CHAPTER

11

THERMAL PROPERTIES OF MATTER

11.1 HEAT

1. *Explain the concept of heat.*

Heat. *Heat is a form of energy which produces in us the sensation of hotness or coldness.* For example, if we touch a piece of ice, heat flows from our body towards ice and we feel cold. Similarly, when we stand near a fire, heat from the fire flows towards our body and we feel hot.

(*a*) ***Caloric theory of heat.*** According to this theory, *heat is an invisible, weightless and odourless fluid called* **caloric**. When some caloric is added to a body, its temperature rises and when some caloric is removed from a body, its temperature falls. However, this theory failed to explain the production of heat by friction. So this was replaced by dynamic theory of heat.

(*b*) ***Dynamic theory of heat.*** According to this theory, all substances (solids, liquids and gases) are made of molecules. These molecules are in a state of continuous random motion.

Depending on temperature and nature of the substance, the molecules may possess *three* types of motion :

(*i*) *Translatory motion.* That is, the motion in a straight line which is common in gases.

(*ii*) *Vibratory motion.* That is, the to and fro motion of the molecules about their mean positions. This is common in liquids and gases.

(*iii*) *Rotatory motion.* That is, the rotation of the molecules about their axis. This occurs usually at high temperature.

When a body is heated, all these molecular motions become fast. The kinetic energy of a molecule due to each type of motion increases. So we can regard *heat as an energy of molecular motion which is equal to the sum total of the kinetic energy possessed by the molecules of a body by virtue of their translational, vibrational and rotational motions.*

2. *What is meant by the statement that heat is the energy in transit ?*

Heat is the energy in transit. The energy associated with the configuration and random motion of the molecules in a body is called internal energy. The part of this internal energy that is transferred from one body to another due to temperature difference between them is called heat. Clearly, the word 'heat' is meaningful only as long as the energy is being transferred. The expressions like 'heat in a body' or 'heat of a body' are meaningless. So we define *heat as the energy in transit that flows from one body to another due to temperature difference between them.* Once heat is transferred to a body, it becomes a part of its internal energy.

11.2 UNITS OF HEAT

3. *What are the CGS and SI units of heat ? How are they related to one another ?*

CGS unit of heat. The CGS unit of heat is ***calorie*** (cal). *One calorie is defined as the heat energy required to* *raise the temperature of one gram of water through 1°C (from 14.5 to 15.5°C).*

SI unit of heat. Like all other forms of energy, the SI unit of heat is joule (J).

$$1 \text{ calorie} = 4.186 \text{ joule}$$

11.3 JOULE'S MECHANICAL EQUIVALENT OF HEAT

4. *State Joule's law of equivalence between work and heat. Hence define mechanical equivalent of heat.*

Joule's mechanical equivalent of heat. From experiments, Joule established a relation between the work done and heat produced. He showed that whenever a given amount of work (W) is converted into heat, always the same amount of heat (Q) is produced, thus

$$W \propto Q \quad \text{or} \quad W = JQ$$

or
$$J = \frac{W}{Q}$$

If $\quad Q = 1$, then $J = W$

The proportionality constant J is called *Joule's mechanical equivalent of heat. It may be defined as the amount of work that must be done to produce a unit quantity of heat.*

$$J = 4.186 \text{ J cal}^{-1} = 4.186 \times 10^7 \text{ erg cal}^{-1}.$$

Note J is not a physical quantity. It just a conversion factor.

11.4 TEMPERATURE

5. *Explain the concept of temperature.*

Temperature. *Temperature is the degree of hotness or coldness of a body.* When two bodies are placed in contact, the heat flows from the body at higher temperature to the body at lower temperature. Thus *temperature may be defined as the thermal state of a body which decides the direction of flow of heat energy from one body to another when they are placed in thermal contact with each other.*

Kinetic interpretation of temperature. *The temperature of a body is the measure of the average kinetic energy of its molecules.* When a body is heated, its molecules move faster. Their average K.E. increases. This increases the temperature of the body.

In thermodynamics the concept of temperature follows from the zeroth law of thermodynamics. It shall be discussed in the next chapter.

11.5 HEAT VS. TEMPERATURE

6. *Give some points of differences between heat and temperature.*

	Heat	Temperature
1.	Heat is a form of energy which produces in us the sensation of hotness or coldness.	Temperature is the degree of hotness or coldness of a body.
2.	It is a cause, when some heat is supplied to a body, its temperature increases.	It is an effect.
3.	It represents the total kinetic energy of the molecules of a body.	It represents the average kinetic energy possessed by the molecules of a body.
4.	Heat flows from high temperature side to low temperature side irrespective of the amounts of heat possessed by the bodies in contact.	Temperature decides the direction of flow of heat from one body to another.
5.	It is measured in cal, kcal or joule.	It is measured in °C, °F or K.

11.6 THERMOMETRY

7. *Define the term thermometry.*

Thermometry. *The branch of physics that deals with the measurement of temperature is called thermometry.*

8. *What is a thermometer ? ~~What is its principle ?~~*

Thermometer. *Any device used to measure the temperature of a body is called a thermometer.* It is named so because *thermo* is a Latin word which means *heat* and *meter* means a measuring device.

