = \dfrac{3}{4} V \times 10^3 \times g$

$\because$ Weight of body = Weight of water displaced

or $V\rho g = \dfrac{3}{4} V \times 10^3 g \quad \therefore \quad \rho = \mathbf{750 \text{ kg m}^{-3}}$.

6. Density of wood, $\rho = 0.25 \times 10^3 \text{ kgm}^{-3}$

Density of oil, $\rho' = 0.81 \times 10^3 \text{ kg m}^{-3}$

According to the law of floatation,

Weight of the piece of wood = Weight of liquid displaced or $V\rho g = V'\rho' g$

or $\frac{V'}{V} = \frac{\rho}{\rho'} = \frac{0.25 \times 10^3}{0.18 \times 10^3} = 0.31.$

i.e. fraction of volume of the wood submerged under the oil = 0.31

$\therefore$ Fraction of volume of the wood above the surface of the oil = 1 – 0.31 = **0.69.**

7. Fraction of volume above water surface

$= 1 - 0.90 = 0.10$

Weight of man = Weight of water displaced by boat when the man gets in

or $m \times 9.8 = (3 \times 2 \times 0.01) \times 10^3 \times 9.8$

Hence $m = $ **60 kg.**

8. R.D. of ice = 0.9

$\therefore$ Density of ice $= 0.9 \times 10^3$ kgm^{-3}

Let mass of iceberg $= m$ kg

Volume of iceberg, $V = \frac{\text{Mass}}{\text{Density}} = \frac{m}{0.9 \times 10^3}$ m^3

When this volume just sinks, mass of sea-water displaced

$= V \times$ Density of sea-water

$= \frac{m}{0.9 \times 10^3} \times 1.02 \times 10^3 = \frac{102}{90}$ m

According to Archimedes' principle,

$m + 240 = \frac{102}{90} m$ or $m\left(\frac{102}{90} - 1\right) = 240$

or $m = \frac{240 \times 90}{12} = $ **1800 kg.**

9. Volume of block $= (10.0 \text{ cm})^3 = 1000 \text{ cm}^3$

Volume of block in water

$= 10.0 \times 10.0 \times 4.0 = 400 \text{ cm}^3$

Volume of block in oil $= 1000 - 400 = 600 \text{ cm}^3$

According to Archimedes' principle,

Weight of the block = Weight of water displaced + Weight of oil displaced

$mg = 400 \times 1 \times g + 600 \times 0.6 \times g$

or $m = 400 + 360 = $ **760 g.**

10. Let m be the mass of copper. Then the mass of zinc in the alloy $= (12.9 - m)$ g

Volume of copper in the alloy $= \frac{m}{8.9}$

Volume of zinc in the alloy $= \frac{(12.9 - m)}{7.1}$

Total volume of the alloy $= \frac{m}{8.9} + \frac{(12.9 - m)}{7.1}$

Apparent loss of weight of alloy

$= 12.9 - 11.3 = 1.6$ g

$\therefore$ Volume of water displaced by the alloy

$= 1.6 \text{ cm}^3$

Total volume of the alloy = Total volume of water displaced

$$\frac{m}{8.9} + \frac{(12.9 - m)}{7.1} = 1.6$$

On solving, $m = $ **7.61 g.**

11. Fraction of iron piece that remains inside mercury,

$\frac{V'}{V} = \frac{\rho}{\rho'} = \frac{7.8 \times 10^3}{13.6 \times 10^3} = 0.57$

Fraction of iron piece that remains outside mercury

$= 1 - 0.57 = $ **0.43.**

12. Tension in the thread = Weight in air – Upthrust of liquid

$= (0.05)^3 \times 9 \times 10^3 \times 9.8 - (0.05)^3 \times 1.2 \times 10^3 \times 9.8$

$= 7.8 \times (0.5)^3 \times 10^3 \times 9.8 = $ **9.56 N.**

13. In each case, mass of cube

= mass of liquid displaced

$m = V\rho_A = \frac{2}{4} V\rho_B$ or $\frac{\rho_A}{\rho_B} = \frac{2}{4} = $ **1 : 2.**

14. Loss of weight of ball in water = 1 – 0.6 = 0.4 kg f

Loss of weight of ball

= Weight of water displaced = 0.4 kg f

$= V \times 1000$ kg f

Volume of iron ball with air space,

$V = \frac{0.4}{1000} = 0.4 \times 10^{-3}$ m^3

Volume of iron alone

$= \frac{\text{Mass}}{\text{Density}} = \frac{1}{7200} = 0.138 \times 10^{-3}$ m^3

$\therefore$ Volume of air space

= Volume of iron ball with air space – Volume of iron alone

$= 0.4 \times 10^{-3} - 0.138 \times 10^{-3}$

$= $ **0.262×10^{-3} m^3.**

10.18 VISCOSITY

29. *What is viscosity ? Explain the cause of viscosity.*

Viscosity. *Viscosity is the property of fluid by virtue of which an internal force of friction comes into play when a fluid is in motion and which opposes the relative motion between its different layers.* The backward dragging force, called ***viscous drag*** or ***viscous force,*** acts tangentially on the layers of the fluid in motion and tends to destroy its motion.

Cause of viscosity. Consider a liquid moving slowly and steadily over a fixed horizontal surface. Each layer moves parallel to the fixed surface. The layer in contact with the fixed surface is at rest and the velocity of the every other layer increases uniformly upwards, as shown by arrows of increasing lengths in Fig. 10.21.

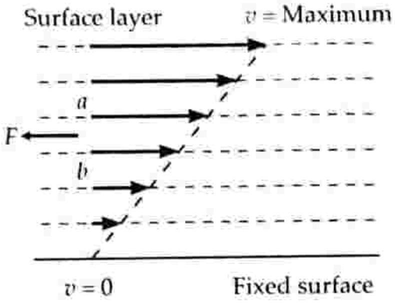


Fig. 10.21 Adjacent layers of a liquid in motion.

Consider any two adjacent layers a and b. The upper fast moving layer a tends to accelerate the lower slow moving layer b while the slow moving layer b tends to retard the fast moving layer a. As a result, a backward dragging tangential force F, called viscous drag comes into play which tends to destroy the relative motion. To maintain the motion, an external force has to be applied to overcome the backward viscous force.

30. *Give some examples in which the effect of viscosity can be easily seen.*

Examples of viscosity :

(*i*) When we stir a liquid contained in a beaker with a glass rod, it starts rotating in coaxial cylindrical layers as shown in Fig. 10.22. When we stop stirring, the speed of different layers gradually decreases and finally the water comes to rest, showing that an internal friction comes into play which destroys the relative motion between different layers.

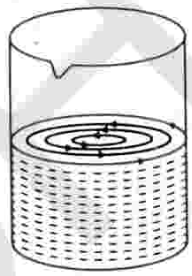


Fig. 10.22 Cylindrical layers in a stirred liquid.

(*ii*) When we swim in a pool of water, we experience some resistance to our motion. This is on account of viscous forces of water.

(*iii*) If we pour water and honey in separate funnels, water comes out readily from the hole in the funnel while honey trickles down drop by drop very slowly. This is because honey is much more viscous than water. The relative motion between the layers of honey is strongly opposed.

(*iv*) The cloud particles fall down very slowly on account of the viscosity of air and hence seen floating in the sky.

(*v*) We can walk fast in air, but not in water. This is because viscosity of air is much smaller than that of water.

10.19 COEFFICIENT OF VISCOSITY

31. *What is meant by coefficient of viscosity ? Give its dimensions and units.*

Coefficient of viscosity. As shown in Fig. 10.23, suppose a liquid is flowing steadily in the form of parallel layers on a fixed horizontal surface. Consider two layers P and Q at distances x and $x + dx$ from the solid surface and moving with velocities v and $v + dv$ respectively. Then $\frac{dv}{dx}$ is the *rate of change of velocity with distance in the direction of increasing distance* and is called **velocity gradient**.

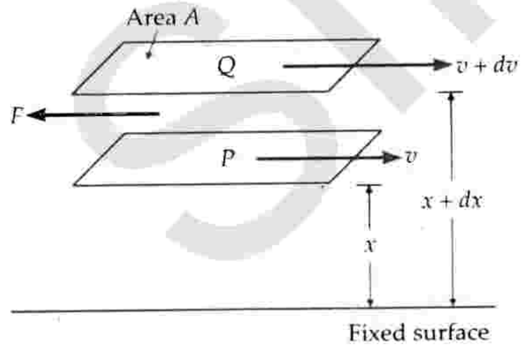


Fig. 10.23 Coefficient of viscosity.

According to Newton, a force of viscosity F acting tangentially between two layers is

(*i*) proportional to the area A of the layers in contact.

$$F \propto A$$

(*ii*) proportional to the velocity gradient $\frac{dv}{dx}$ between the two layers

$$F \propto \frac{dv}{dx}$$

$$\therefore \qquad F \propto A \frac{dv}{dx}$$

$$\text{or} \qquad F = -\eta A \frac{dv}{dx}$$

where η is the coefficient of viscosity of the liquid. It depends on the nature of the liquid and gives a measure of viscosity. Negative sign shows that the viscous force acts in a direction opposite to the direction of motion of the liquid.

If $\quad A = 1 \quad$ and $\quad \frac{dv}{dx} = 1$

then $\quad F = \eta \quad$ (numerically)

Hence *coefficient of viscosity of a liquid may be defined as the tangential viscous force required to maintain a unit velocity gradient between its two parallel layers each of unit area.*

Dimensions of η. Clearly,

$$\eta = \frac{F}{A} \cdot \frac{dx}{dv} \quad \therefore \quad [\eta] = \frac{MLT^{-2} \cdot L}{L^2 \cdot LT^{-1}} = [ML^{-1}T^{-1}].$$

Units of coefficient of viscosity. (*i*) The CGS unit of η is dyne s cm^{-2} or g $cm^{-1}s^{-1}$ and is called ***poise***.

$$1 \text{ poise} = \frac{1 \text{ dyne}}{1 \text{ cm}^2} \cdot \frac{1 \text{ cm}}{1 \text{ cm s}^{-1}} = 1 \text{ dyne s cm}^{-2}$$

The coefficient of viscosity a liquid is said to be 1 poise if a tangential force of 1 dyne cm^{-2} of the surface is required to maintain a relative velocity of 1 cm s^{-1} between two layers of the liquid 1 cm apart.

(*ii*) The SI unit of η is N s m^{-2} or kg m^{-1} s^{-1} and is called ***decapoise*** or ***poiseuille***.

$$1 \text{ poiseuille} = \frac{1 \text{ N}}{1 \text{ m}^2} \cdot \frac{1 \text{ m}}{1 \text{ m s}^{-1}} = 1 \text{ N s m}^{-2}$$

The coefficient of viscosity of a liquid is said to be 1 poiseuille or decapoise if a tangential force of 1 Nm^{-2} of the surface is required to maintain a relative velocity of 1 ms^{-1} between two layers of the liquid 1 m apart.

Relation between poiseuille and poise.

$$1 \text{ poiseuille or } 1 \text{ decapoise} = 1 \text{ Ns m}^{-2}$$
$$= (10^5 \text{ dyne}) \times s \times (10^2 \text{ cm})^{-2}$$
$$= 10 \text{ dyne s cm}^{-2} = 10 \text{ poise.}$$

For Your Knowledge

- Viscosity is like friction and converts kinetic energy into heat energy.
- No fluid has zero viscosity.
- Thin liquids like water, alcohol etc. ; are less viscous than thick liquids coal tar, blood, honey, glycerine etc.
- Unlike solids, the strain in a flowing liquid increases with time continuously. So for solids with elastic modulus of rigidity, the shearing stress is proportional to shear strain, while for fluids it is proportional to the *time rate of change of strain* or *strain rate*. The coefficient of viscosity of a fluid can be defined as the ratio of shearing stress to the strain rate. More over,

Coefficient of viscosity,

$$\eta = \frac{F/A}{v/x} = \frac{F/A}{\frac{\Delta x/x}{t}}$$

$$= \frac{\text{Shearing stress}}{\text{Shear strain}/t} = \frac{\text{Shearing stress}}{\text{Strain rate}}$$

$$\text{Modulus of rigidity, } \eta = \frac{F/A}{\Delta x/x} = \frac{\text{Shearing stress}}{\text{Shear strain}}$$

Thus *the coefficient of viscosity of liquids is analogous to the modulus of rigidity of solids.*

Examples based on Coefficient of Viscosity

FORMULAE USED

1. Velocity gradient $= \frac{dv}{dx}$
2. Newton's formula for viscous force between two parallel layers is $F = -\eta A \frac{dv}{dx}$

UNITS USED

In CGS system, coefficient of viscosity η is in poise, velocity gradient dv/dx in cm s^{-1}/cm, area A in cm^2 and force F in dyne. In SI, η is in decapoise or Pascal second (Pa s), A in m^2 and F in newton.

1 decapoise = 1 Pa s = 10 poise.

EXAMPLE 28. *A metal plate 5 cm × 5 cm rests on a layer of castor oil 1 mm thick whose coefficient of viscosity is 1.55 Nsm^{-2}. Find the horizontal force required to move the plate with a speed of 2 cms^{-1}.*

Solution. Here $A = 25 \text{ cm}^2 = 25 \times 10^{-4} \text{ m}^2$,

$dx = 1 \text{ mm} = 10^{-3} \text{ m}, \eta = 1.55 \text{ Nsm}^{-2}, dv = 2 \times 10^{-2} \text{ ms}^{-1}$

$$F = \eta A \frac{dv}{dx} = 1.55 \times 25 \times 10^{-4} \times \frac{2 \times 10^{-2}}{1 \times 10^{-3}} = \mathbf{0.0775 \text{ N.}}$$

EXAMPLE 29. *A square metal plate of 10 cm side moves parallel to another plate with a velocity of 10 cms^{-1}, both plates immersed in water. If the viscous force is 200 dyne and viscosity of water is 0.01 poise, what is their distance apart ?* **[NCERT]**

Solution. Here $A = 10 \times 10 = 100 \text{ cm}^2$,

$\eta = 0.01$ poise, $F = 200$ dyne, $dv = 10 \text{ cms}^{-1}$

As $F = \eta A \frac{dv}{dx}$

$$\therefore \quad dx = \frac{\eta A \, dv}{F} = \frac{0.01 \times 100 \times 10}{200} = \mathbf{0.05 \text{ cm.}}$$

EXAMPLE 30. *A flat square plate of side 20 cm moves over another similar plate with a thin layer of 0.4 cm of a liquid between them. If a force of one kg wt moves one of the plates uniformly with a velocity of 1 ms^{-1}, calculate the coefficient of viscosity of the liquid.*

Solution. Here

$A = 20 \times 20 = 400 \text{ cm}^2 = 400 \times 10^{-4} \text{ m}^2$,

$dx = 0.4 \text{ cm} = 0.4 \times 10^{-2} \text{ m}$

$F = 1 \text{ kg wt} = 9.8 \text{ N}, \; dv = 1 \text{ ms}^{-1}$

As $F = \eta A \frac{dv}{dx}$

$$\therefore \quad \eta = \frac{F}{A} \cdot \frac{dx}{dv} = \frac{9.8 \times 0.4 \times 10^{-2}}{400 \times 10^{-4} \times 1} = \mathbf{0.98 \text{ Pa s.}}$$

EXAMPLE 31. *The velocity of water in a river is 180 kmh^{-1} near the surface. If the river is 5 m deep, find the shearing stress between horizontal layers of water. Coefficient of viscosity of water $= 10^{-2}$ poise.*

Solution. As the velocity of water at the bottom of the river is zero,

$$dv = 18\ \text{kmh}^{-1} = 18 \times \frac{5}{18} = 5\ \text{ms}^{-1}$$

Also $\quad dx = 5$ m, $\eta = 10^{-2}$ poise $= 10^{-3}$ Pa s

Force of viscosity,

$$F = \eta\, A \frac{dv}{dx}$$

$\therefore$ Shearing stress

$$= \frac{F}{A} = \eta \frac{dv}{dx} = \frac{10^{-3} \times 5}{5} = \mathbf{10^{-3}\ Nm^{-2}}.$$

EXAMPLE 32. *A metal plate of area 0.10 m^2 is connected to a 0.01 kg mass via a string that passes over an ideal pulley (considered massless and frictionless), as shown in Fig. 10.24. A liquid with a film thickness of 0.3 mm is placed between the plate and the table. When released the plate moves to the right with a constant speed of 0.085 ms^{-1}. Find the coefficient of viscosity of the liquid.* **[NCERT]**

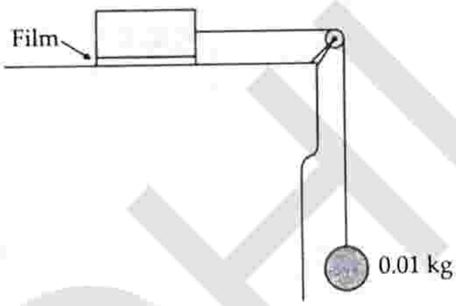


Fig. 10.24

Solution. Here $A = 0.10\ \text{m}^2$, $m = 0.01$ kg,

$dx = 0.3\ \text{mm} = 0.3 \times 10^{-3}$ m, $\quad dv = 0.085\ \text{ms}^{-1}$.

The metal plate moves towards right due to the tension T in the string which is equal to the weight of the suspended mass m. Assuming that the mass m moves with uniform velocity or zero acceleration, then the force of viscosity will be

$$F = T = mg = 0.01 \times 9.8 = 9.8 \times 10^{-2}\ \text{N}.$$

Taking velocity gradient to be uniform, then

$$\eta = \frac{F}{A} \cdot \frac{dx}{dv} = \frac{9.8 \times 10^{-2} \times 0.3 \times 10^{-3}}{0.10 \times 0.085}$$

$$= \mathbf{3.45 \times 10^{-3}\ Pa\ s}.$$

PROBLEMS FOR PRACTICE

1. The relative velocity between two parallel layers of water is 8 cm s^{-1} and the perpendicular distance between them is 0.1 cm. Calculate the velocity gradient. **(Ans.** 80 cm s^{-1} / cm)
2. A circular metal plate of radius 5 cm, rests on a layer of caster oil 2 mm thick, whose coefficient of viscosity is 15.5 poise. Calculate the horizontal force required to move the plate with a speed of 5 cm s^{-1}. **(Ans.** 3.04×10^4 dyne)
3. A metal plate of area 5 cm^2 is placed on a 0.5 mm thick castor oil layer. If a force of 22,500 dyne is needed to move the plate with a velocity of 3 cms^{-1}, calculate the coefficient of viscosity of castor oil. **(Ans.** 750 poise)
4. A metal plate of area 0.02 m^2 is lying on a liquid layer of thickness 10^{-3} m and coefficient of viscosity 120 poise. Calculate the horizontal force required to move the plate with a speed of 0.025 ms^{-1}. **(Ans.** 6 N)

HINTS

2. Here $A = \pi r^2 = \frac{22}{7} \times 25 = \frac{550}{7}\ \text{cm}^2$,

$dx = 2\ \text{mm} = 0.2\ \text{cm}$, $dv = 5\ \text{cm s}^{-1}$, $\eta = 15.5$ poise

$$F = \eta\, A \frac{dv}{dx} = 1.5 \times \frac{550}{7} \times \frac{5}{0.2} = \mathbf{3.04 \times 10^4\ dyne}$$

4. Here $A = 0.02\ \text{m}^2$, $dx = 10^{-3}$ m, $dv = 0.025\ \text{ms}^{-1}$,

$\eta = 120$ poise $= 12$ Pa s

$$\therefore \quad F = \eta A \frac{dv}{dx} = \frac{12 \times 0.02 \times 0.025}{10^{-3}} = \mathbf{6\ N}.$$

10.20 COMPARISON BETWEEN VISCOUS FORCE AND SOLID FRICTION

32. *Give some points of similarity and differences between viscous force and solid friction.*

Points of similarity :

(*i*) Both viscous force and solid friction come into play whenever there is relative motion.

(*ii*) Both oppose the motion.

(*iii*) Both are due to molecular attractions.

Points of differences :

	Viscous force	*Solid friction*
1.	Viscous force is directly proportional to the area of layers in contact.	Solid friction is independent of the area of the surfaces in contact.
2.	It is directly proportional to the relative velocity between the two liquid layers.	It is independent of the relative velocity between two solid surfaces.
3.	It is independent of the normal reaction between the two liquid layers.	It is directly proportional to the normal reaction between the surfaces in contact.

10.21 VARIATION OF VISCOSITY WITH TEMPERATURE AND PRESSURE

33. *Discuss the variation of fluid viscosity with temperature and pressure.*

Effect of temperature on viscosity. (*i*) When a liquid is heated, the kinetic energy of its molecules increases and the intermolecular attractions become weaker. Hence *the viscosity of a liquid decreases with the increase in its temperature.*

Slotle's empirical formula for the variation of viscosity of a liquid with temperature is

$$\eta_t = \frac{\eta_0}{1+\alpha t+\beta t^2}$$

where η_t and η_0 are the coefficients of viscosity at t°C and 0°C respectively, and α and β are constants.

(*ii*) Viscosity of gases is due to the diffusion of molecules from one moving layer to another. But the rate of diffusion of a gas is directly proportional to the square root of its absolute temperature, so viscosity of a gas increases with temperature as

$$\eta \propto \sqrt{T}$$

Effect of pressure. (*i*) Except water the viscosity of liquids increases with the increase in pressure. In case of water, viscosity decreases with the increase in pressure for first few hundred atmospheres of pressure.

(*ii*) The viscosity of gases is independent of pressure.

10.22 PRACTICAL APPLICATIONS OF THE KNOWLEDGE OF VISCOSITY

34. *Mention some important practical applications of the knowledge of viscosity in daily life.*

Practical applications of the knowledge of viscosity :

(*i*) The knowledge of viscosity and its variation with temperature helps us to select a suitable lubricant for a given machine in different seasons.

(*ii*) Liquids of high viscosity are used as buffers for absorbing shocks during the shunting of trains.

(*iii*) The knowledge of viscosity is used in determining the shape and molecular weight of some organic liquids like proteins, cellulose, etc.

(*iv*) The phenomenon of viscosity plays an important role in the circulation of blood through arteries and veins of human body.

(*v*) Millikan used the knowledge of viscosity in determining the charge on an electron.

10.23 POISEUILLE'S FORMULA

35. *State Poiseuille's formula. What are the assumptions used in the derivation of this formula ? Derive this formula on the basis of dimensional considerations.*

Poiseuille's formula. The volume of a liquid flowing out per second through a horizontal capillary tube of length l, radius r, under a pressure difference p applied across its ends is given by

$$Q = \frac{V}{t} = \frac{\pi p r^4}{8 \eta l}$$

This formula is called *Poiseuille's formula.*

Assumptions used in the derivation of Poiseuille's formula :

(*i*) The flow of the liquid is steady and parallel to the axis of the tube.

(*ii*) The pressure is constant over any cross-section of the tube.

(*iii*) The liquid velocity is zero at the walls of the tube and increases towards the axis of the tube.

(*iv*) The tube is held horizontal so that gravity does not influence the flow of liquid.

Derivation of Poiseuille's formula on the basis of dimensional analysis. The volume Q of liquid flowing out per second through a capillary tube depends on

(*i*) coefficient of viscosity η of the liquid,

(*ii*) radius r of the tube,

(*iii*) pressure gradient (p/l) set up along the capillary tube.

Let $\quad Q \propto \eta^a r^b \left(\frac{p}{l}\right)^c \quad$ or $\quad Q = k\,\eta^a r^b \left(\frac{p}{l}\right)^c \qquad$ ...(1)

where k is a dimensionless constant. The dimensions of various quantities are

$$[Q] = \frac{\text{Volume}}{\text{Time}} = \frac{[L^3]}{[T]} = [L^3T^{-1}],$$

$$\left[\frac{p}{l}\right] = \frac{[ML^{-1}T^{-2}]}{[L]} = [ML^{-2}T^{-2}]$$

$$[\eta] = [ML^{-1}T^{-1}], \quad [r] = L$$

Substituting these dimensions in equation (1), we get

$$[L^3T^{-1}] = [ML^{-1}T^{-1}]^a [L]^b [ML^{-2}T^{-2}]^c$$

or $\quad [M^0L^3T^{-1}] = [M^{a+c}L^{-a+b-2c}T^{-a-2c}]$

Equating the powers of M, L and T on both sides, we get

$$a + c = 0$$
$$-a + b - 2c = 3$$
$$-a - 2c = -1$$

On solving, we get $a = -1$, $b = 4$, and $c = 1$

$$\therefore \quad Q = k\,\eta^{-1} r^4 \left[\frac{p}{l}\right]^1 = \frac{kpr^4}{\eta l}$$

Experimentally k is found to be $\pi/8$.

$$\therefore \quad Q = \frac{\pi p r^4}{8 \eta l}$$

This is Poiseuille's formula for the flow of a liquid through a capillary tube.

Examples based on Poiseuille's Formula

FORMULA USED

Poiseuille's formula for the volume of a liquid flowing out per second through a narrow pipe is

$$Q = \frac{V}{t} = \frac{\pi p r^4}{8 \eta l}$$

UNITS USED

In CGS system, coefficient of viscosity η is in poise, length in cm, radius r in cm and pressure p in dyne cm^{-2}. In SI, η is in decapoise or pascal second (Pa s), length l in m, r in metre and p in Nm^{-2}.

1 decapoise = 1 Pa s = 10 poise

EXAMPLE 33. *Check the dimensional consistency of the Poiseuille's formula for the laminar flow in a tube :*

$$Q = \frac{\pi R^4 (p_1 - p_2)}{8 \eta l}$$

Solution. LHS = Q = Volume of liquid flowing out per second $= \frac{\text{Volume}}{\text{Time}}$

$\therefore$ Dimensions of LHS $= L^3T^{-1}$

Dimensions of RHS

$$= \left[\frac{\pi R^4 (p_1 - p_2)}{8 \eta l}\right] = \frac{L^4 \times ML^{-1}T^{-2}}{ML^{-1}T^{-1} \times L} = L^3T^{-1}$$

$\therefore$ Dimensions of LHS = Dimensions of RHS

Hence the Poiseuille's formula is dimensionally consistent.

EXAMPLE 34. *A capillary tube 1 mm in diameter and 20 cm in length is fitted horizontally to a vessel kept full of alcohol. The depth of the centre of capillary tube below the surface of alcohol is 20 cm. If the viscosity and density of alcohol are 0.012 cgs unit and 0.8 g cm^{-3} respectively, find the amount of the alcohol that will flow out in 5 minutes. Given that $g = 980\ cms^{-2}$.*

Solution. Here $r = \frac{1}{2}$ mm = 0.05 cm, $l = 20$ cm,

$\rho = 0.8\ gcm^{-3}$, $t = 5$ min $= 300$ s,

$\eta = 0.012$ cgs unit, pressure head = 20 cm of alcohol

$\therefore \quad p = h \rho g = 20 \times 0.8 \times 980$ dyne cm^{-2}

Volume of alcohol flowing out per second,

$$Q = \frac{\pi p r^4}{8 \eta l} = \frac{3.142 \times 20 \times 0.8 \times 980 \times (0.05)^4}{8 \times 0.012 \times 20} = 0.16\ cm^3$$

Mass of alcohol that flows out in 5 minutes

$= V \times \rho \times t = 0.16 \times 0.8 \times 300 =$ **38.4 g.**

EXAMPLE 35. *In giving a patient a blood transfusion, the bottle is set up so that the level of blood is 1.3 m above needle, which has an internal diameter of 0.36 mm and is 3 cm in length. If 4.5 cm^3 of blood passes through the needle in one minute, calculate the viscosity of blood. The density of blood is 1020 kgm^{-3}.*

Solution. Length of needle, $l = 3$ cm

Radius of needle, $r = \frac{0.36}{2}$ mm $= 0.018$ cm

Volume of blood flowing out per second,

$$Q = \frac{\text{Total Volume}}{\text{Time}} = \frac{4.5}{60} = 0.075\ cm^3\ s^{-1}$$

Density of blood,

$\rho = 1020\ kg\ m^{-3} = 1020 \times 10^{-3}\ g\ cm^{-3} = 1.02\ g\ cm^{-3}$

Pressure difference,

$p = 1.3$ m column of blood

$= 1.3 \times 100 \times 1.02 \times 980$ dyne cm^{-2}

$$\eta = \frac{\pi p r^4}{8 Q l} = \frac{3.142 \times 1.3 \times 100 \times 1.02 \times 980 \times (0.018)^4}{8 \times 0.075 \times 3}$$

$=$ **0.238 poise**

EXAMPLE 36. *A liquid flows through a pipe of 1.0 mm radius and 10 cm length under a pressure 10^4 dyne cm^{-2}. Calculate the rate of flow and the speed of the liquid coming out of the tube. The coefficient of viscosity of the liquid is 1.25 centipoise.*

Solution. Here $r = 1.0$ mm $= 0.1$ cm, $l = 10$ cm, $p = 10^4$ dyne cm^{-2}, $\eta = 1.25$ centipoise $= 0.0125$ poise

Rate of flow,

$$Q = \frac{\pi p r^4}{8 \eta l} = \frac{3.142 \times 10^4 \times (0.1)^4}{8 \times 0.0125 \times 10}$$

$=$ **3.142 cm^3s^{-1}.**

Speed of liquid,

$$v = \frac{Q}{\text{Cross - sectional area}} = \frac{3.142}{\pi r^2} = \frac{1}{r^2} = \frac{1}{(0.1)^2}$$

$= 100\ cms^{-1} =$ **1 ms^{-1}.**

EXAMPLE 37. *Two tubes A and B of lengths 100 cm and 50 cm have radii 0.1 mm and 0.2 mm respectively. If a liquid passing through the two tubes is entering A at a pressure of 80 cm of mercury and leaving B at a pressure of 76 cm of mercury, determine the pressure at the junction of A and B.*

Solution. Length of tube A, $l_A = 100$ cm

Radius of tube A, $r_A = 0.1$ mm $= 0.01$ cm

Pressure at which liquid enters tube A,

$p_A = 80$ cm of Hg

Let pressure at the junction of A and $B = p_j$

Rate of flow through tube A,

$$Q_A = \frac{\pi(p_A - p_j) r_A^4}{8\eta\, l_A} = \frac{\pi(80 - p_j)(0.01)^4}{8\eta \times 100} \text{ cm}^3 \text{ s}^{-1}$$

Length of tube B, $l_B = 50$ cm

Radius of tube B, $r_B = 0.2$ mm $= 0.02$ cm

Pressure at which liquid leaves tube B,

$$p_B = 76 \text{ cm of Hg}$$

Rate of flow through tube B,

$$Q_B = \frac{\pi(p_j - p_B) r_B^4}{8\eta\, l_B} = \frac{\pi(p_j - 76)(0.02)^4}{8\eta \times 50} \text{ cm}^3 \text{ s}^{-1}$$

But $Q_A = Q_B$

$$\therefore \quad \frac{\pi(80 - p_j)(0.01)^4}{8\eta \times 100} = \frac{\pi(p_j - 76)(0.02)^4}{8\eta \times 50}$$

or $(80 - p_j)(0.01)^4 = 2(p_j - 76)(0.02)^4$

or $80 - p_j = 32(p_j - 76)$

or $p_j = \mathbf{76.12 \text{ cm of Hg.}}$

EXAMPLE 38. *Two capillary tubes AB and BC are joined end to end at B. AB is 16 cm long and of diameter 4 mm whereas BC is 4 cm long and of diameter 2 mm. The composite tube is held horizontally with A connected to a vessel of water giving a constant head of 3 cm and C is open to the air. Calculate the pressure difference between B and C.*

Solution. For tube AB :

$$l_1 = 16 \text{ cm}, \; r_1 = 2 \text{ mm} = 0.2 \text{ cm}$$

For tube BC :

$$l_2 = 4 \text{ cm}, \; r_2 = 1 \text{ mm} = 0.1 \text{ cm}$$

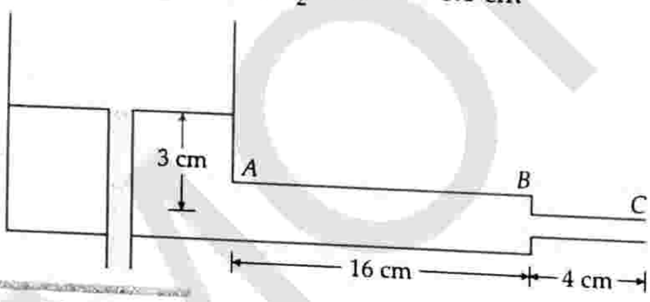


Fig. 10.25

Let h be the head of pressure at B over C, then it will be $(3 - h)$ at A over B.

Rate of flow through AB,

$$Q_1 = \frac{\pi p_1 r_1^4}{8\eta\, l_1} = \frac{\pi(3-h)(0.2)^2}{8\eta \times 16} \text{ cm}^3 \text{ s}^{-1}$$

Rate of flow through BC,

$$Q_2 = \frac{\pi p_2 r_2^4}{8\eta l_2} = \frac{\pi h (0.1)^2}{8\eta \times 4} \text{ cm}^3 \text{ s}^{-1}$$

But $Q_1 = Q_2$

$$\frac{\pi(3-h)(0.2)^4}{8\eta \times 16} = \frac{\pi h (0.1)^4}{8\eta \times 4}$$

or $12 - 4h = h$ or $5h = 12$

or $h = \mathbf{2.4 \text{ cm.}}$

EXAMPLE 39. *A large bottle is fitted with a siphon made of capillary glass tubing. Compare the times taken to empty the bottle when it is filled (i) with water (ii) with petrol of density 0.8 cgs units. The viscosity of water and petrol are 0.01 and 0.02 cgs units respectively.*

Solution. The volume of liquid flowing in time t through a capillary tube is given by

$$V = Qt = \frac{\pi P r^4 t}{8\eta l} = \frac{\pi h \rho g r^4 t}{8\eta l}$$

$\therefore$ For water, $V_1 = \dfrac{\pi h \rho_1 g r^4 t_1}{8 l \eta_1}$

For petrol, $V_2 = \dfrac{\pi h \rho_2 g r^4 t_2}{8 l \eta_2}$

But $V_1 = V_2$

$$\therefore \quad \frac{\pi h \rho_1 g r^4 t_1}{8 l \eta_1} = \frac{\pi h \rho_2 g r^4 t_2}{8 l \eta_2}$$

or $\dfrac{t_1}{t_2} = \dfrac{\eta_1}{\eta_2} \times \dfrac{\rho_2}{\rho_1} = \dfrac{0.01}{0.02} \times \dfrac{0.8}{1.0} = \mathbf{0.4}$

EXAMPLE 40. *The level of liquid in a cylindrical vessel is kept constant at 30 cm. It has three identical horizontal tubes of length 39 cm, each coming out at heights 0, 4 and 8 cm respectively. Calculate the length of a single overflow tube of the same radius as that of identical tubes which can replace the three when placed horizontally at bottom of the cylinder.*

Solution. Pressure head for tube A,

$$h_1 = 30 \text{ cm of water column}$$

Pressure head for tube B,

$$h_2 = 30 - 4 = 26 \text{ cm of water column}$$

Pressure head for tube C,

$$h_3 = 30 - 8 = 22 \text{ cm of water column}$$

Let radius of each tube $= r$ cm

Length of each tube, $l = 39$ cm

Rate of flow of liquid through different tubes will be

$$Q_1 = \frac{\pi p_1 r_1^4}{8\eta l} = \frac{\pi \times h_1 \rho g \times r^4}{8\eta l} = \frac{\pi \times 30 \times \rho g \times r^4}{8\eta \times 39}$$

Similarly,

$$Q_2 = \frac{\pi \times 26 \times \rho g \times r^4}{8\eta \times 39}$$

and

$$Q_3 = \frac{\pi \times 22 \times \rho g \times r^4}{8\eta \times 39}$$

Total volume of liquid flowing through the three tubes per second is

$$Q = Q_1 + Q_2 + Q_3$$

$$= \frac{\pi\rho g r^4}{8\eta \times 39}(30 + 26 + 22) = \frac{\pi\rho g r^4}{4\eta} \quad ...(i)$$

Let l' be the length of the single tube that flows out the same volume per second. Then

$$Q = \frac{\pi h\rho g r^4}{8\eta l'} = \frac{\pi \times 30 \times \rho g r^4}{8\eta l'} \quad ...(ii)$$

From (*i*) and (*ii*), we have

$$\frac{\pi \rho g r^4}{4\eta} = \frac{\pi \times 30 \times \rho g r^4}{8\eta l'} \quad \text{or} \quad 2l' = 30 \text{ cm}$$

Hence $\quad l' = \mathbf{15\ cm.}$

EXAMPLE 41. *Three capillary tubes of the same radius r but of lengths l_1, l_2 and l_3 are fitted horizontally to the bottom of a tall vessel containing a liquid at constant head and flowing through these tubes. Calculate the length of a single outflow tube of the same radius r which can replace the three capillaries.*

Solution. Let p be the liquid pressure at the bottom of the vessel. Then the rates of flow through the three tubes will be

$$Q_1 = \frac{\pi p r^4}{8\eta l_1}, \quad Q_2 = \frac{\pi p r^4}{8\eta l_2}, \quad Q_3 = \frac{\pi p r^4}{8\eta l_3}.$$

Let l be the length of the single tube of radius r which can replace the three tubes. The rate of flow through it will be

$$Q = \frac{\pi p r^4}{8\eta l}.$$

But $\quad Q = Q_1 + Q_2 + Q_3$

$$\therefore \quad \frac{\pi p r^4}{8\eta l} = \frac{\pi p r^4}{8\eta l_1} + \frac{\pi p r^4}{8\eta l_2} + \frac{\pi p r^4}{8\eta l_3}$$

or
$$\frac{1}{l} = \frac{1}{l_1} + \frac{1}{l_2} + \frac{1}{l_3} = \frac{l_2 l_3 + l_1 l_3 + l_1 l_2}{l_1 l_2 l_3}$$

or
$$l = \frac{l_1 l_2 l_3}{l_2 l_3 + l_1 l_3 + l_1 l_2}.$$

PROBLEMS FOR PRACTICE

1. In an experiment with Poiseuille's apparatus, the following observations were noted :

   Volume of liquid collected per minute = 15 cm^3

   Head of liquid = 30 cm ; Length of tube = 25 cm

   Diameter of tube = 0.2 cm ;

   Density of liquid = 2.3 gcm^{-3}

   Find the coefficient of viscosity of the liquid. (**Ans.** 0.425 poise)

2. Water at 20° is escaping from a cistern by way of a horizontal capillary tube 10 cm long and 0.4 mm in diameter, at a distance of 50 cm below the free surface of water in the cistern. Calculate the rate at which the water is escaping. Coefficient of viscosity of water is 0.001 decapoise. (**Ans.** $3.08 \times 10^{-8}\ m^3 s^{-1}$)

3. Water is conveyed through a horizontal tube 8 cm in diameter and 4 kilometer in length at the rate of 20 litres/s. Assuming only viscous resistance, calculate the pressure required to maintain the flow. Coefficient of viscosity of water is 0.001 Pa s. (**Ans.** $7.96 \times 10^4\ Nm^{-2}$)

4. Alcohol flows through two capillary tubes under a constant pressure head. The diameters of the two tubes are in the ratio of 4 : 1 and the lengths are in the ratio 4 : 1. Compare the rates of flow of alcohols through the two tubes. (**Ans.** 1024 : 1)

5. Show that if two capillaries of radii r_1 and r_2 having lengths l_1 and l_2 respectively are set in series, the rate of flow Q is given by

$$Q = \frac{\pi p}{8\eta}\left[\frac{l_1}{r_1^4} + \frac{l_2}{r_2^4}\right]^{-1}$$

   where p is the pressure difference across the arrangement and η is the coefficient of viscosity of the liquid.

6. Three capillaries of lengths l, $2l$ and $l/2$ are connected in series. Their radii are r, $r/2$ and $r/3$ respectively. If stream-line flow is maintained and the pressure difference across the first capillary tube is p_1, find the pressure difference across (*i*) the second and (*ii*) the third capillary tube. (**Ans.** 32 p_1, 40.5 p_1)

HINTS

2. $$Q = \frac{\pi p r^4}{8\eta\rho} = \frac{\pi h\rho g r^4}{8\eta l}$$

$$= \frac{3.14 \times 0.50 \times 10^3 \times 9.8 \times (0.2 \times 10^{-3})^4}{8 \times 0.001 \times 0.1}$$

$$= \mathbf{3.08 \times 10^{-8}\ m^3\ s^{-1}}.$$

3. $$p = \frac{8\eta Q l}{\pi r^4} = \frac{8 \times 0.001 \times 20 \times 10^{-3} \times 4 \times 10^3}{3.142 \times (4 \times 10^{-2})^4}$$

$$= \mathbf{7.96 \times 10^4\ Nm^{-2}}.$$

4. $$\frac{r_1}{r_2} = \frac{d_1}{d_2} = \frac{4}{1} \text{ and } \frac{l_1}{l_2} = \frac{1}{4}$$

   Now $Q = \dfrac{\pi p r^4}{8\eta l}$. For constant η and p, $Q \propto \dfrac{r^4}{l}$

$$\therefore \quad \frac{Q_1}{Q_2} = \frac{r_1^4}{l_1} \times \frac{l_2}{r_2^4} = \left[\frac{r_1}{r_2}\right]^4 \times \frac{l_2}{l_1}$$

$$= \left[\frac{4}{1}\right]^4 \times \frac{4}{1} = \mathbf{1024 : 1.}$$

5. As $Q = \frac{\pi p r^4}{8\eta l}$ or $p = \frac{8\eta Q l}{\pi r^4}$ or $p = k\frac{l}{r^4}$

$$p = p_1 + p_2 \text{ or } \frac{kl}{r^4} = \frac{kl_1}{r_1^4} + \frac{kl_2}{r_2^4}$$

or $$Q = \frac{\pi p}{8\eta}\left[\frac{l_1}{r_1^4} + \frac{l_2}{r_1^4}\right]^{-1}.$$

6. As the three tubes are connected in series, so the rate of flow of liquid through all the three tubes is same, say V. Let p_1, p_2 and p_3 be the pressure differences across the three tubes. Then

$$V = \frac{\pi p_1 r_1^4}{8\eta l_1} = \frac{\pi p_2 r_2^4}{8\eta l_2} = \frac{\pi p_3 r_3^4}{8\eta l_3}$$

But $l_1 = l,\ l_2 = 2l,\ l_3 = l/2,\ r_1 = r,\ r_2 = r/2,\ r_3 = r/3$

$$\therefore \quad \frac{p_1 r^4}{l} = \frac{p_2\left(\frac{r}{2}\right)^4}{2l} = \frac{p_3\left(\frac{r}{3}\right)^4}{l/2}$$

or $$p_1 = \frac{p_2}{32} = \frac{2p_3}{81}$$

Hence $p_2 = \mathbf{32}\, \boldsymbol{p_1}$ and $p_3 = \frac{81}{2} p_1 = \mathbf{40.5}\, \boldsymbol{p_1}$.

10.24 STOKES' LAW

36. *Explain the origin of viscous drag on a body falling through a fluid.*

Viscous drag on a body falling through a fluid. When a body falls through a viscous fluid, the layer of the fluid in contact with the body moves with its velocity. However, the fluid at large distance from it remains at rest. This produces relative motion between different layers of the fluid. As a result, the body experiences a viscous force which tends to retard its motion. This retarding force increases with the increase in velocity of the body.

37. *State Stokes' law. Deduce Stokes' law on the basis of dimensional considerations. State the conditions under which Stokes' law is valid.*

Stokes' law. *According to Stokes' law, the backward viscous force acting on a small spherical body of radius r moving with uniform velocity v through fluid of viscosity η is given by*

$$F = 6\pi\eta rv$$

Derivation of Stokes' law. The viscous force F acting on a sphere moving through a fluid may depend on

(*i*) coefficient of viscosity η of the fluid

(*ii*) radius r of the spherical body

(*iii*) velocity v of the body

Let $$F = k\eta^a r^b v^c \qquad ...(1)$$

where k is dimensionless constant. The dimensions of various quantities are

$$[F] = [MLT^{-2}], \quad \eta = [ML^{-1}T^{-1}]$$
$$[r] = [L], \qquad [v] = [LT^{-1}]$$

Substituting these dimensions in equation (1), we get

$$[MLT^{-2}] = [ML^{-1}T^{-1}]^a\,[L]^b\,[LT^{-1}]^c$$
$$= [M^a L^{-a+b+c} T^{-a-c}]$$

Equating the powers M, L and T on both sides, we get

$$a = 1$$
$$-a + b + c = 1$$
$$-a - c = -2$$

On solving, $a = b = c = 1$

$$\therefore \quad F = k\eta rv$$

For a small sphere, k is found to be equal to 6π.

Hence $F = 6\pi\eta rv$

This proves Stokes' law.

Conditions under which Stokes' law is valid :

(*i*) The fluid through which the body moves has infinite extension.

(*ii*) The body is perfectly rigid and smooth.

(*iii*) There is no slip between the body and fluid.

(*iv*) The motion of the body does not give rise to turbulent motion and eddies. Hence motion is streamlined.

(*v*) The size of the body is small but it is larger than the distance between the molecules of the liquid. Thus the medium is homogeneous and continuous for such a body.

10.25 TERMINAL VELOCITY

38. *Explain how does a body attain a terminal velocity when it is dropped from rest in a viscous medium. Derive an expression for the terminal velocity of a small spherical body falling through a viscous medium. Also discuss the result.*

Terminal velocity. When a body falls through a viscous fluid, it produces relative motion between its different layers. As a result, the body experiences a viscous force which tends to retard its motion. As the velocity of the body increases, the viscous force ($F = 6\pi\eta rv$) also increases. A stage is reached, when the weight of the body becomes just equal to the sum of the upthrust and viscous force. Then no net force acts on the body and it begins to move with a constant velocity. *The maximum constant velocity acquired by a body while falling through a viscous medium is called its terminal velocity.*

Expression for terminal velocity. Consider a spherical body of radius r falling through a viscous liquid of density σ and coefficient of viscosity η. Let ρ be the density of the body.

As the body falls, the various forces acting on the body are as shown in Fig. 10.26. These are

(*i*) Weight of the body acting vertically downwards.

$$W = mg = \frac{4}{3}\pi r^3 \rho g$$

(*ii*) Upward thrust equal to the weight of the liquid displaced.

$$U = \frac{4}{3}\pi r^3 \sigma g$$

(*iii*) Force of viscosity F acting in the upward direction. According to Stokes' law,

$$F = 6\pi\eta r v$$

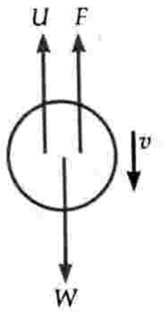


Fig. 10.26 Forces on a sphere falling in a viscous medium.

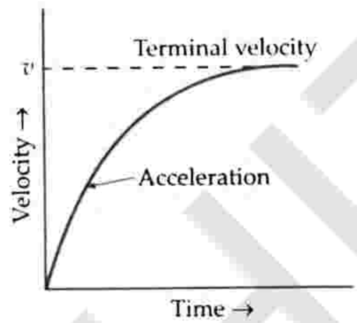


Fig. 10.27 Variation of v with t.

Clearly, the force of viscosity increases as the velocity of the body increases. A stage is reached, when the weight of the body becomes just equal to the sum of the upthrust and the viscous force. Then the body begins to fall with a constant maximum velocity, called *terminal velocity*.

When the body attains terminal velocity v,

$$U + F = W$$

$$\frac{4}{3}\pi r^3 \sigma g + 6\pi\eta rv = \frac{4}{3}\pi r^3 \rho g$$

or $$6\pi\eta rv = \frac{4}{3}\pi r^3 (\rho - \sigma) g$$

or $$v = \frac{2}{9} \cdot \frac{r^2(\rho - \sigma) g}{\eta}$$

This is the expression for terminal velocity.

Discussion of the result :

(*i*) Fig. 10.27 shows how the velocity of a small sphere dropped from rest into a viscous medium varies with time. Initially the body is accelerated and after some time, it acquires terminal velocity v.

(*ii*) *The terminal velocity is directly proportional to the square of the radius of the body.* That is why bigger rain drops fall with a larger velocity compared to the smaller rain drops.

(*iii*) The terminal velocity is directly proportional the difference of the densities of the body and the fluid, *i.e.*, $(\rho - \sigma)$.

(*a*) If $\rho > \sigma$, the body will attain terminal velocity in the downward direction.

(*b*) If $\rho < \sigma$, the terminal velocity will be negative *i.e.*, the body will rise through the fluid. That is why, air bubble in a liquid and clouds in a sky are seen to move in the upward direction.

(*c*) If $\rho = \sigma$, the body remains suspended in the fluid.

(*iv*) The *terminal velocity is inversely proportional to the coefficient of viscosity of the fluid.* The more viscous the fluid, the smaller the terminal velocity attained by a body.

(*v*) The *terminal velocity is independent of the height through which a body is dropped.*

(*vi*) Knowing the values of ρ, σ, r and v, we can determine the coefficient of viscosity η as follows :

$$\eta = \frac{2}{9}\frac{r^2(\rho - \sigma) g}{v}.$$

Examples based on Stokes' Law and Terminal Velocity

FORMULAE USED

1. According to Stokes' law, force of viscosity acting on a spherical body of radius r moving with velocity v through a fluid of viscosity η is

$$F = 6\pi\eta rv$$

2. Terminal velocity of a spherical body of density ρ and radius r moving through a liquid of density ρ' is

$$v = \frac{2}{9}\frac{r^2}{\eta}(\rho - \rho') g$$

UNITS USED

In SI, Force F is in newton, radius r in metre, velocity v in ms^{-1}, viscosity η in decapoise or Pa s, and density ρ in kgm^{-3}. In CGS system, F is in dyne, r in cm, v in cms^{-1}, η in poise and density ρ in gcm^{-3}.

EXAMPLE 42. *A rain drop of radius 0.3 mm falls through air with a terminal velocity of 1 ms^{-1}. The viscosity of air is 18×10^{-5} poise. Find the viscous force on the rain drop.*

Solution. Here $r = 0.3$ mm $= 0.03$ cm,

$$v = 1 \text{ ms}^{-1} = 100 \text{ cms}^{-1}, \ \eta = 18 \times 10^{-5} \text{ poise.}$$

According to Stokes' law, force of viscosity on rain drop is

$$F = 6\pi\eta r v$$
$$= 6 \times 3.142 \times 18 \times 10^{-5} \times 0.03 \times 100 \text{ dyne}$$
$$= \mathbf{1.018 \times 10^{-2}\ dyne.}$$

Example 43. *An iron ball of radius 0.3 cm falls through a column of oil of density 0.94 g cm^{-3}. It is found to attain a terminal velocity of 0.5 cms^{-1}. Determine the viscosity of the oil. Given that density of iron is 7.8 g cm^{-3}.*

Solution. Here $r = 0.3$ cm, $v = 0.5$ cms^{-1},

$$\rho = 7.8 \text{ gcm}^{-3}, \ \rho' = 0.94 \text{ gcm}^{-3}$$

As $v = \dfrac{2}{9}\dfrac{r^2}{\eta}(\rho - \rho')g$

$$\therefore \quad \eta = \frac{2}{9}\frac{r^2}{v}(\rho - \rho')g$$
$$= \frac{2 \times (0.3)^2 \times (7.8 - 0.94) \times 980}{9 \times 0.5}$$
$$= \frac{2 \times 0.09 \times 6.86 \times 980}{9 \times 0.5} = \mathbf{268.9\ poise.}$$

Example 44. *With what terminal velocity will an air bubble of density 1 kgm^{-3} and 0.8 mm in diameter rise in a liquid of viscosity 0.15 Nsm^{-2} and specific gravity 0.9 ? What is the terminal velocity of same bubble in water of $\eta = 1 \times 10^{-3}$ Nsm^{-2} ?*

Solution. Here $r = \dfrac{0.8}{2} = 0.4$ mm $= 0.4 \times 10^{-3}$ m,

$\eta = 0.15$ Nsm^{-2}, $g = 9.8$ ms^{-2}

Specific gravity of liquid $= 0.9$

Density of liquid (medium),

$$\rho' = 0.9 \times 10^3 \text{ kg m}^{-3} = 900 \text{ kg m}^{-3}$$

Density of air bubble (spherical object),

$$\rho = 1 \text{ kg m}^{-3}$$

Terminal velocity of air bubble,

$$v = \frac{2r^2 g(\rho - \rho')}{9\eta} = \frac{2 \times (0.4 \times 10^{-3})^2 \times 9.8 \times (1 - 900)}{9 \times 0.15}$$
$$= \mathbf{-0.0021\ ms^{-1}.}$$

The negative sign shows that the air bubble will rise up.

Terminal velocity of air bubble in water :

Here . $\rho' = 1000$ kg m^{-3}, $\eta = 10^{-3}$ Nsm^{-2}

$$\therefore \quad v = \frac{2 \times (0.4 \times 10^{-3})^2 \times 9.8 \times (1 - 1000)}{9 \times 10^{-3}}$$
$$= \mathbf{-0.348\ ms^{-1}.}$$

Example 45. *Eight rain drops of radius 1 mm each falling down with terminal velocity of 5 cms^{-1} coalesce to form a bigger drop. Find the terminal velocity of the bigger drop.* **[Delhi 95]**

Solution. Radius of each small drop,

$$r = 1 \text{ mm} = 0.1 \text{ cm}$$

Terminal velocity of each small drop,

$$v = 5 \text{ cms}^{-1}$$

Volume of bigger drop = Volume of 8 small drops

$$\frac{4}{3}\pi R^3 = 8 \times \frac{4}{3}\pi r^3$$

or $\quad R = 2r = 2 \times 0.1 \text{ cm} = 0.2 \text{ cm}$

Terminal velocity of each small drop is given by

$$v = \frac{2}{9}\frac{r^2}{\eta}(\rho - \rho')g \quad ...(i)$$

Terminal velocity of bigger drop is given by

$$V = \frac{2}{9}\frac{R^2}{\eta}(\rho - \rho')g \quad ...(ii)$$

Dividing equation (*ii*) by (*i*), we get

$$\frac{V}{v} = \frac{R^2}{r^2}$$

or $\quad V = v \times \dfrac{R^2}{r^2} = 5 \times \dfrac{(0.2)^2}{(0.1)^2}$

$$= 5 \times 4 = \mathbf{20\ cms^{-1}.}$$

Example 46. *Show that if n equal rain droplets falling through air with equal steady velocity of 10 cms^{-1} coalesce, the resultant drop attains a new terminal velocity of 10 $n^{2/3}$ cms^{-1}.*

Solution. Volume of a bigger drop

$$= n \times \text{Volume of a smaller droplet}$$

or $\quad \frac{4}{3}\pi R^3 = n \times \frac{4}{3}\pi r^3 \quad$ or $\quad R^3 = nr^3$

or $\quad R = n^{1/3} r$

Terminal velocity of a small droplet is given by

$$v = \frac{2}{9}\frac{r^2}{\eta}(\rho - \rho')g \quad ...(i)$$

Terminal velocity of a bigger drop is given by

$$V = \frac{2}{9}\frac{R^2}{\eta}(\rho - \rho')g \quad ...(ii)$$

Dividing equation (*ii*) by (*i*), we get

$$\frac{V}{v} = \frac{R^2}{r^2}$$

But $\quad R = n^{1/3} r \quad$ and $\quad v = 10$ cms^{-1}

$$\therefore \quad V = v \times \frac{R^2}{r^2} = 10 \times \frac{n^{2/3} r^2}{r^2} = \mathbf{10\, n^{2/3}\ cms^{-1}.}$$

EXAMPLE 47. Fine particles of sand are shaken up in water contained in a tall cylinder. If the depth of water in the cylinder is 24 cm, calculate the size of the largest particle of sand that can remain suspended after the expiry of 40 minutes. Given density of sand $= 2.6\ g\ cm^{-3}$ and viscosity of water $= 0.01$ poise. Assume that all the particles are spherical and are of different sizes.

Solution. Largest particle which remains suspended is that which just covers 24 cm in 40 minutes. So terminal velocity of largest particle is

$$v = \frac{\text{Distance}}{\text{Time}} = \frac{24\ \text{cm}}{40 \times 60\ \text{s}} = 0.01\ \text{cms}^{-1}.$$

Also $\eta = 0.01$ poise, $\rho = 2.6\ \text{g cm}^{-3}$, $\rho' = 1\ \text{g cm}^{-3}$, $g = 980\ \text{cm s}^{-2}$

As $v = \frac{2}{9}\frac{r^2}{\eta}(\rho - \rho')g$ or $r^2 = \frac{9}{2}\frac{\eta v}{(\rho - \rho')g}$

$$\therefore \quad r = \sqrt{\frac{9\eta v}{2(\rho - \rho')g}} = \sqrt{\frac{9 \times 0.01 \times 0.01}{2 \times (2.6 - 1) \times 980}}$$

$$= \frac{3 \times 0.01}{2 \times 4 \times 7} = \mathbf{5.357 \times 10^{-4}\ cm.}$$

EXAMPLE 48. A sphere is dropped under gravity through a fluid of viscosity η. Taking the average acceleration as half of the initial acceleration, show that the time taken to attain the terminal velocity is independent of the fluid density. **[NCERT]**

Solution. Suppose a sphere of radius r and density ρ falls in a fluid of density ρ' and viscosity η. When the sphere just enters the fluid, the net downward force on it is

F = Weight of the sphere – Weight of the fluid displaced

$$= \frac{4}{3}\pi r^3 \rho g - \frac{4}{3}\pi r^3 \rho' g = \frac{4}{3}\pi r^3 (\rho - \rho')g$$

∴ Initial acceleration,

$$a = \frac{F}{m} = \frac{\frac{4}{3}\pi r^3 (\rho - \rho')g}{\frac{4}{3}\pi r^3 \rho} = \left(\frac{\rho - \rho'}{\rho}\right)g$$

When the sphere attains terminal velocity, its acceleration becomes zero.

$$\therefore \text{ Average acceleration} = \frac{a + 0}{2} = \left(\frac{\rho - \rho'}{2\rho}\right)g$$

Let the sphere take time t to attain the terminal velocity,

$$v = \frac{2}{9}\frac{r^2}{\eta}(\rho - \rho')g$$

Initial velocity, $u = 0$

Hence by using first equation of motion,

$$v = u + at$$

$$\frac{2}{9}\frac{r^2}{\eta}(\rho - \rho')g = 0 + \left(\frac{\rho - \rho'}{2\rho}\right)gt$$

or
$$t = \frac{4}{9} \cdot \frac{r^2 \rho}{\eta}.$$

✖ PROBLEMS FOR PRACTICE

1. Find the terminal velocity of a steel ball 2 mm in diameter falling through glycerine. Relative density of steel = 8, relative density of glycerine = 1.3 and viscosity of glycerine = 8.3 poise. **[Delhi 98]**
(**Ans.** $1.758\ \text{cm s}^{-1}$)

2. A gas bubble of diameter 2 cm rises steadily at the rate of $2.5\ \text{mm s}^{-1}$ through a solution of density $2.25\ \text{g cm}^{-3}$. Calculate the coefficient of viscosity of the liquid. Neglect the density of the gas.
(**Ans.** 1960 poise)

3. A drop of water of radius 0.0015 mm is falling in air. If the coefficient of viscosity of air is 1.8×10^{-5} decapoise, what will be the terminal velocity of the drop ? Given density of water $= 10^3\ \text{kgm}^{-3}$ and $g = 9.8\ \text{ms}^{-2}$. Density of air can be neglected.
(**Ans.** $2.72 \times 10^{-4}\ \text{ms}^{-1}$)

4. The terminal velocity of a copper ball of radius 2.0 mm falling through a tank of oil at 20°C is $6.5\ \text{cms}^{-1}$. Compute the viscosity of the oil at 20°C. Density of oil $= 1.5 \times 10^3\ \text{kg m}^{-3}$, density of copper $= 8.9 \times 10^3\ \text{kg m}^{-3}$. **[NCERT]**
(**Ans.** 0.992 decapoise)

5. A spherical glass ball of mass 1.34×10^{-4} kg and diameter 4.4×10^{-3} m takes 6.4 s to fall steadily through a height of 0.381 m inside a large volume of oil of specific gravity 0.943. Calculate the viscosity of oil. (**Ans.** $0.8025\ \text{Nsm}^{-2}$)

6. Two exactly similar rain drops falling with terminal velocity of $(2)^{1/3}\ \text{ms}^{-1}$ coalesce to form a bigger drop. Find the new terminal velocity of the bigger drop. (**Ans.** $2\ \text{ms}^{-1}$)

✖ HINTS

1. $v = \frac{2}{9}\frac{r^2}{\eta}(\rho - \rho')g = \frac{2}{9}\frac{(0.1)^2}{8.3}(8 - 1.3) \times 980$

$= \mathbf{1.758\ cm\ s^{-1}}.$

2. $\eta = \frac{2}{9}\frac{r^2}{v}(\rho - \rho')g = \frac{2}{9}\frac{(1)^2 \times (0 - 2.25) \times 980}{-0.25}$

$= \mathbf{1960\ poise.}$

3. $v = \frac{2}{9} \times \frac{(1.5 \times 10^{-6})^2 (10^3 - 0) \times 9.8}{1.8 \times 10^{-5}}$

$= \mathbf{2.72 \times 10^{-4}\ ms^{-1}}.$

4. $\eta = \frac{2}{9} \frac{r^2}{v} (\rho - \rho') g$

$= \frac{2 \times (2.0 \times 10^{-3})^2 \times (8.9 \times 10^3 - 1.5 \times 10^3) \times 9.8}{9 \times 6.5 \times 10^{-2}}$

= **0.992 decapoise**

5. Density, $\rho = \frac{\text{Mass}}{\text{Volume}} = \frac{m}{\frac{4}{3}\pi r^3}$

$= \frac{3 \times 1.34 \times 10^{-4}}{4 \times \pi \times (2.2 \times 10^{-3})^3} = 3003.1245\ \text{kgm}^{-3}$

Oil density, $\rho' = 0.943 \times 10^3 = 943\ \text{kgm}^{-3}$

Terminal velocity, $v = \frac{0.381}{6.4} = 0.05753\ \text{ms}^{-1}$

$\eta = \frac{2}{9} \frac{r^2}{v} (\rho - \rho') g$

$= \frac{2}{9} \times \frac{(2.2 \times 10^{-3})^2 (3003.1245 - 943) \times 9.8}{0.05953}$

$= \mathbf{0.8025\ Nsm^{-2}}.$

6. Volume of bigger drop = Volume of two smaller drops

$\frac{4}{3}\pi R^3 = 2 \times \frac{4}{3}\pi r^3$ or $R^3 = 2r^3$ or $\frac{R}{r} = 2^{1/3}$

Terminal velocity of a smaller drop,

$v_1 = \frac{2}{9} \frac{r^2}{\eta} (\rho - \rho') g$

Terminal velocity of the bigger drop,

$v_2 = \frac{2}{9} \frac{R^2}{\eta} (\rho - \rho') g$

$\therefore \quad \frac{v_2}{v_1} = \frac{R^2}{r^2} = (2^{1/3})^2 = 2^{2/3}$

or $\quad v_2 = 2^{2/3} v_1 = 2^{2/3} \times 2^{1/3} = \mathbf{2\ ms^{-1}}.$

10.26 STREAMLINE AND TURBULENT FLOWS

39. *Distinguish between streamline and turbulent flows. What do you understand by a streamline and tube of flow ? Give important properties of streamlines.*

Streamline flow. *When a liquid flows such that each particle of the liquid passing a given point moves along the same path and has the same velocity as its predecessor, the flow is called streamline flow or steady flow.*

Consider the flow of the liquid along the path ABC; where A, B and C are the points inside the liquid. If every successive particle passes through point A with constant velocity $\vec{v}_A$ directed along tangent at A, then through point B with constant velocity $\vec{v}_B$ directed along tangent at B and then through C with constant velocity $\vec{v}_C$, the flow is said to be *steady, orderly or streamlined.* The path ABC along which the particles move one after another is called a *streamline.* The particle velocity at a particular point remains constant with time but velocities at different points may or may not be the same. Streamline flow is possible only if the liquid velocity does not exceed a limiting value, called ***critical velocity***. The fixed path followed by an orderly procession of particles in the steady flow is called a streamline. In Fig. 10.28(*a*), the curve ABC represents a streamline. *A streamline may be defined as the path, the tangent to which at any point gives the direction of the flow of liquid at that point.*

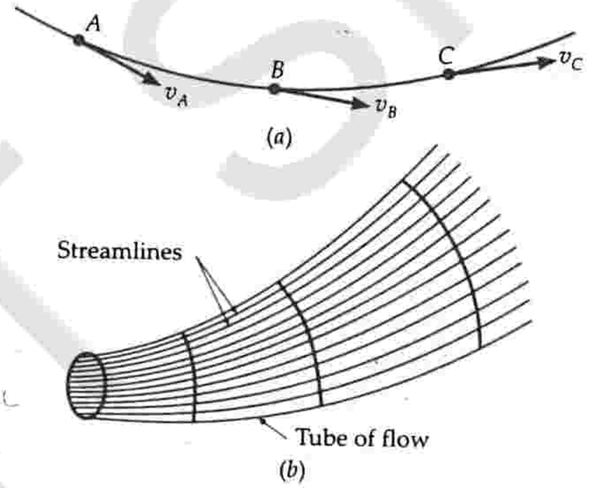


Fig. 10.28 (*a*) A streamline. (*b*) A tube of flow.

Tube of flow. *A bundle of streamlines forming a tubular region is called a tube of flow.* The boundary of such a tube is always parallel to the velocity of fluid particles. No fluid can cross the boundaries of a tube of flow, and the tube behaves somewhat like a tube. In a steady flow, the shape of the flow tube does not change with time.

Turbulent flow. When the liquid velocity exceeds a certain limiting value, called *critical velocity,* the liquid flow becomes zig-zag. The path and the velocity of a liquid particle changes continuously, haphazardly.

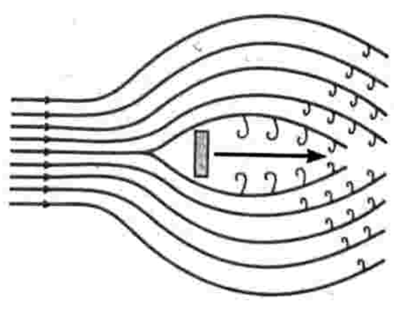


Fig. 10.29 Streamlines for a turbulent flow.

This flow is called *turbulent flow*. It is accompanied by random, irregular, local circular currents called vortices. As shown in Fig. 10.29, a jet of air striking a flat plate placed perpendicular to it causes a turbulent flow.

Properties of streamlines :

(*i*) In a steady flow, *no two streamlines can cross each other*. If they do so, the fluid particle at the point of intersection will have two different directions of flow. This will destroy the steady nature of the fluid flow.

(*ii*) The tangent at any point on the streamline gives the direction of velocity of fluid particle at that point.

(*iii*) Greater the number of streamlines passing normally through a section of the fluid, larger is the fluid velocity at that section.

(*iv*) Fluid velocity remains constant at any point of a streamline, but it may be different at different points of the same streamline.

10.27 LAMINAR FLOW

40. ***What is laminar flow of a liquid ? Distinguish between the velocity profiles of non-viscous and viscous liquids.***

Laminar flow. When the velocity of the flow of a liquid is less than its critical velocity, the liquid flows steadily. Each layer of the liquid slides over the other layer. It behaves as if different laminae are sliding over one another. Such a flow is called laminar flow.

The surface obtained by joining the heads of the velocity vectors for the particles in a section of a flowing liquid is called a *velocity profile*.

(*i*) **Velocity profile for a non-viscous liquid.** In case of a non-viscous liquid, the velocity of all the particles at any section of a pipe is same, so the velocity profile is plane as shown in Fig. 10.30(*a*).

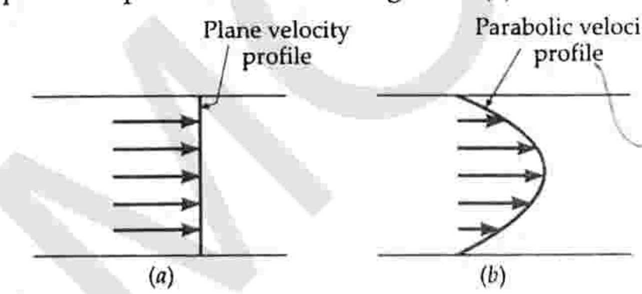


Fig. 10.30 Flow of (*a*) Non-viscous (*b*) Viscous liquid through a pipe.

(*ii*) **Velocity profile of a viscous liquid.** When a viscous liquid flows through a pipe, the velocity of layer at the axis is maximum, the velocity decreases as we go towards the wall of the pipe and becomes zero for the layer in contact with the pipe. Hence the velocity profile for a viscous liquid is parabolic, as shown in Fig. 10.30(*b*).

10.28 CRITICAL VELOCITY

41. ***What do you mean by critical velocity of a liquid ? Derive an expression for it on the basis of dimensional considerations.***

Critical velocity. *The critical velocity of a liquid is that limiting value of its velocity of flow upto which the flow is streamlined and above which the flow becomes turbulent.*

The critical velocity v_c of a liquid flowing through a tube depends on

(*i*) coefficient of viscosity of the liquid (η)

(*ii*) density of the liquid (ρ)

(*iii*) diameter of the tube (D)

Let $\quad v_c = k\,\eta^a\,\rho^b\,D^c$

where k is a dimensionless constant. Writing the above equation in dimensional form, we get

$$[M^0LT^{-1}] = [ML^{-1}T^{-1}]^a\,[ML^{-3}]^b\,[L]^c$$

$$[M^0LT^{-1}] = [M^{a+b}L^{-a-3b+c}T^{-a}]$$

Equating powers of M, L and T, we get

$$a + b = 0$$

$$-a - 3b + c = 1$$

$$-a = -1$$

On solving, we get $a = 1,\ b = -1,\ c = -1$

$$\therefore\ v_c = k\,\eta\,\rho^{-1}\,D^{-1} = \frac{k\eta}{\rho D}.$$

Clearly, the critical velocity v_c will be large if η is large, and ρ and D are small. So we can conclude that

(*i*) The flow of liquids of higher viscosity and lower density through narrow pipes tends to be streamlined.

(*ii*) The flow of liquids of lower viscosity and higher density through broad pipes tends to become turbulent, because in that case the critical velocity will be very small.

10.29 REYNOLD'S NUMBER

42. ***What is Reynold's number ? What is its importance ?***

Reynold's number. *It is dimensionless parameter whose value decides the nature of flow of a liquid through a pipe. It is given by*

$$R_e = \frac{\rho v D}{\eta}$$

where ρ = density of the liquid

v = velocity of the liquid

η = coefficient of viscosity of the liquid

D = diameter of the pipe.

Importance of Reynold's number. If R_e lies between 0 and 2000, the liquid flow is streamlined or laminar. If $R_e > 3000$, the liquid flow is turbulent. If R_e

lies between 2000 and 3000, the flow of liquid is unstable, it may change from laminar to turbulent and *vice-versa*. The exact value at which turbulence sets in a fluid is called *critical Reynold's number*.

43. *Show that the Reynold's number represents the ratio of the inertial force per unit area to the viscous force per unit area.*

Physical significance of Reynold's number. Consider a narrow tube having a cross-sectional area A. Suppose a fluid flows through it with a velocity v for a time interval Δt.

$$\text{Length of the fluid} = \text{Velocity} \times \text{time} = v\,\Delta t$$

Volume of the fluid flowing through the tube in time $\Delta t = Av\,\Delta t$

Mass of the fluid,

$$\Delta m = \text{Volume} \times \text{density} = Av\,\Delta t \times \rho$$

Inertial force acting per unit area of the fluid

$$= \frac{F}{A} = \frac{\text{Rate of change of momentum}}{A}$$

$$= \frac{\Delta m \times v}{\Delta t \times A} = \frac{Av\,\Delta t\,\rho \times v}{\Delta t \times A} = \rho v^2$$

Viscous force per unit area of the fluid

$$= \eta \times \text{velocity gradient} = \eta\,\frac{v}{D}$$

$$\frac{\text{Inertial force per unit area}}{\text{Viscous force per unit area}} = \frac{\rho v^2}{\eta\, v / D} = \frac{\rho\, vD}{\eta} = R_e$$

Thus Reynold's number represents the ratio of the inertial force per unit area to the viscous force per unit area.

Examples based on Reynold's Number

FORMULAE USED

1. For a liquid of viscosity η, density ρ and flowing through a pipe of diameter D, Reynold's number is given by
$$R_e = \frac{\rho v D}{\eta}$$
2. Flow is laminar for R_e between 0 and 2000. The fluid velocity corresponding to $R_e = 2000$ is called critical velocity.
$$v_c = \frac{2000 \times \eta}{\rho\, D}$$
3. Flow is turbulent for R_e above 3000.
4. Flow is unstable for R_e between 2000 and 3000.

UNITS USED

Density ρ is in kg m^{-3}, diameter D in metre, viscosity η in Pa s and Reynold's number is dimensionless.

EXAMPLE 49. *Verify that the quantity $\frac{\rho v D}{\eta}$ (Reynold's number) is dimensionless.*

Solution. Dimensions of various quantities are

$$[\rho] = ML^{-3}, [v] = LT^{-1}, [D] = L, [\eta] = ML^{-1}T^{-1}$$

$$\therefore \quad \left[\frac{\rho v D}{\eta}\right] = \frac{ML^{-3}.\, LT^{-1}.\, L}{ML^{-1}T^{-1}} = M^{\circ}L^{\circ}T^{\circ}$$

Hence Reynold's number $\rho v D/\eta$ is dimensionless.

EXAMPLE 50. *What should be the average velocity of water in a tube of radius 0.005 m so that the flow is just turbulent ? The viscosity of water is 0.001 Pa s.*

Solution. Here $D = 0.010$ m,

$$\eta = 0.001 \text{ Pa s}, \quad \rho = 1000 \text{ kgm}^{-3}$$

For flow to be just turbulent, $R_e = 3000$

$$\therefore \quad v = \frac{R_e\,\eta}{\rho\, D} = \frac{3000 \times 0.001}{1000 \times 0.010} = \mathbf{0.3\ ms^{-1}}.$$

EXAMPLE 51. *The flow rate of water from a tap of diameter 1.25 cm is 0.48 L/min. The coefficient of viscosity of water is 10^{-3} Pa s. After some time the flow rate is increased to 3 L/min. Characteristise the flow for both the flow rates.* **[NCERT]**

Solution. $D = 1.25 \text{ cm} = 1.25 \times 10^{-2}$ m,

$$\eta = 10^{-3} \text{ Pa s}, \rho = 10^3 \text{ kg m}^{-3}$$

The volume of water flowing out per second is

$$Q = va = v \times \frac{\pi D^2}{4}$$

$$\therefore \quad \text{Speed of flow,} \quad v = \frac{4\,Q}{\pi\, D^2}$$

Reynold's number,

$$R_e = \frac{\rho\, vD}{\eta} = \frac{\rho\, D}{\eta} \cdot \frac{4\,Q}{\pi\, D^2} = \frac{4\,\rho Q}{\pi\, D\eta}$$

When $\quad Q = 0.48 \text{ L/min}$

$$= \frac{0.48 \times 10^{-3} \text{ m}^3}{60 \text{ s}} = 8 \times 10^{-6} \text{ m}^3 \text{ s}^{-1}$$

$$R_e = \frac{4 \times 10^3 \times 8 \times 10^{-6}}{3.14 \times 1.25 \times 10^{-2} \times 10^{-3}} = 815$$

As $R_e < 1000$, the flow is steady.

When $\quad Q = 3\text{L/min} = \dfrac{3 \times 10^{-3} \text{ m}^3}{60 \text{ s}}$

$$= 5 \times 10^{-5} \text{ m}^3 \text{ s}^{-1}$$

$$R_e = \frac{4 \times 10^3 \times 5 \times 10^{-5}}{3.14 \times 1.25 \times 10^{-2} \times 10^{-3}} \simeq 5096$$

As $R_e > 3000$, so the flow will be turbulent.

PROBLEMS FOR PRACTICE

1. What should be the maximum average velocity of water in a tube of diameter 0.5 cm so that the flow is laminar ? The viscosity of water is 0.00125 Ns m^{-2}.
(**Ans.** 0.5 ms^{-1})

2. Water is flowing in a pipe of radius 1.5 cm with an average velocity of 15 cms^{-1}. What is the nature of flow ? Given coefficient of viscosity of water is 10^{-3} kg m^{-1} s^{-1} and its density is 10^3 kgm^{-3}.
(**Ans.** $R_c = 4500 > 3000$, flow is turbulent)

3. Water flows at a speed of 6 cms^{-1} through a pipe of tube of radius 1 cm. Coefficient of viscosity of water at room temperature is 0.01 poise. What is the nature of flow ?
(**Ans.** $R_c = 1200 < 2000$, so flow is laminar)

4. Find the critical velocity for air flowing through a tube of 2 cm diameter. For air, $\rho = 1.3 \times 10^{-3}$ g cm^{-3} and $\eta = 181 \times 10^{-6}$ poise. (**Ans.** 140 cms^{-1})

HINTS

1. $v_c = \frac{R_c \eta}{\rho D} = \frac{2000 \times 0.00125}{1000 \times 0.5 \times 10^{-2}} = \mathbf{0.5\ ms^{-1}}$.

2. Here $D = 2 \times 1.5$ cm $= 3.0 \times 10^{-2}$ m

$v = 15$ cm $s^{-1} = 0.15$ ms^{-1}, $\eta = 10^{-3}$ kg m^{-1} s^{-1}

$$R_c = \frac{\rho v D}{\eta} = \frac{10^3 \times 0.15 \times 3.0 \times 10^{-2}}{10^{-3}} = 4500.$$

10.30 IDEAL FLUID

44. *What is meant by an ideal fluid ?*

Ideal fluid. The motion of real fluids is very complicated. To understand fluid dynamics in a simpler manner, we assume that the fluid is ideal. *An ideal fluid is one which is non-viscous, incompressible, and its flow is steady and irrotational.* Thus an ideal fluid has the following features connected with its flow :

(*i*) **Steady flow.** In a steady flow, the fluid velocity at each point does not change with time, either in magnitude or direction.

(*ii*) **Incompressible flow.** The density of the fluid remains constant during its flow.

(*iii*) **Non-viscous flow.** The fluid offers no internal friction. An object moving through this fluid does not experience a retarding force.

(*iv*) **Irrotational flow.** This means that there is no angular momentum of the fluid about any point. A very small wheel placed at any point inside such a fluid does not rotate about its centre of mass.

10.31 EQUATION OF CONTINUITY

45. *Obtain the equation of continuity for the incompressible non-viscous fluid having a steady flow through a pipe.*

Equation of continuity. Consider a non-viscous and incompressible liquid flowing steadily between the sections A and B of a pipe of varying cross-section. Let a_1 be the area of cross-section, v_1 fluid velocity, ρ_1 fluid density at section A ; and the values of corresponding quantities at section B be a_2, v_2 and ρ_2.

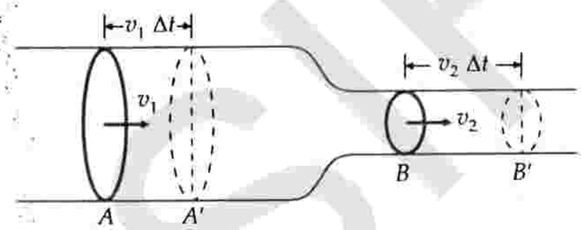


Fig. 10.31 Equation of continuity.

As m = Volume × density
= Area of cross-section × length × density

∴ Mass of fluid that flows through section A in time Δt,

$$m_1 = a_1 v_1 \Delta t \rho_1$$

Mass of fluid that flows through section B in time Δt,

$$m_2 = a_2 v_2 \Delta t \rho_2$$

By conservation of mass,

$$m_1 = m_2$$

or $$a_1 v_1 \Delta t \rho_1 = a_2 v_2 \Delta t \rho_2$$

As the fluid is incompressible, so $\rho_1 = \rho_2$, and hence

$$a_1 v_1 = a_2 v_2 \quad \text{or} \quad av = \text{constant}.$$

This is *the* ***equation of continuity.*** *It states that during the streamlined flow of the non-viscous and incompressible fluid through a pipe of varying cross-section, the product of area of cross-section and the normal fluid velocity (av) remains constant throughout the flow.*

For Your Knowledge

- The equation of continuity is a special case of the law of conservation of mass.
- The equation of continuity shows that $v \propto 1/a$, *i.e., the liquid velocity at any section of the pipe is inversely proportional to the area of cross-section of the pipe at that section.* This explains why the speed of water emerging from a PVC pipe increases when we press its outlet with our fingers and hence decrease its area of cross-section.

46. *Why does deep water run slow ?*

Deep water runs slowly. As the depth of water in a river or a stream increases, the area of cross-section available to the flowing water increases. Consequently, velocity decreases in accordance with the equation of continuity. Thus deep water runs slowly.

10.32 ENERGY OF A FLUID IN A STEADY FLOW

47. *What are different forms of energy possessed by a flowing liquid ? Write expressions for them.*

Energies possessed by a flowing liquid. A liquid in a steady flow can have three kinds of energy (*i*) kinetic energy (*ii*) potential energy and (*iii*) pressure energy.

(*i*) ***Kinetic energy.*** *The energy possessed by a liquid by virtue of its motion is called its kinetic energy.*

$$\text{K.E.} = \frac{1}{2} mv^2$$

where m is the mass of the liquid and v is the velocity of the liquid.

K.E. per unit mass of the liquid $= \frac{1}{2} v^2$

The kinetic energy per unit weight of the liquid is known as *the velocity head.*

$$\therefore \quad \text{Velocity head} = \frac{v^2}{2g}$$

$$\text{K.E. per unit volume} = \frac{1}{2}\frac{mv^2}{V} = \frac{1}{2}\rho v^2$$

(*ii*) ***Potential energy.*** *The energy possessed by a liquid by virtue of its position above the earth's surface is called its potential energy.*

$$\text{P.E.} = mgh$$

where h is the average height of the liquid from the ground level.

P.E. per unit mass of the liquid $= gh$

The potential energy per unit weight of the liquid is known as the *potential head.*

$$\therefore \quad \text{Potential head} = \frac{mgh}{mg} = h$$

$$\text{P.E. per unit volume} = \frac{mgh}{V} = \rho gh$$

(*iii*) ***Pressure energy.*** *The energy possessed by a liquid by virtue of its pressure is called its pressure energy.* A liquid under pressure can do work and so possesses energy.

For example, a liquid in a cylinder can drive a piston as shown in Fig. 10.32. Let P be the pressure exerted by the liquid on a frictionless piston of area a. Suppose the piston moves through distance x under the pressure P.

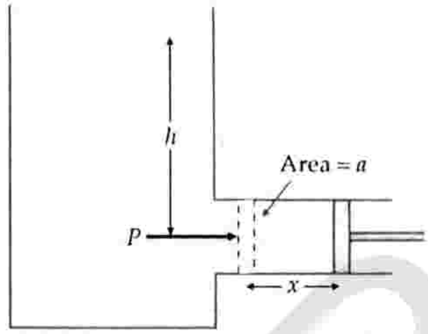


Fig. 10.32 Pressure energy of a liquid.

The work done is

W = Force × distance = Pressure × area × distance

$= Pax = PV$

where $V = ax$ = volume swept by the piston.

This work done is stored as the pressure energy of liquid of volume V.

$\therefore$ Pressure energy of volume $V = PV$

Pressure energy per unit volume

$$= \frac{PV}{V} = P = \text{Excess pressure}$$

$$\text{Pressure energy per unit mass} = \frac{PV}{m} = \frac{P}{\rho}$$

Pressure energy per unit weight of the liquid is called *pressure head.*

$$\text{Pressure head} = \frac{P}{\rho g}$$

10.33 BERNOULLI'S PRINCIPLE

48. *State and prove Bernoulli's principle for the flow of non-viscous fluids. Give its limitations.*

Bernoulli's principle. The Swiss physicist *Daniel Bernoulli* first derived an expression relating the pressure to fluid speed and height in 1738. His result, called *Bernoulli's principle* is based on the law of conservation of energy and applies to ideal fluids.

Bernoulli's principle *states that the sum of pressure energy, kinetic energy and potential energy per unit volume of an incompressible, non-viscous fluid in a streamlined irrotational flow remains constant along a streamline.*

Mathematically, it can be expressed as

$$P + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}$$

Proof. Consider a non-viscous and incompressible fluid flowing steadily between the sections A and B of a pipe of varying cross-section. Let a_1 be the area of cross-section at A, v_1 the fluid velocity, P_1 the fluid pressure, and h_1 the mean height above the ground level. Let a_2, v_2, P_2 and h_2 be the values of the corresponding quantities at B.

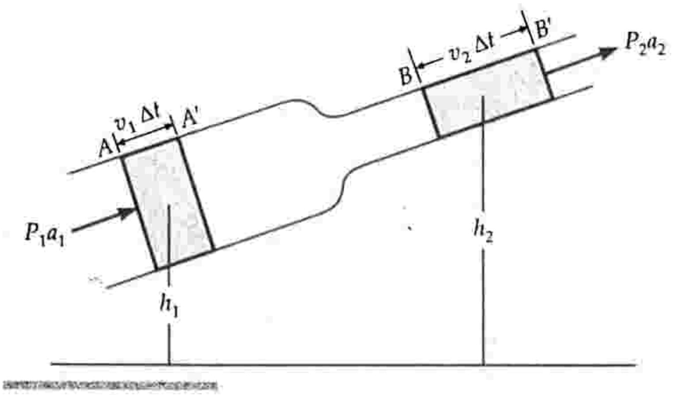


Fig. 10.33 Derivation of Bernoulli's principle.

Let ρ be the density of the fluid. As the fluid is incompressible, so whatever mass of fluid enters the pipe at section A in time Δt, an equal mass of fluid flows out at section B in time Δt. This mass is given by

m = Volume × density

= Area of cross-section × length × density

or $\quad m = a_1\, v_1\, \Delta t\, \rho = a_2\, v_2\, \Delta t\, \rho$ ...(1)

or $\quad a_1 v_1 = a_2 v_2$ ...(2)

∴ Change in K.E. of the fluid

= K.E. at B − K.E. at A

$= \frac{1}{2} m(v_2^2 - v_1^2) = \frac{1}{2}\, a_1\, v_1\, \Delta t\, \rho\, (v_2^2 - v_1^2)$

[Using (1)]

Change in P.E. of the fluid

= P.E. at B − P.E. at A

$= mg\,(h_2 - h_1) = a_1\, v_1\, \Delta t\, \rho\, g\,(h_2 - h_1)$ [Using (1)]

Net work done on the fluid

= Work done on the fluid at A

− Work done by the fluid at B

$= P_1\, a_1 \times v_1\, \Delta t - P_2\, a_2 \times v_2\, \Delta t$

$= P_1\, a_1\, v_1\, \Delta t - P_2\, a_1\, v_1\, \Delta t$ [Using (2)]

$= a_1\, v_1\, \Delta t\, (P_1 - P_2)$

By conservation of energy,

Net work done on the fluid

= Change in K.E. of the fluid

+ Change in P.E. of the fluid

$\therefore\ a_1\, v_1\, \Delta t\, (P_1 - P_2)$

$= \frac{1}{2} a_1 v_1 \Delta t\, \rho\, (v_2^2 - v_1^2) + a_1 v_1 \Delta t \rho g\, (h_2 - h_1)$

Dividing both sides by $a_1 v_1\, \Delta t$, we get

$$P_1 - P_2 = \frac{1}{2}\rho\, v_2^2 - \frac{1}{2}\rho\, v_1^2 + \rho\, g h_2 - \rho\, g h_1$$

or $$P_1 + \frac{1}{2}\rho\, v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho\, v_2^2 + \rho\, g\, h_2$$

or $$P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant} \qquad ...(3)$$

This proves Bernoulli's principle according to which the total energy per unit volume remains constant. Equation (3) can also be written as

$$\frac{P}{\rho g} + \frac{1}{2}\frac{v^2}{g} + h = \text{constant.}$$

This is another form of *Bernoulli's principle according to which the sum of pressure head, velocity head and gravitational head remains constant in the streamline flow of an ideal fluid.*

Limitations of Bernoulli's equation :

1. Bernoulli's equation ideally applies to fluids with zero viscosity or non-viscous fluids. In case of viscous fluids, we need to take into account the work done against viscous drag.
2. Bernoulli's equation has been derived on the assumption that there is no loss of energy due to friction. But in practice, when fluids flow, some of their kinetic energy gets converted into heat due to the work done against the internal forces of friction or viscous forces.
3. Bernoulli's equation is applicable only to incompressible fluids because it does not take into account the elastic energy of the fluids.
4. Bernoulli's equation is applicable only to streamline flow of a fluid and not when the flow is turbulent.
5. Bernoulli's equation does not take into consideration the angular momentum of the fluid. So it cannot be applied when the fluid flows along a curved path.

For Your Knowledge

- Bernoulli's principle is a fundamental principle of fluid dynamics based on the law of conservation of energy.
- In Bernoulli's equation : $P + \rho g h + \frac{1}{2}\rho v^2$ = constant, the term $(P + \rho g h)$ is called *static pressure*, because it is the pressure of the fluid even if it is at rest, and the term $\frac{1}{2}\rho v^2$ is the *dynamic pressure* of the fluid which is the pressure by virtue of its velocity v. So Bernoulli's equation can be written as

  Static pressure + Dynamic pressure = Constant
- If a liquid is flowing through a horizontal tube, h remains constant and we can write

  $$P + \frac{1}{2}\rho v^2 = \text{constant}$$

  This shows that if v increases, P decreases and vice versa. Thus for the *streamline flow of an ideal liquid flowing horizontally, the pressure decreases where velocity increases and vice versa*. This is an important aspect of Bernoulli's principle which finds many applications.

We now consider some useful applications and phenomena which are based on Bernoulli's principle.

10.34 TORRICELLI'S LAW OF EFFLUX

49. *Apply Bernoulli's principle to determine the speed of efflux from the side of a container both when its top is closed and open. Hence derive Torricelli's law.*

Speed of efflux. The word efflux means the outflow of a fluid. As shown in Fig. 10.34, consider a tank containing a liquid of density ρ with a small hole on its side at a height y_1 from the bottom. Let y_2 be the height of the liquid surface from the bottom and P be the air pressure above the liquid surface.

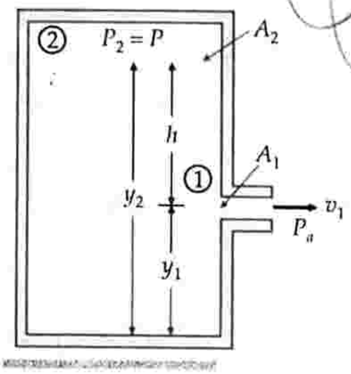


Fig. 10.34 Torricelli's law.

If A_1 and A_2 are the cross-sectional areas of the side hole and the tank respectively, and v_1 and v_2 are the liquid velocities at points 1 and 2, then from the equation of continuity, we get

$$A_1 v_1 = A_2 v_2 \quad \text{or} \quad v_2 = \frac{A_1}{A_2} v_1$$

As $A_2 >> A_1$, so the liquid may be taken at rest at the top, *i.e.*, $v_2 \simeq 0$. Applying Bernoulli's equation at points 1 and 2, we get

$$P_a + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P + \rho g y_2$$

or $$\frac{1}{2}\rho v_1^2 = \rho g (y_2 - y_1) + (P - P_a)$$

If we take $y_2 - y_1 = h$, then

$$\frac{1}{2}\rho v_1^2 = \rho g h + (P - P_a)$$

or $$v_1 = \sqrt{2gh + \frac{2(P - P_a)}{\rho}}$$

Special cases (*i*) When $P >> P_a$, the term $2gh$ may be ignored.

$$v_1 = \sqrt{\frac{2(P - P_a)}{\rho}}.$$

Thus the speed of efflux is determined by container pressure P. Such a situation exists in rocket propulsion.

(*ii*) *When the tank is open to the atmosphere,*

$$P = P_a \quad \text{and} \quad v_1 = \sqrt{2gh}$$

Thus, *the velocity of efflux of a liquid is equal to the velocity which a body acquires in falling freely from the free liquid surface to the orifice. This result is called* ***Torricelli's law.***

10.35 THE VENTURIMETER

50. *What is a venturimeter ? Describe its construction and working.*

Venturimeter. *It is a device used to measure the rate of flow of a liquid through a pipe. It is an application of Bernoulli's principle.* It is also called **flow meter** or **venturi tube**.