Principle of a thermometer. *A thermometer makes use of some measurable property (called thermometric property) of a substance which changes linearly with temperature.*

The thermometric properties of different substances and the corresponding thermometers are as follows :

(*i*) Length of a liquid column in a capillary (Mercury thermometer).

(*ii*) Pressure of a gas at constant volume (Constant volume gas thermometer).

(*iii*) Volume of a gas at constant pressure (Constant pressure gas thermometer).

(*iv*) Electrical resistance of a metal wire (Platinum resistance thermometer).

(*v*) Thermoelectrical e.m.f. (Thermoelectric or thermocouple thermometer).

(*vi*) Radiated power (Pyrometers).

9. *What are two fixed points on a temperature scale ?*

Fixed points on a temperature scale. To construct a temperature scale, two fixed points (two well-defined thermodynamic states) are chosen and are assigned two arbitrary numbers for their temperature. One number fixes the origin of the scale and the other fixes the size of the unit of the scale. The temperature at which pure ice melts at standard pressure (ice-liquid water equilibrium state) is usually chosen as the *lower fixed point*. The temperature at which pure water boils at atmospheric pressure (liquid water-vapour equilibrium state) is chosen as the *upper fixed point*.

10. *Describe the different types of temperature scales commonly used. Write the relation between temperatures on different scales.*

Thermometric scales. The range between the two fixed temperatures is called *fundamental interval*, which when divided into a suitable number of equal divisions forms a thermometric scale. The commonly used temperature scales are as follows :

(*i*) ***The Celsius scale.*** On this scale, the lower fixed point (ice point) is taken as 0°C and the upper fixed point (steam point) as 100°C. The interval between the two fixed points is divided into hundred equal parts (hence the name centigrade) and each part is called 1°C.

(*ii*) ***The Fahrenheit scale.*** On this scale, the lower fixed point is taken as 32°F and the upper fixed point as 212°F. The interval between them is divided into 180 equal parts and each part represents 1°F.

(*iii*) ***The Reaumer scale.*** On this scale, the lower fixed point is taken as 0°R and the upper fixed point as 80°R. The interval between them is divided into 80 equal parts and each part represents 1°R.

(*iv*) ***The Kelvin scale.*** On this scale, the lower fixed point is taken as 273.15 K and the upper fixed point as 373.15 K. The interval between the two fixed points is divided into 100 equal parts. The SI unit of temperature is kelvin (K).

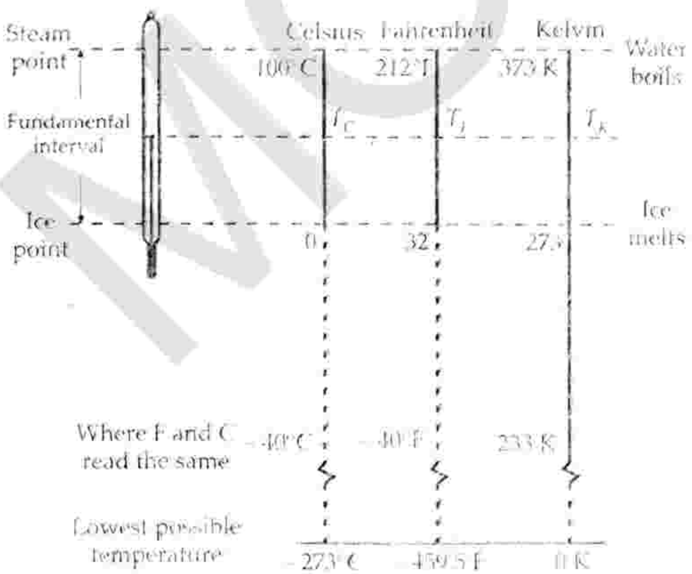


Fig. 11.1 Different temperature scales.

As the fundamental intervals of both Celsius and Kelvin scales have 100 divisions each, so the size of one division on Celsius scale is equal to the size of one division on Kelvin scale.

Conversion of temperature from one scale to another. To convert the temperature from one scale to another, the following relation is used :

$$\frac{\text{Temperature on one scale} - \text{Lower fixed point}}{\text{Upper fixed point} - \text{Lower fixed point}} = \frac{\text{Temperature on other scale} - \text{Lower fixed point}}{\text{Upper fixed point} - \text{Lower fixed point}}$$

As shown in the above figure, if the temperature of body is measured as T_C, T_F, T_R and T_K on Celsius, Fahrenheit, Reaumer and Kelvin scales respectively, then

$$\frac{T_C - 0}{100 - 0} = \frac{T_F - 32}{212 - 32} = \frac{T_R - 0}{80 - 0} = \frac{T_K - 273.15}{373.15 - 273.15}$$

or $$\frac{T_C - 0}{100} = \frac{T_F - 32}{180} = \frac{T_R - 0}{80} = \frac{T_K - 273.15}{100}.$$

11.7 ABSOLUTE SCALE OF TEMPERATURE

11. *What do you mean by absolute zero and absolute scale of temperature ?*

Absolute zero and absolute scale of temperature. According to Charles' law, if V_t and V_0 are the volumes at t°C and 0°C respectively of a given mass of a gas at constant pressure P, then

$$V_t = V_0\left(1 + \frac{t}{273.15}\right)$$

Clearly, the volume of the gas below 0°C will be less than V_0. For example, volume of the gas at $-t$°C is

$$V_{-t} = V_0\left(1 - \frac{t}{273.15}\right)$$

The decrease of temperature results in the decrease in volume of the gas. This has been shown graphically in Fig. 11.2 by plotting the volume of a given mass of a gas against temperature at constant pressure.