Construction. It consists of a horizontal tube having wider opening of cross-section a_1 and a narrow neck of cross-section a_2. These two regions of the horizontal tube are connected to a manometer, containing a liquid of density ρ_m.

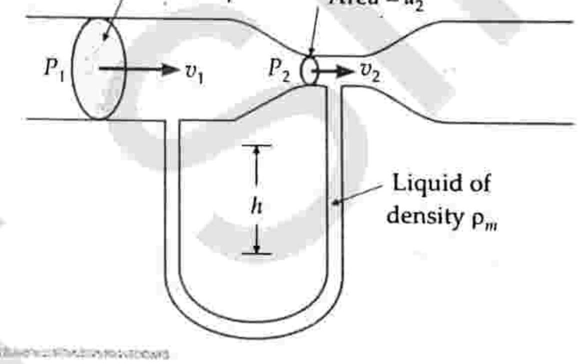


Fig. 10.35 Venturi tube.

Working. Let the liquid velocities be v_1 and v_2 at the wider and the narrow portions. Let P_1 and P_2 be the liquid pressures at these regions. By the equation of continuity,

$$a_1 v_1 = a_2 v_2 \quad \text{or} \quad \frac{a_1}{a_2} = \frac{v_2}{v_1}$$

If the liquid has density ρ and is flowing horizontally, then from Bernoulli's equation,

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

or $$P_1 - P_2 = \frac{1}{2}\rho (v_2^2 - v_1^2) = \frac{1}{2}\rho v_1^2 \left(\frac{v_2^2}{v_1^2} - 1\right)$$

$$= \frac{1}{2}\rho v_1^2 \left(\frac{a_1^2}{a_2^2} - 1\right) \qquad \left[\because \frac{v_2}{v_1} = \frac{a_1}{a_2}\right]$$

$$= \frac{1}{2}\rho v_1^2 \left(\frac{a_1^2 - a_2^2}{a_2^2}\right)$$

If h is the height difference in the two arms of the manometer tube, then

$$P_1 - P_2 = h \rho_m g$$

$$\therefore \quad h \rho_m g = \frac{1}{2}\rho v_1^2 \left(\frac{a_1^2 - a_2^2}{a_2^2}\right)$$

$$\therefore \quad v_1 = \sqrt{\frac{2h\rho_m g}{\rho} \times \frac{a_2^2}{a_1^2 - a_2^2}}$$

Volume of the liquid flowing out per second,

$$Q = a_1 v_1 = a_1 a_2 \sqrt{\frac{2h\rho_m g}{\rho (a_1^2 - a_2^2)}}.$$

10.36 ATOMIZER OR SPRAYER

51. *Briefly describe the working of an atomizer on the basis of Bernoulli's principle.*

Atomizer. The working of an atomizer which is used to spray liquids is based on Bernoulli's principle. Fig. 10.36 shows the essential parts of an atomizer. When the rubber balloon is pressed, the air rushes out

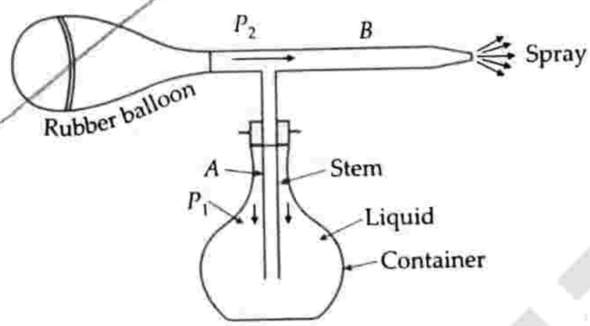


Fig. 10.36 Atomizer.

of the horizontal tube B decreasing the pressure to P_2 which is less than the atmospheric pressure P_1 in the container. As a result, the liquid rises up in the vertical tube A. When it collides with the high speed air in tube B, it breaks up into a fine spray.

10.37 DYNAMIC LIFT

52. *What is dynamic lift ? If a ball is thrown and given a spin, then the path of the ball is curved more than in a usual spin free ball. Why ?*

Dynamic lift. *Dynamic lift is the force that acts on a body*, such as aeroplane wing, a hydrofoil or a spinning ball, *by virtue of its motion through a fluid.* It is responsible for the curved path of a spinning ball and the lift of an aircraft wing.

Curved path of a spinning ball : Magnus effect. When a ball is thrown horizontally with a large velocity and at the same time given a twisting motion to cause a spin, it deviates from its usual parabolic trajectory of spin free motion. This deviation can be explained on the basis of Bernoulli's principle.

When the ball spins about an axis perpendicular to its horizontal motion, it carries with itself an air of layer due to viscous drag. The streamlines around it are in the form of concentric circles, as shown in Fig. 10.37(a). When the ball moves forward with velocity v, the air ahead of the ball rushes backward with velocity v to fill the space left empty by the ball. Thus the streamlines in air due to translatory motion of the ball are of the form shown in Fig. 10.37(b). The layer above the ball moves in a direction opposite to that of the spinning ball, so the resultant velocity decreases and hence pressure increases in accordance with Bernoulli's principle. The layer below the ball moves in the direction of spin, the resultant velocity increases and hence pressure decreases. Due to the difference of pressure on the two sides of the ball, the ball curves downwards in the direction of spin, as shown in Fig. 10.37(c).

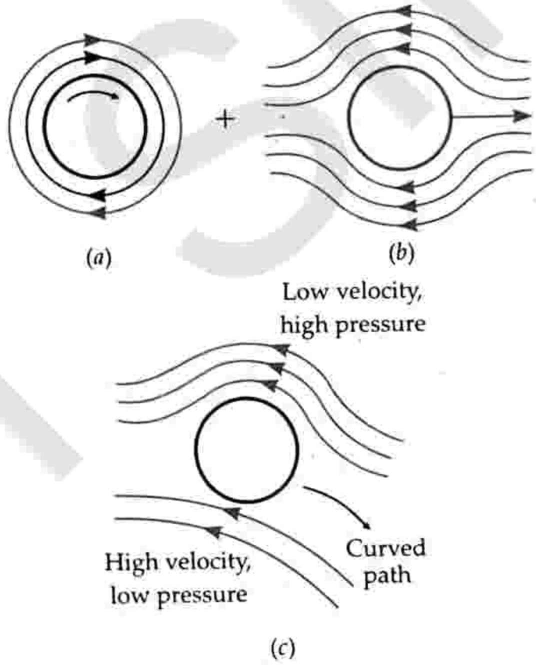


Fig. 10.37

The difference in lateral pressure, which causes a spinning ball to take a curved path which is convex towards the greater pressure side, is called magnus effect. This effect was first noticed by German scientist H.G. Magnus in the mid-nineteenth century. The rougher the surface, the thicker is the layer of air dragged along by the spinning ball, and more curved the path.

53. *On the basis of Bernoulli's principle, explain the lift of an aircraft wing.*

Aerofoil : Lift of an aircraft wing. *Aerofoil is the name given to a solid object shaped to provide an upward vertical force as it moves horizontally through air.* This upward force (dynamic lift) makes aeroplanes fly.

As shown in Fig. 10.38, the cross-section of the wing of an aeroplane looks like an aerofoil. The wing is so designed that its upper surface is more curved (and hence longer) than the lower surface and the front edge is broader than the rear edge. As the aircraft moves, the air moves faster over the upper surface of the wing than on the bottom. According to Bernoulli's principle, the air pressure above the upper surface decreases below the atmospheric pressure and that on the lower surface increases above the atmospheric pressure. The difference in pressure provides an upward lift, called *dynamic lift*, to the aircraft.

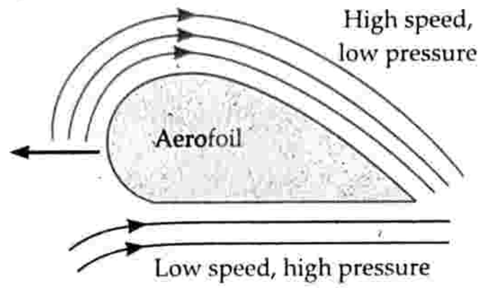


Fig. 10.38 Aerofoil.

10.38 BLOOD FLOW AND HEART ATTACK

54. *How does Bernoulli's principle help in explaining vascular flutter and heart attack ?*

Blood flow and heart attack. In persons suffering with advanced heart condition, the artery gets constricted due to the accumulation of plaque on its inner walls. In order to drive the blood through this constriction, a greater demand is placed on the activity of the heart. The speed of blood flow increases in this region. From Bernoulli's principle, the inside pressure drops and the artery may collapse due to external pressure. The heart exerts further pressure to open this artery and forces the blood through. As the blood rushes through the opening, the internal pressure once again drops leading to a repeat collapse. This phenomenon is called vascular flutter which can be heard on a stethoscope. This may result in a heart attack.

10.39 BLOWING OFF THE ROOFS DURING WIND STORM

55. *Why are the roofs of some houses blown off during a wind storm ?*

Blowing off the roof during wind storm. During certain wind storm or cyclone, the roofs of some houses are blown off without damaging the other parts of the house. The high wind blowing over the roof creates a low pressure P_2 in accordance with Bernoulli's principle. The pressure P_1 below the roof is equal to the atmospheric pressure which is larger than P_2. The difference of pressure $(P_1 - P_2)$ causes an upward thrust and the roof is lifted up. Once the roof is lifted up, it is blown off with the wind.

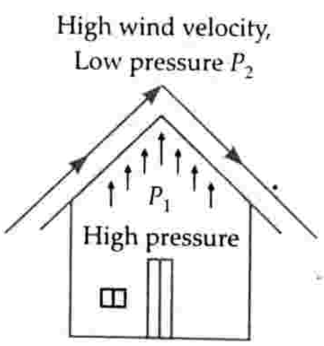


Fig. 10.39 Blowing off the roof during a wind storm.

Examples based on Equation of Continuity and Bernoulli's Theorem

FORMULAE USED

1. Volume of a liquid flowing per second through a pipe of cross-section a with velocity v, $Q = av$
2. Equation of continuity, $av = \text{constant}$

   or $a_1 v_1 = a_2 v_2$
3. First form of Bernoulli's theorem,
   $$\frac{P}{\rho} + gh + \frac{1}{2}v^2 = \text{constant}$$
   or Pressure energy per unit mass + P.E. per unit mass + K.E. per unit mass = constant.
4. Second form of Bernoulli's theorem
   $$\frac{P}{\rho g} + h + \frac{v^2}{2g} = \text{constant}$$
   or Pressure head + Gravitational head + Velocity head = constant.
5. Volume of a liquid flowing out per second through a venturimeter,
   $$Q = a_1 a_2 \sqrt{\frac{2gh}{a_1^2 - a_2^2}}$$
   where a_1 and a_2 are the areas of cross-sections of bigger and smaller tubes respectively.
6. Torricell's theorem, velocity of efflux of a liquid through an orifice at depth h from the liquid surface, $v = \sqrt{2gh}$

UNITS USED

Rate of flow Q is in $m^3 s^{-1}$, velocity v in ms^{-1}, area of cross-section a in m^2, pressure P in Nm^{-2}, density ρ in kgm^{-3} and height h in metre.

EXAMPLE 52. *Water flows through a horizontal pipe of varying area of cross-section at the rate of 10 cubic metre per minute. Determine the velocity of water at a point where radius of pipe is 10 cm.*

Solution. Rate of flow,

$$Q = \frac{\text{Volume}}{\text{Time}} = \frac{10\text{ m}^3}{60\text{ s}} = \frac{1}{6}\text{ m}^3\text{ s}^{-1}$$

Radius, $r = 10\text{ cm} = 0.1\text{ m}$

As $Q = av = \pi r^2 v$

$\therefore$ $$v = \frac{Q}{\pi r^2} = \frac{1 \times 7}{6 \times 22 \times (0.1)^2} = \mathbf{5.303\ ms^{-1}}.$$

EXAMPLE 53. *Water flows through a horizontal pipe whose internal diameter is 2.0 cm, at a speed of 1.0 ms^{-1}. What should be the diameter of the nozzle, if the water is to emerge at a speed of 4.0 ms^{-1}?* **[Central Schools 04]**

Solution. Here $d_1 = 2\text{ cm} = 0.02\text{ m}$, $v_1 = 1\text{ ms}^{-1}$, $v_2 = 4\text{ ms}^{-1}$, $d_2 = ?$

Using equation of continuity,

$$a_1 v_1 = a_2 v_2$$

or $$\frac{\pi d_1^2}{4} \times v_1 = \frac{\pi d_2^2}{4} \times v_2$$

or $$d_2^2 = \frac{v_1}{v_2} \times d_1^2 = \frac{1}{4} \times (0.02)^2 = (0.01)^2$$

or $$d_2 = 0.01\text{ m} = \mathbf{1.0\ cm.}$$

EXAMPLE 54. *At what speed will the velocity head of stream of water be 40 cm ?*

Solution. Here $h = 40$ cm, $g = 980\text{ cms}^{-2}$

Velocity head, $h = \dfrac{v^2}{2g}$

$$\therefore \quad v = \sqrt{2gh} = \sqrt{2 \times 980 \times 40} = \mathbf{280\ cms^{-1}.}$$

EXAMPLE 55. *At what speed will the velocity of a stream of water be equal to 20 cm of mercury column ? Take $g = 10\ ms^{-2}$.*

Solution. Velocity head $= 20$ cm of Hg

$= 20 \times 13.6$ cm of water

But velocity head $= \dfrac{v^2}{2g}$

$$\therefore \quad 20 \times 13.6 = \frac{v^2}{2 \times 1000}$$

or $$v = \sqrt{20 \times 13.6 \times 2 \times 1000} = 737.56\text{ cms}^{-1}$$

$$= \mathbf{7.3756\ ms^{-1}.}$$

EXAMPLE 56. *Calculate the total energy possessed by one kg of water at a point where the pressure is 20 gf / mm^2, velocity is 0.1 ms^{-1} and the height is 50 cm above the ground level.*

Solution. Here

$$P = 20\text{ gf mm}^{-2} = \frac{20}{1000} \times (10^{-3})^{-2}\text{ kg f m}^{-2}$$

$$= 20 \times 10^3 \times 9.8\text{ Nm}^{-2} = 19.6 \times 10^4\text{ Nm}^{-2}$$

$v = 0.1\text{ ms}^{-1}$, $h = 50\text{ cm} = 0.50\text{ m}$,

$\rho = 10^3\text{ kg m}^{-3}$

Pressure energy per kg $= \dfrac{P}{\rho} = \dfrac{19.6 \times 10^4}{10^3} = 196$ J

Gravitational P.E. per kg $= gh = 9.8 \times 0.50 = 4.90$ J

K.E. per kg $= \dfrac{1}{2} v^2 = \dfrac{1}{2} \times (0.1)^2 = 0.005$ J

Total energy possessed by per kg of water

$$= \frac{P}{\rho} + gh + \frac{1}{2} v^2 = 196 + 4.90 + 0.005 = \mathbf{200.905\ J.}$$

EXAMPLE 57. *The reading of pressure meter attached with a closed pipe is $3.5 \times 10^5\ Nm^{-2}$. On opening the valve of the pipe, the reading of the pressure meter is reduced to $3.0 \times 10^5\ Nm^{-2}$. Calculate the speed of the water flowing in the pipe.*

Solution. *Before opening the valve :*

$$p_1 = 3.5 \times 10^5\text{ Nm}^{-2}, \quad v_1 = 0$$

After opening the valve :

$$p_2 = 3.0 \times 10^5\text{ Nm}^{-2}, \quad v_2 = ?$$

In horizontal flow, P.E. remains unchanged. So Bernoulli's theorem can be written as

$$p_2 + \frac{1}{2}\rho v_2^2 = p_1 + \frac{1}{2}\rho v_1^2$$

$$3.0 \times 10^5 + \frac{1}{2} \times 10^3 \times v_2^2 = 3.5 \times 10^5 + \frac{1}{2} \times 10^3 \times (0)^2$$

$$\frac{1}{2} \times 10^3 \times v_2^2 = (3.5 - 3.0) \times 10^5 = 0.5 \times 10^5$$

or $$v_2^2 = 2 \times 0.5 \times 10^2 = 100$$

or $$v_2 = \mathbf{10\ ms^{-1}.}$$

EXAMPLE 58. *A fully loaded Boeing aircraft 747 has a mass of 3.3×10^5 kg. Its total wing area is 500 m^2. It is in level flight with a speed of 960 km/h. (a) Estimate the pressure difference between the lower and upper surfaces of the wings. (b) Estimate the fractional increase in the speed of the air on the upper surface of the wing relative to the lower surface. The density of air is $\rho = 1.2\ kg\ m^{-3}$ and $g = 9.81\ ms^{-2}$.* **[NCERT]**

Solution. (*a*) For the Boeing aircraft in level flight, upward force due to the pressure difference = weight of the aircraft

or $$\Delta p \times A = mg$$

or $$\Delta p = \frac{mg}{A} = \frac{3.3 \times 10^5 \times 9.81}{500}$$

$$= 6.47 \times 10^3\text{ Nm}^{-2} \approx \mathbf{6.5 \times 10^3\ Nm^{-2}.}$$

(*b*) If v_1 and v_2 are the speeds of air on the lower and the upper surfaces of the wings of the aircraft and p_1 and p_2 are the corresponding pressures, then from Bernoulli's principle, we have

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

or $$p_1 - p_2 = \frac{\rho}{2}(v_2^2 - v_1^2)$$

or $$\Delta p = \rho\left(\frac{v_2 + v_1}{2}\right)(v_2 - v_1) = \rho\ v_{av}\ (v_2 - v_1)$$

Here $v_{av} = \frac{v_2 + v_1}{2} = 960 \text{ km h}^{-1} = 267 \text{ ms}^{-1}$

$$\therefore \quad \frac{v_2 - v_1}{v_{av}} = \frac{\Delta p}{\rho\, v_{av}^2} = \frac{6.5 \times 10^3}{1.2 \times (267)^2} \simeq 0.08 = 8\%.$$

Thus the speed of air on the upper surface of the wing is about 8% higher than that below the lower surface.

EXAMPLE 59. *Calculate the minimum pressure required to force the blood from the heart to the top of the head (vertical distance = 50 cm). Assume the density of blood to be 1.04 g cm^{-3}. Friction is to be neglected.*

Solution. Here $h_2 - h_1 = 50$ cm, $\rho = 1.04$ g cm^{-3}

According to Bernouilli's theorem,

$$p_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$$

or
$$p_1 - p_2 = \rho g (h_2 - h_1) + \frac{1}{2}\rho (v_2^2 - v_1^2)$$

If $v_2 = v_1$,

then
$$p_1 - p_2 = \rho g (h_2 - h_1)$$
$$= 1.04 \times 981 \times 50 \text{ dyne cm}^{-2}$$
$$= \mathbf{5.1 \times 10^4 \text{ dyne cm}^{-2}}.$$

EXAMPLE 60. *Water is flowing through two horizontal pipes of different diameters which are connected together. In the first pipe the speed of water is 4 ms^{-1} and the pressure is 2.0×10^4 Nm^{-2}. Calculate the speed and pressure of water in the second pipe. The diameters of the pipes are 3 cm and 6 cm respectively.*

Solution. According to equation of continuity,

$$a_1 v_1 = a_2 v_2$$

or
$$\pi r_1^2 v_1 = \pi r_2^2 v_2 \quad \text{or} \quad v_2 = \left(\frac{r_1}{r_2}\right)^2 v_1$$

Here $r_1 = 1.5$ cm $= 1.5 \times 10^{-2}$ m,

$r_2 = 3$ cm $= 3 \times 10^{-2}$ m and $v_1 = 4$ ms^{-1}

$$\therefore \quad v_2 = \left(\frac{1.5 \times 10^{-2}}{3 \times 10^{-2}}\right)^2 \times 4 = \mathbf{1 \text{ ms}^{-1}}$$

Applying Bernouilli's theorem for horizontal flow,

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

or
$$p_2 = p_1 + \frac{1}{2}\rho (v_1^2 - v_2^2)$$

But ρ (water) $= 10^3$ kgm^{-3}, $v_1 = 4$ ms^{-1}, $v_2 = 1$ ms^{-1}, $p_1 = 2.0 \times 10^4$ Nm^{-2}

$$\therefore \quad p_2 = 2.0 \times 10^4 + \frac{1}{2} \times 10^3 \times (4^2 - 1^2)$$
$$= \mathbf{2.75 \times 10^4 \text{ Nm}^{-2}}.$$

EXAMPLE 61. *The cross-sectional area of water pipe entering the basement is 4×10^{-4} m^2. The pressure at this point is 3×10^5 Nm^{-2} and the speed of water is 2 ms^{-1}. This pipe tapers to a cross-sectional area of 2×10^{-4} m^2 when it reaches the second floor 8 m above. Calculate the speed and pressure at the second floor.*

Solution. Here $v_1 = 2$ ms^{-1}, $a_1 = 4 \times 10^{-4}$ m^2, $p_1 = 3 \times 10^5$ Nm^{-2} and $a_2 = 2 \times 10^{-4}$ m^2, $v_2 = ?$, $p_2 = ?$ $\rho = 1000$ kgm^{-3}, $(h_2 - h_1) = 8$ m

Using continuity equation,

$$a_1 v_1 = a_2 v_2$$

or
$$v_2 = \frac{a_1 v_1}{a_2} = \frac{4 \times 10^{-4} \times 2}{2 \times 10^{-4}} = \mathbf{4 \text{ ms}^{-1}}.$$

According to Bernouilli's theorem

$$\frac{p_1}{\rho} + \frac{1}{2}v_1^2 + g h_1 = \frac{p_2}{\rho} + g h_2 + \frac{1}{2}v_2^2$$

$$\frac{p_2}{\rho} = \frac{p_1}{\rho} - g(h_2 - h_1) - \frac{1}{2}(v_2^2 - v_1^2)$$

or
$$p_2 = p_1 - g\rho (h_2 - h_1) - \tfrac{1}{2}\rho (v_2^2 - v_1^2)$$
$$= 3 \times 10^5 - 9.8 \times 1000 \times 8 - \tfrac{1}{2} \times 1000(16 - 4)$$
$$= 3 \times 10^5 - 78.4 \times 1000 - \tfrac{1}{2} \times 12 \times 1000$$
$$= (3 - 0.784 - 0.06) \times 10^5 = \mathbf{2.156 \times 10^5 \text{ Nm}^{-2}}.$$

EXAMPLE 62. *The pressure difference between two points along a horizontal pipe, through which water is flowing, is 1.4 cm of mercury. If, due to non-uniform cross-section, the speed of flow of water at the point of greater cross-section is 60 cm s^{-1}, calculate the speed at the other point.*

Solution. Using Bernouilli's theorem for horizontal flow,

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

or
$$v_2^2 - v_1^2 = \frac{2}{\rho}(p_1 - p_2)$$

The speed of water will be greater at the place where the cross-section is smaller.

$$\therefore \quad v_2^2 = \frac{2}{\rho}(p_1 - p_2) + v_1^2$$

But $p_1 - p_2 = 1.4$ cm of mercury $= h\rho' g$
$$= 1.4 \times 10^{-2} \times 13.6 \times 10^3 \times 9.8$$
$$= 1.866 \times 10^3 \text{ Nm}^{-2}$$

Also

ρ (water) $= 10^3$ kgm^{-3}, $v_1 = 60$ cms^{-1} $= 0.6$ ms^{-1}

$$\therefore \quad v_2^2 = \frac{2}{10^3} \times 1.866 \times 10^3 + (0.6)^2 = 4.092$$

or $v_2 \simeq \mathbf{2 \text{ ms}^{-1}}$.

EXAMPLE 63. *A pitot tube is mounted on an aeroplane wing to measure the speed of the plane. The tube contains alcohol and shows a level difference of 40 cm. What is the speed of the plane relative to air ? (sp. gr. of alcohol = 0.8 and density of air = 1 kg m^{-3}).*

Solution. The plane of aperture at A is parallel to the direction of air flow, so velocity of air at A is same as that of air in main pipe (which is equal to the velocity v of the plane w.r.t. air. The plane of aperture at B is perpendicular to the direction of air flow, so velocity of air entering the tube is reduced to zero. Applying Bernoulli's theorem at points A and B,

$$p_A + \frac{1}{2}\rho v_A^2 = p_B + \frac{1}{2}\rho v_B^2$$

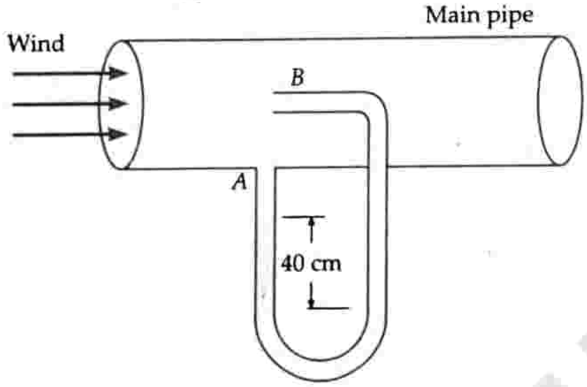


Fig. 10.40

But $v_1 = v$ and $v_B = 0$, so

$$\frac{1}{2}\rho v^2 = p_B - p_A = h\rho' g$$

where ρ' is the density of the liquid in the U-tube.

Hence $v = \sqrt{\dfrac{2h\rho' g}{\rho}}$

Now $h = 40\text{ cm} = 0.4\text{ m}$, $\rho' = 0.8\times 10^3\text{ kgm}^{-3}$, $\rho = 1\text{ kgm}^{-3}$

$$\therefore \quad v = \sqrt{\frac{2\times 0.4\times 0.8\times 10^3\times 9.8}{1}}$$

$$= 56\sqrt{2} = 56\times 1.414 = \mathbf{79.18\ ms^{-1}}.$$

EXAMPLE 64. *A pitot tube is fixed in a main pipe of diameter 20 cm and difference of pressure indicated by the gauge is 5 cm of water column. Find the volume of water passing through the main pipe in one minute.*

Solution. As both main pipe and U-tube contain water, so from the above example, we have

$$\frac{1}{2}\rho v^2 = h\rho g$$

or $$v = \sqrt{2gh} = \sqrt{2\times 9.8\times 0.05} = 0.7\sqrt{2}\text{ ms}^{-1}$$

Volume of water flowing per second,

$$Q = av = \pi r^2 v$$
$$= \frac{22}{7}\times(0.10)^2\times 0.7\sqrt{2} = 0.0311\text{ m}^3\text{ s}^{-1}$$

Volume of water passing in one minute,

$$V = Q\,t = 0.0311\times 60 = \mathbf{1.866\ m^3}.$$

EXAMPLE 65. *A cylinder of height 20 m is completely filled with water. Find the velocity of efflux of water (in ms^{-1}) through a small hole on the side wall of the cylinder near its bottom. Given $g = 10\ ms^{-2}$.* **[AIEEE 02]**

Solution. Here $h = 20$ m, $g = 10\text{ ms}^{-2}$

Velocity of efflux,

$$v = \sqrt{2gh} = \sqrt{2\times 10\times 20} = \mathbf{20\ ms^{-1}}.$$

EXAMPLE 66. *At what velocity does water emerge from an orifice in a tank in which gauge pressure is $3\times 10^5\ Nm^{-2}$ before the flow starts ? Density of water = 1000 kgm^{-3}.*

Solution. Here $p = 3\times 10^5\text{ Nm}^{-2}, \rho = 1000\text{ kgm}^{-3}$, $g = 9.8\text{ ms}^{-2}$

As $p = h\rho g$

$$\therefore \quad h = \frac{p}{\rho g} = \frac{3\times 10^5}{1000\times 9.8}\text{ m}$$

Velocity of efflux,

$$v = \sqrt{2gh} = \sqrt{\frac{2\times 9.8\times 3\times 10^5}{1000\times 9.8}}$$

$$= \sqrt{600} = \mathbf{24.495\ ms^{-1}}.$$

EXAMPLE 67. *A boat strikes an under water rock which punctures a hole 5 cm in diameter in its hull which is 1.5 m below the water line. At what rate in litre per second does water enter ?*

Solution. Here $r = \frac{5}{2}$ cm = 2.5 cm, $h = 1.5$ m = 150 cm

Velocity of efflux,

$$v = \sqrt{2gh} = \sqrt{2\times 980\times 150} = 140\sqrt{15}\text{ cms}^{-1}.$$

Rate at which water enters,

$$Q = av = \pi r^2 v = \frac{22}{7}\times(2.5)^2\times 140\sqrt{15}\text{ cm}^3\text{ s}^{-1}$$

$$= 2750\times 3.873\times 10^{-3}\text{ litres s}^{-1}$$

$$= \mathbf{10.65\ litres\ s^{-1}}.$$

EXAMPLE 68. *A drum of 30 cm radius has a capacity of 220 dm^3 of water. It contains 198 dm^3 of water and is placed on a solid block of exactly the same size as of drum. If a small hole is made at lower end of drum perpendicular to its length, find the horizontal range of water on the ground in the beginning. Given $g = 980\ cm\,s^{-2}$.*

Solution. Radius of the drum, $r = 30$ cm

Volume of the drum = 220 $\text{dm}^3 = 2.2\times 10^5\text{ cm}^3$

Let l be the height of the drum. Then

$$\pi r^2 l = 2.2\times 10^5$$

or $$l = \frac{2.2\times 10^5\times 7}{22\times 30\times 30} = \frac{700}{9}\text{ cm}$$

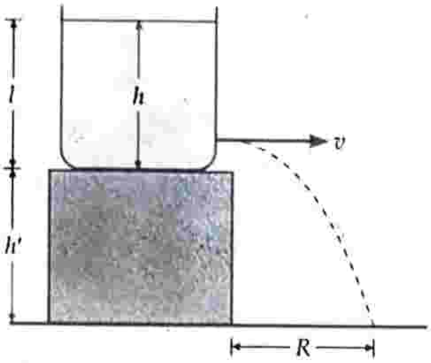


Fig. 10.41

Height of block $= h' = l = \frac{700}{9}$ cm

Volume of water $= 198\ \text{dm}^3 = 1.98 \times 10^5\ \text{cm}^3$

Let h be height of the water column in the drum. Then

$$\pi r^2 h = 1.98 \times 10^5$$

or $$h = \frac{1.98 \times 10^5 \times 7}{22 \times 30 \times 30}\ \text{cm} = 70\ \text{cm}$$

Time taken by water to reach the ground,

$$t = \sqrt{\frac{2h'}{g}}$$

Efflux velocity of water $= \sqrt{2gh}$

If R be the horizontal range, then

R = Time taken by water to reach the ground × Efflux velocity of water

$$= \sqrt{\frac{2h'}{g}} \times \sqrt{2gh} = 2\sqrt{hh'}$$

$$= 2\sqrt{70 \times \frac{700}{9}} = \mathbf{147.6\ cm.}$$

EXAMPLE 69. **Blood velocity :** *The flow of blood in a large artery of an anesthetized dog is diverted through a Venturi meter. The wider part of the meter has a cross-sectional area equal to that of the artery, $A = 8\ mm^2$. The narrower part has an area $a = 4\ mm^2$. The pressure drop in the artery is 24 Pa. What is the speed of the blood in the artery ?* **[NCERT]**

Solution. The Bernoulli's equation for the horizontal flow is

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

By equation of continuity,

$$Av_1 = av_2 \quad \text{or} \quad v_2 = Av_1 / a$$

$$\therefore \quad p_1 - p_2 = \frac{1}{2}\rho v_1^2 - \frac{1}{2}\frac{\rho A^2 v_1^2}{a^2}$$

$$= \frac{1}{2}\rho v_1^2\left(\frac{A^2}{a^2} - 1\right)$$

Here $p_1 - p_2 = 24$ Pa,

ρ (blood) $= 1.06 \times 10^3\ \text{kg m}^{-3}$, $A/a = 8/4 = 2$

$$\therefore \quad v_1 = \sqrt{\frac{2(p_1 - p_2)}{\rho\left(\frac{A^2}{a^2} - 1\right)}}$$

$$= \sqrt{\frac{2 \times 24}{1.06 \times 10^3 \times (2^2 - 1)}} = \mathbf{0.125\ ms^{-1}}.$$

EXAMPLE 70. *A horizontal tube has different cross-sectional areas at points A and B. The diameter of A is 4 cm and that of B is 2 cm. Two manometer limbs are attached at A and B. When a liquid of density 8.0 g cm^{-3} flows through the tube, the pressure difference between the limbs of the manometer is 8 cm. Calculate the rate of flow of the liquid in the tube.* **[ISM Dhanbad 82]**

Solution. Here $a_1 = \pi r_1^2 = \pi \times \left(\frac{4}{2}\right)^2 = 4\pi\ \text{cm}^2$,

$$a_2 = \pi r_2^2 = \pi\left(\frac{2}{2}\right)^2 = \pi\ \text{cm}^2,\ h = 8\ \text{cm},\ g = 980\ \text{cms}^{-2}$$

Rate of flow of the liquid,

$$Q = a_1 a_2 \sqrt{\frac{2gh}{a_1^2 - a_2^2}} = 4\pi \times \pi \sqrt{\frac{2 \times 980 \times 8}{(4\pi)^2 - (\pi)^2}}$$

$$= 4\pi \sqrt{\frac{2 \times 980 \times 8}{15}} = 4 \times 3.14 \times 32.3$$

$$= \mathbf{406\ cm^3\ ms^{-1}}.$$

EXAMPLE 71. *Water is filled in a cylindrical container to a height of 3 m, as shown in Fig. 10.42. The ratio of the cross-sectional area of the orifice and the beaker is 0.1. Find the speed of the liquid coming out from the orifice. Given $g = 10\ ms^{-2}$.* **[IIT 05]**

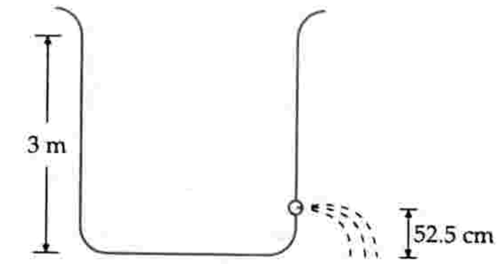


Fig. 10.42

Solution. Let a_1 and a_2 be the area of cross-sections, and v_1 and v_2 be the liquid velocities for the beaker and the orifice respectively.

According to the equation of continuity,

$$a_1 v_1 = a_2 v_2$$

or $$v_1 = \frac{a_2}{a_1} v_2$$

According to Bernoulli's equation,

$$p_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$$

But $p_1 = p_2$ = atmospheric pressure. Therefore

$$g h_1 + \frac{1}{2} v_1^2 = g h_2 + \frac{1}{2} v_2^2$$

or $$g h_1 + \frac{1}{2}\left(\frac{a_2}{a_1} v_2\right)^2 = g h_2 + \frac{1}{2} v_2^2$$

or $$v_2^2\left[1 - \left(\frac{a_2}{a_1}\right)^2\right] = 2g\,(h_1 - h_2)$$

or $$v_2 = \sqrt{\frac{2g\,(h_1 - h_2)}{1 - \left(\frac{a_2}{a_1}\right)^2}}$$

Here $\frac{a_2}{a_1} = 0.1$, $g = 10\ \text{ms}^{-2}$, $h_1 = 3$ m

$$h_2 = 52.5\ \text{cm} = 0.525\ \text{m}$$

$$\therefore \quad h_1 - h_2 = 3 - 0.525 = 2.475\ \text{m}$$

$$v_2 = \sqrt{\frac{2 \times 10 \times 2.475}{1 - (0.1)^2}} = \sqrt{50} = \mathbf{7.07\ ms^{-1}}.$$

PROBLEMS FOR PRACTICE

1. Water flows through a horizontal pipe of varying cross-section at the rate of 20 litres per minute. Determine the velocity of water at a point where diameter is 4 cm. (**Ans.** 2639 ms^{-1})
2. A garden hose having an internal diameter 2.0 cm is connected to a lawn sprinkler that consists of an enclosure with 24 holes, each 0.125 cm in diameter. If water in the hose has a speed of 90.0 cms^{-1}, find the speed of the water leaving the sprinkler holes. (**Ans.** 9.6 cms^{-1})
3. In a normal adult, the average speed of the blood through the aorta (which has a radius of 0.9 cm) is 0.33 ms^{-1}. From the aorta, the blood goes into major arteries, which are 30 in number, each of radius 0.5 cm. Calculate the speed of blood through the arteries. (**Ans.** 0.036 ms^{-1})
4. Velocity of flow of water in a horizontal pipe is 4.9 ms^{-1}. Find the velocity head of water. (**Ans.** 1.225 m)
5. Water flows into a horizontal pipe whose one end is closed with a valve and the reading of a pressure gauge attached to the pipe is $3 \times 10^5\ \text{Nm}^{-2}$. This reading of the pressure gauge falls to $1 \times 10^5\ \text{Nm}^{-2}$ when the valve is opened. Calculate the speed of water flowing into the pipe. **[MNREC 91]** (**Ans.** 20 ms^{-1})
6. Water enters at one end of a horizontal pipe of non-uniform cross-section with a velocity of 0.4 ms^{-1} and leaves the other end with a velocity of 0.6 ms^{-1}. The pressure of water at the first end is 1500 Nm^{-2}. Calculate the pressure at the other end. Density of water = 1000 kgm^{-3}. (**Ans.** 1400 Nm^{-2})
7. Water is flowing with a speed of 2 m/s in a horizontal pipe with cross-sectional area decreasing from $2 \times 10^{-2}\ \text{m}^2$ to 0.01 m^2 at pressure 4×10^4 Pa. What will be the pressure at small cross-section ? **[Central Schools 05]** (**Ans.** 3.4×10^4 Pa)
8. A tank containing water has an orifice 10 m below the surface of water in the tank. If there is no wastage of energy, find the speed of discharge. (**Ans.** 14 ms^{-1})
9. Calculate speed of efflux of kerosene from an orifice of a tank in which pressure is 4 atmosphere. Density of kerosene is 0.72 kg per litre. One atmosphere = 1 kg f cm^{-2}. (**Ans.** 33.5 ms^{-1})
10. Water flows at the rate of 4 litres per second through an orifice at the bottom of tank which contains water 720 cm deep. Find the rate of escape of water if additional pressure of 16 kg f / cm^2 is applied at the surface of water. (**Ans.** 19.28 litre s^{-1})
11. The diameter of a pipe at two points, where a venturi meter is connected is 8 cm and 5 cm and the difference of levels in it is 4 cm. Calculate the volume of water flowing through the pipe per second. (**Ans.** 1889 cm^3s^{-1})
12. A venturi meter is 37.5 cm in diameter in mains and 15 cm diameter in throat. The difference between the pressure of water in the mains and the throat is 23 cm of Hg. Find the discharge in litres per minute. Sp. gravity of Hg = 13.56. (**Ans.** 8400 litre min^{-1})

HINTS

1. Rate of flow,

$$Q = \frac{20\ \text{litre}}{1\ \text{minute}} = \frac{20 \times 10^{-3}\ \text{m}^3}{60\ \text{s}} = \frac{1}{3} \times 10^{-3}\ \text{m}^3\ \text{s}^{-1}$$

$$v = \frac{Q}{a} = \frac{Q}{\pi r^2} = \frac{7 \times 10^{-3}}{3 \times 22 \times (0.02)^2} = \mathbf{0.2639\ ms^{-1}}.$$

2. Volume of water crossing hose per second = Volume of water leaving the holes per second

$$\frac{\pi D^2}{4} \times V = n\frac{\pi d^2}{4} \times v$$

or $$v = \frac{D^2 V}{n d^2} = \frac{2^2 \times 90.0}{24 \times (0.125)^2} = \mathbf{9.6\ cms^{-1}}.$$

3. $a_1 v_1 = 30 a_2 v_2$ or $\pi r_1^2 v_1 = 30 \pi r_2^2 v_2$

$$\therefore \quad v_2 = \left(\frac{r_1}{r_2}\right)^2 \frac{v_1}{30} = \left(\frac{0.9}{0.5}\right)^2 \times \frac{0.33}{30} = \mathbf{0.036\ ms^{-1}}.$$

4. Velocity head $= \dfrac{v^2}{2g} = \dfrac{(4.9)^2}{2 \times 9.8} = \mathbf{1.225\ m.}$

5. Applying Bernoulli's theorem for horizontal flow, we get

$$p_2 + \frac{1}{2}\rho v_2^2 = p_1 + \frac{1}{2}\rho v_1^2$$

$$\therefore \quad 1 \times 10^5 + \frac{1}{2} \times 1000 \times v_2^2 = 3 \times 10^5 + \frac{1}{2} \times 1000 \times 0$$

or $$v_2^2 = \frac{(3 \times 10^5 - 1 \times 10^5) \times 2}{1000}$$

$$= \frac{2 \times 10^5 \times 2}{1000} = 400$$

or $$v_2 = \mathbf{20\ ms^{-1}}.$$

7. Here $v_1 = 2\ ms^{-1}$, $a_1 = 2 \times 10^{-2}\ m^2$, $a_2 = 0.01\ m^2$

$$p_1 = 3 \times 10^4\ Pa, \quad p_2 = ?, \quad \rho = 10^3\ kg\ m^{-3}$$

$$v_2 = \frac{a_1 v_1}{a_2} = \frac{2 \times 10^{-2} \times 2}{0.01} = 4\ ms^{-1}$$

As $$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

$$\therefore \quad p_2 = p_1 - \frac{1}{2}\rho(v_1^2 - v_2^2)$$

$$= 4 \times 10^4 - \frac{1}{2} \times 10^3 (2^2 - 4^2)$$

$$= \mathbf{3.4 \times 10^4\ Pa.}$$

9. Here $p = 4$ atmosphere $= 4 \times 1.03$ kg f cm^{-2}

$$= 4 \times 1.03 \times 9.8 \times 10^4\ Nm^{-2}.$$

Density, $\rho = 0.72$ kg $litre^{-1}$ $= 0.72 \times 1000$ kg m^{-3}

If the orifice be at depth h below the surface of oil, then

$$p = h\rho g \quad \text{or} \quad gh = \frac{p}{\rho}$$

$$\therefore \quad v = \sqrt{2gh} = \sqrt{\frac{2p}{\rho}}$$

$$= \sqrt{\frac{2 \times 4 \times 1.03 \times 9.8 \times 10^4}{0.72 \times 1000}} = \mathbf{33.5\ ms^{-1}}.$$

10. Here $h = 720$ cm

$$\therefore \quad v = \sqrt{2gh} = \sqrt{2 \times 980 \times 720}\ cms^{-1}$$

Additional pressure $= 16$ kg f $cm^{-2} = 1600$ gf cm^{-2}

$$= \frac{16000 \times 980}{980}\ \text{cm of water}$$

$$= 16000\ \text{cm of water}$$

New pressure head, $h_1 = 16000 + 720 = 16720$ cm

New velocity,

$$v_1 = \sqrt{2gh} = \sqrt{2 \times 980 \times 16720}\ cms^{-1}$$

Rate of flow, $Q = av$ and $Q_1 = av_1$

$$Q_1 = \frac{v_1}{v} Q = \sqrt{\frac{16720}{720}} \times 4\ \text{litre}\ s^{-1}$$

$$= \mathbf{19.28\ litre\ s^{-1}}.$$

11. Here $a_1 = \pi r_1^2 = \pi (4)^2 = 16\pi\ cm^2$,

$$a_2 = \pi r_2^2 = \pi (2.5)^2 = 6.25\ \pi\ cm^2$$

$$Q = a_1 a_2 \sqrt{\frac{2gh}{a_1^2 - a_2^2}}$$

$$= 6.25\ \pi \times 16\ \pi \sqrt{\frac{2 \times 980 \times 4}{(16\ \pi)^2 - (6.25\ \pi)^2}}$$

$$= \mathbf{1889\ cm^3\ s^{-1}}.$$

12. Here $h = 23$ cm of Hg $= 23 \times 13.56$ cm of water

$$r_1 = \frac{37.5}{2} = 18.75\ cm, \quad r_2 = \frac{15}{2} = 7.5\ cm$$

$$a_1 = \pi r_1^2 = \pi (18.75)^2\ cm^2, \quad r_2 = \pi r_2^2 = \pi (7.5)^2\ cm^2$$

Rate of flow,

$$Q = a_1 a_2 \sqrt{\frac{2gh}{a_1^2 - a_2^2}}$$

$$= \pi (18.75)^2 \times \pi (7.5)^2 \sqrt{\frac{2 \times 980 \times 23 \times 13.56}{[\pi (18.75)^2]^2 - [\pi (7.5)^2]^2}}$$

$$= \pi \times (18.75)^2 \times (7.5)^2 \sqrt{\frac{2 \times 980 \times 23 \times 13.56}{(18.75)^4 - (7.5)^4}}\ cm^3 s^{-1}$$

$$= \pi \times (18.75)^2 \times (7.5)^2 \sqrt{\frac{2 \times 980 \times 23 \times 13.56}{120432.13}}$$

$$= 1.4 \times 10^5\ cm^3\ s^{-1}$$

$$= \frac{1.4 \times 10^5 \times 60}{1000} = \mathbf{8400\ litre\ min^{-1}}.$$

10.40 COHESIVE AND ADHESIVE FORCES

56. *What are cohesive and adhesive forces ? Give examples.*

(*i*) **Cohesive force.** *It is the force of attraction between the molecules of the same substance.*

Example. Solids have definite shape and size due to strong forces of cohesion amongst their molecules.

(*ii*) **Adhesive force.** *It is the force of attraction between the molecules of two different substances.*

Example. It is due to force of adhesion that ink sticks to paper while writing.

Water wets the walls of its glass container because the force of adhesion between water and glass is greater than the force of cohesion between the water molecules.

On the contrary, mercury does not wet glass because the force of cohesion between the mercury molecules is much greater than the force of adhesion between mercury and glass.

10.41 MOLECULAR RANGE

57. *Define the terms molecular range, sphere of influence and surface film.*

Molecular range. *It is the maximum distance upto which a molecule can exert some appreciable force of attraction on other molecules.* It is of the order of 10^{-9} m in solids and liquids.

Sphere of influence. *A sphere drawn around a molecule as centre and with a radius equal to the molecular range is called the sphere of influence of the molecule.* The molecule at the centre attracts all the molecules lying in its sphere of influence.

While studying the behaviour of a molecule under the influence of cohesive forces, we need to consider only the molecules lying in its sphere of influence.

Surface film. *A thin film of liquid near its surface having thickness equal to the molecular range for that liquid is called surface film.*

10.42 SURFACE TENSION

58. *Define surface tension. Give its units and dimensions.*

Surface tension. A steel needle may be made to float on water though the steel is more dense than water. This is because the water surface acts as a stretched elastic membrane and supports the needle. This property of a liquid is called surface tension.

***Surface tension** is the property by virtue of which the free surface of a liquid at rest behaves like an elastic stretched membrane tending to contract so as to occupy minimum surface area.*

As shown in Fig. 10.43, imagine a line AB on the free surface of a liquid. The small elements of the surface on this line are in equilibrium because they are acted upon by equal and opposite forces, acting perpendicular to the line from either side. The force acting on this line is proportional to the length of this line. If l is the length of the imaginary line and F the total force on either side of the line, then

$$F \propto l \quad \text{or} \quad F = \sigma l$$

or $$\sigma = \frac{F}{l}$$

or Surface tension $= \dfrac{\text{Force}}{\text{Length}}$

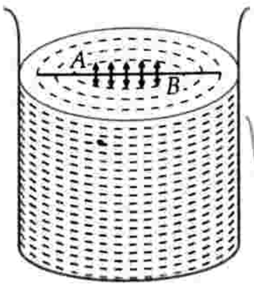


Fig. 10.43 Definition of surface tension.

Surface tension is measured as the force acting per unit length of an imaginary line drawn on the liquid surface, the direction of force being perpendicular to this line and tangential to the liquid surface.

Units and dimensions of surface tension :

SI unit of surface tension $= \text{Nm}^{-1}$

CGS unit of surface tension = dyne cm^{-1}

Dimensions of surface tension

$$= \frac{[\text{Force}]}{[\text{Length}]} = \frac{\text{MLT}^{-2}}{\text{L}} = [\text{MT}^{-2}].$$

10.43 MOLECULAR THEORY OF SURFACE TENSION

59. *Explain surface tension on the basics of molecular theory.*

Molecular theory of surface tension. In Fig. 10.44, $PQRS$ is the surface film of a liquid. Consider the molecule A well inside the liquid. It is attracted equally in all directions by the molecules lying in its sphere of influence. Net force on such a molecule is zero.

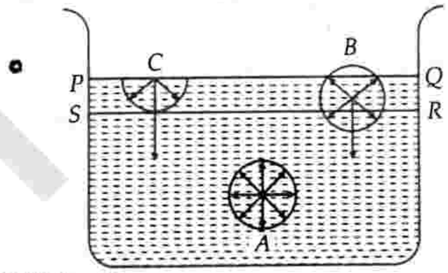


Fig. 10.44 Cohesive force acting on a molecule of water.

Now consider molecule B lying inside the surface film. Its sphere of influence lies partly outside. This molecule experiences less force upward and more force downward by the molecules in its sphere of influence. For molecule C, half its sphere of influence lies above the surface. The resultant downward force on such a molecule is maximum. Due to this downward force, the potential energy of the molecules of the surface film is higher than those lying well inside the liquid. For a system to be stable, potential energy must be minimum. For the surface film to have minimum energy, the number of molecules in it must be minimum. Thus the surface film tends to have minimum surface area. As a result, the free surface of a liquid at rest behaves like an elastic stretched membrane.

10.44 SOME PHENOMENA BASED ON SURFACE TENSION

60. *Explain some examples which illustrate the existence of surface tension.*

Examples to illustrate surface tension :

(*i*) **Needle supported on water surface.** Take a greased needle of steel on a piece of blotting paper and place it gently over the water surface. Blotting paper soaks water and soon sinks down but the needle keeps floating. The floating needle causes a little depression. The forces F, F due to surface tension of the curved surface are inclined as shown in Fig. 10.45. The vertical components of these two forces support the weight of the needle.

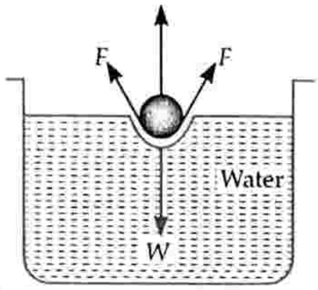


Fig. 10.45 A needle floating on water.

(*ii*) **Endless wet thread on a soap film.** If we take a circular frame of a stiff wire and dip it into a soap solution, a thin soap film is formed on the frame. If a wet endless thread loop is gently placed over the film, it takes any irregular shape. But when the film is pricked at the centre, the loop is stretched outwards and takes a symmetrical circular shape. This is because for a given length a circle has the maximum surface area and so the outer liquid film tries to occupy minimum possible area like a stretched elastic membrane.

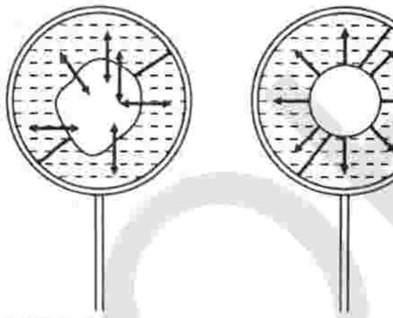


Fig. 10.46 A soap film in a circular frame.

(*iii*) **Rain drops are generally spherical in shape.** Due to surface tension, the rain drops tend to minimise their surface area and the surface area of a sphere is minimum for a given volume.

(*iv*) **Small mercury droplets are spherical and larger ones tend to flattened.** Small mercury droplets are spherical because the forces of surface tension tend to reduce their area to a minimum value and a sphere has minimum surface area for a given volume.

Fig. 10.47 Flattering of large mercury drops.

Larger drops of mercury are flattened due to the large gravitational force acting on them. Here the shape is such that the sum of the gravitational potential energy and the surface potential energy must be minimum. Hence the centre of gravity moves down as low as possible. This explains flattening of the larger drops.

(*v*) **The hair of a painting brush cling together when taken out of water.** This is because the water films formed on them tend to contract to minimum area.

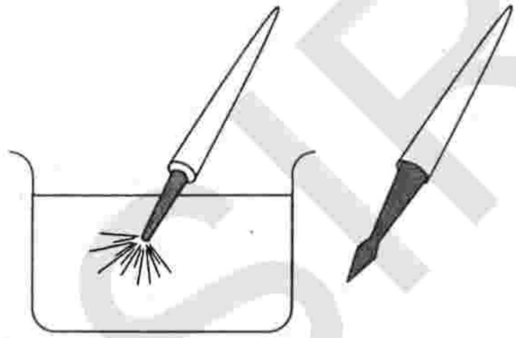


Fig. 10.48 Hair cling due to surface tension.

(*vi*) **A bug floats on water due to surface tension.** As shown in Fig. 10.49, a bug bends its legs on the surface of water such that the deformed surface gives rise to forces of surface tension which act tangential to the deformed surfaces. The weight of the bug is balanced by the upward components of these forces of surface tension.

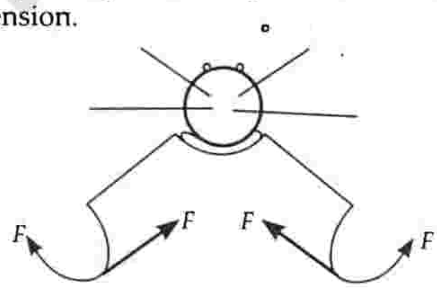


Fig. 10.49 A bug supported on water surface by surface tension.

(*vii*) **Plateau's experiment.** This experiment demonstrates beautifully that a liquid drop assumes spherical shape in the absence of gravitational forces. Prepare a mixture of alcohol and water such that its density is equal to that of olive oil and put it in a glass beaker. Introduce a large drop of olive oil in the

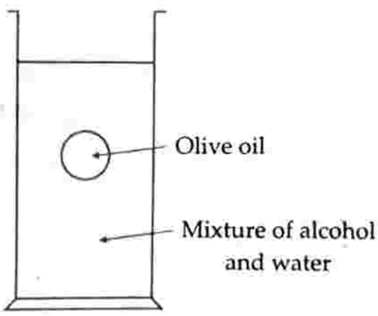


Fig. 10.50 Spherical shape in the absence of gravity.

mixture. The drop floats in the mixture as a spherical ball. The upthrust on the drop becomes equal to its weight. So gravitational forces get eliminated. Even a sufficiently large drop is also spherical.

(*viii*) **Oil spreads on cold water but remains as a drop on hot water.** This is because the surface tension of oil is less than that of the cold water but it is greater than that of the hot water.

10.45 SURFACE ENERGY

61. *Define surface energy. Prove that it is numerically equal to the surface tension.*

Surface energy. The free surface of a liquid possesses minimum area due to surface tension. To increase the surface area, molecules have to be brought from interior to the surface. Work has to be done against the forces of attraction. This work is stored as the potential energy of the molecules on the surface. So the molecules at the surface have extra energy compared to the molecules in the interior.

The extra energy possessed by the molecules of surface film of unit area compared to the molecules in the interior is called surface energy. It is equal to the work done in increasing the area of the surface film by unit amount.

$$\text{Surface energy} = \frac{\text{Work done}}{\text{Increase in surface area}}$$

The SI unit of surface energy is Jm^{-2}.

The relation between surface energy and surface tension. Consider a rectangular frame $ABCD$ in which the wire AB is movable. Dip the frame in soap solution. A film is formed which pulls the wire AB inward due to surface tension with a force,

$$F = 2\,\sigma \times l$$

Here the factor 2 is taken because the soap film has two free surfaces.

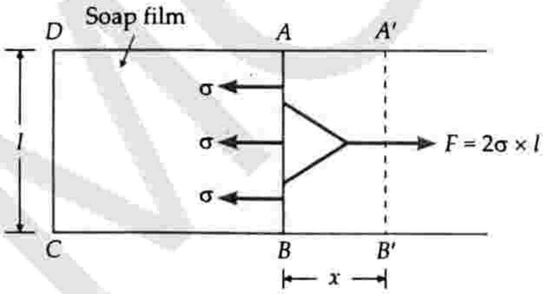


Fig. 10.51 Surface energy.

Suppose AB is moved out through distance x to the position $A'B'$. Then

$$\text{Work done} = \text{Force} \times \text{distance} = 2\sigma \times l \times x$$

Increase in surface area of film $= 2lx$

$$\therefore \quad \text{Surface energy} = \frac{\text{Work done}}{\text{Increase in surface area}} = \frac{2\sigma\, lx}{2lx} = \sigma$$

Thus surface energy of liquid is numerically equal to its surface tension.

10.46 EXPERIMENTAL MEASUREMENT OF SURFACE TENSION

62. *Describe a simple experiment for measuring the surface tension of a liquid.*

Measurement of surface tension. As shown in Fig. 10.52, suspend a rectangular glass plate from one arm of a sensitive balance. Place a beaker containing some liquid below it. The plate is balanced by weights on the other side, with its lower edge just above water.

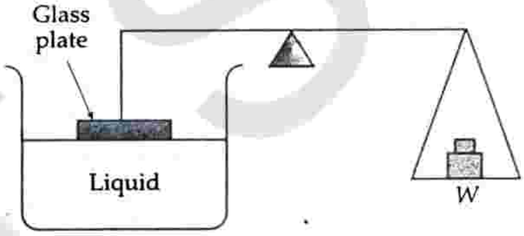


Fig. 10.52 Measurement of surface tension.

The beaker is raised slightly till the liquid just touches the glass plate and pulls it down a little because of surface tension. Weights are added on the other side till the glass plate just leaves the water surface. If the additional weight required is W, then the surface tension of the liquid-air interface will be

$$\sigma_{la} = \frac{W}{2l} = \frac{mg}{2l}$$

where m is the extra mass added and l is the length of the plate edge.

Examples based on Surface Tension and Surface Energy

FORMULAE USED

1. Surface tension $= \dfrac{\text{Force}}{\text{Length}}$ or $\sigma = \dfrac{F}{l}$
2. Increase in surface energy or work done,
   W = Surface tension × increase in area of the liquid surface.

UNITS USED

The unit of surface tension is Nm^{-1} and that of increase in surface energy or work done is joule.

EXAMPLE 72. *A wire ring of 3 cm radius is rested on the surface of a liquid and then raised. The pull required is 3.03 g more before the film breaks than it is afterwards. Find the surface tension of the liquid.*

Solution. The additional pull F of 3.03 g wt is equal to the force of surface tension.

$$F = 3.03 \text{ g wt} = 3.03 \times 981 \text{ dyne}$$

As the liquid touches the ring both along the inner and outer circumference, so force on the ring due to surface tension,

$$F = 2 \times 2\pi r \times \sigma = 4\pi r \sigma$$

$$\therefore \quad 4\pi r\sigma = 3.03 \times 981$$

$$\sigma = \frac{3.03 \times 981}{4\pi r} = \frac{3.03 \times 981}{4 \times 3.14 \times 3}$$

$$= \mathbf{78.84 \text{ dyne cm}^{-1}}.$$

EXAMPLE 73. *Calculate the work done in blowing a soap bubble from a radius of 2 cm to 3 cm. The surface tension of the soap solution is 30 dyne cm^{-1}.* **[Delhi 11]**

Solution. Here $r_1 = 2$ cm, $r_2 = 3$ cm, $\sigma = 30$ dyne cm^{-1}

Increase in surface area

$$= 2 \times 4\pi(r_2^2 - r_1^2) = 8\pi(3^2 - 2^2) = 40\pi \text{ cm}^2$$

Work done $= \sigma \times$ Increase in surface area

$$= 30 \times 40 \times 3.142 = \mathbf{3770.4 \text{ erg}}.$$

EXAMPLE 74. *The surface tension of a soap solution is 0.03 Nm^{-1}. How much work is done to produce a soap bubble of radius 0.05 m?*

Solution. Work done = Total surface area × surface tension

$$= 2 \times 4\pi r^2 \times \sigma = 2 \times 4 \times 3.14 \times (0.05)^2 \times 0.03$$

$$= \mathbf{1.884 \times 10^{-3} \text{ J}}.$$

EXAMPLE 75. *A liquid drop of diameter D breaks up into 27 tiny drops. Find the resulting change in energy. Take surface tension of the liquid as σ.* **[Central Schools 09]**

Solution. Radius of larger drop $= D/2$

Let radius of each small drop $= r$

Now volume of 27 small drops

= Volume of the larger drop

$$27 \times \frac{4}{3} \times \pi r^3 = \frac{4}{3}\pi\left(\frac{D}{2}\right)^3 \quad \text{or} \quad r = \frac{D}{6}$$

Initial surface area of larger drop

$$= 4\pi R^2 = 4\pi\left(\frac{D}{2}\right)^2 = \pi D^2$$

Final surface area of 27 small drops $= 27 \times 4\pi r^2$

$$= 27 \times 4\pi\left(\frac{D}{6}\right)^2 = 3\pi D^2$$

$\therefore$ Increase in surface area $= 3\pi D^2 - \pi D^2 = 2\pi D^2$

Change in energy = Increase in surface area × Surface tension

$$= \mathbf{2\pi D^2 \sigma}.$$

EXAMPLE 76. *A mercury drop of radius 1.0 cm is sprayed into 10^6 droplets of equal size. Calculate the energy expended. Surface tension of mercury $= 32 \times 10^{-2}$ Nm^{-1}.* **[Roorkee 84]**

Solution. Volume of 10^6 droplets

= Volume of larger drop

$$10^6 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$$

$$r = 10^{-2} R = 10^{-2} \times 1.0 = 10^{-2} \text{ cm} = 10^{-4} \text{ m}$$

Surface area of larger drop

$$= 4\pi R^2 = 4\pi \times (10^{-2})^2 = 4\pi \times 10^{-4} \text{ m}^2$$

Surface area of 10^6 droplets

$$= 4\pi r^2 \times 10^6 = 4\pi \times (10^{-4})^2 \times 10^6$$

$$= 4\pi \times 10^{-2} \text{ m}^2$$

$\therefore$ Increase in surface area

$$= 4\pi \times 10^{-4}(100 - 1) = 4\pi \times 99 \times 10^{-4} \text{ m}^2$$

$\therefore$ Work done in spraying a spherical drop of mercury

= Surface tension × increase in surface area

$$= 32 \times 10^{-2} \times 4\pi \times 99 \times 10^{-4} = \mathbf{3.98 \times 10^{-2} \text{ J}}.$$

EXAMPLE 77. *A liquid drop of diameter 4 mm breaks into 1000 droplets of equal size. Calculate the resultant change in surface energy, the surface tension of the liquid is 0.07 Nm^{-1}.* **[Central Schools 05]**

Solution. Volume of 1000 droplets = Volume of larger drop

$$1000 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$$

$$r = \frac{R}{10} = \frac{2 \times 10^{-3} \text{ m}}{10} = 2 \times 10^{-4} \text{ m}$$

Surface area of larger drop

$$= 4\pi R^2 = 4\pi \times (2 \times 10^{-3})^2 = 16\pi \times 10^{-6} \text{ m}^2$$

Surface area of 1000 droplets

$$= 4\pi r^2 \times 1000 = 4\pi \times (2 \times 10^{-4})^2 \times 1000$$

$$= 16\pi \times 10^{-5} \text{ m}^2$$

$\therefore$ Increase in surface area

$$= 16\pi \times 10^{-6}(10 - 1) = 144\pi \times 10^{-6} \text{ m}^2$$

The resultant increase in surface energy

= Surface tension × increase in surface area

$$= 0.07 \times 144 \times \frac{22}{7} \times 10^{-6} = \mathbf{3168 \times 10^{-8} \text{ J}}.$$

EXAMPLE 78. *Two soap bubbles in vacuum having radii 3 cm and 4 cm respectively coalesce under isothermal conditions to form a single bubble. What is the radius of the new bubble?*

Solution. Surface energy of first bubble

= Surface tension × surface area

$$= 2 \times 4\pi r_1^2 \sigma = 8\pi r_1^2 \sigma$$

Similarly, surface energy of second bubble

$$=8\pi r_2^2\,\sigma$$

Let r be the radius of the coalesced bubble. Then,

surface energy of coalesced bubble $=8\pi r^2\sigma$

By the conservation of energy,

$$8\pi r^2\sigma=8\pi r_1^2\,\sigma+8\pi r_2^2\,\sigma=8\pi\,(r_1^2+r_2^2)\,\sigma$$

or $$r^2=r_1^2+r_2^2=3^2+4^2=25$$

or $$r=\mathbf{5\ cm.}$$

EXAMPLE 79. *If 500 erg of work is done in blowing a soap bubble to a radius r, what additional work is required to be done to blow it to a radius equal to 3r ?*

Solution. Work done in blowing the soap bubble from radius 0 to r is

$$W=\sigma\times 2\times 4\pi\,r^2 \qquad ...(i)$$

Additional work required to increase the radius from r to $3r$ will be

$$W'=\sigma\times \text{Increase in surface area}$$
$$=\sigma\times 2\times 4\pi\,[(3r)^2-r^2]$$
$$W'=\sigma\times 2\times 4\pi\times 8r^2 \qquad ...(ii)$$

Dividing equation (*ii*) by (*i*), we get

$$\frac{W'}{W}=8$$

or $$W'=8W=8\times 500\text{ erg}=\mathbf{4000\ erg.}$$

EXAMPLE 80. *Soapy water drips from a capillary. When the drop breaks away, the diameter of its neck is 1 mm. The mass of the drop is 0.0129 g. Find the surface tension of soapy water.*

Solution. When the drop breaks away from the capillary, weight of drop

= Force of surface tension acting on the capillary

or $mg=\pi D\times\sigma,$

where D = diameter of the drop

or $$\sigma=\frac{mg}{\pi D}=\frac{1.29\times 10^{-5}\times 9.8}{3.14\times 1\times 10^{-3}}$$

$$=\mathbf{4.03\times 10^{-2}\ Nm^{-1}}.$$

EXAMPLE 81. *A glass plate of length 10 cm, breadth 4 cm and thickness 0.4 cm, weighs 20 g in air. It is held vertically with long side horizontal and half the plate immersed in water. What will be its apparent weight ? Surface tension of water = 70 dyne cm^{-1}.*

Solution. Here $l=10$ cm, $b=4$ cm,

$t=0.4$ cm, $m=20$ g, $\sigma=70$ dyne cm^{-1}

Various forces acting on the plate are

(*i*) Weight of the plate acting vertically downwards,

$$=mg=20\times 980\text{ dyne}=20\text{ g f}$$

(*ii*) Force due to surface tension acting vertically downwards,

$$F=\sigma\times \text{Length of plate in contact with water}$$
$$=\sigma\times 2\ (\text{length}+\text{thickness})$$
$$=70\times 2\ (10+0.4)=70\times 20.8\text{ dyne}$$
$$=\frac{70\times 20.8}{980}\text{ g f}=1.4857\text{ g f}$$

(*iii*) Upwards thrust due to liquid

= Weight of the liquid displaced

= Volume of liquid displaced × density × g

$$=(l\times b/2\times t)\times\rho\times g$$
$$=(10\times 4/2\times 0.4)\,1\times 980\text{ dyne}$$
$$=\frac{8\times 980}{980}\text{ g f}=8\text{g f.}$$

$\therefore$ Apparent weight $=20+1.4857-8=\mathbf{13.4857\ g\ f.}$

EXAMPLE 82. *If a number of little droplets of water of surface tension σ, all of the same radius r combine to form a single drop of radius R and the energy released is converted into kinetic energy, find the velocity acquired by the bigger drop.*

Solution. Volume of bigger drop

= Volume of n smaller drops

$$\frac{4}{3}\pi R^3=n\times\frac{4}{3}\pi r^3 \text{ or } n=\frac{R^3}{r^3}$$

Mass of bigger drop,

$$m=\text{Volume}\times\text{density}$$
$$=\frac{4}{3}\pi R^3\times 1=\frac{4}{3}\pi R^3$$

Energy released,

$$W=\text{S.T.}\times\text{Decrease in surface area}$$
$$=\sigma\times 4\pi\,(nr^2-R^2)=4\pi\sigma\left(\frac{R^3}{r^3}r^2-R^2\right)$$
$$=4\pi\,R^3\sigma\left(\frac{1}{r}-\frac{1}{R}\right)=3\times\frac{4}{3}\pi R^3\,\sigma\left(\frac{R-r}{rR}\right)$$
$$=3m\sigma\left(\frac{R-r}{rR}\right)$$

But K.E. produced $=W$

$$\therefore\quad \frac{1}{2}mv^2=3m\sigma\left(\frac{R-r}{rR}\right)\quad\text{or}\quad v=\sqrt{\frac{6\sigma\,(R-r)}{rR}}.$$

EXAMPLE 83. *If a number of little droplets of water, each of radius r, coalesce to form a single drop of radius R, show that the rise in temperature will be given by*

$$\Delta\theta=\frac{3\sigma}{J}\left(\frac{1}{r}-\frac{1}{R}\right)$$

where σ is the surface tension of water and J is the mechanical equivalent of heat.

Solution. Let n be the number of little droplets which coalesce to form single drop. Then

Volume of n little droplets

$$= \text{Volume of single drop}$$

or $$n \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3 \quad \text{or} \quad nr^3 = R^3$$

Decrease in surface area $= n \times 4\pi r^2 - 4\pi R^2$

$$= 4\pi[nr^2 - R^2] = 4\pi\left[\frac{nr^3}{r} - R^2\right]$$

$$= 4\pi\left[\frac{R^3}{r} - R^2\right] = 4\pi R^3\left[\frac{1}{r} - \frac{1}{R}\right] \quad [\because nr^3 = R^3]$$

Energy evolved,

W = Surface tension × decrease in surface area

$$= 4\pi\sigma R^3\left[\frac{1}{r} - \frac{1}{R}\right]$$

Heat produced,

$$Q = \frac{W}{J} = \frac{4\pi\sigma R^3}{J}\left[\frac{1}{r} - \frac{1}{R}\right]$$

But $Q = ms\Delta\theta$

= Volume of single drop × density of water × specific heat of water × $\Delta\theta$

$$= \frac{4}{3}\pi R^3 \times 1 \times 1 \times \Delta\theta$$

Hence

$$\frac{4}{3}\pi R^3\Delta\theta = \frac{4\pi\sigma R^3}{J}\left[\frac{1}{r} - \frac{1}{R}\right]$$

or $$\Delta\theta = \frac{3\sigma}{J}\left[\frac{1}{r} - \frac{1}{R}\right].$$

Problems For Practice

1. A soap film is formed on a rectangular frame of length 7 cm dipping in soap solution. The frame hangs from the arm of a balance. An extra weight of 0.38 g is to be placed in the opposite pan to balance the pull on the frame. Calculate the surface tension of the soap solution. Given $g = 980 \text{ cms}^{-2}$. (**Ans.** 26.6 dyne cm^{-1})
2. A soap film is on a rectangular wire ring of size 5 cm × 4 cm. If the size of the film is changed to 5 cm × 5 cm, then calculate the work done in this process. The surface tension of soap film is $5 \times 10^{-2} \text{ Nm}^{-1}$. (**Ans.** 5×10^{-5} J)
3. A soap bubble is blown to a diameter of 7 cm. If 36,960 erg of work is done in blowing it further, find the new radius if the surface tension of soap solution is 40 dyne cm^{-1}. (**Ans.** 7 cm)
4. A soap bubble of radius $1/\sqrt{\pi}$ cm is expanded to radius $2/\sqrt{\pi}$ cm. Calculate the work done. Surface tension of soap solution = 30 dyne cm^{-1}. (**Ans.** 720 erg)
5. What amount of energy will be liberated if 1000 droplets of water, each of diameter 10^{-8} cm, coalesce to form a bigger drop ? Surface tension of water is 0.072 Nm^{-1}. (**Ans.** 2.035×10^{-14} J)
6. Calculate the force required to take away a flat plate of radius 5 cm from the surface of water. Given surface tension of water = 72 dyne cm^{-1}. (**Ans.** 2260.8 dyne)
7. A thin wire is bent in the form of a ring of diameter 3.0 cm. The ring is placed horizontally on the surface of soap solution and then raised up slowly. How much upward force is necessary to break the vertical film formed between the ring and the solution ? Surface tension of a soap solution $= 3.0 \times 10^{-2} \text{ Nm}^{-1}$. (**Ans.** 5.65×10^{-3} N.)
8. The length of a needle floating on water is 2.5 cm. How much minimum force, in addition to the weight of the needle, will be needed to lift the needle above the surface of water ? Surface tension of water $= 7.2 \times 10^{-4} \text{ Ncm}^{-1}$. (**Ans.** 3.6×10^{-3} N)
9. A rectangular plate of dimensions 6 cm × 4 cm and thickness 2 mm is placed with its largest face flat on the surface of water.
   (*i*) What is the downward force on the plate due to surface tension ? Surface tension of water $= 7.0 \times 10^{-2} \text{ Nm}^{-1}$
   (*ii*) If the plate is placed vertical so that the longest side just touches the water surface, find the downward force on the plate.
   (**Ans.** 1.4×10^{-2} N, 8.68×10^{-3} N)

Hints

1. As soap film has two free surfaces, so

$$F = \sigma \times 2l \quad \text{or} \quad 0.38g = \sigma \times 2 \times 7$$

or $$\sigma = \frac{0.38 \times 980}{14} = \mathbf{26.6 \text{ dyne cm}^{-1}}.$$

3. Initial radius, $r = \frac{7}{2}$ cm = 3.5 cm

Initial surface area of soap bubble

$$= 2 \times 4\pi r^2 = 8\pi \times (3.5)^2 \text{ cm}^2.$$

Let new radius of the soap bubble = R cm

Then final surface area $= 2 \times 4\pi R^2 = 8\pi R^2 \text{ cm}^2$

Increase in surface area $= 8\pi[R^2 - (3.5)^2] \text{ cm}^2$

Now Work done = Surface tension × Increase in surface area

$\therefore \quad 36960 = 40 \times 8\pi [R^2 - (3.5)^2]$

or $\quad R^2 - 12.25 = \dfrac{36960}{40 \times 8 \times 3.14} = 36.75$

or $\quad R^2 = 36.75 + 12.25 = 49$

or $\quad R = \mathbf{7\ cm.}$

4. Work done = S.T. × Increase in surface area

$$= 30 \times 2 \times 4\pi \left[\left(\frac{2}{\sqrt{\pi}}\right)^2 - \left(\frac{1}{\sqrt{\pi}}\right)^2\right] = \mathbf{720\ erg.}$$

5. Radius of a droplet,

$$r = \frac{10^{-8}}{2} = 0.5 \times 10^{-8} \text{ m}$$

Volume of bigger drop = Volume of 1000 droplets

$$\frac{4}{3}\pi R^3 = 1000 \times \frac{4}{3}\pi r^3$$

or $\quad R = 10\,r = 10 \times 0.5 \times 10^{-8} = 5 \times 10^{-8}$ m

Decrease in surface area

$= 1000 \times 4\pi r^2 - 4\pi R^2 = 4\pi\,[1000\, r^2 - R^2]$

$= 4 \times 3.14[1000 \times (0.5 \times 10^{-8})^2 - (5 \times 10^{-8})^2]$

$= 4 \times 3.14 \times 225 \times 10^{-16}\ \text{m}^2 = 2826 \times 10^{-16}\ \text{m}^2$

Energy liberated

= Surface tension × Decrease in surface area

$= 0.072 \times 2826 \times 10^{-16} = \mathbf{2.035 \times 10^{-14}\ J.}$

6. Required force = $2\pi r$ × Surface tension

$= 2 \times 3.14 \times 5 \times 72$

$= \mathbf{2260.8\ dyne.}$

7. $F = 2 \times 2\pi r \times \sigma$

$= 4 \times 3.14 \times 1.5 \times 10^{-2} \times 3.0 \times 10^{-2}$

$= \mathbf{5.65 \times 10^{-3}\ N.}$

8. $F = \sigma \times 2l$

$= 7.2 \times 10^{-4} \times 2 \times 2.5$

$= \mathbf{3.6 \times 10^{-3}\ N.}$

9. (*i*) $F = \sigma \times 2(l + b)$

$= 7 \times 10^{-2} \times 2 \times (0.06 + 0.04)$

$= \mathbf{1.4 \times 10^{-2}\ N.}$

(*ii*) $F = \sigma \times 2(l + t)$

$= 7 \times 10^{-2} \times 2 \times (0.06 + 0.002)$

$= \mathbf{8.68 \times 10^{-3}\ N.}$

10.47 PRESSURE DIFFERENCE ACROSS A CURVED LIQUID SURFACE

63. *Show that a pressure difference exists between the two sides of a curved liquid surface.*

Pressure difference across a curved liquid surface. When the free surface of a liquid is curved, there is a difference of pressure between the liquid side and the vapour side of the surface. We consider the three possible liquid surfaces :

(*i*) As shown in Fig. 10.53(*a*), if the surface is plane, the molecule A on the surface is attracted equally in all directions. The resultant force due to surface tension is zero. Pressure on both sides of the surface is same *i.e.*,

$$P_L = P_V.$$

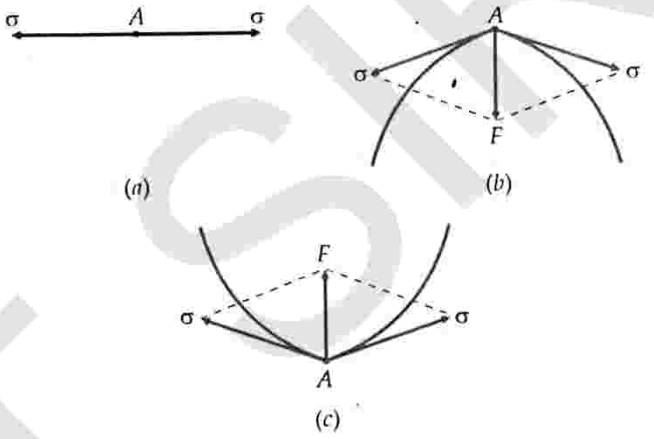


Fig. 10.53 Excess pressure across a curved surface.

(*ii*) As shown in Fig. 10.53(*b*), if the surface is convex, there is a resultant downward force F on molecule A. For the surface to remain in equilibrium, the pressure on the liquid side must be greater than the pressure on the vapour side *i.e.*,

$$P_L > P_V.$$

(*iii*) As shown in Fig. 10.53(*c*), if the surface is concave, there is a resultant upward force F due to surface tension on the molecule A. For the surface to remain in equilibrium, the pressure on the vapour side must be greater than the pressure on the liquid side *i.e.*,

$$P_V > P_L.$$

Thus we find that *whenever a liquid surface is curved, the pressure on its concave side is greater than the pressure on the convex side.*

10.48 EXCESS PRESSURE INSIDE A LIQUID DROP

64. *Derive an expression for the excess pressure inside a liquid drop.*

Excess pressure inside a liquid drop. Consider a spherical liquid drop of radius R. Let σ be the surface tension of the liquid. Due to its spherical shape, there is an excess pressure p inside the drop over that on outside. This excess pressure acts normally outwards. Let the radius of the drop increase from R to $R + dR$ under the excess pressure p.

Initial surface area $= 4\pi R^2$

Final surface area

$$= 4\pi (R + dR)^2 = 4\pi (R^2 + 2R\,dR + dR^2)$$

$$= 4\pi R^2 + 8\pi R\,dR$$

dR^2 is neglected as it is small.

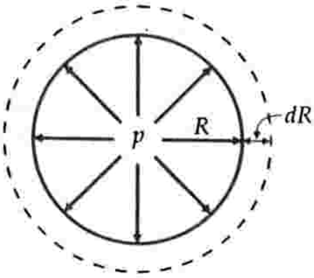


Fig. 10.54 Excess pressure inside a liquid drop.

Increase in surface area

$$= 4\pi R^2 + 8\pi R\,dR - 4\pi R^2 = 8\pi R\,dR$$

Work done in enlarging the drop

= Increase in surface energy

= Increase in surface area × Surface tension

$$= 8\pi R\,dR\,\sigma$$

But work done = Force × Distance

= Pressure × Area × Distance

$$= p \times 4\pi R^2 \times dR$$

Hence, $p \times 4\pi R^2 \times dR = 8\pi R\,dR\,\sigma$

Excess pressure,

$$p = \frac{2\sigma}{R}.$$

65. *Derive an expression for the excess pressure inside a soap bubble.*

Excess pressure inside a soap bubble. Proceeding as in the case of a liquid drop in the above question, we obtain

Increase in surface area $= 8\pi R\,dR$

But a soap bubble has air both inside and outside, so it has two free surfaces.

∴ Effective increase in surface area

$$= 2 \times 8\pi R\,dR = 16\pi R\,dR$$

Work done in enlarging the soap bubble

= Increase in surface energy

= Increase in surface area × Surface tension

$$= 16\pi R\,dR\,\sigma$$

But, Work done = Force × Distance

$$= p \times 4\pi R^2 \times dR$$

Hence

$$p \times 4\pi R^2 \times dR = 16\pi R\,dR\,\sigma$$

or

$$p = \frac{4\sigma}{R}.$$

66. *Write an expression for the excess pressure inside an air bubble.*

Excess pressure inside an air bubble inside a liquid. An air bubble inside a liquid is similar to a liquid drop in air. It has only one free spherical surface. Hence excess pressure is given by

$$p = \frac{2\sigma}{R}$$

For Your Knowledge

- The smaller the radius of a liquid drop, the greater is the excess of pressure inside the drop. It is due to this excess of pressure inside the tiny fog droplets that they are rigid enough to behave like solids and resist fairly large deforming forces.
- When an ice-skater slides over the surface of smooth ice, some ice melts due to the pressure exerted by the sharp metal edges of the skates. The tiny water droplets act as rigid ball-bearings and help the skaters to run along smoothly.
- When an air bubble of radius R lies at a depth h below the free surface of a liquid of density ρ and surface tension σ, the excess pressure inside the bubble will be

$$p = \frac{2\sigma}{R} + h\rho g$$

Examples based on Excess Pressure in Drops & Bubbles

FORMULAE USED

1. Excess pressure inside a liquid drop,
$$p = \frac{2\sigma}{R} \quad \text{(with one free surface)}$$
2. Excess pressure inside a soap bubble,
$$p = \frac{4\sigma}{R} \quad \text{(with two free surfaces)}$$
3. Excess pressure in an air bubble,
$$p = \frac{2\sigma}{R} \quad \text{(with one free surface)}$$

UNITS USED

Surface tension σ is in Nm^{-1}, pressure p in Nm^{-2} or Pa and radius R in metre

EXAMPLE 84. *What should be the pressure inside a small air bubble of 0.1 mm radius, situated just below the surface? Surface tension of water $= 7.2 \times 10^{-2}$ Nm^{-1} and atmospheric pressure $= 1.013 \times 10^5$ Nm^{-2}.* **[Chandigarh 04]**

Solution. Here $R = 0.1 \text{ mm} = 0.1 \times 10^{-3}$ m,

$\sigma = 7.2 \times 10^{-2}$ Nm^{-1}

Excess pressure,

$$p = \frac{2\sigma}{R} = \frac{2 \times 7.2 \times 10^{-2}}{0.1 \times 10^{-3}} = 1.44 \times 10^3 \text{ Nm}^{-2}$$

Pressure inside the bubble

= Atmospheric pressure + Excess pressure

$= 1.013 \times 10^5 + 1.44 \times 10^3 = \mathbf{1.027 \times 10^5 \text{ Nm}^{-2}}$.

EXAMPLE 85. *The excess pressure inside a soap bubble of radius 6 mm is balanced by 2 mm column of oil of specific gravity 0.8. Find the surface tension of soap solution.*

Solution. Here $R = 6 \text{ mm} = 6 \times 10^{-3}$ m,

$h = 2 \text{ mm} = 2 \times 10^{-3}$ m, $\rho = 0.8 \times 10^3 \text{ kgm}^{-3}$

Excess pressure inside soap bubble

= Pressure exerted by 2 mm oil column

or $\frac{4\sigma}{R} = h\rho g \quad \therefore \quad \sigma = \frac{1}{4} hR\rho g$

$$= \frac{1}{4} \times 2 \times 10^{-3} \times 6 \times 10^{-3} \times 0.8 \times 10^3 \times 9.8$$

$= \mathbf{2.35 \times 10^{-2} \text{Nm}^{-1}}$.

EXAMPLE 86. *Two soap bubbles have radii in the ratio 2 : 3. Compare the excess of pressure inside these bubbles. Also compare the works done in blowing these bubbles.*

Solution. If R_1 and R_2 are the radii of the two bubbles, then $\frac{R_1}{R_2} = \frac{2}{3}$

Let σ be the surface tension of the soap solution.

Excess pressure inside the bubble of radius R_1,

$$p_1 = \frac{4\sigma}{R_1}$$

Excess pressure inside the bubble of radius R_2,

$$p_2 = \frac{4\sigma}{R_2}$$

$$\therefore \quad \frac{p_1}{p_2} = \frac{4\sigma}{R_1} \times \frac{R_2}{4\sigma} = \frac{R_2}{R_1} = \frac{3}{2} = \mathbf{3 : 2}$$

Work done in blowing up the two soap bubbles is

$$W_1 = 2 \times 4\pi R_1^2 \times \sigma$$

and $W_2 = 2 \times 4\pi R_2^2 \times \sigma$

$$\therefore \quad \frac{W_1}{W_2} = \frac{R_1^2}{R_2^2} = \left(\frac{2}{3}\right)^2 = \mathbf{4 : 9}.$$

EXAMPLE 87. *A small hollow sphere having a small hole in it is immersed into water to a depth of 20 cm before any water penetrates into it. If the surface tension of water is 73 dyne cm^{-1}, find the radius of the hole.*

Solution. Here $\sigma = 73$ dyne cm^{-1}, $h = 20$ cm

At equilibrium position,

$$\frac{2\sigma}{R} = h\rho g$$

$$\therefore \quad R = \frac{2\sigma}{h\rho g} = \frac{2 \times 73}{20 \times 1 \times 980} = \mathbf{0.007449 \text{ cm.}}$$

EXAMPLE 88. *A glass tube of 1 mm bore is dipped vertically into a container of mercury, with its lower end 2 cm below the mercury surface. What must be the gauge pressure of air in the tube to blow a hemispherical bubble at its lower end ? Given density of mercury $= 13600 \text{ kg m}^{-3}$ and surface tension of mercury $= 35 \times 10^{-3} \text{ Nm}^{-1}$.*

Solution.

Here $R = \frac{1}{2} \text{ mm} = 0.5 \text{ mm} = 5 \times 10^{-4}$ m,

$\sigma = 35 \times 10^{-3} \text{ Nm}^{-1}$,

$\rho = 13600 \text{ kgm}^{-3}$, $h = 2 \text{ cm} = 0.02$ m

Pressure of air

$$= h\rho g + \frac{2\sigma}{R} = 0.02 \times 13600 \times 9.8 + \frac{2 \times 35 \times 10^{-3}}{5 \times 10^{-4}}$$

$= 2665.5 + 140 = \mathbf{2805.6 \text{ Nm}^{-2}}$.

EXAMPLE 89. *The lower end of a capillary tube of diameter 2.00 mm is dipped 8.00 cm below the surface of water in a beaker. What is the pressure required in the tube in order to blow a hemispherical bubble at its end in water ? The surface tension of water at the temperature of the experiment is $7.30 \times 10^{-2} \text{ Nm}^{-1}$. 1 atmospheric pressure $= 1.01 \times 10^5$ Pa, density of water $= 1000 \text{ kgm}^{-3}$, $g = 9.80 \text{ ms}^{-2}$. Also calculate the excess pressure.* **[NCERT]**

Solution. Here $R = \frac{2.00}{2} = 1.00 \text{ mm} = 1.00 \times 10^{-3}$ m,

$h = 8.00 \times 10^{-2}$ m, $\sigma = 7.30 \times 10^{-2} \text{ Nm}^{-1}$,

$P = 1.01 \times 10^5$ Pa, $\rho = 1000 \text{ kgm}^{-3}$, $g = 9.80 \text{ ms}^{-2}$

Excess pressure inside the bubble is

$$P = \frac{2\sigma}{r} = \frac{2 \times 7.30 \times 10^{-2}}{1.00 \times 10^{-3}} = \mathbf{146 \text{ Pa.}}$$

Pressure outside the bubble

= Atmospheric pressure
+ Pressure due to 8.00 cm of water column

$= P + h\rho g$

$= 1.01 \times 10^5 + 8.00 \times 10^{-2} \times 1000 \times 9.80$

$= 1.01 \times 10^5 + 0.00784 \times 10^5 = 1.01784 \times 10^5$ Pa

Pressure inside the bubble

= Pressure outside the bubble + Excess pressure

$= 1.01784 \times 10^5 + 146$

$= \mathbf{1.0193 \times 10^5 \text{ Pa} \simeq 1.02 \times 10^3 \text{ Pa}}$.

PROBLEMS FOR PRACTICE

1. What would be the gauge pressure inside an air bubble of 0.2 mm radius situated just below the surface of water ? Surface tension of water is $0.07\ \text{Nm}^{-1}$ (**Ans.** $700\ \text{Nm}^{-2}$)
2. The pressure of air in a soap bubble of 0.7 cm diameter is 8 mm of water above the atmospheric pressure. Calculate the surface tension of soap solution. Take $g = 9.8\ \text{ms}^{-2}$. (**Ans.** $6.86 \times 10^{-2}\ \text{Nm}^{-1}$)
3. Calculate the total pressure inside a spherical bubble of radius 0.2 mm formed inside water at a depth of 10 cm. Surface tension of water at a depth of 30 cm is $70\ \text{dyne cm}^{-1}$, barometric pressure is 76 cm, density of mercury is $13.6\ \text{g cm}^{-3}$ and $g = 980\ \text{cms}^{-2}$. (**Ans.** $1029728\ \text{dyne cm}^{-2}$)
4. Calculate the total pressure inside a spherical air bubble of radius 0.1 mm at a depth of 10 cm below the surface of a liquid of density $1.1\ \text{g cm}^{-3}$ and surface tension $50\ \text{dyne cm}^{-1}$. Height of Hg barometer = 76 cm. (**Ans.** $1.0337 \times 10^{6}\ \text{dyne cm}^{-2}$)
5. Find the difference in excess pressure on the inside and outside of a rain drop if its diameter changes from 0.03 cm to 0.0002 cm by evaporation. Surface tension of water is $72\ \text{dyne cm}^{-1}$. (**Ans.** $1430400\ \text{dyne cm}^{-2}$)
6. What is the pressure inside a vapour bubble of radius 10^{-3} m formed in boiling water ? Surface tension of water at 100 is $0.059\ \text{Nm}^{-1}$ and $1\ \text{atm} = 101325\ \text{Nm}^{-2}$. (**Ans.** $101443\ \text{Nm}^{-2}$)
7. There is an air bubble of radius 1.0 mm in a liquid of surface tension $0.075\ \text{Nm}^{-1}$ and density $10^3\ \text{kgm}^{-3}$. The bubble is at a depth of 10.0 cm below the free surface of a liquid. By what amount is the pressure inside the bubble greater than the atmospheric pressure ? (**Ans.** $1130\ \text{Nm}^{-2}$)
8. An ancient building has a dome of 5 m radius and uniform but small thickness. The surface tension of its masionary structure is about $500\ \text{Nm}^{-1}$. Treated as hemisphere, find the maximum load that dome can support. (**Ans.** 31420 N)

HINTS

1. Here $\sigma = 0.07\ \text{Nm}^{-1}$, $R = 0.2\ \text{mm} = 2 \times 10^{-4}$ m.

   Gauge pressure, $p = \dfrac{2\sigma}{R} = \dfrac{2 \times 0.07}{2 \times 10^{-4}} = \mathbf{700\ Nm^{-2}}$.

2. Excess pressure in soap bubble

   = Pressure exerted by 8 mm column of water

   or $\dfrac{4\sigma}{R} = h\rho g$ or $\sigma = \dfrac{1}{4} h R \rho g$

   $\therefore \quad \sigma = \dfrac{1}{4} \times 8 \times 10^{-3} \times 0.35 \times 10^{-2} \times 1000 \times 9.8$
   $= \mathbf{6.86 \times 10^{-2}\ Nm^{-1}}$.

3. Total pressure

   = Atmospheric pressure + Pressure due to liquid column of 10 cm + Excess pressure

   $= h\rho g + h'\rho' g + \dfrac{2\sigma}{R}$

   $= 76 \times 13.6 \times 980 + 10 \times 1 \times 980 + \dfrac{2 \times 70}{0.02}$

   $= 1012928 + 9800 + 7000$

   $= \mathbf{1029728\ dyne\ cm^{-2}}$.