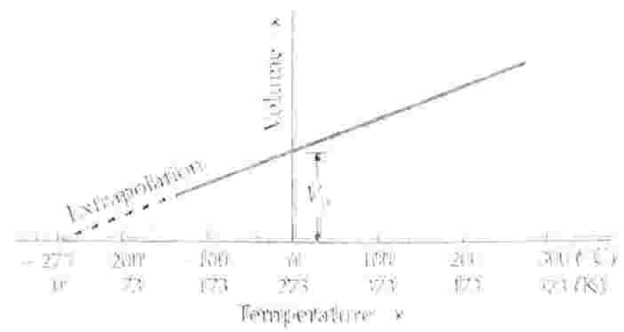


Fig. 11.2 Volume of a gas as a function of temperature.

The graph is a straight line. If we extrapolate the straight line, it meets the temperature axis at $-273.15°C$. Thus a gas occupies zero or no volume at $-273.15°C$. Clearly, a temperature below $-273.15°C$ is impossible because then the volume of the gas would be negative which is meaningless.

Moreover, according to the kinetic theory of gases, all molecular motion stops at $-273.15°C$. Hence *the lowest temperature of $-273.15°C$ at which a gas is supposed to have zero volume (and zero pressure) and at which entire molecular motion stops is called the absolute zero of temperature.* In practice, all gases condense to liquids and solids before this temperature is reached.

Lord Kelvin suggested new scale of temperature starting with −273.15 as its zero. This scale of temperature is known as Kelvin scale or absolute scale. The size of degree on Kelvin scale is same as that on Celsius scale. Therefore,

$$T\,(K) = t\,(°C) + 273.15$$

Thus ice point (°C) on absolute scale is 273.15 K and the steam point (100°C) is 373.15 K. The absolute scale of temperature is also called thermodynamic scale of temperature.

11.8 TRIPLE POINT OF WATER

12. *What is meant by triple point of water ? What is the advantage of taking triple point of water as the fixed point for a temperature scale ?*

Triple point of water. *The triple point of water is the state at which the three phases of water namely ice, liquid water and water vapour are equally stable and co-exist in equilibrium. It is unique because it occurs at a specific temperature of 273.16 K and a specific pressure of 0.46 cm of Hg column.* Thus for water,

$$P_{tr} = 0.46 \text{ cm of Hg}$$
$$T_{tr} = 273.16 \text{ K} \text{ or } 0.01°C$$

In modern thermometry, the triple point of water is chosen to be one of the fixed points. As it is characterised by a unique temperature and pressure, so it is preferred over the conventional fixed points namely the melting point of ice and boiling point of water. The melting point of ice and boiling point of water both change with pressure. Moreover, the presence of impurities changes their values. But the triple point of water is independent of the external factors.

In the absolute Kelvin scale, the triple point of water is assigned the value 273.16 K. The absolute zero is taken as the other fixed point on this scale.

11.9 CONSTANT VOLUME GAS THERMOMETER*

13. *Describe the construction and working of a constant volume gas thermometer. Give its advantages and disadvantages.*

Constant volume gas thermometer. A gas thermometer is an ideal thermometer because the increase of volume or pressure of a gas with temperature is independent of the nature of the gas.

Principle. *If the volume is kept constant, the pressure of a given mass of a gas increases linearly with the rise of temperature.* This is called Gay Lussac's law and is illustrated graphically in Fig. 11.3.

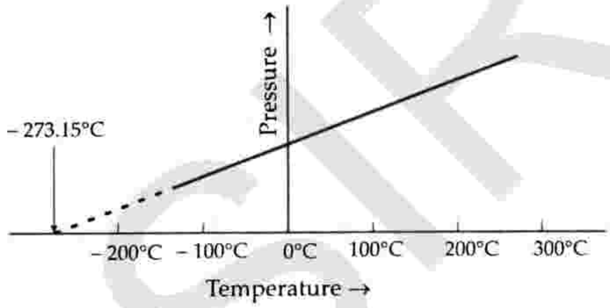


Fig. 11.3 Pressure versus temperature of a low density gas kept at constant volume.

Construction. Fig. 11.4 shows a schematic diagram of a constant volume gas thermometer. A fixed mass of gas is enclosed in a bulb *A* made of glass, quartz or platinum depending on the temperature range required to be measured. The bulb *A* is connected to a capillary *BC* which is connected to a manometer *CD* containing mercury. The end *D* of the manometer is open to atmosphere. The mercury of the manometer is connected to a mercury reservoir through a rubber tube. The volume of the gas is kept constant by raising or lowering the mercury reservoir *F* until the mercury level in the left part of the manometer coincides with *C*. A vertical scale *E* measures the difference *h* of mercury levels in the manometer tube. The pressure of the gas is then the atmospheric pressure plus the pressure due to the mercury column of height *h*.

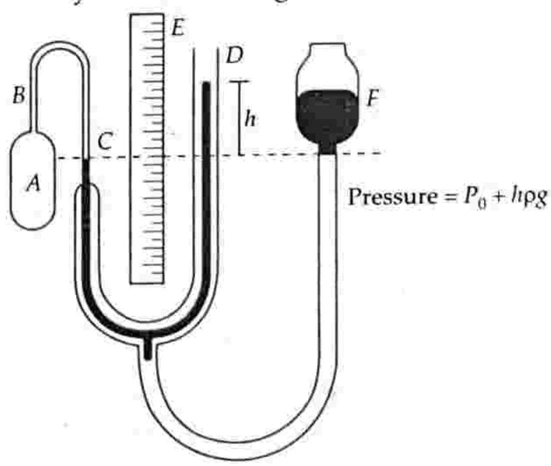


Fig. 11.4 Constant volume gas thermometer.

Working. The bulb is first immersed in the system (*e.g.*, boiling oil) whose temperature is to be determined.

The pressure P of the gas is measured. Next, the bulb is surrounded by water at the triple point, and the level of mercury is again brought to C in the left side of the manometer. The pressure P_{tr} of the gas is measured. According to Gay Lussac's law :

$$\frac{P_{tr}}{T_{tr}} = \frac{P}{T} \quad \text{or} \quad T = T_{tr}\left(\frac{P}{P_{tr}}\right)$$

But $T_{tr} = 273.16$ K

$$\therefore \quad T = 273.16\left(\frac{P}{P_{tr}}\right)$$

Advantages :

(*i*) As the expansion of the gas is large, so the gas thermometers are very sensitive.

(*ii*) The gases expand uniformly and regularly over a wide range of temperature.

(*iii*) The expansion coefficient of all gases is nearly the same, so thermometers using different gases give same reading.

(*iv*) Gas thermometers can be used for a wide range of temperature. With helium gas low temperature of about $-270°C$ and using nitrogen gas high temperature of about $1600°C$ can be measured.

Disadvantages :

(*i*) The correction has to be applied against the increase in the volume of the glass bulb.

(*ii*) The gas in the bulb is not at the same temperature as the gas in the capillary tube. Correction has to be applied for this purpose.

(*iii*) The gas is not ideal.

(*iv*) The thermometer is large and hence inconvenient to use.

(*v*) It is not a direct reading thermometer.

(*vi*) It requires a great deal of time to know the unknown temperature.

11.10 IDEAL GAS TEMPERATURE*

14. *What do you mean by ideal gas temperature ? Does it depend on nature of the gas ?*

Ideal gas temperature. If P and P_{tr} are the pressures of a gas (at constant volume) at constant temperature T and at the triple point (273.16 K) respectively, then from Gay Lussac's law, we have

$$T = 273.16\left(\frac{P}{P_{tr}}\right)$$

The temperature defined by the above equation depends slightly on the nature of the gas and its pressure. But at low pressures and high temperatures the real gases approach the ideal gas behaviour. If the pressure in the bulb of the gas thermometer is taken to be smaller and smaller (*i.e.*, in the limit $P_{tr} \to 0$), all the different gas thermometers give the same value of temperature for a given system. So we define a temperature by the equation

$$T = \lim_{P_{tr} \to 0} 273.16\left(\frac{P}{P_{tr}}\right)$$

The temperature defined by the above equation is known as *ideal gas temperature* on the Kelvin scale and is independent of the nature of the gas. However, it depends on the properties of the gases in general. In thermodynamics, it is possible to define a truly universal *absolute temperature scale* which does not depend on any property of the thermometric substance and in which the unit of temperature is Kelvin (K). The ideal gas thermometer scale is found to be identical with this absolute scale.

11.11 OTHER THERMOMETERS

15. *What is a liquid thermometer ? What are the advantages of using mercury as a thermometric substance over other liquids ?*

Liquid thermometer. Its working is based on the fact that *liquids expand uniformly and regularly on being heated*. Mercury thermometer is the most common liquid thermometer. It consists of a glass capillary tube of uniform bore having a bulb at its one end. The bulb is filled with mercury and the tube is sealed at the top after taking out air from it. The ice point and steam point are respectively marked as its lower and upper fixed points respectively. The use of capillary tube results in an easily observable rise in the level of mercury even for of a small rise in temperature.

If l_0, l_{100} and l_t be the lengths of mercury column at 0°C, 100°C and an unknown temperature t°C respectively, then the unknown temperature measured by the mercury thermometer is

$$t = \frac{l_t - l_0}{l_{100} - l_0} \times 100°C$$

Advantages of using mercury as a thermometric substance as compared to other liquids :

(*i*) Mercury has a uniform coefficient of expansion over a wide range of temperature.

(*ii*) Mercury is opaque and bright, so its level can be seen easily in a glass tube.

(*iii*) It does not stick to the walls of the glass tube.

(*iv*) It is a good conductor of heat and so attains the temperature of the hot body quickly.

(*v*) It has low specific heat, it absorbs very small amount of heat from the body whose temperature is to be measured.

(*vi*) The range of mercury thermometer is quite large because of its low freezing point (– 39°C) and high boiling point (357°C).

(*vii*) Mercury is non-volatile.