5. Here $R_1 = \dfrac{0.03}{2} = 0.015\ \text{cm}$,

   $R_2 = \dfrac{0.0002}{2} = 0.0001\ \text{cm}$

   $\therefore \quad p_2 - p_1 = \dfrac{2\sigma}{R_2} - \dfrac{2\sigma}{R_1} = 2 \times 72 \times \left[\dfrac{1}{0.0001} - \dfrac{1}{0.015}\right]$

   $= \mathbf{1430400\ dyne\ cm^{-2}}$.

6. Pressure inside the bubble

   = Excess pressure + Atmospheric pressure

   $= \dfrac{2 \times 0.059}{10^{-3}} + 101325 = 118 + 101325$

   $= \mathbf{101443\ Nm^{-2}}$.

7. Pressure inside the air bubble greater than atmospheric pressure

   $= \dfrac{2\sigma}{R} + h\rho g = \dfrac{2 \times 0.075}{1 \times 10^{-3}} + 0.10 \times 10^{3} \times 9.8$

   $= \mathbf{1130\ Nm^{-2}}$.

8. Excess pressure inside the dome, $p = \dfrac{4\sigma}{R}$

   Maximum load that the dome can support is

   $F = p \times 4\pi R^2 = \dfrac{4\sigma}{R} \times 4\pi R^2 = 4\pi R\sigma$

   $= 4 \times 3.142 \times 5 \times 500 = \mathbf{31420\ N}$.

10.49 ANGLE OF CONTACT

67. *Define the term angle of contact. On what factors does it depend ?*

Angle of contact. The liquid surface is usually curved when it is in contact with a solid. The particular shape that it takes depends on the relative strengths of cohesive and adhesive forces. If

Adhesive force > Cohesive force : Liquid wets the solid surface and has concave meniscus

Adhesive force < Cohesive force : Liquid does not wet the solid surface and has a convex meniscus

Adhesive force = Cohesive force : Liquid surface is plane

***Angle of contact** is defined as the angle θ between the tangent to the liquid surface at the point of contact and the solid surface inside the liquid.*

The value of angle of contact depends on the following factors :

(i) Nature of the solid and the liquid in contact.
(ii) Cleanliness of the surface in contact.
(iii) Medium above the free surface of the liquid.
(iv) Temperature of the liquid.

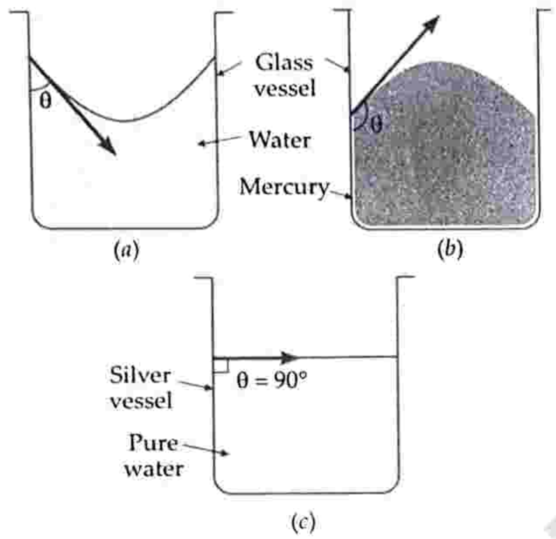


Fig. 10.55 Defining angle of contact. (a) Concave (b) Convex (c) Plane, menisci.

For those liquids which wet the walls of the vessel, the angle of contact is acute. For the liquids which do not wet the walls of the vessel, the angle of contact is obtuse.

The angle of contact for water and glass is about 8°, for mercury and glass it is 138° and for pure water and silver, angle of contact is 90°.

10.50 SHAPE OF LIQUID MENISCUS IN A NARROW TUBE

68. *Explain what determines the shape of liquid meniscus in a narrow tube.*

Shape of liquid meniscus in a narrow tube. Consider a molecule O on the surface of the liquid in contact with the solid wall of the vessel. The various forces acting at the boundary of the three surfaces are as follows :

(i) Surface tension σ_{LV} of the liquid-vapour surface acting tangentially to the liquid surface.
(ii) Surface tension σ_{SV} of the solid-vapour surface acting parallel to the walls of the vessel.
(iii) Surface tension σ_{SL} of the solid-liquid surface acting parallel to the wall of the vessel directed into the liquid.
(iv) Adhesive force F_a between the molecules of the vessel and the liquid acting normal to the wall of the container.

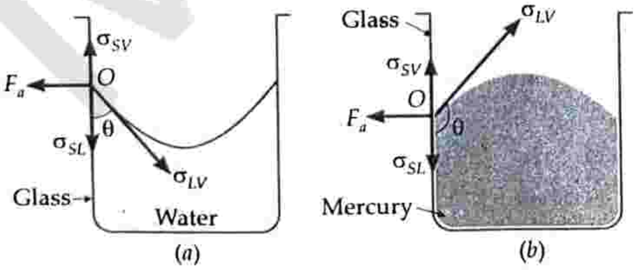


Fig. 10.56 Forces of surface tension at the boundaries of three phases.

For equilibrium, no forces should act on molecule O in any direction. Let θ be the angle of contact. Then the components of σ_{LV} parallel and perpendicular to water surface are $\sigma_{LV}\sin\theta$ and $\sigma_{LV}\cos\theta$ respectively. For equilibrium, we must have

$$F_a = \sigma_{LV}\cos\theta$$

and $$\sigma_{SV} = \sigma_{SL} + \sigma_{LV}\cos\theta$$

or $$\cos\theta = \frac{\sigma_{SV} - \sigma_{SL}}{\sigma_{LV}}$$

The following three cases are possible :

(i) If $\sigma_{SV} > \sigma_{SL}$, cos θ is positive and $\theta < 90°$ *i.e.,* angle of contact is acute. The liquid meniscus is concave upwards. This happens in the case of water taken in a glass vessel [Fig. 10.56(a)].
(ii) If $\sigma_{SV} < \sigma_{SL}$, cos θ is negative and $\theta > 90°$ *i.e.,* angle of contact is obtuse. The liquid meniscus is convex, upwards. This happens in the case of mercury taken in a glass vessel [Fig. 10.56(b)].
(iii) When $\sigma_{SV} = \sigma_{SL}$, $\cos\theta = 0$ and $\theta = 90°$. The liquid meniscus is plane. This happens in the case of pure water taken in a silver vessel.

10.51 CAPILLARITY

69. *What do you understand by the term capillarity ? Give some examples of capillarity from daily life.*

Capillarity. The Latin word *capilla means hair. A tube of very fine (hair-like) bore is called a capillary tube.*

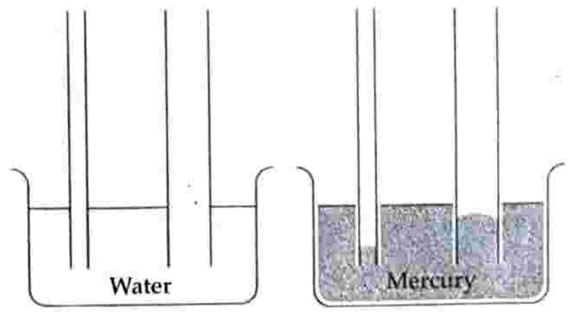


Fig. 10.57 Capillarity.

When a capillary tube of glass open at both ends is dipped in liquid which wets its walls (*e.g.*, water, alcohol), the liquid rises in the tube. But when the capillary tube is dipped in a liquid which does not wet its walls (*e.g.*, mercury), the level of liquid is depressed in the tube.

The phenomenon of rise or fall of a liquid in a capillary tube in comparison to the surrounding is called capillarity.

Some examples of capillarity from daily life :

(*i*) A blotting paper soaks ink by capillary action. The pores of blotting paper act as capillaries.

(*ii*) Oil rises in the long narrow spaces between the threads of a wick, the narrow spaces act as capillary tubes.

(*iii*) We use towels made of a coarse cloth for drying our skin after taking bath.

(*iv*) Sap rises from the roots of a plant to its leaves and branches due to capillarity action.

(*v*) The tip of the nib of a pen is split to provide capillary action for the ink to rise.

10.52 RISE OF LIQUID IN A CAPILLARY TUBE : ASCENT FORMULA

70. *Derive an expression for the rise of liquid in a capillary tube and show that the height of the liquid column supported is inversely proportional to the radius of the tube.*

Ascent formula. Consider a capillary tube of radius r dipped in a liquid of surface tension σ and density ρ. Suppose the liquid wets the sides of the tube. Then its meniscus will be concave. The shape of the meniscus of water will be nearly spherical if the capillary tube is of sufficiently narrow bore.

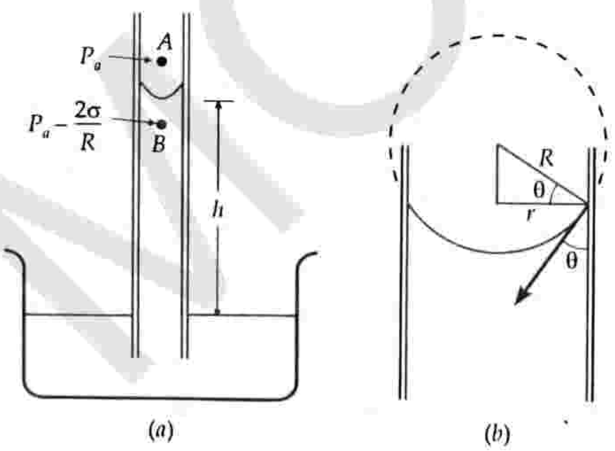


Fig. 10.58 (*a*) Rise of liquid in a capillary tube. (*b*) Enlarged view.

As the pressure is greater on the concave side of a liquid surface, so excess of pressure at a point A just above the meniscus compared to point B just below the meniscus is

$$p = \frac{2\sigma}{R}$$

where R = radius of curvature of the concave meniscus. If θ is the angle of contact, then from the right angled triangle shown in Fig. 10.58(*b*), we have

$$\frac{r}{R} = \cos\theta$$

or $$R = \frac{r}{\cos\theta}$$

$$\therefore \quad p = \frac{2\sigma\cos\theta}{r}$$

Due to this excess pressure p, the liquid rises in the capillary tube to height h when the hydrostatic pressure exerted by the liquid column becomes equal to the excess pressure p. Therefore, at equilibrium we have

$$p = h\rho g$$

or $$\frac{2\sigma\cos\theta}{r} = h\rho g$$

or $$h = \frac{2\sigma\cos\theta}{r\rho g}$$

This is the ***ascent formula*** for the rise of liquid in a capillary tube. If we take into account the volume of the liquid contained in the meniscus, then the above formula gets modified as

$$h = \frac{2\sigma\cos\theta}{r\rho g} - \frac{r}{3}$$

However, the factor $r/3$ can be neglected for a narrow tube.

The ascent formula shows that the height h to which a liquid rises in the capillary tube is

(*i*) *inversely proportional to the radius of the tube.*

(*ii*) *inversely proportional to the density of the liquid.*

(*iii*) *directly proportional to the surface tension of the liquid.*

Hence a liquid rises more in a narrower tube than in wider tube.

10.53 RISE OF LIQUID IN A CAPILLARY TUBE OF INSUFFICIENT HEIGHT

71. *Explain what happens when the length of a capillary tube is less than the height upto which the liquid may rise in it.*

Rise of liquid in a tube of insufficient height. The height to which a liquid rises in a capillary tube is given by

$$h = \frac{2\sigma\cos\theta}{r\rho g}$$

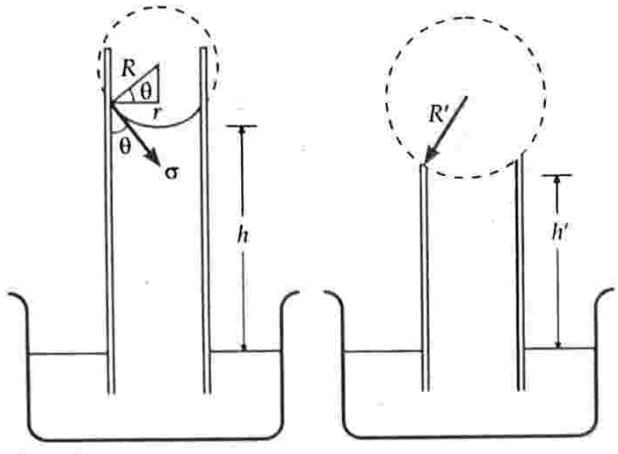


Fig. 10.59 Rise of liquid in a tube of insufficient height.

The radius r of the capillary tube and radius of curvature R of the liquid meniscus are related by $r = R \cos\theta$. Therefore,

$$h = \frac{2\,\sigma \cos\theta}{R \cos\theta\, \rho\, g} = \frac{2\,\sigma}{R\, \rho\, g}$$

As σ, ρ and g are constants, so

$$hR = \frac{2\,\sigma}{\rho\, g} = \text{a constant}$$

$$\therefore \quad hR = h'\, R'$$

where R' is the radius of curvature of the new meniscus at a height h'.

As $h' < h$, so $R' > R$

Hence in a capillary tube of insufficient height, the liquid rises to the top and spreads out to a new radius of curvature R' given by

$$R' = \frac{hR}{h'}$$

But the liquid will not overflow.

Examples based on Capillarity : Ascent Formula

FORMULAE USED

1. When a capillary tube of radius r is dipped in a liquid of density ρ and surface tension σ, the liquid rises or falls through a distance,

$$h = \frac{2\sigma \cos\theta}{r\rho g}$$

where θ is the angle of contact.

UNITS USED

Radius r is in metre, density ρ in kgm^{-3}, surface tension σ in Nm^{-1} and height h in metre.

EXAMPLE 90. *Calculate the height to which water will rise in capillary tube of 1.5 mm diameter. Surface tension of water is 7.4×10^{-3} Nm^{-1}.*

Solution.

Here $r = \frac{1.5}{2} = 0.75 \text{ mm} = 0.75 \times 10^{-3}$ m,

$\sigma = 7.4 \times 10^{-3}$ Nm^{-1}.

For water, $\rho = 10^3$ kg m^{-3},

Angle of contact $\theta = 0°$

$$\therefore \quad h = \frac{2\sigma\cos\theta}{r\rho g} = 2 \times \frac{7.4 \times 10^{-3} \times \cos 0°}{0.75 \times 10^{-3} \times 10^3 \times 9.8}$$

$$= \mathbf{0.002014\ m.}$$

EXAMPLE 91. *A liquid rises to a height of 7.0 cm in a capillary tube of radius 0.1 mm. The density of the liquid is 0.8×10^3 kg m^{-3}. If the angle of contact between the liquid and the surface of the tube be zero, calculate the surface tension of the liquid. Given $g = 10$ ms^{-2}.*

Solution. Here $h = 7.0 \text{ cm} = 7.0 \times 10^{-2}$ m,

$r = 0.1 \text{ mm} = 10^{-4}$ m, $\rho = 0.8 \times 10^3$ kgm^{-3}, $\theta = 0°$

$g = 10$ ms^{-2}

$$\sigma = \frac{hr\rho g}{2\cos\theta} = \frac{7.0 \times 10^{-2} \times 10^{-4} \times 0.8 \times 10^3 \times 10}{2 \times \cos 0°}$$

$$= \mathbf{2.8 \times 10^{-2}\ Nm^{-1}}.$$

EXAMPLE 92. *Water rises up in a glass capillary upto a height of 9.0 cm, while mercury falls down by 3.4 cm in the same capillary. Assume angles of contact for water-glass and mercury-glass as 0° and 135° respectively. Determine the ratio of surface tensions of mercury and water. Take $\cos 135° = -0.71$.*

Solution. *For water :* $h_1 = 9 \text{ cm} = 0.09$ m,

$\rho_1 = 10^3$ kgm^{-3}, $\theta_1 = 0°$

For mercury : $h_2 = -3.4 \text{ cm} = -0.034$ m,

$\rho_2 = 13.6 \times 10^3$ kgm^{-3}, $\theta_2 = 135°$

Let σ_w and σ_m be the surface tensions of water and mercury respectively. Then

$$\sigma_w = \frac{h_1\, r\rho_1\, g}{2\cos\theta_1} \quad \text{and} \quad \sigma_m = \frac{h_2\, r\rho_2\, g}{2\cos\theta_2}$$

$$\frac{\sigma_m}{\sigma_w} = \frac{h_2\rho_2 \cos\theta_1}{h_1\rho_1 \cos\theta_2}$$

$$= \frac{-0.034 \times 13.6 \times 10^3 \times \cos 0°}{0.09 \times 10^3 \times \cos 135°}$$

$$= \frac{-0.034 \times 13.6 \times 10^3 \times 1}{0.09 \times 10^3 \times (-0.71)} = \mathbf{7.2 : 1.}$$

EXAMPLE 93. *Water rises in a capillary tube to a height of 2.0 cm. In another capillary whose radius is one-third of it, how much the water will rise ? If the first capillary is inclined at an angle of 60° with the vertical, then what will be the position of water in the tube ?*

Solution. Ascent of a liquid in a capillary tube is given by

$$h = \frac{2\sigma\cos\theta}{r\rho g}$$

$\therefore$ For a given liquid,

$$hr = \frac{2\sigma\cos\theta}{\rho g} = \text{constant}$$

$$[\because \sigma, \theta, \rho, g \text{ are constants}]$$

or $\qquad h'r' = hr$

For a capillary tube of radius $r/3$, we have

$$h' = \frac{hr}{r'} = \frac{2.0\text{ cm} \times r}{r/3} = \mathbf{6.0\text{ cm.}}$$

When the first capillary is inclined at an angle of 60° to the vertical, the vertical height h (=2.0 cm) of the liquid will remain the same. Thus if the length of water in the capillary be l cm, then from Fig. 10.60, we have

$$l = \frac{h}{\cos 60°} = \frac{2.0\text{ cm}}{0.5} = \mathbf{4.0\text{ cm.}}$$

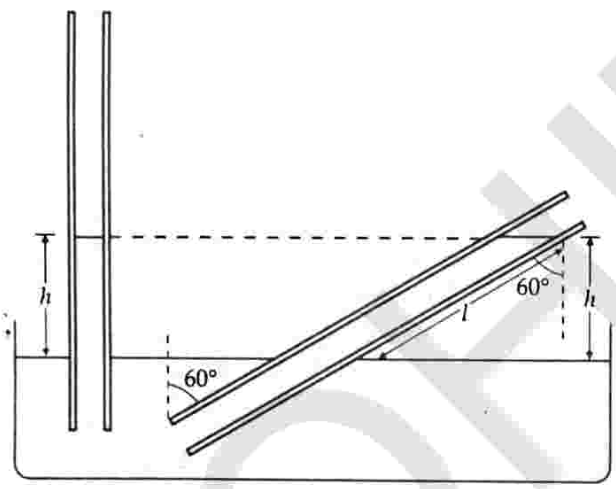


Fig. 10.60

Example 94. *If a 5 cm long capillary tube with 0.1 mm internal diameter opens at both ends is slightly dipped in water having surface tension 75 dyne cm^{-1}, state whether (i) water will rise half way in the capillary (ii) water will rise up to the upper end of capillary (iii) water will overflow out of the upper end of capillary. Explain your answer.*

Solution.

Radius, $r = \frac{0.1}{2}$ mm $= 0.05$ mm $= 0.005$ cm

Surface tension, $\sigma = 75$ dyne cm^{-1}

density, $\rho = 1$ gcm^{-3} ; angle of contact, $\theta = 0°$

Let h be the height to which water rises in the capillary tube. Then

$$h = \frac{2\sigma\cos\theta}{r\rho g} = \frac{2 \times 75 \times \cos 0°}{0.005 \times 1 \times 981} = \mathbf{30.58\text{ cm.}}$$

Given length of capillary tube, $h' = 5$ cm

(*i*) As $h > \frac{h'}{2}$, so the first possibility is ruled out.

(*ii*) As the tube is of insufficient length, so the water will rise upto the upper end of the tube.

(*iii*) The water will not overflow out of the upper end of the capillary. It will rise only upto the upper end of the capillary. The liquid meniscus will adjust its radius of curvature R' in such a way that

$$R'h' = Rh \qquad \left[\because hR = \frac{2\sigma}{\rho g} = \text{constant}\right]$$

where R is the radius of curvature that the liquid meniscus would possess if the capillary tube were of sufficient length.

$$\therefore \quad R' = \frac{Rh}{h'} = \frac{rh}{h'} = \frac{0.005 \times 30.58}{5} = \mathbf{0.0306\text{ cm.}}$$

$$\left[\because R = \frac{r}{\cos\theta} = \frac{r}{\cos 0°} = r\right]$$

Example 95. *A glass U-tube is such that the diameter of one limb is 3.0 mm and that of the other is 6.00 mm. The tube is inverted vertically with the open ends below the surface of water in a beaker. What is the difference between the heights to which water rises in the two limbs ? Surface tension of water is 0.07 Nm^{-1}. Assume that the angle of contact between water and glass is 0°.*

Solution. Let P_A, P_B, P_C and P_D be the pressures at points A, B, C and D respectively. The pressure on the concave side of the liquid surface is greater than that on the other side by $2\sigma/R$.

As angle of contact θ is 0°, so

$$R\cos 0° = r \quad \text{or} \quad R = r$$

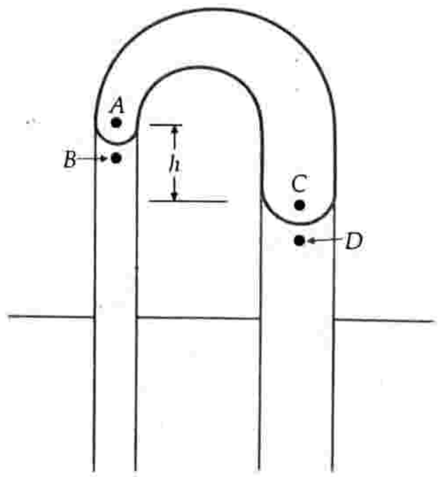


Fig. 10.61

$$\therefore \quad P_A = P_B + \frac{2\sigma}{r_1} \text{ and } P_C = P_D + \frac{2\sigma}{r_2}$$

where r_1 and r_2 are the radii of the two limbs.

But $\quad P_A = P_C$

$$\therefore \quad P_B + \frac{2\sigma}{r_1} = P_D + \frac{2\sigma}{r_2}$$

or $\quad P_D - P_B = 2\sigma\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$

or $\quad h\rho g = 2\sigma\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$

or $\quad h = \frac{2\sigma}{\rho g}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$

Given $\quad \sigma = 0.07\ \text{Nm}^{-1}, \quad \rho = 1000\ \text{kgm}^{-3},$

$r_1 = 1.5 \times 10^{-3}$ m, $r_2 = 3 \times 10^{-3}$ m

$$\therefore \quad h = \frac{2 \times 0.07}{1000 \times 9.8}\left(\frac{1}{1.5 \times 10^{-3}} - \frac{1}{3 \times 10^{-3}}\right) \text{m}$$

$= 4.76 \times 10^{-3}$ m $=$ **4.76 mm.**

Problems For Practice

1. The radius of a capillary tube is 0.025 mm. It is held vertically in a liquid whose density is 0.8×10^3 kg m^{-3}, surface tension is 3.0×10^{-2} Nm^{-1} and for which the cosine of the angle of contact is 0.3. Determine the height upto which the liquid will rise in the tube. Given $g = 10$ ms^{-2}. (**Ans.** 9 cm)
2. A capillary tube of inner diameter 0.5 mm is dipped in a liquid of specific gravity 13.6, surface tension 545 dyne cm^{-1} and angle of contact 130°. Find the depression or elevation in the tube. (**Ans.** – 2.1 cm)
3. Calculate the diameter of a capillary tube in which mercury is depressed by 1.21 cm. Given surface tension for mercury is 540×10^{-3} Nm^{-1}, the angle of contact with glass is 140° and density of mercury is 13.6×10^3 kg m^{-3}. (**Ans.** 1.026×10^{-3} m)
4. The tube of mercury barometer is 5 mm in diameter. How much error does the surface tension cause in the reading ? S.T. of mercury $= 540 \times 10^{-3}$ Nm^{-1}. Angle of contact = 135°. (**Ans.** -0.2293×10^{-2} m)
5. Water rises to a height of 9 cm in a certain capillary tube. If in the same tube, level of Hg is depressed by 3 cm, compare the surface tension of water and mercury. Specific gravity of Hg is 13.6, the angle of contact for water is zero and that for Hg is 135°. (**Ans.** 0.152)
6. A capillary tube whose inside radius is 0.5 mm is dipped in water of surface tension 75 dyne cm^{-1}. To what height is the water raised by the capillary action above the normal level ? What is the weight of water raised ? (**Ans.** 3.061 cm, 23.55 dyne or 0.024 g wt)
7. A U-tube is made up of capillaries of bore 1 mm and 2 mm respectively. The tube is held vertically and partially filled with a liquid of surface tension 49 dyne cm^{-1} and zero contact angle. Calculate the density of the liquid, if the difference in the levels of the meniscus is 1.25 cm. Take $g = 980$ cms^{-2}. (**Ans.** 0.8 g cm^{-3})

Hints

3. Here $h = -1.21\text{ cm} = -1.21 \times 10^{-2}$ m, $\theta = 140°$

$\sigma = 540 \times 10^{-3}$ Nm^{-1},

$\cos 140° = \cos(180° - 40°) = -\cos 40° = -0.7660$

Diameter,

$$2r = \frac{4\sigma\cos\theta}{h\rho g} = \frac{4 \times 540 \times 10^{-3} \times (-0.7660)}{-1.21 \times 10^{-2} \times 13.6 \times 10^3 \times 9.8}$$

$= \mathbf{1.026 \times 10^{-3}}$ **m.**

4. Error in the barometer reading

= Depression of mercury level due to surface tension

$$= \frac{2\sigma\cos\theta}{r\rho g} = \frac{2 \times 540 \times 10^{-3} \times \cos 135°}{2.5 \times 10^{-3} \times 13.6 \times 10^3 \times 9.8}$$

$= \mathbf{-0.2293 \times 10^{-2}}$ **m.**

5. $$\frac{\sigma_w}{\sigma_m} = \frac{h_1 \rho_1 \cos\theta_2}{h_2 \rho_2 \cos\theta_1} = \frac{10 \times 1 \times \cos 135°}{-3.42 \times 13.6 \times \cos 0°}$$

$$= \frac{10 \times (-0.7071)}{-3.42 \times 13.6} = \mathbf{0.152.}$$

6. Here $r = 0.5$ mm $= 0.05$ cm, $\sigma = 75$ dyne cm^{-1},

$\rho = 1$ g cm^{-3}, $\theta = 0°$, $g = 980$ dyne cm^{-1}.

$$h = \frac{2\sigma\cos\theta}{r\rho g} = \frac{2 \times 75 \times 1}{0.05 \times 1 \times 980} = \mathbf{3.061\ cm}$$

Weight of water raised,

$W = mg =$ Volume $\times$ Density $\times g = \pi r^2 h \rho g$

$= 3.14 \times (0.05)^2 \times 3.061 \times 980 =$ **23.55 dyne**

$=$ **0.024 g wt.**

7. Here $\mathbf{h_1 = \frac{2\sigma\cos\theta}{r_1 \rho g}}$ and $\mathbf{h_2 = \frac{2\sigma\cos\theta}{r_2 \rho g}}$

$$\therefore \quad h_1 - h_2 = \frac{2\sigma\cos\theta}{\rho g}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$\rho = \frac{2\sigma\cos\theta}{(h_1 - h_2)\, g}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

Now $r_1 = \frac{1}{2}$ mm $= 0.05$ cm,

$r_2 = \frac{2}{2}$ mm $= 0.1$ cm,

$\sigma = 49$ dyne cm^{-1},

$h_1 - h_2 = 1.25$ cm, $\theta = 0°$, $g = 980$ cm s^{-2}

$$\therefore \quad \rho = \frac{2 \times 49 \times \cos 0°}{1.25 \times 980}\left[\frac{1}{0.05} - \frac{1}{0.1}\right]$$

$$= \frac{2 \times 49 \times 1}{1.25 \times 980} \times 10 = \mathbf{0.8\ g\ cm^{-3}}.$$

10.54 FACTORS AFFECTING SURFACE TENSION

72. *Describe the various factors which affect the surface tension of a liquid.*

Factors affecting surface tension of a liquid :

(*i*) **Effect of contamination.** If water surface has dust, grease or oil, the surface tension of water reduces. A small piece of camphor put in clear water dances vigorously due to decrease of surface tension of water.

(*ii*) **Effect of solute.** (*a*) A highly soluble substance like sodium chloride increases the surface tension of water. (*b*) A sparingly soluble substance like phenol or soap reduces the surface tension of water.

(*iii*) **Effect of temperature.** The surface tension of liquids decreases with increase of temperature. The surface tension of a liquid becomes zero at a particular temperature, called *critical temperature* of that liquid. For small temperature differences, surface tension decreases almost linearly as

$$\sigma_t = \sigma_0 (1 - \alpha t)$$

where σ_t and σ_0 are the surface tensions at t°C and 0°C respectively, and α is the temperature coefficient of surface tension.

10.55 DETERGENTS AND SURFACE TENSION

73. *Describe the cleansing action of detergents.*

Cleansing action of detergents. Oil stains and grease on dirty clothes cannot be removed by simply washing the clothes with water because water does not wet them. By adding detergent or soap to water, the greasy dirt can be easily removed. The cleansing action of detergents can be explained as follows :

(*i*) Soap or detergent molecules have the shape of a hairpin.

(*ii*) When detergent is dissolved in water, the heads of its hairpin shape molecules get attracted to water surface.

(*iii*) When clothes with greasy stains are dipped in water containing detergent, the pointed ends of detergent molecules get attached to the molecules of grease. So a water-grease interface is formed. Thus surface tension is greatly reduced. The greasy dirt is held suspended.

(*iv*) When the clothes are rinsed in water, the greasy dirt is washed away by running water.

So when detergent is added to water, the surface tension of water is reduced, its area of contact with grease is increased and hence its cleansing ability is increased.

Very Short Answer Conceptual Problems

Problem 1. We can cut an apple easily with a sharp knife as compared to with a blunt knife. Explain why ? *[Himachal 09]*

Solution. The area of a sharp edge is much less than the area of a blunt edge. For the same total force, the effective force per unit area (or pressure) is more for the sharp edge than the blunt one. Hence a sharp knife cuts easily than a blunt knife.

Problem 2. Why are the bags and suitcases provided with broad handles ?

Solution. Broad handles have large area. So the force per unit area or the pressure exerted on the hand will be small while carrying the bag or the suitcase.

Problem 3. Railway tracks are laid on large sized wooden sleepers. Why ?

Solution. This spreads force due to the weight of the train on a larger area and hence reduces the pressure considerably. This, in turn, prevents yielding of the ground under the weight of the train.

Problem 4. It is painful to walk barefooted on the ground covered with edged pebbles. Why ?

Solution. While walking, when entire weight of our body gets supported on the sharp edge of any pebble, it will exert a large pressure on our feet due to reaction. This causes considerable pain on our feet.

Problem 5. It is difficult for a man to walk on sand. Why ?

Solution. This is because sand yields under the weight of the man. This difficulty can be overcome by placing wooden plank on the sand. Then the weight of the man will act on larger area and hence pressure exerted on sand decreases. The sand does not yield.

Problem 6. Water does not come out of a dropper unless its rubber bulb is pressed hard. Why ?

Solution. Water is held inside the dropper against the atmospheric pressure. When the rubber bulb is pressed, pressure on water becomes greater than the atmospheric pressure and so water comes out.

Problem 7. Why two holes are made to empty an oil tin ?

Solution. When oil comes out through a tin with one hole, the pressure inside the tin becomes less than the atmospheric pressure, soon the oil stops flowing out. When two holes are made in the tin, air keeps on entering the tin through the other hole and maintains pressure inside.

Problem 8. Why is a slight blow on a cork of bottle filled with a liquid sufficient to break the bottle ?

Solution. The blow on the cork exerts a pressure ($= f/a$) on the liquid. According to Pascal's law, this pressure is transmitted to the entire bottle through the liquid. Since the surface area (A) of the bottle is large, the large force ($= fA/a$) so developed is sufficient to break the bottle.

Problem 9. What is the force on a man due to atmospheric pressure ? Why one does not feel it ?

Solution. Atmospheric pressure on the surface of the earth is about 1 kg f per square centimetre. The surface area of the body of average man is about $2 m^2$ and hence, a total force equal to about 2×10^5 N, acts on the body of a man due to atmospheric pressure. But one does not feel this enormous force because his blood exerts a pressure slightly greater than the atmospheric pressure.

Problem 10. How does the boiling point of a liquid vary with pressure ?

Solution. The boiling point of a liquid increases with pressure. For example, if the pressure is more than the atmospheric pressure, water boils at a temperature higher than 100°C.

Problem 11. Why the boiling point of a liquid varies with pressure ?

Solution. At the boiling point, vapour pressure of the liquid is equal to the atmospheric pressure. Hence when the atmospheric pressure on the surface of the liquid increases, the liquid boils at higher temperature to generate greater vapour pressure.

Problem 12. Why the food is cooked faster in the pressure cooker ? Why it becomes difficult to cook food at the mountains ?

Solution. The pressure inside the pressure cooker is very high. This raises the boiling point of water and the temperature inside the cooker is higher than 100°C which results in faster cooking of food. At the mountains, the pressure is less, so the boiling point of water is less than 100°C. This makes the cooking of food difficult.

Problem 13. Why the passengers are advised to remove the ink from their pens while going up in an aeroplane ?

Solution. We know that atmospheric pressure decreases with height. Since ink inside the pen is filled at the atmospheric pressure existing on the surface of earth, it tends to come out to equalise the pressure. This can spoil the clothes of the passengers, so they are advised to remove the ink from the pen.

Problem 14. Why is it difficult to stop bleeding from a cut in human body at high altitudes ?

Solution. The atmospheric pressure is low at high altitudes. Due to greater pressure difference in blood pressure and the atmospheric pressure, it is difficult to stop bleeding from a cut in the body.

Problem 15. Why are straws used to suck soft drinks ?

Solution. When we suck through the straw, the pressure inside the straw becomes less than the atmospheric pressure. Due to the pressure difference, the soft drink rises in the straw and we are able to take the soft drink easily.

Problem 16. If a drop of water vapour is introduced in a mercury barometer, how will the barometric height change ?

Solution. Due to pressure exerted by water vapour, the barometric height decreases.

Problem 17. Why water is not used in barometers ?

Solution. Water is not used in barometer due to following reasons :

(*i*) It sticks to the walls of the barometer tube.

(*ii*) It has low density. A barometer with water will require a tube length of about 11 m which is unmanageable.

Problem 18. Why is mercury used in barometers ?

Solution. Mercury is used in barometers due to the following reasons :

(*i*) It does not stick to the walls of the barometric tube.

(*ii*) Its density is high, so the length of the tube used is conveniently small.

(*iii*) Its vapour pressure is quite small.

Problem 19. Why is the reading of a mercury barometer always less than actual pressure ?

Solution. Due to property of surface tension, mercury in the tube gets depressed. Consequently, the observed height of mercury in the barometer tube is less than its actual height. Thus, the reading of mercury barometer is always less than the actual pressure.

Problem 20. How can we check whether the barometric tube contains air or not ?

Solution. If on raising or lowering the barometric tube in a trough of mercury, the height of mercury column changes, then air is present in the barometric tube.

Problem 21. The dams of water reservoir are made thick near the bottom. Why ?

Solution. Pressure exerted by a liquid column of height $h = h\rho g$. As the value of h is greatest near the bottom, so pressure exerted by water is greatest near the bottom. So dams are made thick near the bottom.

Problem 22. Why an air bubble in water rises from bottom to top and grows in size ?

Solution. The fluids move from higher pressure to lower pressure and a fluid pressure increases with depth. Hence pressure at the top is less than that at the bottom and so the air bubble will rise from bottom to top. When bubble moves from bottom to top, pressure decreases and in accordance with Boyle's law volume V will increase *i.e.*, bubble will grow in size.

Problem 23. A beaker containing a liquid is kept inside a big closed jar. If the air inside the jar is continuously pumped out, how will the pressure change inside the liquid near the bottom ?

Solution. The total pressure at a point near the bottom is equal to the sum of the pressure due to liquid column and due to air inside the jar. When air of the jar is pumped out, the pressure of air inside the jar decreases. So pressure near the bottom of the liquid also decreases.

Problem 24. Why does a siphon fail to work in vacuum ?

Solution. As siphon works on account of atmospheric pressure, hence it fails to work in vacuum.

Problem 25. A barometer kept in an elevator accelerating upwards reads 76 cm of Hg. What will be the possible air pressure inside the elevator ?

Solution. The net acceleration of the elevator accelerating upwards $= g + a$

$\therefore$ Pressure inside the elevator

$$= h\rho\,(g + a) = \frac{76 \times 13.6 \times (g + a)}{13.6 \times g} \text{ cm of Hg}$$

Clearly, this pressure will be greater than 76 cm of Hg.

Problem 26. A barometer accelerating downwards reads 76 cm of Hg. What will be the possible air pressure inside the jar ?

Solution. The net acceleration of the elevator accelerating downwards $= g - a$

$\therefore$ Pressure inside the elevator

$$= h\rho\,(g - a) = \frac{76 \times 13.6 \times (g - a)}{13.6 \times g} \text{ cm of Hg}$$

Clearly, this pressure will be less than 76 cm of Hg.

Problem 27. In a mercury barometer, at sea level, the normal pressure of the air (one atmosphere) acting on the mercury in the dish supports a 76 cm column of mercury in a closed tube. If you go up in the air, until the density has fallen to half its sea level value, what height of mercury column would you expect ?

Solution. Pressure exerted by a gas is directly proportional to its density. When we go high up in air at a point where the density of air falls to half its sea level value, the pressure also reduces to half its sea-level value. Hence the height of mercury column is also halved *i.e.*, it becomes 38 cm.

Problem 28. A liquid cannot withstand a shear stress. How does this imply that the surface of a liquid at rest must be level, *i.e.*, normal to the gravitational force ?

Solution. If the free surface of the liquid is not perpendicular to the gravitational force, then there will be a component of force along the surface. The liquid will not be at rest in that case.

Problem 29. A wooden block is on the bottom of a tank when water is poured in. The contact between the block and the tank is so good that no water gets between them. Is there a buoyant force on the block ?

Solution. Since there is no water under the block to exert an upward force on it, therefore, there is no buoyant force.

Problem 30. A piece of iron sinks in water, but a ship made of iron floats in water. Why ?

Solution. The weight of water displaced by iron piece is less than its own weight, so it sinks. On the other hand, the ship displaces water more than its own weight, so it floats.

Problem 31. A man is sitting in a boat, which is floating in a pond. If the man drinks some water from the pond, will the level of water in the pond fall ?

Solution. No. When the man drinks water, say m kg, he displaces m kg of water and hence the level tends to increase. But m kg of water has already gone inside his stomach. So the level remains the same.

Problem 32. An ice cube floats in a glass of water filled to the brim. What happens when the ice melts ?

Solution. The water level remains unchanged. The ice cube displaces a weight of water equal to its own weight. When the ice cube melts, the volume of water produced equals the volume of water it displaced when frozen.

Problem 33. An ice piece with an air bubble in it is floating in a vessel containing water. What will happen to the level of water when the ice melts completely ?

Solution. The level of water does not change. The mass of air bubble is negligible. So the volume of water produced during melting is equal to the volume of water displaced by ice piece with air bubble.

Problem 34. Ice floats in water with about nine-tenths of its volume submerged. What is the fractional volume submerged for an iceberg floating on a fresh water lake of a (hypothetical) planet whose gravity is ten times that of earth ?

Solution. The fractional volume submerged does not depend upon the value of acceleration due to gravity. So, on the new planet, the ice will float in water with nearly nine-tenths of its volume submerged.

Problem 35. Does the Archimedes' principle hold in a vessel in free fall ?

Solution. No. The vessel in free fall is in a state of weightlessness *i.e.*, the value of g is zero. The buoyant force does not exist. Hence Archimedes' principle does not hold good.

Problem 36. What is the fractional volume submerged of an ice cube in a pail of water placed in an enclosure which is falling freely under gravity ?

Solution. Since the pail of water is falling freely, therefore, it will be in state of weightlessness. Both the weight of the ice cube and the upthrust would be zero. So, the ice cube can float with any value of fractional volume submerged in water.

Problem 37. A piece of cork is floating in water contained in a beaker. What is the apparent weight of the cork piece ?

Solution. The weight of the cork piece acting vertically downwards is balanced by the upthrust due to water. So the apparent weight of the floating cork piece is zero.

Problem 38. One small and one big piece of cork are pushed below the surface of water. Which has greater tendency to rise swiftly ?

Solution. The upthrust on a piece of cork is equal to the weight of water displaced by it. So upthrust is greater on the bigger piece of cork and it has greater tendency to rise swiftly.

Problem 39. Why is it easier to swim in sea water than in river water ?

Solution. The sea water has many salts dissolved in it. So the density of sea water is greater than that of river water. Consequently, the sea water exerts greater upthrust on the swimmer than the river water. Hence it is easier to swim in sea water than in river water.

Problem 40. Two bodies of equal weight and volume and having the same shape, except that one has an opening at the bottom and the other is sealed, are immersed to the same depth in water. Is less work required to immerse one than the other ? If so, which one and why ?

Solution. More work is required in case of the body having hole at its bottom. As the liquid enters the hole, more work is required in compressing the air, less work is required in case of the sealed body.

Problem 41. A wooden cylinder floats in a vessel with its axis vertical. How will the level of water in the vessel change if the cylinder floats with its axis horizontal ?

Solution. The level of water will not change because in both cases, the cylinder displaces equal volume of water.

Problem 42. A block of ice is floating in a liquid of specific gravity 1.2 contained in a beaker. When the ice melts completely will the level in the beaker change ?

Solution. The level of liquid in the beaker will rise. This is because the density of water formed by melting of ice is less than the density of liquid in the beaker. Consequently, the volume of water formed by melting of ice will be more than the volume of the portion of the liquid displaced by ice while floating.

Problem 43. A boy is carrying a fish in one hand and a bucket full of water in the other hand. He then places the fish in the bucket and thinks that in accordance with Archimedes' principle he is now carrying less weight as weight of fish will reduce due to upthrust. Is he right ?

Solution. No. When he places the fish in the water in the bucket, the weight of fish is reduced due to upthrust but the weight of water is increased by the same amount so that the total weight carried by the boy remains the same.

Problem 44. A solid body floats on mercury with a part of its volume below the surface. Will the fractional volume of the body immersed in mercury increase or decrease, if a layer of water poured on the top of mercury covers the body completely ?

Solution. In the first case, the weight of the body is balanced by the weight of mercury displaced. But when water is poured to cover the body completely, the weight of (mercury + water) displaced by the body is equal to the weight of the body. The weight of mercury displaced is now less than that in the first case. So the fractional volume of the body inside the mercury will decrease.

Problem 45. A bucket of water is suspended from a spring balance. Does the reading of balance change (*a*) When a piece of stone suspended from a string is immersed in water without touching the bucket ? (*b*) When a piece of iron or cork is put in water in the bucket ?

Solution. (*a*) Yes, the reading of balance will increase but the increase in weight will be equal to the loss in weight of stone and not the weight of stone.

(*b*) Yes, the reading of balance will increase but increase in weight will be equal to weight of iron or cork piece.

Problem 46. Why a sinking ship often turns over as it becomes immersed in water ?

Solution. When the ship is floating, the meta-centre of the ship is above the centre of gravity. While sinking, the ship takes in water. As a result, the centre of gravity is raised above the meta-centre. The ship turns over due to the couple formed by the weight and the buoyant force.

Problem 47. A soft plastic bag weighs the same when empty as when filled with air at atmospheric pressure. Why ?

Solution. The weight of air displaced by the bag is same as the weight of air inside it. The increase in weight due to the filled air gets cancelled by the upthrust of air. So weight remains same when air is filled in the bag.

Problem 48. Stirred liquid comes to rest after some time. Why ?

Solution. Different layers of a stirred liquid destroy the relative motion among themselves due to internal viscous force.

Problem 49. What is the reason that a constant driving force is always required for the maintenance of the flow of oil through the pipe lines in the oil refineries ?

Solution. Due to viscous force between the layers of oil, the motion of liquids gets retarded after a certain distance. Hence, a constant driving force is required for maintaining the flow of oil through the pipe lines.

Problem 50. What is the effect of temperature on the viscosity of liquids and gases ?

Solution. The viscosity of liquids decreases with rise in temperature while that of gases increases with an increase in temperature.

Problem 51. Hotter liquids move faster than colder ones. Why ? *[Central Schools 12]*

Solution. The viscosity or the internal force of friction of a liquid decreases with the increase in temperature. Hence hotter liquids move faster than colder ones.

Problem 52. Why oils of different viscosity are used in different seasons ?

Solution. Due to the rise of temperature in summer, the viscosity of oil decreases. Hence more viscous oils are used in summer than in winter.

Problem 53. Why machine parts are jammed in the winter ?

Solution. In winter season, the coefficient of viscosity of the lubricating oil gets increased due to fall in temperature. As a result of this, oil becomes more thick and therefore various machine parts are jammed.

Problem 54. One flask contains glycerine and other contains water. Both are stirred vigorously and kept on the table. Which liquid will come rest to earlier than the other ?

Solution. As the coefficient of viscosity of glycerine is greater than that of water, so the relative motion between different layers of glycerine is more strongly opposed compared to water. Hence glycerine comes to rest earlier than water.

Problem 55. Why should the lubricant oil be of high viscosity ?

Solution. Lubricant oil is used to reduce the dry friction between various machine parts. Due to high viscosity, lubricant oil sticks to the machine parts and cannot be squeezed out during the working of the machine.

Problem 56. Why does an object entering the earth's atmosphere at high velocity catch fire ?

Solution. When an object enters the earth's atmosphere at high velocity, its motion is strongly opposed by the viscous drag of air. As a result, the kinetic energy of the object gets converted into heat energy. The heat produced may be so large that the object catches fire.

Problem 57. The velocity of water in a river is less on the bank and large in the middle. Why ?

Solution. Due to large adhesive force between the water stream and the bank of the river, the velocity of water is very small near the bank. It increases towards the middle of the river due to decrease in the adhesive force.

Problem 58. Which fall faster – big rain drops or small rain drops ?