16. *Describe the working principle of a platinum resistance thermometer.*

Platinum resistance thermometer. The electric resistance of a metal wire increases linearly with temperature as

$$R_t = R_0(1 + \alpha t)$$

where α is the temperature coefficient of resistance.

So electric resistance may be used as a thermometric property to define a temperature scale. A platinum wire is often used in this thermometer because platinum has high melting point and its α is constant and large. If R_0 and R_{100} denote the resistance of a platinum wire at ice point and steam point respectively, then temperature t_R of a body for which the corresponding resistance is R_t, is given by

$$t_R = \frac{R_t - R_0}{R_{100} - R_0} \times 100°C$$

Platinum resistance thermometer can be used in the temperature range – 170°C to 200°C. For measuring lower temperatures, a *germanium resistance thermometer* is used. Germanium is a semiconductor whose resistance increases with the decrease in temperature.

17. *Briefly describe the working principle of a thermoelectric thermometer.*

Thermoelectric thermometer. This thermometer is based on **Seebeck effect**. According to this effect, *when wires of two different metals (Bi – Sb or Cu – Fe) are joined to form a closed circuit called thermocouple and their junctions are maintained at different temperatures, an e.m.f. is produced and a current flows in the circuit.* If one junction is at 0°C (cold junction) and the other at t°C (hot junction), then the thermoelectric e.m.f. is given by

$$\mathcal{E} = at + bt^2$$

where a and b are constants for the given pair of metals. To measure an unknown temperature, each individual thermoelectric thermometer is first calibrated by drawing a curve between the temperature of hot junction and the e.m.f. generated.

For the linear part of the thermo e.m.f., the unknown temperature is given by

$$t_\varepsilon = \frac{\mathcal{E}_t - \mathcal{E}_0}{\mathcal{E}_{100} - \mathcal{E}_0} \times 100 \text{ degrees.}$$

The normal range of a thermoelectric thermometer is – 200°C to 1600°C.

Examples based on Measurement of Temperature

FORMULAE USED

1. If T_C, T_F, T_R and T are the temperatures of a body on Celsius, Fahrenheit, Reaumer and Kelvin scales respectively, then

$$\frac{T_C - 0}{100 - 0} = \frac{T_F - 32}{212 - 32} = \frac{T_R - 0}{80 - 0} = \frac{T - 273.15}{100}$$

or $$\frac{T_C}{5} = \frac{T_F - 32}{9} = \frac{T_R}{4} = \frac{T - 273.15}{5}$$

2. $T_C = \frac{5}{9}(T_F - 32)$, $T_F = \frac{9}{5}T_C + 32$

3. $T = T_C + 273.15$, $T_C = T - 273.15$

4. $T_F = \frac{9}{5}(T - 273.15) + 32 = \frac{9}{5}T - 459.67$

or $T = \frac{5}{9}T_F + 255.37$

5. For a constant volume air thermometer,

$$T = T_0 \times \frac{p}{p_0}$$

In terms of triple point of water, $T = T_{tr} \times \frac{p}{p_{tr}}$

6. For a platinum resistance thermometer, resistance of platinum at t°C, $R = R_0(1 + \alpha t)$

Temperature coefficient of resistance, $\alpha = \frac{R - R_0}{R_0 \times t}$

UNITS USED

Pressure is in pascal (Pa), resistance in ohm (Ω) and temperature coefficient of resistance (α) in °C^{-1}.

Example 1. *A faulty thermometer has its fixed points marked as 5° and 95°. Temperature of a body as measured by the faulty thermometer is 59°. Find the correct temperature of the body on Celsius scale.*

Solution. $$\frac{T_C - 0}{100 - 0} = \frac{\text{Temp. on faulty scale} - \text{Lower fixed point}}{\text{Upper fixed point} - \text{Lower fixed point}}$$

or $$\frac{T_C - 0}{100} = \frac{59 - 5}{95 - 5} = \frac{54}{90} \quad \text{or} \quad T_C = 60°C.$$

Example 2. *A thermometer has wrong calibration. It reads the melting point of ice – 10°C. It reads 60°C in place of 50°. Calculate the temperature of boiling point of water on this scale.*

Solution. Let

θ_1 = Lower fixed point on faulty thermometer

θ_2 = Reading on faulty thermometer

n = number of divisions between upper and lower fixed points

Now $\frac{C}{100} = \frac{\theta_2 - \theta_1}{n}$ ...(1)

In first case,

$$\theta_2 = -10°C, \qquad C = 0°C$$

$$\therefore \quad 0 = \frac{-10 - \theta_1}{n} \quad \text{or} \quad \theta_1 = -10°C$$

In second case,

$$\theta_2 = 60°C, \qquad C = 50°C$$

$$\therefore \quad \frac{50}{100} = \frac{60 - \theta_1}{n} \quad \text{or} \quad \frac{1}{2} = \frac{60 - (-10)}{n} = \frac{70}{n}$$

or $$n = 140$$

As boiling point of water on Celsius scale is 100°C, so putting $C = 100$ in equation (1), we get

$$\frac{100}{100} = \frac{\theta_2 - (-10)}{140} \quad \text{or} \quad \theta_2 = 130°C$$

$\therefore$ Boiling point of water on faulty thermometer

$$= \mathbf{130°C}$$

EXAMPLE 3. *At what temperature, do the readings of Celcius and Fahrenheit scales coincide ?* [Himachal 07]

Solution. Let $T_C = T_F = x$

As $$\frac{T_C - 0}{100} = \frac{T_F - 32}{180}$$

$$\therefore \quad \frac{x}{100} = \frac{x - 32}{180}$$

or $$180x = 100x - 3200 \quad \text{or} \quad 80x = -3200$$

or $$x = -40°$$

Hence $-40°C$ and $-40°F$ are identical temperatures.

EXAMPLE 4. *A constant volume gas thermometer using helium records a pressure of 20.0 kPa at the triple-point of water, and pressure of 14.3 kPa at the temperature of 'dry ice' (solid CO_2). What is the temperature of 'dry ice' ?*

Solution. Here $T_{tr} = 273.16$ K, $P_{tr} = 20.0$ k Pa,

$$P = 14.3 \text{ k Pa}$$

Temperature of dry ice,

$$T = \frac{P}{P_{tr}} \times T_{tr} = \frac{14.3 \times 273.16}{20} = \mathbf{195.30\ K.}$$

EXAMPLE 5. *A constant volume thermometer using helium gas records a pressure of 1.75×10^4 Pa at normal freezing point of water, and a pressure of 2.39×10^4 Pa at normal boiling point of water. Obtain from these observations the temperature of absolute zero on the Celsius scale.* [NCERT]

Solution. Let T_1 and T_2 be the absolute temperatures corresponding to normal freezing and normal boiling points (*i.e.*, under 1 atmospheric pressure) of water.

For a constant volume thermometer,

$$\frac{T_2}{T_1} = \frac{P_2}{P_1} = \frac{2.39 \times 10^4 \text{ Pa}}{1.75 \times 10^4 \text{ Pa}} = \frac{239}{175}$$

Now the temperature t_c on Celsius scale and temperature T on the Kelvin scale are related as

$$T = t_c + T_0$$

where T_0 is a constant. As the normal freezing point and normal boiling point of water are 0°C and 100°C respectively, so from the above equation, we get

$$T_1 = 0 + T_0 \quad \text{and} \quad T_2 = 100 + T_0$$

$$\therefore \quad \frac{T_0 + 100}{T_0} = \frac{239}{175} \quad \text{or} \quad \frac{100}{T_0} = \frac{239}{175} - 1 = \frac{64}{175}$$

or $$T_0 = \frac{100 \times 175}{64} = 273$$

Hence the absolute zero ($T = 0$) on the Celsius scale will be

$$t_c = T - T_0 = 0 - 273 = -273.$$

The exact value of absolute zero is -273.15.

EXAMPLE 6. *A plantinum wire has resistance of 10 Ω at 0°C and 20 Ω at 273°C. Find the value of coefficient of resistance.* [NCERT]

Solution. Here $R_0 = 10\ \Omega, \quad R = 20\ \Omega,$

$$\theta = 273° - 0° = 273°C$$

Temperature coefficient of resistance,

$$\alpha = \frac{R - R_0}{R_0 \times \theta} = \frac{20 - 10}{10 \times 273} = \frac{1}{273}\ °C^{-1}.$$

PROBLEMS FOR PRACTICE

1. A faulty thermometer reads 5°C in melting ice and 99°C in steam. Find the correct temperature in °F when the faulty thermometer reads 52°C. (Ans. 122°F)
2. An ungraduated thermometer of uniform bore is attached to a centimeter scale and is found to read 10.3 cm in melting ice, 26.8 cm in boiling water and 6.5 cm in freezing mixture. Calculate the temperature of the freezing mixture. (Ans. − 23.03°C)
3. Normal temperature of the human body is 98.4°F. Find the temperature on Celsius and absolute scale. (Ans. 36.88°F, 310.03 K)
4. What is the triple point of water on a Fahrenheit scale ? What is the absolute zero on this scale ? (Ans. 32.018°F, − 459°F)
5. When a thermometer is taken from the melting ice to a warm liquid, the mercury level rises to 2/5th of

the distance between the lower and the upper fixed points. Find the temperature of liquid in °C and K.

(**Ans.** 40°C, 313.15 K)

6. The pressure of air in the bulb of constant volume air thermometer is 75 cm of mercury at 0°C, 100 cm at 100°C and 80 cm at the room temperature. Calculate the room temperature. (**Ans.** 20°C)

7. At what temperature is the Fahrenheit scale reading equal to twice of Celsius scale reading ?

(**Ans.** 160° C or 320°F)

8. At what temperature is the Fahrenheit scale reading equal to half of Celsius scale reading ?

(**Ans.** – 24.6°C)

9. A constant volume gas thermometer using sulphur records a pressure of 2×10^4 Pa at the triple point of water and 2.87×10^4 Pa at temperature of melting sulphur. Calculate the melting point of sulphur.