Solution. The terminal velocity of rain drops

$$v_t = \frac{2}{9}\frac{r^2(\rho-\sigma)}{\eta}g \quad i.e., \; v_t \propto r^2.$$

Hence big drops fall faster.

Problem 59. Why rain drops falling under gravity do not gain very high velocity ?

Solution. When the rain drops fall freely under gravity, their motion is opposed by the viscous drag of air. They attain terminal velocity when the viscous force due to air becomes equal to the weight of the drops.

Terminal velocity, $v \propto r^2$

As radii of rain drops are small, so they do not acquire very high velocity.

Problem 60. Why dust generally settles down in a closed room ?

Solution. Dust particles are spheres of small radii. After acquiring the terminal velocity, they start falling through air with uniform speed. As the terminal velocity for small dust particles will be very small, they will settle down in a closed room after some time.

Problem 61. Why do clouds seen floating in the sky ?

Solution. We know that terminal velocity of a spherical body falling through a viscous medium is directly proportional to the square of its radius. Hence terminal velocity of a small drop of water is very small. The small drops of water acquire this terminal velocity much before reaching the earth, and are seen to float in the sky in the form of clouds.

Problem 62. Explain why parachute is invariably used, while jumping from an aeroplane.

Solution. A parachute experiences a large viscous force due to its huge structure. Hence, it descends through the air with a very small velocity and the person using the parachute does not get hurt.

Problem 63. Fog particles appear suspended in the atmosphere. Give reason. *[Delhi 11]*

Solution. Fog particles have very small sizes. So their terminal velocity ($v_t \propto r^2$) through air is very small. They appear suspended in the atmosphere.

Problem 64. The sides of a horizontal pipe carrying dirty water get dirty. Why ?

Solution. While flowing through the horizontal pipe, the velocity of dirty water is maximum along the axis of the pipe and minimum near the walls of the pipe. The viscous force opposes the relative motion between adjacent layers. Due to this, the dirt particles move outwards and get struck on the walls of the pipe. This makes the pipe dirty.

Problem 65. Why two streamlines cannot cross each other ? *[Himachal 04]*

Solution. If two streamlines cross each other, there will be two directions of flow at the point of intersection which is impossible.

Problem 66. What happens when the velocity of a liquid becomes greater than its critical velocity ?

Solution. The flow of liquid changes from streamline to turbulent.

Problem 67. What happens to the external energy maintaining the flow of a liquid when the flow becomes turbulent ?

Solution. In turbulent flow of a liquid, most of the external energy maintaining the flow is used in setting up the eddies in the liquid.

Problem 68. Why does the velocity increase when water flowing in a broader pipe enters a narrow pipe ?

Solution. This is due to equation of continuity :

$$a_1 v_1 = a_2 v_2$$

As $a_2 < a_1$, so $v_2 > v_1$.

Problem 69. Why still water runs deep ?

Solution. In case of a deep water, the cross-sectional area, through which it flows, is quite large. Hence according to equation of continuity, its velocity is small and therefore it appears to be still.

Problem 70. Why it is dangerous to stand near the edge of the platform when a fast train is crossing it ?

Solution. When a fast train crosses the platform, the air dragged along with the train also moves with a high velocity. In accordance with Bernouilli's equation, the pressure in the region of high velocity gets decreased. If a person stands near the edge of the platform he may be pushed towards the train due to high pressures outside.

Problem 71. Why two boats moving in parallel directions close to each other get attracted ? *[Delhi 11]*

Solution. When the two boats come closer to each other, the velocity of water between the narrow gap increases and so pressure decreases in accordance with Bernouilli's equation. The pressure on the outer surfaces of both the row boats becomes greater than the pressure in the gap. Therefore, the two boats are pulled towards each other.

Problem 72. Why does the speed of a liquid increase and its pressure decrease, when the liquid passes through a narrow constriction in a pipe ?

Solution. When the liquid passes through a narrow constriction, its velocity increases ($v \propto 1/a$) in accordance with the equation of continuity. The Bernouilli's equation for the horizontal flow is

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

As the velocity is high at the narrow constriction, so the pressure is low.

Problem 73. Why does a flag flutter, when strong winds are blowing on a certain day ?

Solution. When strong winds blow over the top of the flag, the kinetic energy of the wind at the top is more and hence pressure decreases. Due to difference in pressure above and below the flag, the flag flutters.

Problem 74. On which principle does a carburettor work ?

Solution. Carburettor is that chamber of an internal combustion engine in which air is mixed with petrol vapours. Air is allowed to enter through a nozzle with high velocity. The pressure is lowered at that point and petrol is sucked up into the chamber. The petrol vaporises quickly. The mixed vapours of petrol and air are then fed into the cylinder of internal combustion engine.

Problem 75. Roofs of the huts are blown up during stormy days. Why ? *[Delhi 01 ; Central Schools 09]*

Solution. The velocity of the wind is high above the roof. So the pressure is low in accordance with Bernouilli's principle. But the pressure below the roof is the atmospheric pressure which is high. So the roof is blown up.

Problem 76. When air is blown between two balls suspended close to each other, they are attracted towards each other. Why ? *[Delhi 10]*

Solution. When air is blown between the two balls, the velocity is increased and hence pressure is decreased (Bernolli's principle). On the outer sides of the balls, the pressure is high and hence the two balls get attracted.

Problem 77. An aeroplane runs for some distance on the runway before taking off. Why ?

Solution. Generally, air does not strike an aeroplane with a large velocity. To get the lift, the aeroplane runs for some distance on the runway before taking off. Due to its special shape, the velocity of air above the plane increases and hence pressure decreases. The aeroplane gets an uplift.

Problem 78. The accumulation of snow on the aeroplane may reduce the lift. Explain.

Solution. Due to the accumulation of snow on the wings of the aeroplane, the shape of the wings no longer remains that of the aerofoil. This reduces the path difference and hence the velocity difference between the layer of air on the two sides of the wings. Hence the pressure difference on the two sides of the wings is reduced. This reduces the uplift on the aeroplane.

Problem 79. Why bullets are given cylindrical shape ?

Solution. The magnus effect is absent if the spinning cylinder is moving linearly in the direction parallel to spin axis. That is why the bullets are made cylindrical instead of spherical. They do not deviate from the linear path.

Problem 80. If air is blown very fast into the vertical hole of a spool of thread, a card laid flat against the other end does not fall. Why ?

Solution. Refer to Fig. 10.62. Due to large velocity of air between the spool and the card, the pressure between them reduces below the atmospheric pressure. The card is held by the higher atmospheric pressure below it.

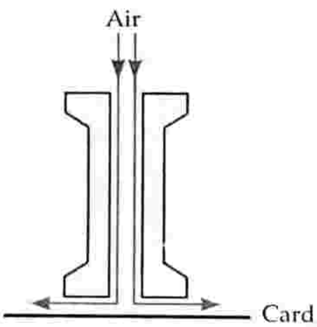


Fig. 10.62

Problem 81. Explain why we cannot remove a filter paper from a funnel by blowing air into its narrow end.

Solution. Refer to Fig. 10.63. When air is blown into the narrow end of the funnel, the velocity of air in the region between the filter paper and the curved wall of the funnel increases. This decreases the pressure. The filter paper gets more firmly held with the wall of the funnel. So it is not possible to remove the filter paper from the funnel by blowing air into its narrow end.

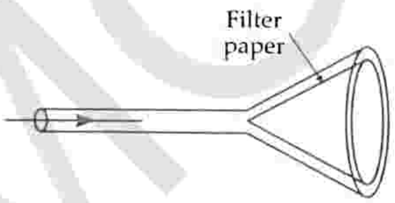


Fig. 10.63

Problem 82. According to Bernoulli's theorem, the pressure of water in a horizontal pipe of uniform diameter should remain constant. But actually it goes on decreasing with the increase in length of the pipe. Why ?

Solution. Bernoulli's theorem is valid only for non-viscous liquids. But water is a viscous liquid. A part of the pressure energy of water is used in doing work against the viscous force. So the pressure of water decreases.

Problem 83. What is the effect on the equilibrium of a physical balance when air is blown below one pan ?

Solution. Due to increase in velocity of air below the pan, the pressure decreases. So the pan goes down.

Problem 84. In case of an emergency, a vacuum brake is used to stop the train. How does this brake work ?

Solution. Steam at high pressure is allowed to enter the cylinder of the vacuum brake. Due to high velocity, the pressure decreases in accordance with Bernoulli's principle. The reduction of pressure lifts up the piston. This in turn lifts up the brake.

Problem 85. Is Bernoulli's theorem valid for viscous liquid ?

Solution. No, it should be modified to take into account the work done against viscous drag.

Problem 86. Two cylindrical vessels placed on a horizontal table contain water and mercury respectively up to the same heights. There is a small hole in the walls of each of the vessels at half the height of liquids in the vessels. Find out the ratio of the velocities of efflux of water and mercury from the holes. Which of the two jets of liquid will fall at a greater distance on the table from the vessel ? Relative density of mercury with respect to water = 13.6.

Solution. The velocity of efflux ($v = \sqrt{2gh}$) is independent of density of liquid. So, both the jets of liquid will fall at the same distance.

Problem 87. If a small ping pong ball is placed in a vertical jet of water or air, it will rise to a certain height above the nozzle and stay at that level. Explain.

Solution. Due to the high velocity of the jet of water, the pressure between the ball and the jet decreases. The greater (atmospheric) pressure on the other side of the ball pushes it against the jet and the ball remains suspended. The high velocity of water takes the ball upwards along with it and makes it to spin.

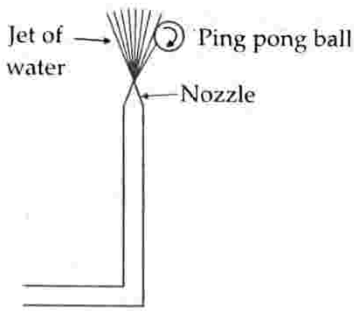


Fig. 10.64 A ping pong ball supported on a jet of water.

Problem 88. Bernoulli's theorem holds for incompressible, non-viscous fluids. How is this relationship changed when the viscosity of the fluid is not negligible ?

Solution. If the viscosity of the fluid is not negligible, a part of the mechanical energy of the fluid is spent in doing work against forces of viscosity. So the total energy : $P+\rho gh+\frac{1}{2}\rho v^2$ of the fluid goes on decreasing along the direction of flow of the fluid.

Problem 89. Why are the cars and aeroplanes given streamline shape ?

Solution. Cars and aeroplanes are given streamline shape to minimise the backward drag of atmosphere.

Problem 90. Why a glass rod coated with wax does not become wet when dipped in water ?

Solution. The reason is that force of adhesion between water and wax molecules is smaller than force of cohesion between water molecules.

Problem 91. Mercury does not cling to glass. Why ? *[Himachal 04]*

Solution. The force of adhesion between mercury and glass is smaller than the force of cohesion between the mercury molecules.

Problem 92. Why is it not possible to separate two pieces of paper joined by glue or gum ?

Solution. The adhesive force between the molecules of glue and paper is much more than the cohesive force between the glue molecules. As a result, the two papers stick together with a large force and cannot be separated.

Problem 93. Why does the free surface a liquid behave like an elastic stretched membrane ?

Solution. The liquid molecules on the surface experience a downward force. So they have greater potential energy. In order to have minimum energy, the free surface tends to contract to minimum area and hence behaves like an elastic stretched membrane.

Problem 94. Why does mercury collect itself into drops when placed on a clean glass plate ?

Solution. The adhesive force between mercury and glass molecules is less than the cohesive force between mercury molecules. So instead of sticking to glass, mercury collects itself into drops.

Problem 95. How dental plate clinges to the roof of the mouth ?

Solution. The dental plate is made exactly parallel to the roof of the mouth at all the points. A small amount of sliva gets enclosed between dental plate and the roof of the mouth. Due to the force of adhesion between the sliva and dental plate, the plate clinges to the mouth.

Problem 96. What shape does a liquid take when it weighs nothing ?

Solution. When a liquid weighs nothing, surface tension is the only force acting on it. Due to surface tension, the liquid surface tends to occupy minimum surface area. For a given volume, the surface area of a sphere is minimum, so the liquid assumes spherical shape.

Problem 97. One can form a fairly large vertical film of soap solution but not of pure water. Why ?

Solution. This is because the surface tension of soap solution is much smaller than that of pure water. When we try to make a vertical film of pure water, it breaks due to the high surface tension of water.

Problem 98. Small insects can move about on the surface of water. Why ? *[Central Schools 08]*

Solution. Due to surface tension, the free surface of water behaves like stretched elastic membrane which is able to support the small weight of insects and they can move on the surface of water.

Problem 99. Why ends of a glass tube become rounded on heating ?

Solution. When glass is heated, it melts to a liquid. The surface of this liquid tends to have a minimum area. Now, for a given volume, the area of the surface of a sphere is minimum. This is why the ends of a glass tube become rounded on heating.

Problem 100. Antiseptics have low surface tension. Why ? *[Himachal 03]*

Solution. Low surface tension helps the antiseptic to spread over large area over the wounds.

Problem 101. Why hot soup tastes better than cold soup ?

Solution. Hot soup has comparatively less surface tension than cold soup. So it spreads over larger area of the tongue and tastes better than cold soup.

Problem 102. Oil spreads over the surface of water whereas water does not spread over the surface of oil. Why ?

Solution. The surface tension of the water is more than that of oil. Therefore, when oil is poured over water, the greater value of surface tension of water pulls oil in all directions, and as such it spreads on the water. On the other hand, when water is poured over oil, it does not spread over it because the surface tension of oil being less than that of water, it is not able to pull water over it.

Problem 103. Why does a stream of water from a faucet become narrow as it falls ?

Solution. This is due to surface tension. The molecules of water come close to each other due to surface tension and surface area tends to be minimum making the stream narrow.

Problem 104. Why do the hair of a shaving brush cling together when taking out of water ?

Solution. When the brush is taken out of water, thin water film is formed at the tips of the hair. It contracts due to surface tension and so the hair cling together.

Problem 105. A needle floats on the surface of pure water but goes down when detergent is added to water. Why ?

Solution. Due to addition of detergent, the surface tension and hence the reaction of surface tension decreases. Hence the needle sinks due to smaller upward reaction.

Problem 106. Why it becomes easier to spray the water in which some soap is dissolved ?

Solution. When soap is dissolved in water, the surface tension of water decreases and so less energy is needed for spraying the water *i.e.*, spraying of water in which soap is dissolved becomes easier.

Problem 107. The clothes are better cleaned with hot water than with cold water. Why ? *[Delhi 1996]*

Solution. Surface tension decreases with the increase of temperature. Lesser the surface tension, more is the wetting (and hence the washing) power of water.

Problem 108. How does soap help us to remove dirt better in washing clothes ? *[Delhi 1999]*

Solution. With the addition of soap, the surface tension of water decreases. The decrease in surface tension results in greater wetting and hence washing power.

Problem 109. A oil drop on a hot cup of soup spreads over when the temperature of the soup falls. Why ?

Solution. The surface tension of hot water is less than that of oil and hence oil drop does not spread over hot water. When water is cooled, its surface tension decreases. At low temperature, the surface tension of water becomes greater than that of oil and hence oil drop starts spreading over it.

Problem 110. Glass marbles are made by heating the end of a glass rod until drops of molten glass fall. Explain.

Solution. When drops of molten glass fall freely, they are in state of weightlessness. As only the force of surface tension acts on them, so they acquire spherical shape which on solidification become glass marbles.

Problem 111. Oil is sprinkled on sea waves to calm them. Why ?

Solution. When oil is sprinkled, the breeze spreads the oil on the sea-water in its own direction. The surface tension of sea-water (without oil) is greater than oily water. Hence the water without oil pulls the oily water against the direction of breeze, and the sea waves become calm.

Problem 112. A tiny liquid drop is spherical but a larger drop has oval shape. Why ?

Solution. In the case of a tiny drop, the force of surface tension is large compared to its weight, so the drop has spherical shape. A large drop has oval shape because the force of gravity (weight) exceeds the force of surface tension.

Problem 113. Why does a small piece of camphor dance about on the water surface ?

Solution. Due to its irregular shape, the camphor piece dissolves more rapidly at some points than at others. Where it dissolves, the surface tension of water is reduced. As the force of surface tension reduces by different amounts at different points of the camphor piece, a resultant force acts on it which makes it dance about on water surface.

Problem 114. The addition of flux to tin makes soldering easy. Why ?

Solution. The addition of flux reduces surface tension of the molten tin. This helps it spread easily over the area of soldering.

Problem 115. The paints and lubricating oils have low surface tension. Why ?

Solution. The paints and lubricating oils having low surface tension can spread over a large surface area.

Problem 116. Why are the droplets of mercury when brought in contact pulled together to from a bigger drop ? Also state with reason whether the temperature of bigger drop will be the same, or more, or less than the temperature of the smaller drops ?

Solution. Due to surface tension, liquid drops tend to have minimum surface area. When mercury droplets are brought in contact, they form one drop thereby decreasing the surface area. Due to decrease in surface, surface energy is lost by the bigger drop which appears as heat. So its temperature increases.

Problem 117. The angle of contact for a solid and a liquid is less than 90°. Will the liquid wet the solid ? Will the liquid rise in the capillary made of that solid ?

Solution. The liquid will wet the solid and will rise in the capillary tube made of that solid.

Problem 118. Write down formula for excess pressure inside (*i*) a liquid drop (*ii*) a soap bubble.

Solution. (*i*) For a liquid drop : $p = \frac{2\sigma}{R}$.

(*ii*) For a soap bubble : $p = \frac{4\sigma}{R}$,

where σ is surface tension.

Problem 119. Why excess pressure in a soap bubble is twice the excess pressure of a liquid drop of the same radius ?

Solution. A soap bubble has two free surfaces, one internal and another external ; whereas liquid drop has only one outer free surface.

Problem 120. Two soap bubbles of unequal sizes are blown at the ends of a capillary tube. Which one will grow at the expense of the other and what does it show ?

Solution. The bigger one will grow at the expense of the smaller one. This is because excess pressure is inversely proportional to radius and air flows from higher pressure to lower pressure.

Problem 121. What is the importance of (*i*) wetting agents used by dyers, and (*ii*) water proofing agents ?

Solution. (*i*) They are added to decrease the angle of contact between the fabric and the dye so that the dye may penetrate well. (*ii*) They are used to increase the angle of contact between the fabric and water to prevent the water from penetrating the cloth.

Problem 122. Teflon is coated on the surface of non-sticking pans. Why ?

Solution. When the surface of a pan is coated with teflon, the angle of contact between the pan and the oil used for the frying purpose becomes obtuse. Thus the frying pan becomes non-sticking.

Problem 123. What makes water-proof rain coat water-proof ?

Solution. The angle of contact between water and the material of the raincoat is obtuse. So the rainy water does not wet the raincoat *i.e.*, the raincoat is water-proof.

Problem 124. How does the ploughing of fields help in preservation of moisture in the soil ? *[Chandigarh 08]*

Solution. This is done to break the tiny capillaries through which water can rise and finally evaporate. The ploughing of field helps the solid to retain the moisture.

Problem 125. How does the cotton wick in an oil-filled lamp keep on burning ?

Solution. The narrow spaces between the threads of the wick serve as capillary tubes through which oil keeps on rising due to capillary action.

Problem 126. Why sand is drier than clay ?

Solution. Due to narrow pores in clay, water rises in the clay due to capillary action and keeps it damp. Practically no pores or capillaries exist in sand and hence water cannot rise in sand. So sand is drier than clay.

Problem 127. Why undergarments are usually made of cotton ?

Solution. Cotton threads have large number of capillaries between them. These capillaries help to absorb sweat from the surface of the body due to capillarity action.

Problem 128. A piece of chalk immersed in water emits bubbles in all directions. Why ? *[Chandigarh 04]*

Solution. A chalk piece has a large number of capillaries. As it is immersed in water, water rises due to capillary action. The air present in the chalk is expelled out in the form of bubbles in all directions.

Problem 129. Why new earthen pots keep water cooler than the old one ?

Solution. Due to capillary action, water oozes out of the pores of the new earthen pot. This water takes latent heat from the pot to evaporate and so the water in the pot gets cooled. In old pots, most of the capillaries are blocked. So the cooling is not so effective.

Problem 130. Why is the tip of the nib of a pen split ?

Solution. The split in the tip of the nib acts as a capillary tube. The ink rises up in the nib due to capillary action. Hence we are able to write with the pen.

Problem 131. Water rises in a capillary tube, whereas mercury falls in the same tube. Why ?

Solution. In a capillary tube, a liquid rises to a height h given by

$$h = \frac{2\,\sigma \cos\theta}{r\rho g}$$

For water, θ is acute, $\cos\theta$ is positive and hence h is positive. So water rises in the capillary tube. For mercury, θ is obtuse, $\cos\theta$ is negative and hence h is negative. So mercury gets depressed in the capillary tube.

Problem 132. Why is it difficult to make mercury enter a fine thermometer tube ?

Solution. The meniscus of mercury in a glass tube is convex. Since there exists an excess pressure ($p = 2\,\sigma / r$) on the concave side of the curved liquid surface, the pressure below the meniscus is greater than the atmospheric pressure. This excess pressure has a very large value for a tube of fine bore. As the pressure of mercury inside the tube is greater than that outside it, mercury instead of entering into the tube tends to flow out.

Problem 133. Water gets depressed in a glass tube whose inner surface is coated with wax. Why ?

Solution. The angle of contact between water and wax is obtuse. So $\cos\theta$ is negative and hence capillary rise : $h = 2\,\sigma \cos\theta / r\rho g$ is also negative. That is why water gets depressed.

Problem 134. If a capillary tube is immersed at first in cold water and then in hot water, the height of capillary is smaller in the second case. Why ?

Solution. The height upto which a liquid rises in a capillary tube is given by

$$h = \frac{2\,\sigma \cos\theta}{r\rho g}$$

The surface tension (σ) of hot water is less than that of cold water. Moreover, capillary tube expands in hot water, so its radius r increases. So capillary rise h is smaller in hot water than in cold water.

Problem 135. If a capillary tube is put in water in a state of weightlessness how will the rise of water in a capillary tube be different to one observed under normal conditions ?

Solution. In normal conditions, when the force of surface tension (due to which water rises in capillary) becomes equal to the weight of the water column raised in the tube, water stops rising. In the state of weightlessness, the effective weight of water column raised is zero. Hence water will rise upto the other end of the capillary, however long the capillary is. Water will not overflow, its surface will become flat (its radius of curvature will become infinity).

Problem 136. How is the rise of liquid affected, if the top of the capillary tube is closed ?

Solution. As the liquid rises in the capillary tube, the air gets compressed between the top end of the tube and the liquid meniscus. The compressed air opposes the rise of liquid due to surface tension. The liquid rises till the two opposing forces just balance each other. Hence if the top end of the capillary tube is closed, the liquid rises to a smaller height.

Problem 137. A 20 cm long capillary tube is dipped in water. The water rises upto 8 cm. If the entire arrangement is put in a freely falling elevator, what will be the length of water column in the capillary tube ? *[AIEEE 05]*

Solution. In the freely falling elevator, the entire arrangement is in a state of weightlessness, *i.e.*, $g = 0$. So water will rise in the tube $\left(h = \dfrac{2\sigma \cos \theta}{r\rho g} \right)$ to fill the entire 20 cm length of the tube.

Problem 138. Spherical balls of radii R are falling in a viscous fluid of viscosity η with a velocity v. How does the retarding viscous force acting on a spherical ball depend on R and v ? *[AIEEE 04]*

Solution. $F = 6\pi\eta R v$ *i.e.*, retarding viscous force is directly proportional to both R and v.

Problem 139. Why do air bubbles in a liquid move in upward direction ? *[Himachal 07]*

Solution. The density of air bubble is less than that of liquid. Initially, the resultant of upthrust and the viscous force is greater than the weight of the air bubble. So, the air bubble experiences a net upward force. Then bubble soon attains a terminal velocity in the upward direction.

Problem 140. Explain why some oils spread on water, when others float as drops. *[Central Schools 09]*

Solution. If the surface tension of oil is less than that of water, then it spreads on water. If the surface tension of oil is more than that of water, then it floats as drops on water.

Problem 141. What happens when a capillary tube of insufficient length is dipped in a liquid ?

Solution. When a capillary tube of insufficient length is dipped in a liquid, the liquid rises to the top. The radius of curvature of the concave meniscus increases till the pressure on its concave side becomes equal to the pressure exerted by the liquid column of insufficient length. But the liquid does not overflow.

Short Answer Conceptual Problems

Problem 1. Explain why :

(*i*) A balloon filled with helium does not rise in air indefinitely but halts after a certain height (Neglect winds).

(*ii*) The force required by a man to raise his limbs immersed in water is smaller than the force for the same movement in air. *[Delhi 06]*

Solution. (*i*) A balloon filled with helium goes on rising in air so long as the weight of the air displaced by it (*i.e.*, upthrust) is greater than the weight of filled balloon. We know that the density of air decreases with height. Therefore, the balloon halts after attaining a height at which density of air is such that the weight of air displaced just equals the weight of filled balloon.

(*ii*) Water exerts much more upthrust on the limbs of man than air. So the net weight of limbs in water is much less than that in air. Hence the force required by a man to raise his limbs immersed in water is smaller than the force for the same movement in air.

Problem 2. What height of water column produces the same pressure as a 760 mm high column of Hg ? *[Delhi 02]*

Solution. Pressure exerted by h height of water column = Pressure exerted by 760 mm of Hg column

$$\therefore\ h \times 1000 \times 9.8 = 0.760 \times 13.6 \times 10^3 \times 9.8 \qquad [P = h\rho g]$$

or
$$h = \frac{0.760 \times 13.6 \times 10^3}{1000} = \mathbf{10.336\ m}.$$

Problem 3. A small ball of mass m and density ρ is dropped in a viscous liquid of density ρ. After some time, the ball falls with a constant velocity. Calculate the viscous force on the ball.

Solution. Volume of the ball, $V = \dfrac{m}{\rho}$

Mass of the liquid displaced, $m' = V\rho_0 = \dfrac{m}{\rho} \cdot \rho_0$

When the body falls with a constant velocity,

Viscous force = Effective weight of the ball

$$F = \text{Weight of the ball} - \text{Upthrust}$$
$$= mg - m'g$$

or
$$F = mg - \frac{m\rho_0}{\rho} \cdot g = mg\left(1 - \frac{\rho_0}{\rho}\right).$$

Problem 4. A tank filled with fresh water has hole in its bottom and water is flowing out of it. If the size of the hole is increased what will be the change in :

(*a*) Volume of water flowing out per second ?

(*b*) Velocity of the outcoming water ?

(*c*) If in the above tank, the fresh water is replaced by sea water, will the velocity of outcoming water change ?

Solution. (*a*) The volume of water flowing out per sec will increase as its volume depends directly on the size of hole.

(b) The velocity of outflow of water remains unchanged because it depends upon the height of water level and is independent of the size of the hole.

(c) No, though the density of sea water is more than that of fresh water, but the velocity of outflow of water is independent of the density of water.

Problem 5. In a bottle of narrow neck, water is poured with the help of an inclined glass rod. Why ?

Solution. If water is directly poured into a bottle of narrow neck, the stream of water blocks the neck due to the pressure of inside air and the strong force of adhesion between glass and water. If a small part of the glass rod is placed inside the bottle and water is poured along the length of the rod outside the bottle, water molecules cling to the glass rod and the force of gravity pulls down these molecules into the bottle.

Problem 6. The excess pressure inside a soap bubble is thrice the excess pressure inside a second soap bubble. What is the ratio between the volume of the first and the second bubble ? [Chandigarh 03]

Solution. Given : $P_1 = 3P_2$

or $$\frac{4\sigma}{r_1} = \frac{3 \times 4\sigma}{r_2}$$

or $$r_2 = 3r_1$$

$$\therefore \quad \frac{V_1}{V_2} = \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \left(\frac{r_1}{r_2}\right)^3 = \left(\frac{1}{3}\right)^3 = \mathbf{1:27}.$$

Problem 7. The viscous force 'F' acting on a body of radius 'r' moving with a velocity 'v' in a medium of coefficient of viscosity 'η' is given by $F = 6\pi\eta rv$. Check the correctness of the formula. [Chandigarh 03]

Solution. $[F] = [MLT^{-2}]$

$$[6\pi\eta rv] = [ML^{-1}T^{-1}][L][T^{-1}] = [MLT^{-2}]$$

$\therefore$ Dimensions of LHS = Dimensions of RHS.

Hence the given formula for viscous force F is dimensionally correct.

Problem 8. Two soap bubbles of different diameters are in contact with a certain portion common to both the bubbles. What will be the shape of the common boundary as seen from inside the smaller bubble ? Support your answer with a neat diagram. Give reason for your answer. [Delhi 04]

Solution. When seen from inside the smaller bubble, the shape of the common boundary will appear concave, as shown in Fig. 10.65.

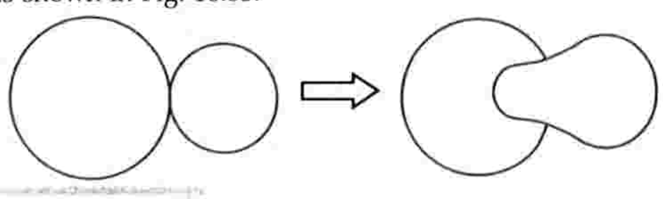


Fig. 10.65

Reasons. (i) For a curved liquid film, the pressure is greater on its concave side.

(ii) Pressure inside the smaller bubble is more than that inside the larger drop, because $p \propto 1/r$.

Problem 9. A big size balloon of mass M is held stationary in air with the help of a small block of mass $M/2$ tied to it by a light string such that both float in mid air. Describe the motion of the balloon and the block when the string is cut. Support your answer with calculations. [Delhi 04]

Solution. The situation is shown in Fig. 10.66.

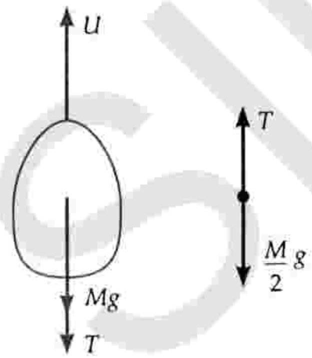


Fig. 10.66

When the balloon is held stationary in air, the forces acting on it get balanced.

Upthrust = Weight of balloon + Tension in string

$$U = Mg + T \quad \text{...(1)}$$

For the small block of mass $M/2$ floating stationary in air,

$$T = \frac{M}{2}g \quad \text{...(2)}$$

From (1) and (2), $U = Mg + \frac{M}{2}g = \frac{3}{2}Mg$

When the string is cut, $T = 0$. The small block will begin to fall freely. The balloon will rise up with an acceleration a such that

$$U - Mg = Ma$$

or $$\frac{3}{2}Mg - Mg = Ma$$

or $$a = \frac{g}{2}, \text{ in the upward direction.}$$

Problem 10. A tarnado consists of rapidly whirling air vortex. Why is the pressure always much lower in the centre than at the outside ? How does this condition account for the destructive power of tarnado ?

Solution. The angular velocity of air in a tarnado increases as it goes towards the centre. As the air moves towards the centre, its moment of inertia (I) decreases and to conserve angular momentum ($L = I\omega$), the angular velocity ω increases. Because of the enormous increase in velocity of inner layers, the air pressure at the centre reduces greatly in accordance with Bernoulli's theorem. This sudden reduction in pressure may prove highly disastrous both for life and property in the vicinity of a tarnado.

HOTS

Problems on Higher Order Thinking Skills

Problem 1. *A piece of ice with a stone frozen in it floats on water taken in a beaker. Will the level of water increase or decrease or remain the same when ice melts completely ?*

Solution. Let M be the mass of ice piece and m that of stone. As this combination of mass $(M + m)$ floats in water, so the mass of water displaced is $(M + m)$. If ρ is the density of water, then the volume of water displaced is

$$V = \frac{M + m}{\rho} \qquad ...(i)$$

When ice melts, we get extra water of mass M and volume M/ρ. The stone sinks and displaces water equal to its own volume m/d, where d is the density of stone. Thus, total the volume of extra water (obtained by melting of ice) and water displaced by stone is

$$V' = \frac{M}{\rho} + \frac{m}{d} \qquad ...(ii)$$

As $d > \rho$, so $\quad \frac{1}{d} < \frac{1}{\rho}$ and $\frac{m}{d} < \frac{m}{\rho}$

$\therefore$ From (*i*) and (*ii*), it is obvious that $V' < V$

Thus the level of water in the beaker will come down.

Problem 2. *An ice block with a cork piece embedded inside floats in water. What will happen to the level of water when ice melts ?*

Solution. Let M be the mass of ice and m that of cork.

If ρ is the density of water, then the volume of water displaced by ice + cork is $V = \frac{M + m}{\rho}$

When ice melts, volume of extra water formed $= \frac{M}{\rho}$

As the cork floats, volume of water displaced by it

$$= \frac{m}{\rho}$$

Total volume of water, $V' = \frac{M}{\rho} + \frac{m}{\rho} = \frac{M + m}{\rho}$

Clearly, $V' = V$

Thus the level of water will not change.

Problem 3. *A boat floating in a water tank is carrying a number of large stones. If the stones are unloaded into water, what will happen to the water level ?* **[IIT]**

Solution. Let M be the mass of the boat and m that of stones. Then total mass $= M + m$. Let ρ be the density of water.

Volume of water displaced by boat and stones together,

$$V = \frac{M + m}{\rho}$$

After the stones are unloaded into water, volume of water displaced by boat alone,

$$V_1 = \frac{M}{\rho}$$

If σ is the density of stones, then volume of water displaced by stones,

$$V_2 = \frac{m}{\sigma}$$

Total volume of water displaced,

$$V' = V_1 + V_2 = \frac{M}{\rho} + \frac{m}{\sigma}$$

As $\sigma > \rho$, so $V > V'$ or $V' < V$

i.e. Volume of water displaced in second case < Volume of water displaced in first case.

Hence the level of water in the water tank will fall down.

Problem 4. *To what height should a cylindrical vessel be filled with a homogeneous liquid to make the force, with which the liquid presses the side of the vessel equal to the force exerted by the liquid on the bottom of the vessel ?* **[IIT]**

Solution. Let h be the height of the liquid column of density ρ taken in the cylindrical vessel of radius r.

Force exerted by the liquid on the bottom of the vessel

$= $ Total weight of the liquid column

$= mg = \pi r^2 h\rho g$

Area of the sides of the vessel $= 2\pi rh$

Average pressure exerted by the liquid on the sides of the vessel

$$= \frac{\text{Pressure at the top} + \text{Pressure at the bottom}}{2}$$

$$= \frac{0 + h\rho g}{2} = \frac{1}{2} h\rho g$$

Force exerted by the liquid on the sides of the vessel

$= \text{Pressure} \times \text{Area} = \frac{1}{2} h\rho g \times 2\pi rh$

As the above two forces are given to be equal, so

$$\frac{1}{2} h\rho g \times 2\pi rh = \pi r^2 h\rho g \quad \text{or} \quad h = r$$

i.e., Height of liquid column

= Radius of the cylindrical vessel.

Problem 5. *A block of wood is floating on water at 0°C with a certain volume V above the water level. The temperature of water is slowly raised from 0°C to 20°C. How will the volume V change with the rise in temperature ?*

Solution. Let V' be the volume of the block of wood and W be the weight of the block. Then by the law of floatation,

Weight of water displaced = Weight of block

$$(V' - V)\rho_t\, g = W$$

where ρ_t is the density of water t °C.

As wood has negligible coefficient of expansion, so V' may be taken constant.

$$\therefore \qquad V = V' - \frac{W}{\rho_t\, g}$$

As the temperature is gradually increased from 0°C, ρ_t increases and so V increases upto 4°C, when the density of water becomes maximum. Above 4°C, ρ_t decreases and so V decreases continuously.

Problem 6. *A ball floats on the surface of water in a container exposed to the atmosphere. Will the ball remain immersed at its initial depth or will it sink or rise somewhat if the container is shifted to the moon ?*

Solution. The gravity on moon is about one-sixth of that on the earth. But gravity has equal effect both on weight of the body and the upthrust. So equilibrium of the floating body is not affected. On the earth, weight of the floating body is balanced by upthrust due to both water and air.

$$\therefore \qquad W = mg = V_w\, \rho_w\, g + V_a\, \rho_a\, g$$

$$\text{or} \qquad m = V_w\, \rho_w + V_a\, \rho_a \qquad \ldots(i)$$

But the moon has no atmosphere. So

$$W = mg = V'_w\, \rho_w\, g$$

$$\text{or} \qquad m = V'_w\, \rho_w \qquad \ldots(ii)$$

From (*i*) and (*ii*), we note that

$$V'_w = V_w + \frac{V_a\, \rho_a}{\rho_w}$$

Clearly, $V'_w > V_w$

That is, the volume of ball immersed in water on the moon is greater than that on earth. Hence ball will sink slightly more in water when taken to the moon.

Problem 7. *A balloon filled with air is weighed so that it barely floats in water, as shown in Fig. 10.67. Explain why it sinks to the bottom when it is submerged more by a small distance.*

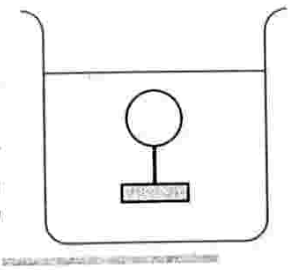


Fig. 10.67

Solution. When the balloon is submerged more by a small distance, the air inside it is slightly compressed. So the buoyant force on it decreases. Now the downward force (weight of sinker + weight of air) exceeds the buoyant force, so it sinks to the bottom.

Problem 8. *A beaker containing water is placed on a spring balance. A stone of weight W is hung and lowered into the water without touching the sides and bottom of the beaker. Explain how the reading will change.*

Solution. Like other forces, buoyant force is also exerted equally on the bodies in contact. When water exerts buoyant force B on the stone in the upward direction, the stone also exerts an equal downward force B on the water. Now the weight W of the vessel + water and the force B on water act downward. So the reaction of the spring scale is

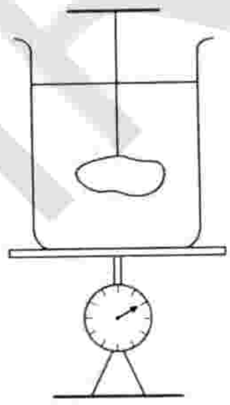


Fig. 10.68

$$R = W + B$$

Hence the reading of the spring scale will increase by an amount equal to the buoyant force.

Problem 9. *When sewing, why does a person often wet the end of a thread before trying to put it through the eye of a needle ?*

Solution. When the end of a thread is made wet with water, a thin film of water is formed over its fibres. The fibres of the thread cling together due to surface tension of water. The area of cross-section of the thread decreases and it becomes easier to put it through the eye of the needle.

Problem 10. *A vessel contains oil (density 0.8 g cm^{-3}) over mercury (density = 13.6 g cm^{-3}). A homogeneous sphere floats with half its volume immersed in mercury and the other half in oil. What is the density of material of sphere ?* **[IIT 88]**

Solution. Let V be the volume of sphere and ρ its density. Let ρ_0 and ρ_m be the density of oil and mercury respectively.

By the law of floatation,

Weight of sphere = Weight of oil displaced + Weight of mercury displaced

$$\text{or} \qquad V\rho g = \frac{V}{2}\rho_0\, g + \frac{V}{2}\rho_m\, g$$

$$\text{or} \qquad \rho = \frac{\rho_0 + \rho_m}{2}$$

$$= \frac{0.8 + 13.6}{2} = \mathbf{7.2\ g\ cm^{-3}}.$$

Problem 11. *An iceberg weighs 400 tonnes. The specific gravity of iceberg is 0.92 and the specific gravity of water is 1.02. What percentage of iceberg is below the water surface ?*

[Roorkee 86]

Solution. Fraction of the iceberg below the water surface will be

$$\frac{\text{Volume of the immersed part}}{\text{Total volume of the iceberg}} = \frac{\text{Density of the substance}}{\text{Density of water}}$$

$$= \frac{0.92}{1.02} = 0.902 = \mathbf{90.2\%}$$

Problem 12. *A hemispherical portion of radius R is removed from the bottom of a cylinder of radius R. The volume of the remaining cylinder is V and its mass is M. It is suspended by a string in a liquid of density ρ where it stays vertical. The upper surface of the cylinder is at a depth h below the liquid surface. How much is the force on the bottom of the cylinder by the liquid ?*

[IIT Screening 01]

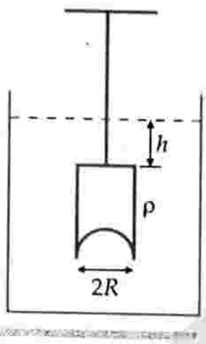


Fig. 10.69

Solution. (Upward) force on the bottom – (downward) force on the top

= Buoyant force on the cylinder

= Weight of liquid displaced

or $$F - (h\rho g)\,\pi R^2 = V\rho g$$

or $$F = V\rho g + (h\rho g)\,\pi R^2 = \rho g\ \mathbf{(V + \pi R^2 h)}.$$

Problem 13. *A large open tank has two holes in the wall. One is a square hole of side L at a depth y from the top and the other is a circular hole of radius R at a depth 4y from the top. When the tank is completely filled with water, the quantities of water flowing out per second from both holes are the same. Then, what is the value of R ?*

[IIT Screening 2K]

Solution. The volume of water flowing out per second

= velocity × area of cross-section of the hole

$= vA.$

Equating the rates of flow,

$$v_1 A_1 = v_2 A_2$$

But $v_1 = \sqrt{2gy},\ A_1 = L^2,\ v_2 = \sqrt{2g \times 4y},\ A_2 = \pi R^2$

$$\therefore \quad \sqrt{2gy} \times L^2 = \sqrt{2g \times 4y} \times \pi R^2$$

or $$L^2 = 2\pi R^2 \quad \text{or} \quad R = \frac{L}{\sqrt{2\pi}}.$$

Problem 14. *A bubble having surface tension T and radius R, is formed on a ring of radius b (b << R). Air is blown inside the tube with velocity v as shown. The air molecule collides perpendicularly with the wall of the bubble and stops. Calculate the radius at which the bubble separates from the ring.* [Fig. 10.70]

[IIT Mains 03]

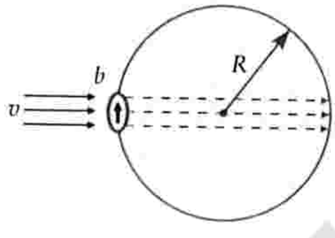


Fig. 10.70

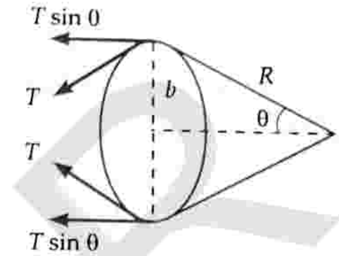


Fig. 10.71

Solution. The bubble will separate from the ring when

$$2\pi b \times 2T \sin\theta = \rho A v^2$$

or $$4\pi bT \times \frac{b}{R} = \rho \times \pi b^2 \times v^2$$

or $$R = \frac{\mathbf{4T}}{\boldsymbol{\rho v^2}}.$$

Problem 15. *A cylindrical vessel of radius 3 cm has at the bottom a horizontal capillary tube of length 20 cm and internal radius 0.4 mm. If the vessel is filled with water, find the time taken by it to empty one half of its contents. Given that the viscosity of water is 0.01 poise.* **[IIT]**

Solution. Let h be the height of water level in the vessel at any instant t and dh be the fall in level in small time dt. Then the rate of flow of water will be

$$Q = -A\frac{dh}{dt} = \frac{\pi p r^4}{8\eta l} \quad \text{[Here } A = \text{Area of the vessel]}$$

or $$-A\frac{dh}{dt} = \frac{\pi h\rho g r^4}{8\eta l} \qquad [\because\ p = h\rho g]$$

$$\therefore \quad dt = -\frac{8\eta\, l\, A}{\pi\rho g r^4}\,\frac{dh}{h}$$

Let H be the initial height of the water in the vessel and T be the time taken by the vessel to become half-empty *i.e.*, water level falls to $H/2$. Then

$$\int_0^T dt = -\frac{8\eta l\, A}{\pi\rho g r^4}\int_H^{H/2}\frac{dh}{h}$$

or $$T = -\frac{8l\eta A}{\pi\rho g r^4}\left[\log_e\frac{H}{2} - \log_e H\right] = \frac{8l\eta A}{\pi\rho g r^4}\log_e 2$$

But $R = 5$ cm, $\quad r = 0.04$ cm, $\quad l = 20$ cm,
$\eta = 0.01$ poise, $\quad A = \pi R^2$

$$\therefore \quad T = \frac{8 \times 20 \times 0.01 \times \pi \times (5)^2}{\pi \times 1 \times 981 \times (0.04)^4} \times 2.303 \times 0.3010$$

$= \mathbf{11041\ s.}$

Problem 16. *A soap bubble of radius 4 cm and surface tension 30 dyne* cm^{-1} *is blown at the end of a tube of length 10 cm and internal radius 0.20 cm. If the viscosity of air is* 189×10^{-4} *poise, find the time taken by the bubble to be reduced to a radius of 2 cm.*

Solution. Let R be the radius of the bubble at any instant. Its volume is

$$V = \frac{4}{3}\pi R^3$$

$\therefore$ Rate of flow of air

$$= \frac{dV}{dt} = \frac{4}{3}\pi \times 3R^2 \frac{dR}{dt} = 4\pi R^2 \frac{dR}{dt}$$

But $\frac{dV}{dt} = \frac{\pi p r^4}{8\eta l}$ and for a soap bubble, $p = \frac{4\sigma}{R}$

$$\therefore \quad \frac{dV}{dt} = \frac{\pi r^4}{8\eta l} \cdot \frac{4\sigma}{R} = 4\pi R^2 \frac{dR}{dt} \quad \text{or} \quad dt = \frac{8l\eta}{\sigma r^4} R^3\, dR$$

Time taken by the bubble when its radius changes from R_1 to R_2 is

$$t = \int dt = \frac{8l\eta}{\sigma r^4} \int_{R_2}^{R_1} R^3\, dR = \frac{8l\eta}{\sigma r^4}\left(\frac{R_1^4 - R_2^4}{4}\right)$$

But $R_1 = 4$ cm, $R_2 = 2$ cm, $\sigma = 30$ dyne cm^{-1}
$l = 10$ cm, $r = 0.2$ cm, $\eta = 1.85 \times 10^{-4}$ poise

$$\therefore \quad t = \frac{8 \times 10 \times 1.85 \times 10^{-4}}{30 \times (0.2)^4} \times \left(\frac{4^4 - 2^2}{4}\right) = \mathbf{296\ s.}$$

Problem 17. *A metallic sphere of radius* 1.0×10^{-3} *m and density* 1.0×10^4 *kg* m^{-3} *enters a tank of water, after a free fall through a distance of h in the earth's gravitational field. If its velocity remains unchanged after entering water, determine the value of h. Given coefficient of viscosity of water* $= 1.0 \times 10^{-3}$ *Nsm*$^{-2}$, $g = 10$ *ms*$^{-2}$ *and density of water* $= 1.0 \times 10^3$ *kg m*$^{-3}$. **[Roorkee 90]**

Solution. The velocity attained by the sphere after falling freely from height h is

$$v = \sqrt{2gh} \quad \text{...(i)}$$

After entering water, the velocity of the sphere does not change. So v is also the terminal velocity of the sphere. Hence

$$v = \frac{2}{9}\frac{r^2}{\eta}(\rho - \rho')\, g$$

But $\rho = 10^4$ kgm^{-3}, $\rho' = 10^3$ kgm^{-3}, $r = 10^{-3}$ m,
$g = 10$ ms^{-2}, $\eta = 10^{-3}$ Nsm^{-2}

$$\therefore \quad v = \frac{2}{9} \times \frac{(10^{-3})^2 \times (10^4 - 10^3) \times 10}{10^{-3}} = 20 \text{ ms}^{-1}$$

From (*i*), $h = \frac{v^2}{2g} = \frac{20 \times 20}{2 \times 10} = \mathbf{20\ m.}$

Problem 18. *Water stands at a height H in a tank whose side walls are vertical. A hole is made in one of the walls at a depth h below the water surface. (i) Find at what distance from the foot of the wall does the emerging stream of water strike the floor ? (ii) For what value of h, this range is maximum ? (iii) Can a hole be made at another depth so that the second stream has the same range ?* **[Roorkee 88]**

Solution. (*i*) According to the Bernoulli's theorem,

Energy per unit volume at free surface = Energy per unit volume at the hole

$$\therefore \quad p + \rho g H + 0 = p + \rho g (H - h) + \frac{1}{2}\rho v^2$$

or $$\frac{1}{2}\rho v^2 = \rho g h \quad \text{or} \quad v = \sqrt{2gh}$$

This is the velocity with which the water comes out of the hole.

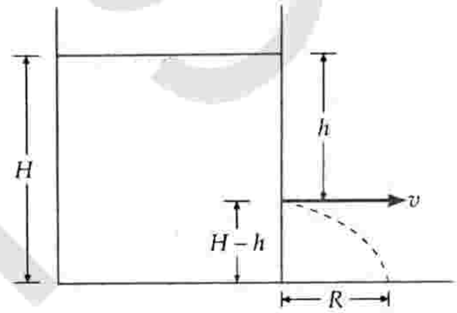


Fig. 10.72

Let t be the time taken by water to fall through height $(H - h)$. Then

$$(H - h) = 0 + \frac{1}{2}gt^2$$

[Initial velocity in the vertical downward direction = 0]

$$\therefore \quad t = \sqrt{\frac{2(H-h)}{g}}$$

The distance from the foot of the wall at which the liquid falls,

$$R = v \times t = \sqrt{\frac{2(H-h)}{g}} \times \sqrt{2gh} = 2\sqrt{h(H-h)}.$$

(*ii*) For R to be maximum, $\frac{dR}{dh} = 0$

or $$\frac{d}{dh}[2(hH - h^2)^{1/2}] = 0 \qquad [\because R = 2\sqrt{h(H-h)}]$$

or $$2 \times \frac{1}{2}(hH - h^2)^{-1/2} \times (H - 2h) = 0$$

or $$\frac{H - 2h}{\sqrt{hH - h}} = 0$$

or $$H - 2h = 0 \quad \text{or} \quad h = H/2$$

Hence R will be maximum when the hole is made in the middle of the wall.

(*iii*) Let x be the depth below the free surface at which range R is same.

$$\therefore \quad 2\sqrt{h(H-h)} = 2\sqrt{x(H-x)}$$

or $$hH - h^2 = xH - x^2$$

or $$x^2 - Hx + (hH - h^2) = 0$$

or $$x = \frac{H \pm \sqrt{H^2 - 4(hH - h^2)}}{2}$$

$$= \frac{H \pm \sqrt{(H-2h)^2}}{2} = h \text{ or } (H-h)$$

Hence another hole for which the range R is same must be at depth $(H-h)$.

Problem 19. *A horizontal pipe line carries water in a streamline flow. At a point along the pipe where the cross-sectional area is 10 cm², the water velocity is 1 ms⁻¹ and the pressure is 2000 Pa. What is the pressure at another point where the cross-sectional area is 5 cm² ?* **[IIT 94]**

Solution. According to the equation of continuity,

$$a_1 v_1 = a_2 v_2 \quad \text{or} \quad 10 \text{ cm}^2 \times 1 \text{ ms}^{-1} = 5 \text{ cm}^2 \times v_2$$

$$\therefore \quad v_2 = 2 \text{ ms}^{-1}$$

Using Bernoulli's theorem for horizontal flow,

$$p_2 + \frac{1}{2}\rho v_2^2 = p_1 + \frac{1}{2}\rho v_1^2$$

or $$p_2 + \frac{1}{2} \times 10^3 \times 2^2 = 2000 + \frac{1}{2} \times 10^3 \times 1^2$$

or $$p_2 = 2000 + 500 - 2000 = \mathbf{500 \text{ Pa.}}$$

Problem 20. *A liquid is kept in a cylindrical vessel which is being rotated about its axis. The liquid rises at the sides. If the radius of the vessel is 0.05 m and the speed of rotation is 2 rps, find the difference in the heights of the liquid at the centre of the vessel and at its sides.* **[Roorkee 87]**

Solution. According to Bernoulli's theorem, we have

$$p + \frac{1}{2}\rho v^2 = \text{constant}$$

When the liquid rotates, the velocity at the sides is higher so the pressure is lower. Since the pressure on a given horizontal level must be same, the liquid rises at the sides to height h to compensate for this drop in pressure.

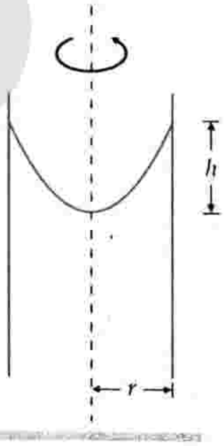


Fig. 10.73

$$\therefore \frac{1}{2}\rho v^2 = h\rho g$$

or $$h = \frac{v^2}{2g} = \frac{(2\pi r \nu)^2}{2g} = \frac{2\pi^2 r^2 \nu^2}{g} \quad [\because v = \omega r = 2\pi \nu r]$$

But $r = 0.05$ m, $\nu = 2$ rps

$$\therefore \quad h = \frac{2 \times 9.87 \times (0.05)^2 \times 2^2}{9.8} = \mathbf{0.02 \text{ m.}}$$

Problem 21. *Calculate the rate of flow of glycerine of density 1.25×10^3 kgm⁻³ through the conical section of a pipe if the radii of its ends are 0.1 m and 0.04 m and pressure-drop across its length is 10 Nm⁻².* **[Roorkee 91]**

Solution. Using Bernoulli's theorem for horizontal flow,

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

or $$v_2^2 - v_1^2 = \frac{2(p_1 - p_2)}{\rho}$$

$$= \frac{2 \times 10 \text{ Nm}^{-2}}{1.25 \times 10^3 \text{ kgm}^{-3}} = 16 \times 10^{-3}$$

According to equation of continuity, $a_1 v_1 = a_2 v_2$

$$\therefore \quad \frac{v_1}{v_2} = \frac{a_2}{a_1} = \frac{\pi r_2^2}{\pi r_1^2} = \frac{r_2^2}{r_1^2} = \frac{(0.04)^2}{(0.1)^2} = 16 \times 10^{-2}$$

or $$v_1 = 16 \times 10^{-2} v_2$$

Hence $$v_2^2 - (16 \times 10^{-2} v_2)^2 = 16 \times 10^{-3}$$

or $$v_2^2 \left[1 - \frac{256}{10000}\right] = 16 \times 10^{-3}$$

or $$v_2^2 \times \frac{9744}{10000} = 16 \times 10^{-3}$$

or $$v_2^2 = \frac{160}{9744} \quad \text{or} \quad v_2 = 0.13 \text{ ms}^{-1}$$

Rate flow of glycerine

$$= a_2 v_2 = 3.14 \times (0.04)^2 \times 0.13 = \mathbf{6.53 \times 10^{-4} \text{ m}^3 \text{ s}^{-1}.}$$

Problem 22. *Water from a tap emerges vertically downward with an initial speed of 1.0 ms⁻¹. The cross-sectional area of the tap is 10^{-4} m². Assume that the pressure is constant throughout the stream of water, and that the flow is steady. What is the cross-sectional area of the stream 0.15 m below the tap ?* **[IIT 98]**

Solution. Here $v_1 = 1.0 \text{ ms}^{-1}$, $a_1 = 10^{-4} \text{ m}^2$,

$$h_1 - h_2 = 0.15 \text{ m}, \quad v_2 = ?, \quad a_2 = ?$$

According to Bernoulli's theorem,

$$P + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

$$[\because P_1 = P_2 = P \text{ (say)}]$$

or $$\frac{1}{2} v_1^2 + g h_1 = \frac{1}{2} v_2^2 + g h_2$$

or $$v_2^2 = v_1^2 + 2g(h_1 - h_2) = (1.0)^2 + 2 \times 10 \times 0.15 = 4$$

or $$v_2 = 2 \text{ ms}^{-1}$$

By equation of continuity, $a_1 v_1 = a_2 v_2$

$$\therefore \quad a_2 = \frac{a_1 v_1}{v_2} = \frac{10^{-4} \times 1}{2} = \mathbf{5 \times 10^{-5} \text{ m}^2.}$$

Guidelines to NCERT Exercises

10.1. *Explain why :*

(*a*) *The blood pressure in humans is greater at the feet than at the brain.* **[Himachal 08 ; Delhi 98]**

(*b*) *Atmospheric pressure at a height of about 6 km decreases to nearly half its value at the sea level, though the height of the atmosphere is more than 100 km.*

(*c*) *Hydrostatic pressure is a scalar quantity even though pressure is force divided by area, and force is a vector.* **[Delhi 06]**

Ans. (*a*) The height of the blood column is quite large at feet than at the brain. Consequently, the blood pressure in humans is greater at the feet than at the brain.

(*b*) The density of air does not decrease linearly with height. The density decreases rapidly upto a height of about 6 km and above 6 km, it decreases rather very slowly. For this reason, the atmospheric pressure at a height of about 6 km decreases to nearly half its value at the sea level.

(*c*) Hydrostatic pressure is transmitted equally in all directions, no definite direction is associated with it. Hence it is a scalar quantity.

10.2. *Explain why :*

(*i*) *The angle of contact of mercury with glass is obtuse, while that of water with glass is acute.*

(*ii*) *Water on a clean glass surface tends to spread out while mercury on the same surface tends to form drops. (Put differently, water wets the glass while mercury does not).*

(*iii*) *Surface tension of a liquid is independent of the area of the surface.*

(*iv*) *Detergents should have small angles of contact.*

(*v*) *A drop of liquid under no external forces is always spherical in shape.* **[Central Schools 08]**

Ans. (*i*) The cohesive force between mercury molecules is greater than the adhesive force between mercury and glass molecules. As a result, meniscus of mercury is convex and hence angle of contact is obtuse. On the other hand, the adhesive force between water and glass molecules is greater than the cohesive force between water molecules. So the meniscus of water is concave and hence angle of contact is acute.

(*ii*) The adhesive force between water and glass molecules is greater than the cohesive force between water molecules. So water tends to spread on a clean glass surface. On the other hand, the cohesive force between mercury molecules is greater than the adhesive force between mercury and glass molecules. So mercury tends to form drops.

(*iii*) Surface tension is defined as the force acting per unit length of a line imagined tangential to a liquid surface at rest. It depends on the nature of the liquid and its temperature and is independent of the area of the liquid surface.

(*iv*) The detergents should have small angle of contact so that they have low surface tension and hence greater ability to wet a surface. Moreover, if θ is small, $\cos\theta$ will be large and hence detergent will penetrate $\left(h = \dfrac{2\sigma\cos\theta}{r\rho g}\right)$ more in the narrow spaces in the cloth and will easily remove the dirt.

(*v*) Due to surface tension, the free surface of a liquid tends to acquire minimum surface area. In the absence of any external force, a liquid drop becomes spherical in shape because for a given volume, a sphere has minimum surface area.

10.3. *Fill in the blanks using the word(s) from the list appended with each statement :*

(*i*) *Surface tension of liquids generally ______ with temperatures (increases/decreases).*

(*ii*) *Viscosity of gases ______ with temperature, whereas viscosity of liquids ______ with temperature (increases/ decreases).*

(*iii*) *For solids with elastic modulus of rigidity, the shearing force is proportional to ______ while for fluids it is proportional to ______*

(shear strain/rate of shear strain).

(*iv*) *For a fluid in steady flow, the increase in flow speed at a constriction follows from ______, while the decrease of pressure there follows from ______*

(conservation of mass/Bernoulli's principle).

(*v*) *For the model of a plane in a wind tunnel, turbulence occurs at a ______ speed than the critical speed for turbulence for an actual plane (greater/smaller).*

Ans. (*i*) decreases (*ii*) increases, decreases (*iii*) shear strain, rate of shear strain (*iv*) conservation of mass, Bernoulli's principle (*v*) greater.

10.4. *Explain why*

(*i*) *To keep a piece of paper horizontal, you should blow over, not under, it.*

(*ii*) *When we try to close a water tap with our fingers, fast jets of water gush through the openings between our fingers.*

(*iii*) *The size of the needle of a syringe controls flow rate better than the thumb pressure exerted by a doctor while administering an injection.*

(iv) A fluid flowing out of a small hole in a vessel results in a backward thrust on the vessel.

(v) A spinning cricket ball in air does not follow a parabolic trajectory.

Ans. (*i*) When we blow over the paper, the velocity of air increases and hence pressure of air decreases in accordance with Bernoulli's principle. But the pressure below the paper (atmospheric pressure) is high which keeps the paper horizontal.

(*ii*) According to the equation of continuity, velocity is inversely proportional to the area of cross-section. Since area between two fingers is very small as compared to the area of water tap above it, hence fast jets of water gush out through openings between the fingers.

(*iii*) According to Bernoulli's principle,

$$p + \rho g h + \frac{1}{2}\rho v^2 = \text{constant}$$

Thus the total energy of the injectable medicine depends upon second power of the velocity and first power of the pressure. It means that total energy of the injectable medicine has greater dependence on its velocity. Therefore, a doctor adjusts the flow rate of the medicine with the help of the size of the needle of the syringe ($a_1 v_1 = a_2 v_2$) rather than the thumb pressure, while administering an injection.

(*iv*) Due to small area of cross-section of the hole, the fluid flows out of the vessel with a large speed and hence with a large linear momentum. As no external force acts on the system, in order to conserve the linear momentum, the vessel acquires a backward momentum and hence a backward thrust acts on the vessel.

(*v*) Refer answer to Q. 52 on page 10.39.

10.5. *A 50 kg girl wearing high heel shoes balances on a single heel. The heel is circular with a diameter 1.0 cm. What is the pressure exerted by the heel on the horizontal floor ?*

Ans. Here $m = 50$ kg , $D = 1.0\,\text{cm} = 10^{-2}$ m, $g = 9.8\ \text{ms}^{-2}$

$$P = \frac{F}{A} = \frac{mg}{\pi (D/2)^2} = \frac{4\,mg}{\pi D^2}$$

$$= \frac{4 \times 50 \times 9.8}{(22/7) \times (10^{-2})^2} = \mathbf{6.2 \times 10^6\ Pa.}$$

10.6. *Toricelli's barometer used mercury. Pascal duplicated it using French wine of density 984. Determine the height of the wine column for normal atmospheric pressure.*

Ans. Pressure exerted by h height of wine column

= Pressure exerted by 76 cm of Hg column

or $h \times 984 \times 9.8 = 0.76 \times 13.6 \times 10^3 \times 9.8$

$$\therefore \quad h = \frac{0.76 \times 13.6 \times 10^3}{984} = \mathbf{10.5\ m.}$$

10.7 *A vertical off-shore structure is built to withstand a maximum stress of 10^9 Pa. Is the structure suitable for putting up on top of an oil well in Bombay High ? Take the depth of the sea to be roughly 3 km, and ignore ocean currents.*

Ans. Here $h = 3\ \text{km} = 3000\ \text{m}$,

$$\rho\,(\text{water}) = 1000\ \text{kgm}^{-3}$$

Pressure due to sea water,

$$P = h\rho g = 3000 \times 1000 \times 9.8 = 2.94 \times 10^7\ \text{Pa}.$$

This pressure is much less than the stress of 10^9 Pa which the structure can withstand, hence the structure is suitable for the required purpose.

10.8. *A hydraulic automobile lift is designed to lift cars with a maximum mass of 3000 kg. The area of cross-section of the piston carrying the load is 425 cm². What maximum pressure would the smaller piston have to bear ?*

Ans. Area of cross-section of larger piston,

$$A = 425\ \text{cm}^2 = 425 \times 10^{-4}\ \text{m}^2$$

Load on larger piston,

$$F = mg = 3000 \times 9.8\ \text{N}$$

Pressure on larger piston,

$$P = \frac{F}{A} = \frac{3000 \times 9.8}{425 \times 10^{-4}} = 6.92 \times 10^5\ \text{Nm}^{-2}$$

As the liquids transmit pressure equally in all directions, therefore the pressure that the smaller piston would have to bear = $\mathbf{6.92 \times 10^5\ Nm^{-2}}$.

10.9 *A U-tube contains water and methylated spirit separated by mercury. The mercury columns in the two arms are in level with 10.0 cm of water in one arm and 12.5 cm of spirit in the other. What is the specific gravity of spirit ?*

Ans. Refer to Fig. 10.74. As the mercury columns in the two arms of the U-tube are at the same level, therefore

Pressure due to water column

= Pressure due to spirit column

$$h_w \rho_w g = h_s \rho_s g \quad \text{or} \quad h_w \rho_w = h_s \rho_s$$

But

$h_w = 10\ \text{cm}$,

$\rho_w = 1\ \text{g cm}^{-3}$

$h_s = 12.5\ \text{cm}$

$\therefore \quad 10 \times 1 = 12.5 \times \rho_s$

or $\quad \rho_s = \dfrac{10}{12.5}$

$= 0.8\ \text{g cm}^{-3}$

Specific gravity of spirit

$= \dfrac{\rho_s}{\rho_w}$

$= \dfrac{0.8\ \text{g cm}^{-3}}{1\ \text{g cm}^{-3}}$

$= \mathbf{0.8.}$

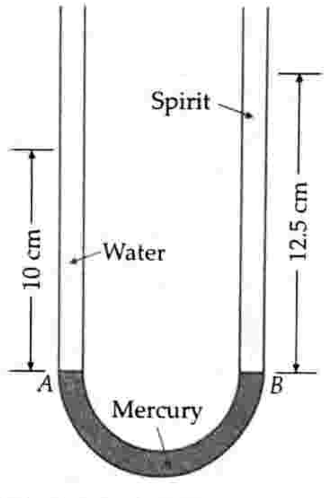


Fig. 10.74

10.10. *In previous exercise, if 15.0 cm of water and spirit each are further poured into the respective arms of the tube, what is the difference in the levels of mercury in the two arms ? Specific gravity of mercury = 13.6.*

Ans. Refer to Fig. 10.75. Pressure on mercury level in one arm due to water,

$$P_1 = h_w \rho_w g$$
$$= (10 + 15) \times 1 \times g$$
$$= 25g$$

Pressure on mercury level in another arm due to spirit,

$$P_2 = h_s \rho_s g$$
$$= (12.5 + 15) \times 0.8 \times g$$
$$= 22g$$

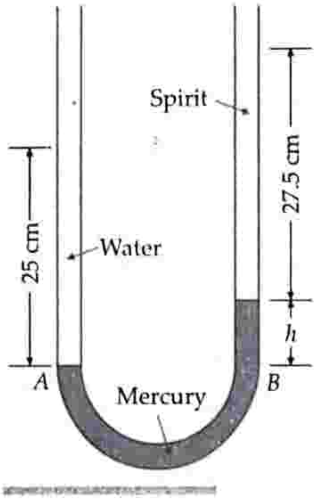


Fig. 10.75

As the pressure in water arm is more, the mercury will rise in spirit arm. If this pressure difference corresponds to height difference h in the two arms, then

$$P_1 - P_2 = h\rho g$$

$$25g - 22g = h \times 13.6 \times g \quad \text{or} \quad h = \frac{3}{13.6} = \mathbf{0.221\ cm.}$$

Thus mercury rises in the arm containing spirit ; the difference in the levels of mercury in the two columns is 0.221 cm.

10.11. *Can Bernoulli's equation be used to describe the flow of water through a rapid in a river ? Explain.*

Ans. No, Bernoulli's equation cannot be used to describe the rapid flow of water in a river because rapid flow is turbulent and not streamlined. The Bernoulli's equation is applicable to streamline flow of a fluid.

10.12. *Does it matter if one uses gauge instead of absolute pressures in applying Bernoulli's equation ? Explain.*

Ans. No, it does not matter if one uses gauge instead of absolute pressure in applying Bernoulli's equation. However, this will be valid only if the atmospheric pressures at the two places of consideration are not appreciably different.

10.13. *Glycerine flows steadily through a horizontal tube of length 1.5 m and radius 1.0 cm. If the amount of glycerine collected per second at one end is 4.0×10^{-3} kgs^{-1}, what is the pressure difference between the two ends of the tube ? Density of glycerine = 1.3×10^3 kg m^{-3} and viscosity of glycerine = 0.83 Nsm^{-2}.*

Ans. Here $l = 1.5\ \text{m},\ r = 1.0\ \text{cm} = 1.0 \times 10^{-2}\ \text{m},$

$$\rho = 1.3 \times 10^3\ \text{kg m}^{-3},\ \eta = 0.83\ \text{Nsm}^{-2}$$

Mass collected per second = 4.0×10^{-3} kg

$\therefore$ Volume of the liquid flowing per second,

$$Q = \frac{\text{Mass collected per second}}{\text{Density}}$$

$$= \frac{4.0 \times 10^{-3}}{1.3 \times 10^3}\ \text{m}^3\text{s}^{-1}$$

But $\quad Q = \dfrac{\pi p r^4}{8 l \eta}$

$$\therefore \quad p = \frac{8 l \eta Q}{\pi r^4} = \frac{8 \times 1.5 \times 0.83}{3.14 \times (1.0 \times 10^{-2})^4} \times \frac{4.0 \times 10^{-3}}{1.3 \times 10^3}$$

$$= \mathbf{9.8 \times 10^2\ Pa.}$$

10.14. *In a test experiment on a model aeroplane in a wind tunnel, the flow speeds on the upper and lower surfaces of the wing are 70 ms^{-1} and 63 ms^{-1} respectively. What is the lift of the wing if its area is 2.5 m^2 ? Density of air = 1.3 kg m^{-3}.*

Ans. Here $\rho = 1.3$ kg m^{-3}, $v_1 = 70$ ms^{-1}, $v_2 = 63$ ms^{-1}

Let p_1 and p_2 be the pressures on the upper and lower surfaces of the wing. Applying Bernoulli's theorem,

$$\frac{p_1}{\rho} + \frac{1}{2}v_1^2 = \frac{p_2}{\rho} + \frac{1}{2}v_2^2$$

or $\quad p_2 - p_1 = \dfrac{1}{2}(v_1^2 - v_2^2)\rho$

$$= \frac{1}{2}(70^2 - 63^2) \times 1.3 = 605.15\ \text{Nm}^{-2}$$

Lift of the wing = Net upward pressure × Area of the wing

$$= (p_2 - p_1)\, A = 605.15 \times 2.5\ \text{N}$$
$$= \mathbf{1512.9\ N.}$$

10.15. *Figs. 10.76(a) and (b) refer to the steady flow of a (non-viscous) liquid. Which of the two figures is incorrect ? Why ?*

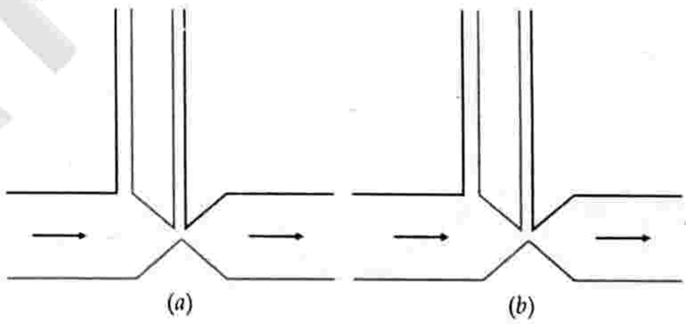


Fig. 10.76

Ans. Fig. 10.76(a) is incorrect. At the constriction, the area of cross-section is small and so the liquid velocity is large. Consequently, the liquid pressure must be small (Bernoulli's principle).

10.16. *The cylindrical tube of a spray pump has a cross-section of 8.0 cm^2, one end of which has 40 fine holes each of diameter 1.0 mm. If the liquid flow inside the tube is 1.5 m min^{-1}, what is the speed of ejection of the liquid through the holes ?*

Ans. Here $a_1 = 8.0\ \text{cm}^2 = 8 \times 10^{-4}\ \text{m}^2,$

$$v = 1.5\ \text{m min}^{-1} = \frac{1.5}{60}\ \text{ms}^{-1}$$

Radius of a hole $= \dfrac{d}{2} = \dfrac{1.0\ \text{mm}}{2} = 0.5 \times 10^{-3}$ m

$\therefore$ Cross-section of a hole $= \pi \times (0.5 \times 10^{-3})^2\ \text{m}^2$

Total cross-section of 40 holes,

$$a_2 = \pi \times (0.5 \times 10^{-3})^2 \times 40 \text{ m}^2$$

If v_2 is the speed of ejection of the liquid through the holes, then

$$a_1 v_1 = a_2 v_2 \qquad \text{(Equation of continuity)}$$

or $$v_2 = \frac{a_1 v_1}{a_2} = \frac{8 \times 10^{-4} \times 1.5}{\pi \times (0.5 \times 10^{-3})^2 \times 40 \times 60}$$

$$= \mathbf{0.637 \text{ ms}^{-1}}$$

10.17. *A U-shaped wire is dipped in a soap solution, and removed. The thin soap film formed between the wire and a light slider supports a weight of 1.5×10^{-2} N (which includes the small weight of the slider). The length of the slider is 30 cm. What is the surface tension of the film ?*

Ans. Here $F = 1.5 \times 10^{-2}$ N, $l = 30$ cm $= 0.3$ m

As the soap film has two free surfaces, so the force F acts over twice the length of the slider. Hence

$$\sigma = \frac{F}{2l} = \frac{1.5 \times 10^{-2}}{2 \times 0.30} = \mathbf{2.5 \times 10^{-2} Nm^{-1}}.$$

10.18. *Fig. 10.77(a) shows a thin liquid film supporting a small weight $= 4.5 \times 10^{-2}$ N. What is the weight supported by a film of the same liquid at the same temperature in Figs. 10.77(b) and (c) ? Explain your answer physically.*

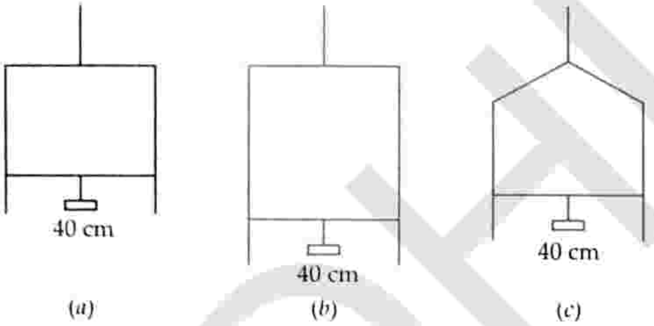


Fig. 10.77

Ans. The weight supported both in (*b*) and (*c*) is 4.5×10^{-2} N *i.e.*, same as in case (*a*). The weight supported $= 2\,\sigma\, l$. As σ and l are same in all cases, so the weight supported is same.

10.19. *What is the pressure inside a drop of mercury of radius 3.00 mm of room temperature ? Surface tension of mercury at that temperature (20 °C) is 4.65×10^{-1} Nm^{-1}. The atmospheric pressure is 1.01×10^5 Pa. Also give the excess pressure inside the drop.*

Ans. Here $R = 3.00$ mm $= 3.00 \times 10^{-3}$ m,

$$\sigma = 4.65 \times 10^{-1} \text{ Nm}^{-1}, \; P_0 = 1.01 \times 10^5 \text{ Pa}$$

Excess pressure inside the drop is

$$p = \frac{2\sigma}{R}$$

$$= \frac{2 \times 4.65 \times 10^{-1}}{3.00 \times 10^{-3}} = 310 \text{ Nm}^{-2}$$

Total pressure inside the drop,

$$P = \text{Atmospheric pressure + Excess pressure}$$
$$= 1.01 \times 10^5 + 310$$
$$= 101000 + 310 = 101310 \text{ Nm}^{-2} = \mathbf{1.013 \times 10^5 \text{ Pa}}.$$

10.20. *What is the excess pressure inside a bubble of soap solution of radius 5.00 mm ? Given that the surface tension of soap solution at the temperature (20° C) is 2.50×10^{-2} Nm^{-1}. If an air bubble of the same dimension were formed at a depth of 40.0 cm inside a container containing the soap solution (of relative density 1.20), what would be the pressure inside the bubble ? (1 atm $= 1.01 \times 10^5$ Pa).*

Ans. Here $r = 5.00$ mm $= 5.00 \times 10^{-3}$ m,

$$\sigma = 2.50 \times 10^{-2} \text{ Nm}^{-1}, \; h = 40 \text{ cm} = 0.4 \text{ m},$$
$$P_0 = 1.01 \times 10^5 \text{ Pa}$$

Excess pressure inside a soap bubble,

$$p = \frac{4\sigma}{R} = \frac{4 \times 2.50 \times 10^{-2}}{5.00 \times 10^{-3}} = \mathbf{20 \text{ Pa}}.$$

Excess pressure inside an air bubble under soap solution

$$p' = \frac{2\sigma}{R} = \frac{2 \times 2.50 \times 10^{-2}}{5.00 \times 10^{-3}} = 10 \text{ Pa}$$

Density of soap solution,

$$\rho = \text{R.D.} \times 1000 = 1.20 \times 1000 = 1200 \text{ kgm}^{-3}$$

Total pressure inside the air bubble

= Atmospheric pressure + Pressure due to 40 cm soap solution + Excess pressure

$$= 1.01 \times 10^5 + h\rho g + p'$$
$$= 1.01 \times 10^5 + 0.40 \times 1200 \times 9.8 + 10$$
$$= 101000 + 4704 + 10 = \mathbf{105714 \text{ Pa}}.$$

10.21. *A tank with a square base of area 1.0 m^2 is divided by a vertical partition in the middle. The bottom of the partition has a small hinged door of area 20 cm^2. The tank is filled with water in one compartment, and an acid (of relative density 1.7) in the other, both to a height of 4.0 m. Compute the force necessary to keep the door closed.*

Ans. *For compartment containing water :*

Height of water column, $h = 4.0$ m

Density of water, $\rho = 10^3$ kgm^{-3}

Pressure due to water at the door at the bottom,

$$P_w = h\rho g = 4.0 \times 10^3 \times 9.8 = 39.2 \times 10^3 \text{ Pa}$$

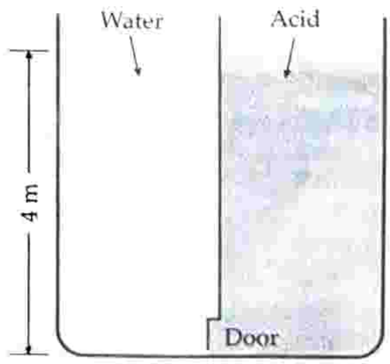


Fig. 10.78

For compartment containing acid :

Height of acid column = 4.0 m

Density of acid,

$$\rho = 1.7 \times 10^3 \text{ kg m}^{-3}$$

Pressure due to acid at the door at the bottom,

$$P_a = h\rho g = 4.0 \times 1.7 \times 10^3 \times 9.8$$
$$= 66.64 \times 10^3 \text{ Pa}$$

$$\therefore \quad P_a - P_w = 66.64 \times 10^3 - 39.2 \times 10^3$$
$$= 27.44 \times 10^3 \text{ Pa}$$

Area of the door, $A = 20 \text{ cm}^2 = 20 \times 10^{-4} \text{ m}^2$

Force on the door due to difference of pressure on its two sides

$$= (P_a - P_w) \times A$$
$$= 27.44 \times 10^3 \times 20 \times 10^{-4} = 54.88 \text{ N}$$

Thus, to keep the door closed, a force of 54.88 N must be applied on it from the water side.

10.22. *A manometer reads the pressure of a gas in an enclosure as shown in Fig. 10.79(a). When some of the gas is removed by a pump, the manometer reads as in Fig. 10.79(b).*

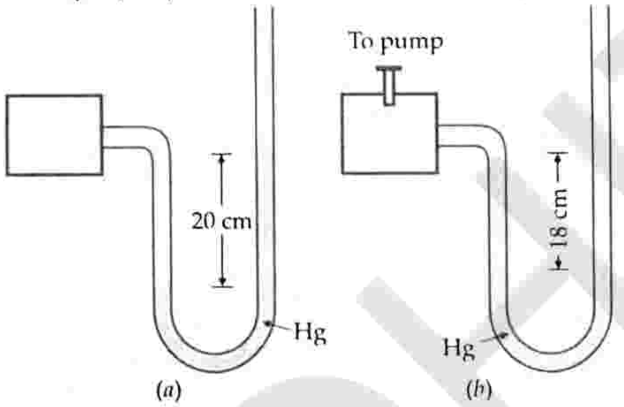


Fig. 10.79

The liquid used in the manometers is mercury and the atmospheric pressure is 76 cm of mercury.

(*i*) *Give the absolute and gauge pressure of the gas in the enclosure for cases (a) and (b) in units of cm of mercury.*

(*ii*) *How would the levels change in case (b) if 13.6 cm of water (immiscible with mercury) are poured into the right limb of the manometer ? (Ignore the small change in volume of the gas).*

Ans. Here atmospheric pressure,

$$P = 76 \text{ cm of Hg}$$

(*i*) *In case (a)*: Pressure head, $h = +\,20$ cm of Hg

Absolute pressure = $P + h = 76 + 20 =$ **96 cm of Hg**

Gauge pressure $= h =$ **20 cm of Hg**

In case (b) : Pressure head, $h = -\,18$ cm of Hg

Absolute pressure

$$= P + h = 76 - 18 = \textbf{58 cm of Hg}$$

Gauge pressure = $h =$ **– 18 cm of Hg.**

(*ii*) As $\quad h_1 \rho_1 g = h_2 \rho_2 g$

$$h_1 \times 13.6 \times g = 13.6 \times 1 \times g$$

or $\quad h_1 = 1$ cm

Therefore, as 13.6 cm of water is poured in right limb, it will displace mercury level by 1 cm in the left limb, so that difference of levels in the two limbs will become **19 cm.**

10.23. *Two vessels have the same base area but different shapes. The first vessel takes twice the volume of water that the second vessel requires to fill upto a particular common height.*

(*i*) *Is the force exerted by the water on the base of the vessel the same in the two cases ?*

(*ii*) *If so, why do the vessels filled with water to that same height give different readings on a weighing scale ?*

Ans. (*i*) As the two vessels have been filled with water upto the same common height, the pressures exerted on the bases of the two vessels are equal. Moreover, the two vessels have the same base area, so forces exerted on the bases of the two vessels will also be equal.

(*ii*) Water exerts force on the sides of the vessel also. This force has a nonzero vertical component when the sides of the vessel are not perfectly normal to the base. This net vertical component of the force exerted by water on the sides of the vessel is greater for the first vessel than the second. Hence the vessels weigh different even when the force on the base is same in the two cases.

10.24. *During blood transfusion the needle is inserted in a vein where the gauge pressure is 2000 Pa. At what height must the blood container be placed so that blood may just enter the vein ? The density of whole blood* $= 1.06 \times 10^3 \text{ kg m}^{-3}$.

Ans. Let h be the height of container at which its blood exerts pressure equal to gauge pressure in vein. Then

$$h\rho g = P_g$$

or $$h = \frac{P_g}{\rho g}$$

$$= \frac{2000}{1.06 \times 10^3 \times 9.8} = 0.1925 \text{ m}$$

The blood will just enter the vein if the blood container is kept at height slightly greater than 0.1925 m *i.e.*, at 0.2 m.

10.25. *In deriving Bernoulli's equation, we equated the work done on the fluid in the tube to its change in the potential and kinetic energy. (a) How does the pressure change as the fluid moves along the tube if dissipative forces are present ? (b) Do the dissipative forces become more important as the fluid velocity increases ? Discuss qualitatively.*

Ans. (*a*) If dissipative forces are present, some of the pressure energy of the fluid is spent in doing work against these forces, so the fluid pressure decreases with the increase in length of the tube.

(*b*) Yes, the dissipative forces become more important when the fluid velocity increases.

10.26. *(a) What is the largest average velocity of blood flow in an artery of radius 2×10^{-3} m if the flow must remain laminar ? (b) What is the corresponding flow rate ? Take viscosity of blood to be 2.084×10^{-3} Pa s and density of blood $= 1.06 \times 10^3$ kg m^{-3}.* **[Delhi 06]**

Ans. (*a*) Here $\rho = 1.06 \times 10^3$ kg m^{-3},

$$D = 2r = 4 \times 10^{-3} \text{ m}, \quad \eta = 2.084 \times 10^{-3} \text{ Pa s}$$

The maximum value of Reynold's number for laminar flow is 2000. Hence the maximum average velocity for laminar flow or critical velocity is given by

$$v_c = \frac{R_e \eta}{\rho D} = \frac{2000 \times 2.084}{1.06 \times 10^3 \times 4 \times 10^{-3}} = \mathbf{0.98 \ ms^{-1}}.$$

(*b*) Volume of blood flowing per second,

$$Q = av_c = \pi r^2 v_c = \frac{22}{7} \times (2 \times 10^{-3})^2 \times 0.98$$

$$= 1.23 \times 10^{-5} \text{m}^3\text{s}^{-1}.$$

10.27. *A plane is in level flight at constant speed and each of its two wings has an area of 25 m^2. If the speed of the air is 180 km/h over the lower wing and 234 km/h over the upper wing surface, determine the plane's mass. Take air density to be 1 kg m^{-3} and $g = 9.81$ ms^{-2}.*

Ans. Here $v_1 = 180$ km h^{-1}

$$= 180 \times \frac{5}{18} = 50 \text{ ms}^{-1}$$

$$v_2 = 234 \text{ km h}^{-1} = 234 \times \frac{5}{18} = 65 \text{ ms}^{-1}$$

Area of the wings,

$$A = 2 \times 25 = 50 \text{ m}^2, \quad \rho = 1 \text{ kg m}^{-3}.$$

For a plane in the level flight, Bernoulli's equation is

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

or

$$p_1 - p_2 = \frac{1}{2}\rho(v_2^2 - v_1^2)$$

$$= \frac{1}{2} \times 1 \times (65^2 - 50^2)$$

$$= 862.5 \text{ Nm}^{-2}$$

Upward force on the plane $= (p_1 - p_2) \times A$

$$= 862.5 \times 50 = 43125 \text{ N}$$

In level flight, the upward force balances the weight of the plane, so

$$mg = 43125 \text{ N}$$

Mass of the plane, $m = \dfrac{43125}{g} = \dfrac{43125}{9.81} = \mathbf{4396 \ kg}$.

10.28. *In Millikan's oil drop experiment, what is the terminal speed of a drop of radius 2.0×10^{-5} m and density 1.2×10^3 kg m^{-3} ? Take the viscosity of air at the temperature of the experiment to be 1.8×10^{-5} Nsm^{-2}. How much is the viscous force on the drop at that speed ? Neglect buoyancy of the drop due to air.*

Ans. Here $r = 2.0 \times 10^{-5}$ m, $\rho = 1.2 \times 10^3$ kg m^{-3},

$$\eta = 1.8 \times 10^{-5} \text{ Nsm}^{-2}$$

If the buoyancy of the drop due to air is neglected, then the terminal speed is given by

$$v = \frac{2}{9} \cdot \frac{r^2 \rho g}{\eta}$$

$$= \frac{2}{9} \times \frac{(2.0 \times 10^{-5})^2 \times 1.2 \times 10^3 \times 9.8}{1.8 \times 10^{-5}}$$

$$= 5.8 \times 10^{-2} \text{ ms}^{-1} = \mathbf{5.8 \ cms^{-1}}$$

Viscous force,

$$F = 6\pi\eta r v$$

$$= 6 \times \frac{22}{7} \times 1.8 \times 10^{-5} \times 2.0 \times 10^{-5} \times 5.8 \times 10^{-2}$$

$$= \mathbf{3.9 \times 10^{-10} \ N}.$$

10.29. *Mercury has an angle of contact equal to 140° with soda lime glass. A narrow tube of radius 1.00 mm made of thin glass is dipped in a trough containing mercury. By what amount does the mercury dip down in the tube relative to the liquid surface outside ? Surface tension of mercury at the temperature of the experiment is 0.465 Nm^{-1}. Density of mercury $= 13.6 \times 10^3$ kgm^{-3}.*

Ans. Here $\theta = 140°$, $r = 1.00$ mm $= 1.00 \times 10^{-3}$ m,

$$\sigma = 0.465 \text{ Nm}^{-1}, \quad \rho = 13.6 \times 10^3 \text{ kgm}^{-3}.$$

$$\cos 140° = \cos(180° - 40°) = -\cos 40° = -0.7660$$

$$\therefore \quad h = \frac{2\sigma\cos\theta}{r\rho g} = \frac{2 \times 0.465 \times \cos 140°}{1.00 \times 10^{-3} \times 13.6 \times 10^3 \times 9.8}$$

$$= \frac{2 \times 0.465 \times (-0.7660)}{13.6 \times 9.8}$$

$$= -5.34 \times 10^{-3} \text{ m} = \mathbf{-5.34 \ mm}.$$

The negative sign indicates that the mercury level is depressed in the capillary tube.

10.30. *The narrow bores of diameters 3.0 mm and 6.0 mm are joined together to form a U-shaped tube open at both ends. If the U-tube contains water, what is the difference in its levels in the two limbs of the tube ? Surface tension of water at the temperature of the experiment is 7.3×10^{-2} Nm^{-1}. Take the angle of contact to be zero, and density of water to be 1.0×10^3 kg m^{-3}. Take $g = 9.8$ ms^{-2}.*

Ans. Here $r_1 = \dfrac{3.0}{2} = 1.5 \text{ mm} = 1.5 \times 10^{-3}$ m,

$$r_2 = \frac{6.0}{2} = 3.0 \text{ mm} = 3.0 \times 10^{-3} \text{ m}$$

$$\sigma = 7.3 \times 10^{-2} \text{ Nm}^{-1}, \quad \theta = 0°$$

$$\rho = 1.0 \times 10^3 \text{ kg m}^{-3}, \; g = 9.8 \text{ ms}^{-2}$$

Let h_1 and h_2 be the heights to which water rises in the two tubes. Then

$$h_1 = \frac{2\sigma\cos\theta}{r_1 \rho g} \quad \text{and} \quad h_2 = \frac{2\sigma\cos\theta}{r_2 \rho g}$$

Therefore,

$$h_1 - h_2 = \frac{2\sigma \cos 0}{\rho g}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$= \frac{2 \times 7.3 \times 10^{-2} \cos 0^\circ}{10^3 \times 9.8}\left[\frac{1}{1.5 \times 10^{-3}} - \frac{1}{3 \times 10^{-2}}\right]$$

$$= \frac{14.6 \times 10^{-2}}{10^3 \times 9.8 \times 10^{-3}}\left[\frac{1}{1.5} - \frac{1}{3}\right]$$

$$= \frac{14.6 \times 10^{-2}}{9.8} \times \frac{1}{3} = 0.5290 \times 10^{-2} \text{ m}$$

$= \mathbf{5.290\ mm.}$

10.31 (*a*) *It is known that density* ρ *of air decreases with height* y *as*

$$\rho = \rho_0 e^{-y/y_0}$$

where $\rho_0 = 1.25\ kg\, m^{-3}$ *is the density at sea level and* y_0 *is a constant. This density variation is called the law of atmospheres. Obtain this law assuming that the temperature of atmosphere remains a constant (isothermal conditions). Also assume that the value of g remains constant.*

(*b*) *A large He balloon of volume 1425* m^3 *is used to lift a pay load of 400 kg. Assume that the balloon maintains constant radius as it rises. How high does it rise ?*

Take $y_0 = 8000\ m$ *and* $\rho_{He} = 0.18\ kg\, m^{-3}$.

Ans. (*a*) Let ρ be the density of the air at a height y above the sea level. The rate of decrease of density with height must be proportional to the density at that height. That is,

$$-\frac{d\rho}{dy} \propto \rho \quad \text{or} \quad \frac{d\rho}{dy} = -k\rho$$

Here k is a constant of proportionality and –ve sign shows that the density of air decreases with the increase in height. Now

$$\frac{d\rho}{\rho} = -k\,dy$$

As height changes from 0 to y, density changes from ρ_0 to ρ. Integrating the above equation within these limits, we get

$$\int_{\rho_0}^{\rho} \frac{1}{\rho}\, d\rho = -k\int_0^y dy$$

or $$[\log_e \rho]_{\rho_0}^{\rho} = -k\,[y]_0^y$$

or $$\log_e \rho - \log_e \rho_0 = -k(y - 0)$$

or $$\log_e \frac{\rho}{\rho_0} = -ky$$

or $$\frac{\rho}{\rho_0} = e^{-ky}$$

or $$\rho = \rho_0 e^{-ky}$$

Taking constant k equal to $1/y_0$, we get

$$\rho = \rho_0 e^{-y/y_0}$$

(*b*) The balloon will rise upto a height, where its density becomes equal to the density of air at that height.