(**Ans.** 190.4 K)

10. The resistance of a resistance thermometer at 19°C is 3.50 Ω and at 99°C is 3.66 Ω. At what temperature will its resistance be 4.30 Ω ? (**Ans.** 419°C)

HINTS

1. As the lowest and the highest points on the faulty thermometer are 5°C and 99°C, so

$$\frac{T_C - 5}{99 - 5} = \frac{T_F - 32}{180} \quad \text{or} \quad \frac{52 - 5}{94} = \frac{T_F - 32}{180}$$

or $\quad T_F = \mathbf{122°F}.$

2. Temperature reading of melting ice = 10.3 cm

Temperature reading of boiling water = 26.8 cm

Temperature reading of freezing mixture = 6.5 cm

Let T_C be the temperature of freezing mixture on Celsium scale. Then

$$\frac{6.5 - 10.3}{26.8 - 10.3} = \frac{T_C - 0}{100 - 0}$$

On solving, $T_C = \mathbf{-23.03°C}$.

3. $T_C = \frac{5}{9}(T_F - 32) = \frac{5}{9}(98.4 - 32) = \frac{5}{9} \times 66.4 = \mathbf{36.88°C}.$

$T = T_C + 273.15 = 36.88 + 273.15 = \mathbf{310.03\ K}.$

5. Let the distance between the lower and upper fixed points be 1 unit. Then

$$\frac{\frac{2}{5} - 0}{1 - 0} = \frac{T_C - 0}{100}$$

or $\quad T_C = \mathbf{40°C}$

and $\quad T = 40 + 273.15 = \mathbf{313.15\ K}$

6. $\frac{T_C - 0}{100 - 0} = \frac{P - P_0}{P_{100} - P_0} = \frac{80 - 75}{100 - 75} \quad \therefore \quad T_C = \mathbf{20°C}.$

7. $T_F = \frac{9}{5} T_C + 32$ and $T_F = 2\,T_C$

$\therefore\ 2\,T_C = \frac{9}{5} T_C + 32$ or $T_C = \mathbf{160°C}$ and $T_F = \mathbf{320°F}.$

8. $T_F = \frac{9}{5} T_C + 32$ and $T_F = \frac{1}{2} T_C$

$\therefore\ \frac{1}{2} T_C = \frac{9}{5} T_C + 32$ or $T_C = -\frac{320}{13} = \mathbf{-24.6°C}.$

10. Let R_0, R_1 and R_2 be the resistances at 0°, θ_1 °C and θ_2 °C and α be the temperature coefficient of resistance. Then

$$R_1 = R_0(1 + \alpha\,\theta_1), \quad R_2 = R_0(1 + \alpha\,\theta_2)$$

On dividing and solving, $\alpha = \frac{R_2 - R_1}{R_1\,\theta_2 - R_2\,\theta_1}$

But $\theta_1 = 19°C$, $R_1 = 3.50\,\Omega$, $\theta_2 = 99°C$, $R_2 = 3.66\,\Omega$

$$\therefore \quad \alpha = \frac{3.66 - 3.50}{3.50 \times 99 - 3.66 \times 19} = \frac{0.16}{276.96} \quad ...(i)$$

Suppose the resistance becomes 4.30 Ω at t °C. Then

$$\alpha = \frac{4.30 - 3.50}{3.50 \times t - 4.30 \times 19} = \frac{0.80}{3.50t - 81.70} \quad ...(ii)$$

From (*i*) and (*ii*),

$$\frac{0.16}{276.96} = \frac{0.80}{3.50t - 81.70}$$

or $\quad 3.50t - 81.70 = 5 \times 276.96$

or $\quad 3.50t = 1384.80 + 81.70 = 1466.50$

$\therefore \quad t = \mathbf{419°C}.$

11.12 THERMAL EXPANSION

18. *What is meant by thermal expansion of a body ? What are the different types of thermal expansion ?*

Thermal expansion. Almost all solids, liquids and gases expand on heating. *The increase in the size of a body when it is heated is called thermal expansion.*

Different types of thermal expansion :

(*i*) ***Linear expansion.*** *It is the increase in the length of a metal rod on heating.*

(*ii*) ***Superficial expansion.*** *It is the increase in the surface area of a metal sheet on heating.*

(*iii*) ***Cubical expansion.*** *It is the increase in the volume of block on heating.*

19. *Why do solids expand on heating ?*

Cause of thermal expansion. All solids consist of atoms and molecules. At any given temperature, these atoms and molecules are held at equilibrium distance by forces of attraction. When a solid is heated, the amplitude of vibration of its atoms and molecules increases. The average interatomic separation increases. This results in the thermal expansion of the solid.

20. *Define coefficient of linear expansion. Write an expression for it. Give its units.*

Coefficient of linear expansion. Suppose a solid rod of length l is heated through a temperature ΔT and its final (increased) length is l'. It is found from experiments that

(*i*) Increase in length $\propto$ rise in temperature

i.e., $l' - l \propto \Delta T$

(*ii*) Increase in length $\propto$ original length

i.e., $l' - l \propto l$

Combining the above two factors, we get

$$l' - l \propto l\,\Delta T \quad \text{or} \quad l' - l = \alpha\, l\,\Delta T$$

The proportionality constant α is called ***coefficient of linear expansion***. Its value depends on the nature of the solid. Clearly,

$$l' = l\,[1 + \alpha\,\Delta T] \quad \text{and} \quad \alpha = \frac{l' - l}{l\,\Delta T} = \frac{\Delta l}{l\,\Delta T}$$

or $$\alpha = \frac{\text{Increase in length}}{\text{Original length} \times \text{Rise in temperature}}$$

Hence *the coefficient of linear expansion of the material of a solid rod is defined as the increase in length per unit original length per degree rise in its temperature.*

The unit of α is $°C^{-1}$ or K^{-1}.