Volume of balloon, $V = 1425\text{ m}^3$

Mass of He gas in the balloon $= 1425 \times 0.18 = 256.5$ kg

Total mass of the balloon including pay-load

$$M = 400 + 256.5 = 656.5 \text{ kg}$$

Density of the balloon,

$$\rho = \frac{M}{V} = \frac{656.5}{1425} = 0.46 \text{ kg m}^{-3}$$

Given $y_0 = 8000$ m, $\rho_0 = 1.25$ kg m^{-3}, $\rho = 0.46$ kg m^{-3}

As $$\rho = \rho_0 e^{-y/y_0}$$

$\therefore$ $$0.46 = 1.25 e^{-y/8000}$$

or $$e^{y/8000} = \frac{1.25}{0.46} = 2.72$$

or $$y = 8000 \log_e 2.72$$

$$= 8000 \times 1 = 8000 \text{ m} = \mathbf{8\ km}$$

Text Based Exercises

Type A : Very Short Answer Questions — 1 Mark Each

1. Define the term fluid.
2. What is the modulus of rigidity of a fluid ?
3. Is thrust a scalar or vector quantity ?
4. Which law deals with the transmission of fluid pressure ?
5. State Pascal's law. **[Central Schools 07]**
6. Name two practical applications of Pascal's law.
7. What is atmospheric pressure ?
8. Does the atmospheric pressure vary with the height above the earth's surface ?
9. What is one torr of pressure ? **[Himachal 07C]**
10. What is meant by one bar of pressure ?
11. Write the relation between torr and millibar.
12. Name the factors on which the atmospheric pressure at a place depends.
13. Name the scientist who first experimentally measured the atmospheric pressure.

14. Express 1 atmosphere in terms of Nm^{-2} and bar.
15. What is a manometer ?
16. If water is used instead of mercury in a barometer, what will be the height of water column ?
17. What does gradual fall of barometric height indicate ?
18. What does sudden fall in barometric height indicate ?
19. What is indicated by gradual increase of atmospheric pressure ?
20. Which practical unit of pressure is used in meteorological science ?
21. What will happen if water is used in place of mercury in a barometer tube ? **[Meghalaya 96]**
22. Define buoyancy.
23. In which case a body will weigh maximum (*i*) in air (*ii*) vacuum or (*iii*) in water ?
24. How much should be systolic blood pressure for a normal human being ?
25. How much should be diastolic blood pressure for a normal human being ?
26. State the law of floatation.
27. A body is just floating in a liquid (of equal density). What happens to the body if it is slightly pressed and released ?
28. In case of stable equilibrium, should the meta-centre be above or below the centre of gravity of the body ?
29. What is the internal force of friction of a fluid known as ?
30. Define viscosity.
31. What is reciprocal of viscosity known as ?
32. Define coefficient of viscosity of a liquid. **[Manipur 97]**
33. Name the CGS and SI units of the coefficient of viscosity.
34. State the dimensional formula of the coefficient of viscosity.
35. Define poise. **[Meghalaya 98]**
36. Define poiseuille or decapoise.
37. Define kinematic viscosity.
38. How does the viscosity of gases depend on temperature ?
39. Water flows through a pipe. Which of its layers moves fastest ?
40. State Stokes' law.
41. Mention two applications of Stokes' law.
42. Out of solid friction and viscous force, which is independent of velocity ?
43. What is the nature of graph between terminal velocity of a spherical body and the square of its radius ?
44. Two balls A and B have radii in the ratio 1 : 2. What will be the ratio of their terminal velocities in a liquid ?
45. What is the net weight of a body when it falls with terminal velocity through a viscous medium ?
46. What do mean by a streamline and a tube of flow ?
47. When does the streamline flow become turbulent ?
48. What is laminar flow of a liquid ?
49. What is Reynold's number ?
50. What is the range of Reynold's number for the laminar flow of a liquid ?
51. Write the dimensional formula of Reynold's number.
52. Which of the following values of the Reynold's number can be true for turbulent flow :
    (*i*) 200 (*ii*) 500 (*iii*) 1500 (*iv*) 3000 ?
53. What is critical velocity of a liquid ?
54. Draw a graph between the velocity of a small sphere dropped from rest into a viscous liquid and time. Also indicate the terminal velocity as v_t on the graph.
55. Which fundamental law forms the basis of equation of continuity ?
56. What will be the nature of graph between the velocity of fluid flow (v) and area of cross-section (a) of the pipe ?
57. What are the different forms of energy possessed by a fluid in a streamline flow ?
58. What is that fundamental principle on which Bernoulli's theorem is based ?
59. What is an ideal fluid ?
60. Write the expressions for pressure head, gravitational head and velocity head.
61. What should be the properties of a liquid to satisfy Bernoulli's theorem ?
62. State Torricelli's theorem.
63. State Bernoulli's theorem. **[Central Schools 05]**
64. What is a Pitot tube ? State the principle on which it is based.
65. Define force of cohesion.
66. Define surface tension. State its SI unit. **[Central Schools 05]**
67. Name a physical quantity that has same dimensions as surface tension.
68. What is meant by the term molecular range ?
69. Define sphere of influence of a liquid molecule.
70. Which part of the liquid is responsible for the phenomenon of surface tension ?
71. How does surface tension change with temperature ?
72. Write down the following liquids in the order of increasing surface tension :

    Water, mercury, soap solution.

73. What is the value of surface tension at the critical temperature ?

74. What is the effect of solute on the surface tension of a liquid ?

75. Define angle of contact.

76. Will the angle of contact be acute or obtuse for liquids which wet the walls of the container ?

77. Will the angle of contact be acute or obtuse for liquids which do not wet the walls of the container ?

78. What are the factors on which the angle of contact depends ?

79. How does the angle of contact of a liquid depend on temperature ?

80. What happens to the surface tension when some impurity is mixed in liquid ? **[Himachal 01]**

81. What is capillarity ? **[Delhi 95]**

82. Name the material in whose capillary water will descend instead of rising.

83. A lead sphere acquires a terminal velocity v when falls in a viscous liquid. What will be the terminal velocity attained by another lead sphere of radius three times in the same liquid ? **[Delhi 96]**

84. Water rises to a height of 20 mm in a capillary. If the radius of the capillary is made $\frac{1}{3}$rd of its previous value, to what height will the water now rise in the tube ? **[Delhi 96]**

85. By which phenomenon, the water rises from roots to leaves of plants ? **[Himachal 04]**

86. State conditions of equilibrium of floating bodies. **[Central Schools 07]**

87. Give relationship between poise and decapose. **[Himachal 07]**

Answers

1. A fluid is a substance that flows. So the term fluid refers to both liquids and gases.

2. Zero. A fluid has no definite shape of its own.

3. Vector quantity.

4. Pascal's law.

5. Pascal's law states that a change in pressure applied to an enclosed incompressible fluid is transmitted undiminished to every point of the fluid and the walls of the containing vessel.

6. Hydraulic press and hydraulic brakes.

7. Atmospheric pressure at any point is equal to weight of air contained in a column of unit cross-sectional area and extending upto top of atmosphere.

8. The atmospheric pressure decreases with the height above the earth's surface.

9. 1 torr = pressure exerted by 1 mm of Hg column $= 10^{-3} \times 13.6 \times 10^3 \times 9.8 = 133.8 \text{ Nm}^{-2}$.

10. $1 \text{ bar} = 10^5 \text{ Nm}^{-2}$.

11. $1 \text{ torr} = 133.8 \text{ Nm}^{-2} = \frac{133.8}{10^5} \text{ bar} = 1.338$ millibar.

12. Atmospheric pressure at a place depends on (*i*) height of atmosphere (*ii*) density of atmosphere and (*iii*) acceleration due to gravity.

13. Torricellie.

14. $1 \text{ atm} = 1.013 \times 10^5 \text{ Nm}^{-2} = 1.013 \times 10^5 \text{ Pa}$
$= 1.013$ bar.

15. Manometer is a device used to measure the pressure of a gas enclosed in a vessel.

16. 10.34 m.

17. The atmospheric pressure falls when the water vapours increase in air. This indicates the possibility of rain.

18. It indicates the possibility of storm.

19. It indicates dry weather.

20. Atmospheric pressure (atm).
$1 \text{ atm} = 1.013 \times 10^5 \text{ Nm}^{-2}$.

21. If water is used as barometric substance, it would require a tube about 11 m long. It is difficult to hold such a tube in a vertical position.

22. When a body is immersed in a fluid, it experiences a thrust. This effect is called buoyancy.

23. In vacuum.

24. Nearly 120 mm of Hg.

25. Nearly 80 mm of Hg.

26. According to the law of floatation, a body will float in a liquid if weight of the liquid displaced by the body is atleast equal to or greater than the weight of the body.

27. The body sinks to the bottom of the liquid.

28. Meta-centre should be above the centre of gravity.

29. Viscosity.

30. Viscosity is the property of the fluid by virtue of which it opposes the relative motion between its different layers.

31. Fluidity.

32. The coefficient of viscosity of a liquid is defined as the tangential viscous force required to maintain a unit velocity gradient between two liquid layers each of unit area.

33. The CGS unit of the coefficient of viscosity is poise and the SI unit is decapoise.
34. The dimensional formula of the coefficient of viscosity is $[ML^{-1}T^{-1}]$.
35. The coefficient of viscosity of a liquid is said to be 1 poise if a tangential force of 1 dyne cm^{-2} of the surface is required to maintain a relative velocity of 1 $cm\,s^{-1}$ between two layers of the liquid 1 cm apart.
36. The coefficient of viscosity of a liquid is said to be 1 poiseuille or decapoise if tangential force of 1 Nm^{-2} of the surface is required to maintain a relative velocity of 1 ms^{-1} between two layers of the liquid 1 m apart.
37. The ratio of coefficient of viscosity η and the density ρ of a liquid is called its kinematic viscosity.

$$\text{Kinematic viscosity} = \frac{\eta}{\rho}$$

38. The viscosity of gases increases with the increase of temperature. For a gas, $\eta \propto \sqrt{T}$
39. The axial layer of water moves fastest.
40. According to Stroke's law the viscous drag on a small body of radius (r) moving with a uniform velocity (v) through a viscous medium of viscosity (η) is given by $F = 6\pi\, \eta\, rv$.
41. Stokes' law can be used to find (*i*) radius of an oil drop and (*ii*) coefficient of viscosity of a liquid.
42. Solid friction.
43. Straight line.
44. As $v \propto r^2$, so the ratio of the terminal velocities of A and B will be 4 : 1.
45. Zero, because the weight of the body acting vertically downwards is balanced by the viscous force and the upthrust due to the medium.
46. A streamline may be defined as the straight or curved path, such that the tangent to it at any point gives the direction of flow of the liquid at that point. A group of streamlines is said to form a tube of flow.
47. The streamline flow becomes the turbulent flow when the velocity of the liquid exceeds the *critical velocity*.
48. The type of flow of a liquid in which its layers slide over one another without mixing is called the laminar flow.
49. Reynold's number is dimensionless combination of four factors which decides the nature of flow of a viscous liquid through a pipe. It is given by

$$R_e = \frac{\rho VD}{\eta}.$$

50. For laminar flow, R_e lies between 0 and 2000.
51. The dimensional formula of Reynold's number is $[M^0L^0T^0]$.
52. 3000.
53. Critical velocity is the maximum velocity of liquid above which the flow of a liquid changes from streamline to turbulent.
54. See Fig. 10.27 on page 10.29.
55. Law of conservation of mass.
56. Rectangular hyperbola.
57. (*i*) Kinetic energy (*ii*) Potential energy (*iii*) Pressure energy.
58. Law of conservation of energy.
59. An ideal fluid is one which is non-viscous, incompressible, and its flow is steady and irrotational.
60. Pressure head $= \frac{P}{\rho g}$, gravitational head $= h$ and velocity head $= \frac{v^2}{2g}$.
61. The liquid should be incompressible and non-viscous and the flow of the liquid should be streamlined and irrotational.
62. Torricelli's theorem states that the velocity with which a liquid flows out through an orifice at a certain depth below the free surface is the same as that attained by another body which falls freely through the same height.
This velocity of efflux is given by $v = \sqrt{2gh}$.
63. Refer to point 36 of Glimpses.
64. It is a device which is used to measure the velocity of the flow at any depth in a flowing liquid. It is based on Bernouilli's theorem.
65. The force of attraction between the molecules of the same substance is called the force of cohesion.
66. The property of a liquid by virtue of which its free surface at rest behaves like a stretched membrane and tends to have the minimum surface area is called surface tension. Its SI unit is Nm^{-1}.
67. Spring constant.
68. The maximum distance upto which a molecule can exert measurable force of attraction on other molecules is called molecular range (= 10^{-9} m).
69. A sphere drawn around a molecule as centre and radius equal to the molecular range is called the sphere of influence of the molecule.
70. The liquid in surface film is responsible for surface tension.
71. The surface tension of a liquid decreases with the rise of temperature.
72. Soap solution, water, mercury.
73. Zero.
74. If the solute (*e.g.*, salt in water) is highly soluble, the surface tension of liquid increases. If the solute (*e.g.*, soap in water) is less soluble, the surface tension decreases.

75. Angle of contact is defined as the angle between the tangent to the liquid surface at the point of contact to the solid surface inside the liquid.
76. Acute.
77. Obtuse.
78. The angle of contact depends on (*i*) Nature of the solid and liquid in contact. (*ii*) Cleanliness of the surface in contact. (*iii*) Medium above the free surface of the liquid. (*iv*) Temperature of the liquid.
79. The angle of contact of a liquid increases with the increase of temperature.
80. If the liquid surface has dust, grease or oil, the surface tension of the liquid decreases.
81. The phenomenon of rise or fall of a liquid in a capillary with respect to the surroundings is called capillarity.
82. Paraffin wax.
83. The terminal of second sphere will be 9 v because $v_t \propto r^2$.
84. As $h \propto \frac{1}{R}$

$$\therefore \quad \frac{h'}{h} = \frac{R}{R'}$$

or $$h' = \frac{R}{R'} \cdot h = \frac{R}{\frac{1}{3}R} h$$

$$= 3 \times 20 \text{ mm} = \mathbf{60\ mm}.$$

85. Capillary action.
86. Refer answer to Q.28 on page 10.15.
87. 1 decapoise = 10 poise.

Type B : Short Answer Questions — 2 or 3 Marks Each

1. Define the terms thrust and pressure. Give their S.I. units.
2. Show that a liquid at rest exerts force perpendicular to the surface of the container at every point.
3. With the help of a suitable diagram, describe a method for measuring fluid pressure at any point inside a fluid.
4. State Pascal's law. How will you experimentally verify this law ?
5. State Pascal's law. Explain the working of hydraulic lift. **[Himachal 06, 07 ; Chandigarh 08]**
6. Show that the pressure exerted by a liquid column is proportional to its height. **[Manipur 98]**
7. Explain hydrostatic paradox with a suitable example.
8. What is atmospheric pressure ? How is atmospheric pressure measured with the help of a mercury barometer ?
9. What is meant by torr ? Calculate the height of the atmosphere. **[Himachal 07]**
10. Describe how an open tube manometer is used to measure the pressure of a gas. Distinguish between absolute pressure and gauge pressure.
11. Pressure of a gas in a closed cylinder is expressed in the following way : $P = P_a + h\rho g$.
    Identify the expressions for :
    (*a*) Absolute pressure of the gas.
    (*b*) Gauge pressure of the gas. **[Central Schools 08, 09]**
12. Define buoyancy and centre of buoyancy.
13. State and prove Archimedes' principle. **[Himachal 06, 07 ; Chandigarh 04]**
14. Explain the laws of floation with all possibilities of a body in a liquid. **[Himachal 06]**
15. State the conditions for the equilibrium of floating bodies. Also discuss the stability of a floating body.
16. Define viscosity. Describe the cause of viscosity.
17. Define coefficient of viscosity. State and define its S.I. unit.
18. Write two factors affecting viscosity. Which one is more viscous : pure water or saline water ? **[Delhi 12]**
19. State Poiseuille's formula. Deduce it on the basis of dimensional considerations.
20. State Stokes' Law. Given the numerical constant in Stokes' law as 6π, obtain this law from the definition of viscosity. **[Central Schools 03]**
21. Explain how does a body attain a terminal velocity when it is dropped from rest in a viscous medium.
22. State Stokes' law. Derive an expression for the terminal velocity of a sphere falling through a viscous fluid. **[Delhi 97 ; Himachal 05]**
23. By using Stokes' law, derive an expression of terminal velocity. On what factors does it depend ? **[Chandigarh 08]**
24. Give an example for a force proportional to velocity. Prove that terminal velocity of a solid object moving in viscous medium is directly proportional to its size and inversely proportional to the viscosity of the medium. **[Central Schools 03]**
25. Distinguish between streamline and turbulent flows.
26. What do you mean by a streamline and tube of flow ? Give two properties of streamlines.
27. What is laminar flow of a liquid ? Draw velocity profiles for the laminar flow of viscous and non-viscous liquids.
28. What is Reynold's number ? What is its importance ? **[Central Schools 12]**

29. Show that the Reynold's number represents the ratio of the inertial force per unit area to the viscous force per unit area.
30. State and prove the equation of continuity. [Himachal 05]
31. State and derive Torricelli's law of efflux.
32. What is a venturi meter ? Explain its construction and working.
33. If a ball is thrown and given a spin, then the path of the ball is more curved than in a usual spin free path. Explain.
34. On the basis of Bernoulli's principle, explain the lift of an aircraft wing.
35. Define angle of contact and surface energy. [Delhi 95]
36. Define surface tension. Derive a relation between surface tension and surface energy. What is the unit of surface tension ? [Delhi 98 ; Himachal 05]
37. How is the surface tension of a liquid explained on the basis of intermolecular forces ? From where the energy comes when a liquid rises against gravity in a capillary tube ? [Manipur 97 ; Himachal 05]
38. Small mercury drops are spherical and larger ones tend to flattened. Explain.
39. Describe a simple experiment for measuring the surface tension of a liquid.
40. Show that a pressure difference exists between the two sides of a curved liquid surface.
41. Derive an expression for the excess pressure inside a liquid drop. [Himachal 05]
42. Derive an expression for the excess of pressure inside a soap bubble. [Delhi 99 ; Himachal 05]
43. Derive excess of pressure inside an air bubble. [Chandigarh 08]
44. State Stokes' law. Derive this law by the method of dimensions. [Delhi 98]
45. What is capillarity ? Derive an expression for the height to which a liquid rises in a capillary tube of radius r. [Chandigarh 07 ; Delhi 02 ; Himachal 03, 05C, 07]
46. Describe the cleansing action of detergents.
47. Define terminal velocity. Derive an expression for it. [Himachal 07]
48. A liquid is in streamlined flow through a pipe of non-uniform cross-section. Prove the sum of its kinetic energy, pressure energy and potential energy per unit volume remains constant. [Delhi 11]

Answers

1. Refer answer to Q. 4 on page 10.1 and Q. 6 on page 10.2.
2. Refer answer to Q. 5 on page 10.2.
3. Refer answer to Q. 7 on page 10.2.
4. Refer answer to Q. 12 on page 10.4.
5. Refer answer to Q. 13 on page 10.5.
6. Refer answer to Q. 15 on page 10.7.
7. Refer answer to Q. 17 on page 10.8.
8. Refer answer to Q. 18 on page 10.8 and Q. 19 on page 10.9.
9. Refer answer to Q. 21 on page 10.9.
10. Refer answer to Q. 20 on page 10.9.
11. (*a*) Total pressure of the gas is the absolute pressure. It is $P = P_a + h\rho g$.

    (*b*) The difference between the absolute pressure and the atomospheric pressure is the gauge pressure. It is : $P_g = P - P_a = h\rho g$.
12. Refer answer to Q. 24 on page 10.14.
13. Refer answer to Q. 25 on page 10.14.
14. Refer answer to Q. 26 on page 10.14
15. Refer answer to Q. 28 on page 10.15.
16. Refer answer to Q. 29 on page 10.20.
17. Refer answer to Q. 31 on page 10.21.
18. The viscosity of a liquid depends on its nature and temperature. Saline water is more viscous than pure water.
19. Refer answer to Q. 35 on page 10.24.
20. Refer answer to Q. 37 on page 10.28.
21. Refer answer to Q. 38 on page 10.28.
22. Refer answers to Q. 37 and Q. 38 on page 10.28.
23. Refer answer to Q. 38 on page 10.28.
24. The backward viscous force acting on a small spherical body moving through a viscous medium is proportional to its velocity.

    For expression of terminal velocity, refer answer to Q. 38 on page 10.28.
25. Refer answer to Q. 39 on page 10.32.
26. Refer answer to Q. 39 on page 10.32.
27. Refer answer to Q. 40 on page 10.33.
28. Refer answer to Q. 42 on page 10.33.
29. Refer answer to Q. 43 on page 10.34.
30. Refer answer to Q. 45 on page 10.35.
31. Refer answer to Q. 49 on page 10.38.
32. Refer answer to Q. 50 on page 10.38.
33. Refer answer to Q. 52 on page 10.39.
34. Refer answer to Q. 53 on page 10.39.
35. Refer to points 46 and 48 of Glimpses.
36. Refer answer to Q. 58 on page 10.47 and Q. 61 on page 10.49.

37. Refer answer to Q. 59 on page 10.47. The surface energy of the liquid is used during its capillary rise.

38. Refer answer to Q. 60 (*iv*) on page 10.48.

39. Refer answer to Q. 62 on page 10.49.

40. Refer answer to Q. 63 on page 10.53.

41. Refer answer to Q. 64 on page 10.53.

42. Refer answer to Q. 65 on page 10.54.

43. Refer answer to Q.66 on page 10.54.

44. Refer answer to Q.37 on page 10.28.

45. Refer answer to Q. 69 on page 10.57 and Q. 70 on page 10.58.

46. Refer answer to Q. 73 on page 10.62.

47. Refer answer to Q. 38 on page 10.28.

48. Refer answer to Q. 38 on page 10.28.

Type C : Long Answer Questions **5 Marks Each**

1. State and prove the Pascal's law of transmission of fluid pressure. **[Himachal 05C, 09C]**

2. Give the principle and explain the working of hydraulic brakes with a suitable diagram. **[Himachal 09C]**

3. State Pascal's law. Discuss its two practical applications. **[Himachal 05]**

4. Discuss the variation of fluid pressure with depth. Hence explain how is Pascal's law affected in the presence of gravity.

5. Define coefficient of viscosity and give its SI unit. On what factors does the terminal velocity of a spherical ball falling through a viscous liquid depend ? Derive the formula

$$v_t = \frac{2r^2 g}{9\eta}(\rho - \rho')$$

where the symbols have their usual meanings. **[Delhi 02, 03C]**

6. Define terminal velocity. Show that the terminal velocity v of a sphere of radius r, density ρ falling vertically through a viscous fluid of density σ and coefficient of viscosity η is given by $v = \frac{2}{9}\frac{(\rho-\sigma)r^2 g}{\eta}$.

Use this formula to explain the observed rise of air bubbles in a liquid. **[Himachal 06]**

7. (*a*) Define streamline.

(*b*) Write any two properties of streamlines.

(*c*) Draw streamlines for a clockwise spinning sphere.

(*d*) Derive equation of continuity. **[Central Schools 08]**

8. What is meant by the term coefficient of viscosity ? State Stokes' law. Define terminal velocity and find an expression for the terminal velocity in case of a sphere falling through a viscous liquid such as glycerine. **[Himachal 05C ; Delhi 03]**

9. State and prove Bernoulli's principle for the flow of non-viscous, incompressible liquid in streamlined flow. Give its limitations. **[Himachal 05 ; Delhi 09, 12]**

10. State Bernoulli's theorem. With the help of suitable diagram, establish Bernoulli's equation for liquid flow. Explain the lifting of aeroplane by it. **[Central Schools 12]**

11. (*i*) State and prove Bernoulli's theorem.

(*ii*) A cylindrical vessel of uniform cross-section contains liquid upto the height 'H'. At a depth 'h' = $H/2$ below the free surface of the liquid there is an orifice. Using Bernoulli's theorem, find the velocity of efflux of liquid. **[Delhi 05]**

12. What is meant by Streamline flow ? State the equation of Continuity. Write the properties of an ideal fluid. Establish Bernoulli's equation as applied to an ideal fuel. **[Delhi 03]**

13. State Bernoulli's theorem. Prove that the total energy possessed by a flowing ideal liquid is conserved, stating assumptions used. **[Himachal 04]**

14. (*a*) State Bernoulli's equation.

(*b*) Name the physical quantity corresponding to each term of this equation.

(*c*) What type of liquid flow obeys this equation ?

(*d*) Show that this equation is same as the equation due to Pascal's law in the presence of gravity if a liquid or gas is at rest. **[Central Schools 08]**

15. (*a*) Explain why sometimes the light roofs of thatched houses are blown off during a storm.

(*b*) Derive Stokes' law dimensionally. **[Central Schools 09]**

16. Define surface tension and surface energy. Write units and dimensions of surface tension. Also prove that surface energy is numerically equal to the surface tension. **[Delhi 08]**

17. Show that there is always an excess pressure on the concave side of the meniscus of a liquid. Obtain expression for the excess pressure inside (*i*) a liquid drop, (*ii*) liquid bubble (*iii*) air bubble inside a liquid. **[Himachal 04]**

18. (*a*) How do the insects run on the surface of water ?

(*b*) Derive an expression for excess pressure inside a soap bubble.

(*c*) Define the term surface energy. Write down its dimensional formula and units. **[Central Schools 08]**

19. (*i*) What is the phenomenon of capillarity ? Derive an expression for the rise of liquid in a capillary tube.

(*ii*) What will happen if the length of the capillary tube is smaller than the height to which the liquid rises ? Explain briefly. **[Delhi 97, 05]**

20. (*a*) Derive an expression for the rise of liquid in capillary tube of uniform diameter and sufficient length.

(*b*) A liquid drop of diameter D breaks up into 27 tiny drops. Find the resulting change in energy. Take surface tension of the liquid as σ. **[Central Schools 09]**

21. What is capillarity ? Derive an expression for the height to which the liquid rises in a capillary tube of radius r with angle of contact θ. Give two examples of capillarity from daily life. **[Central Schools 07, 12]**

22. Explain why

(*a*) A balloon filled with liquid helium does not rise in air indefinitely but halts after a certain height. **[Central Schools 07]**

(*b*) The angle of contact of mercury with glass is obtuse while that of water with glass is acute.

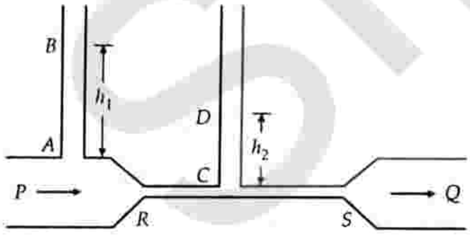
(*c*) A drop of liquid under no external force is always spherical in shape. **[Delhi 03C]**

23. (*a*) Explain the principle and working of Hydraulic lift with the help of a schematic diagram.

(*b*) To keep a piece of paper horizontal, you should blow over, not under it. Why ? **[Central Schools 07]**

24 (*a*) State Pascal's law of fluid pressure with a suitable diagram, explain how is Pascal's law applied in a hydraulic lift ?

(*b*) As shown in the figure, water flows from P to Q. Explain why is height h_1 of column AB of water greater than height h_2 of column CD of water ? **[Delhi 10]**



Answers

1. Refer answer to Q. 11 on page 10.4.

2. Refer answer to Q. 14 on page 10.5.

3. Refer answer to Q. 11 on page 10.4 and Q. 13 and Q. 14 on page 10.5.

4. Refer answer to Q. 16 on page 10.7.

5. Refer answer to Q. 38 on page 10.28.

6. Refer answer to Q. 38 on page 10.28.

7. (*a*) Refer answer to Q.39 on page 10.32.

(*b*) Refer answer to Q.39 on page 10.32.

(*c*) See Fig. 10.37 (*a*) on page 10.39.

(*d*) Refer answer to Q.45 on page 10.35.

8. Refer answer to Q. 36 and Q. 38 on page 10.28.

9. Refer answer to Q. 48 on page 10.36.

10. Refer answer to Q. 48 on page 10.36 and Q. 53 on page 10.39.

11. (*i*) Refer answer to Q. 48 on page 10.36.

(*ii*) Applying Bernoulli's theorem,

$$\frac{p}{\rho} + 0 + gH = \frac{p}{\rho} + \frac{1}{2}v^2 + g(H-h)$$

or $gH = \frac{1}{2}v^2 + g(H - H/2)$ or $v = \sqrt{gH} = \sqrt{2gh}$.

12. Refer to points 28, 33 and 35 of Glimpses.

13. Refer answer to Q. 48 on page 10.36.

14. (*a*) Refer answer to Q. 48 on page 10.36.

$$P + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}.$$

(*b*) P = Pressure energy per unit volume.

$\frac{1}{2}\rho v^2$ = Kinetic energy per unit volume

ρgh = Potential energy per unit volume

(*c*) The liquid flow must be streamlined and irrotational.

(*d*) When the liquid is at rest, $v = 0$.

$P + \rho gh$ = constant. This is Pascal's law.

15. (*a*) Refer to solution of Problem 75 on page 10.67.

(*b*) Refer answer to Q.36 on page 10.28.

16. Refer answer to Q. 58 on page 10.47 and Q. 61 on page 10.49.

17. Refer answers to Q. 64, 65 and 66 on page 10.53 and 10.54.

18. (*a*) Refer to the solution of Problem 98 on page 10.69.

(*b*) Refer answer to Q.65 on page 10.54.

(*c*) Refer answer to Q.61 on page 10.49.

19. Refer answer to Q. 70 and Q. 71 on page 10.58.

20. (*a*) Refer answer to Q.70 on page 10.58.

(*b*) Refer to the solution of Ex. 75 on page 10.58.

21. Refer answer to Q. 69 on page 10.57 and Q. 70 on page 10.58.

22. Refer to solutions of problems 1(*i*) on page 10.72 and NCERT Exercise 10.2(*i*) and (*v*) on page 10.79.

23. (*a*) Refer answer to Q.13 on page 10.5.

(*b*) Refer answer to NCERT Exercise 10.4(*i*) on page 10.79.

24. (*a*) Refer answer to Q. 13 on page 10.5.

(*b*) Cross-sectional area of part RS if the tube less than the remaining part, so speed of flow is large. By Bernoulli's theorem, liquid pressure must be small in part RS *i.e.*, $h_1 > h_2$.

Competition Section

Mechanical Properties of Fluids

GLIMPSES

1. **Fluid.** A fluid is a substance that can flow. The term fluid refers to both liquids and gases.
2. **Fluid statics.** The branch of physics that deals with the study of fluids at rest is called *fluid statics* or *hydrostatics*.
3. **Fluid dynamics.** The branch of physics that deals with the study of fluids in motion is called *fluid dynamics* or *hydrodynamics*.
4. **Thrust.** The total force exerted by a liquid on any surface in contact with it is called thrust. A liquid always exerts force perpendicular to the surface of the container at every point.
5. **Pressure.** The thrust exerted by a liquid per unit area of the surface in contact with it is known as pressure.

$$\text{Pressure} = \frac{\text{Thrust}}{\text{Area}} \quad \text{or} \quad P = \frac{F}{A}$$

Pressure is a scalar quantity.

6. **Units and dimensions of pressure.** The CGS unit of pressure is dyne cm^{-2} and its SI unit is Nm^{-2} which is also called pascal (Pa). The dimensional formula of pressure is $[ML^{-1}T^{-2}]$.
7. **Density.** The density of any material is defined as its mass per unit volume.

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} \quad \text{or} \quad \rho = \frac{M}{V}$$

Density is a positive scalar quantity.

8. **Units and dimensions of density.** The SI unit of density is kg m^{-3} and the CGS unit is g cm^{-3}. The dimensional formula of density is $[ML^{-3}]$.
9. **Specific gravity.** The relative density or *specific gravity* of a substance is defined as the ratio of the density of the substance to the density of water at 4°C.

$$\text{Specific gravity} = \frac{\text{Density of substance}}{\text{Density of water at 4°C}}.$$

Specific gravity is a dimensionless positive scalar quantity.

10. **Pascal's law.** It states that a change in pressure applied to an enclosed incompressible fluid is transmitted undiminished to every point of the fluid and the walls of the containing vessel. Or, the pressure exerted at any point on an enclosed liquid is transmitted equally in all directions.
11. **Hydraulic lift.** It is an application of Pascal's law. It is used to lift heavy objects.

According to Pascal's law,

Pressure applied on smaller piston

= Pressure transmitted to larger piston

$$P = \frac{f}{a} = \frac{F}{A} \quad \text{or} \quad F = P \times A = \frac{f}{a} \times A$$

As $A > a$, so $F > f$.

Thus hydraulic lift acts as a force multiplier.

12. **Hydraulic brakes.** The hydraulic brakes used in automobiles are based on Pascal's law of transmission of pressure in a liquid.
13. **Pressure exerted by a liquid.** A liquid column of height h and density ρ exerts a pressure given by

$$P = h \rho g$$

14. **Effect of gravity on fluid pressure.** The pressure in a fluid varies with depth h according to the expression

$$P = P_a + h \rho g$$

where ρ is the fluid density, assumed uniform.

15. **Hydrostatic paradox.** The pressure exerted by a liquid column depends only on the height of the liquid column and not on the shape of the containing vessel.

16. **Atmospheric pressure.** The pressure exerted by the atmosphere is called atmospheric pressure. At sea-level, we have

Atmospheric pressure

= Pressure exerted by 0.76 m of Hg

$= h\rho g = 0.76 \times 13.6 \times 10^3 \times 9.8 = 1.013 \times 10^5\ \text{Nm}^{-2}$

Other units used for atmospheric pressure are as follows :

$$1\,\text{atm} = 1.013 \times 10^6\ \text{dyne cm}^{-2}$$
$$= 1.013 \times 10^5\ \text{Nm}^{-2}\ (\text{or Pa})$$
$$1\ \text{bar} = 10^6\ \text{dyne cm}^{-2} = 10^5\ \text{Nm}^{-2}$$
$$1\ \text{millibar (m bar)} = 10^{-3}\ \text{bar} = 10^3\ \text{dyne cm}^{-2}$$
$$= 10^2\ \text{Nm}^{-2}$$
$$1\ \text{torr} = 1\ \text{mm Hg}$$
$$1\,\text{atm} = 101.3\ \text{kPa} = 1.013\ \text{bar} = 760\ \text{torr}$$

17. **Absolute pressure and gauge pressure.** The total or actual pressure P at a point is called absolute pressure. Gauge pressure is the difference between the actual pressure (absolute pressure) at a point and the atmospheric pressure. Thus

$$P_g = P - P_a \quad \text{or} \quad P = P_a + P_g$$

Absolute pressure = Atmospheric pressure + Gauge pressure.

18. **Buoyancy and centre of buoyancy.** The upward force acting on a body immersed in a fluid is called upthrust or buoyant force and the phenomenon is called buoyancy. The force of buoyancy acts though the centre of gravity of the displaced fluid which is called centre of buoyancy.

19. **Archemedes' principle.** It states that when a body is immersed partly or wholly in a fluid, it loses some weight. The loss in weight is equal to the weight of the fluid displaced.

Apparent weight of a body in a fluid

= True weight – Weight of fluid displaced

$$W_{app} = W - U = V\sigma g - V\rho g$$
$$= V\sigma g\left(1 - \frac{\rho}{\sigma}\right) = W\left(1 - \frac{\rho}{\sigma}\right)$$

where $W = V\sigma g$ is the weight of the body and σ its density.

20. **Law of floatation.** A body will float in a liquid if weight of the liquid displaced by the body is atleast equal to or greater than the weight of the body.

When a body just floats,

Weight of the body = Weight of liquid displaced

$$V\sigma g = V'\rho g \quad \text{or} \quad \frac{V'}{V} = \frac{\sigma}{\rho}$$

or $\dfrac{\text{Volume of the immersed part}}{\text{Total volume of the body}} = \dfrac{\text{Density of the body}}{\text{Density of liquid}}$

21. **Viscosity.** It is the property of a fluid due to which an opposing force comes into play whenever there is relative motion between its different layers.

22. **Newton's formula for viscous force.** The viscous drag between two parallel layers each of area A and having velocity gradient dv/dx is given by

$$F = -\eta A \frac{dv}{dx}$$

where η is the coefficient of viscosity of the liquid.

23. **Coefficient of viscosity.** It may be defined as the tangential viscous force required to maintain a unit velocity gradient between two liquid layers each of unit area. Its dimensional formula is $[ML^{-1}T^{-1}]$.

24. **Units of η.** The CGS unit of η is ***poise***. The coefficient of viscosity of a liquid is 1 poise if a tangential force of 1 dyne cm^{-2} of the surface is required to maintain a relative velocity of 1 cm s^{-1} between two layers of the liquid 1 cm apart.

$$1\ \text{poise} = 1\,\text{dyne s cm}^{-2} = 1\,\text{g cm}^{-1}\text{s}^{-1}.$$

The SI unit of η is ***decapoise***. The coefficient of viscosity of a liquid is 1 decapoise if a tangential force of 1 Nm^{-2} of the surface is required to maintain a relative velocity of 1 ms^{-1} between two layers of the liquid 1 m apart.

$$1\ \text{poisuille} = 1\,\text{decapoise} = 1\,\text{Nsm}^{-2}$$
$$= 1\ \text{Pa s} = 10\ \text{poise}.$$

25. **Poiseuille's formula.** The volume of a liquid flowing per second through a horizontal capillary tube of length l, radius r under a pressure difference p across its two ends is given by

$$Q = \frac{V}{t} = \frac{\pi p r^4}{8\eta l}.$$

26. **Stokes' law.** It states that the backward dragging force of viscosity acting on a spherical body of radius r moving with velocity v through a fluid of viscosity η is $\quad F = 6\pi\eta rv.$

27. **Terminal velocity.** It is the maximum constant velocity attained by a spherical body while falling through a viscous medium. The terminal velocity of a spherical body of density ρ and radius r moving through a fluid of density ρ' and viscosity η is given by $\quad v = \dfrac{2}{9}\dfrac{r^2}{\eta}(\rho - \rho')g$

28. **Streamline flow and turbulent flow.** It is the flow of liquid in which each particle of the liquid passing through a point travels along the same

path and with the same velocity as the preceding particle passing through the same point.

A liquid possesses streamline motion only when its velocity is less than a certain limiting value, called *critical velocity*. When the velocity of the liquid becomes greater than the critical velocity, the particles follow zig-zag path, such a disordered or irregular motion is called turbulent flow.

29. **Tube of flow.** A tube of flow is a bundle of streamlines having the same velocity of fluid element over any cross-section perpendicular to the direction of flow.

30. **Laminar flow.** The steady flow in which liquid moves in the form of layers is called laminar flow. The velocity of the layer varies from maximum at axis to zero for the layer at the wall of the tube.

31. **Critical velocity.** The critical velocity of a liquid is that limiting value of its velocity of flow upto which the flow is streamlined and above which the flow becomes turbulent. It is given by

$$v_c = \frac{k\eta}{\rho D}$$

32. **Reynold's number.** It is a dimensionless number which determines the nature of the flow of the liquid. For a liquid of viscosity η, density ρ and flowing through a pipe of diameter D, Reynold's number is given by

$$R_e = \frac{\rho\, v\, D}{\eta}$$

If $R_e < 2000$, the flow is laminar.

If $R_e > 3000$, the flow is turbulent.

If $2000 < R_e < 3000$, the flow is unstable. It may change from laminar to turbulent and vice-versa.

$$\text{Reynold's number} = \frac{\text{Inertial force per unit area}}{\text{Viscous force per unit area}}.$$

33. **Ideal fluid.** An ideal fluid is one which is non-viscous, incompressible, and its flow is steady and irrotational.

34. **Rate of flow.** The volume of a liquid flowing per second through a pipe of cross-section a with velocity v is given by

$$Q = \frac{V}{t} = av.$$

35. **Equation of continuity.** If there is no source or sink of the fluid along the length of the pipe, the mass of the fluid crossing any section of the pipe per second is always constant.

$$m = a_1\, v_1\, \rho_1 = a_2 v_2 \rho_2$$

It is called equation of continuity. For an incompressible liquid, $\rho_1 = \rho_2$, then

$$a_1 v_1 = a_2 v_2 \quad \text{or} \quad av = \text{constant}.$$

Thus during the streamlined flow of a non-viscous and incompressible fluid through a pipe of varying cross-section, the product of area of cross-section and the normal fluid velocity remains constant throughout the fluid flow.

36. **Bernoulli's principle.** It states that the sum of pressure energy, kinetic energy and potential energy per unit volume of an incompressible, non-viscous fluid in a streamlined, irrotational flow remains constant along a streamline. Thus

$$P + \frac{1}{2}\rho\, v^2 + \rho\, gh = \text{constant}$$

or

$$\frac{P}{\rho g} + h + \frac{1}{2}\frac{v^2}{g} = \text{constant}$$

The terms $\frac{P}{\rho g}$, h and $\frac{v^2}{2g}$ are called pressure head, gravitational head and velocity head respectively. For the horizontal flow of a liquid (h = constant), Bernoulli's equation takes the form

$$P + \frac{1}{2}\rho v^2 = \text{constant}$$

or

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2.$$

It indicates that velocity increases where pressure decreases and vice-versa.

37. **Torricelli's Law.** It states that the velocity of efflux *i.e.* the velocity with which the liquid flows out of an orifice (a narrow hole) is equal to that which a freely falling body would acquire in falling through a vertical distance equal to the depth of orifice below the free surface of liquid. Hence the velocity of efflux of a liquid through an orifice at depth h from the liquid surface will be

$$v = \sqrt{2gh}.$$

38. **Venturimeter.** It is a device used to measure the rate of flow of a liquid through a pipe. It is an application of Bernoulli's principle. It is also called flow meter or venturi tube. Volume of the liquid flowing out per second is given by

$$Q = a_1 a_2 \sqrt{\frac{2h\,\rho_m\, g}{\rho\,(a_1^2 - a_2^2)}}.$$

39. **Magnus effect.** The difference in lateral pressure, which causes a spinning ball to take a curved path which is convex towards the greater pressure side, is called magnus effect.

40. **Aerofoil.** It is a solid object shaped to provide an upward vertical force as it moves horizontally through air.

41. **Cohesive and adhesive forces.** The force of attraction between the molecules of the same substance is called cohesive force while the force of attraction between the molecules of two different substances is called adhesive force.

42. **Molecular range.** It is the maximum distance upto which a molecule can exert some measurable attraction on other molecules. The order of molecular range is 10^{-9} m in solids and liquids.

43. **Sphere of influence.** It is a sphere drawn with molecule as centre and molecular range as radius.

44. **Surface film.** A thin film of liquid near its surface and having thickness equal to the molecular range for that liquid is called surface film. The molecules present in the surface film possess additional potential energy.

45. **Surface Tension.** It is the property of a liquid by virtue of which, it behaves like an elastic stretched membrane with a tendency to contact so as to occupy a minimum surface area. It is measured as the force per unit length on an imaginary line drawn on the surface of liquid.

$$\text{Surface tension} = \frac{\text{Force}}{\text{Length}} \quad \text{or} \quad \sigma = \frac{F}{l}$$

Its SI unit is Nm^{-1} and CGS unit is dyne cm^{-1}.

46. **Surface energy.** The additional potential energy per unit area of the surface film as compared to the molecules in the interior is called the surface energy.

Surface energy

$$= \frac{\text{Work done in increasing the surface area}}{\text{Increase in surface area}}$$

Surface energy of a liquid is numerically equal to surface tension of the liquid.

47. **Excess pressure inside a drop and bubble.** There is excess of pressure on the concave side of a curved surface.

$$\text{Excess pressure inside a liquid drop} = \frac{2\sigma}{R}$$

(One free surface)

$$\text{Excess pressure inside a liquid bubble} = \frac{4\sigma}{R}$$

(Two free surfaces)

$$\text{Excess pressure inside an air bubble} = \frac{2\sigma}{R}$$

(One free surface)

where R is radius of the liquid drop, liquid bubble or the air bubble.

48. **Angle of contact.** The angle, which the tangent to the free surface of liquid at the point of contact makes with the wall of the containing vessel inside the liquid, is called angle of contact. For the liquids having concave meniscus, the angle of contact is acute and for the liquids having convex meniscus, the angle of contact is obtuse. The liquids, for which the angle of contact is acute, show a rise in the capillary tube; while those for which the angle of contact is obtuse, show a fall.

49. **Capillarity.** A tube of very fine bore is called a capillary tube. The phenomenon of rise or fall of a liquid in capillary tube is known as capillarity.

50. **Ascent formula.** When a capillary tube of radius r is dipped in a liquid of density ρ and surface tension σ, the liquid rises or falls through a distance,

$$h = \frac{2\sigma \cos\theta}{r \rho g}$$

51. **Rise of liquid in a tube of insufficient height.** The radius r of the capillary tube and radius of curvature R of the liquid meniscus are related by $r = R\cos\theta$. Therefore

$$h = \frac{2\sigma\cos\theta}{R\cos\theta \, . \, \rho g} = \frac{2\sigma}{R\rho g}$$

As, σ, ρ and g are constants, so

$$hR = \frac{2\sigma}{\rho g} = \text{a constant}$$

If $h' < h$, then the radius of curvature R increases to R' such that $hR = h'R'$. The liquid rises and spreads out to a new radius $R' = hR / h'$. But the liquid does not overflow.

IIT Entrance Exam

Multiple Choice Questions with One Correct Answer

1. A closed compartment containing gas is moving with some acceleration in horizontal direction. Neglect effect of gravity. Then the pressure in the compartment is

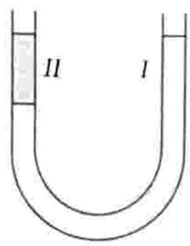
(a) same everywhere

(b) lower in the front side

(c) lower in the rear side

(d) lower in the upper side [IIT 99]

2. A U-tube of uniform cross- section is partially filled with a liquid I. Another liquid II which does not mix with liquid I is poured into one side. It is found that the liquid levels of the two sides of the tube are the same, while the level of liquid I has risen by 2 cm. If the specific gravity of liquid I is 1.1, the specific gravity of liquid II must be



(a) 1.12 (b) 1.1

(c) 1.05 (d) 1.0

[IIT 83]