Table 11.1 *Coefficients of linear expansion for some substances*

Materials	α (10^{-5} K)
Aluminium	2.5
Brass	1.8
Iron	1.2
Copper	1.7
Silver	1.9
Gold	1.4
Glass (pyrex)	0.32
Lead	0.29

Table 11.1 gives the average values of α for some materials in the temperature range 0 – 100°C. Clearly, copper expands 5 times more than glass for the same rise in temperature. Generally, metals expand more and have comparatively high values of α.

21. *Define coefficient of superficial expansion and give its units.*

Coefficient of superficial expansion. Suppose a metal sheet of initial surface area S is heated through temperature ΔT and its final surface area becomes S'.

Then $S' - S \propto \Delta T$ and $S' - S \propto S$

$\therefore$ $S' - S \propto S\,\Delta T$ or $S' - S = \beta\, S\,\Delta T$

The proportionality constant β is called *coefficient of superficial expansion* and its value depends on the nature of the material. Clearly,

$$S' = S\,[1 + \beta\,\Delta T] \quad \text{and} \quad \beta = \frac{S' - S}{S\,\Delta T} = \frac{\Delta S}{S\,\Delta T}$$

or $$\beta = \frac{\text{Increase in surface area}}{\text{Original surface area} \times \text{Rise in temperature}}$$

Hence the *coefficient of superficial expansion of a metal sheet is defined as the increase in its surface area per unit original surface area per degree rise in its temperature.*

The unit of β is $°C^{-1}$ or K^{-1}.

22. *Define coefficient of cubical expansion. Write an expression for it. Give its units.*

Coefficient of cubical expansion. Suppose a solid block of initial volume V is heated through a temperature ΔT and its final volume is V'.

Then $V' - V \propto \Delta T$

and $V' - V \propto V$ or $V' - V = \gamma\, V\,\Delta T$

The proportionality constant γ is called the **coefficient of cubical expansion** which depends on the nature of the material of the solid. Clearly,

$$V' = V\,[1 + \gamma\,\Delta T] \quad \text{and} \quad \gamma = \frac{V' - V}{V\,\Delta T} = \frac{\Delta V}{V\,\Delta T}$$

or $$\gamma = \frac{\text{Increase in volume}}{\text{Original volume} \times \text{Rise in temperature}}$$

Hence *the coefficient of cubical expansion of a substance is defined as the increase in volume per unit original volume per degree rise in its temperature.*

The unit of γ is $°C^{-1}$ or K^{-1}.

23. *How does the coefficient of cubical expansion of a substance vary with temperature ? Draw γ versus T curve for copper.*

Variation of γ with temperature. For a given substance, γ varies with temperature. Fig. 11.5 shows

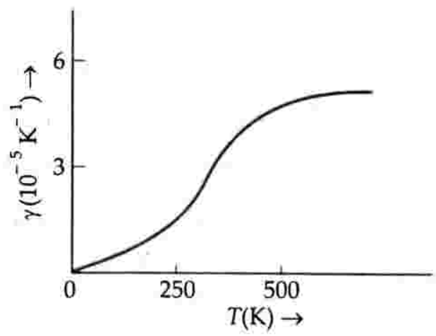


Fig. 11.5 Variation of γ of copper with temperature.

the variation of the coefficient of cubical expansion of copper with temperature. The value of γ first increases with temperature and then becomes constant at a high temperature (above 500 K).

Table 11.2 Coefficients of volume expansion for some substances

Materials	$\gamma\ (K^{-1})$	Materials	$\gamma\ (K^{-1})$
Aluminium	7×10^{-5}	Hard rubber	2.4×10^{-4}
Brass	6×10^{-5}	Invar	2×10^{-6}
Iron	3.55×10^{-5}	Mercury	18.2×10^{-5}
Paraffin	58.8×10^{-5}	Water	20.7×10^{-5}
Ordinary Glass	2.5×10^{-5}	Ethyl alcohol	110×10^{-5}
Pyrex Glass	1×10^{-5}		

Table 11.2 gives the average values of γ for some common substances in the temperature range 0 – 100°C. It can be noted that solids and liquids have small values of γ. The materials pyrex glass and invar (an alloy of iron and nickel) have still smaller values of γ. Ethyl alcohol has a higher value of γ than mercury and expands more than mercury for the same rise of temperature.

11.13 COEFFICIENT OF CUBICAL EXPANSION OF AN IDEAL GAS

24. *Show that the coefficient of cubical expansion of an ideal gas at constant pressure is equal to the reciprocal of its absolute temperature.*

Coefficient of cubical expansion of an ideal gas at constant pressure. For an ideal gas,

$$PV = nRT \quad ...(i)$$

At constant pressure,

$$P\,\Delta V = nR\,\Delta T \quad ...(ii)$$

[$\because$ n and R are constants]

Dividing (*ii*) by (*i*), we get

$$\frac{\Delta V}{V} = \frac{\Delta T}{T} \quad \text{or} \quad \frac{\Delta V}{V\,\Delta T} = \frac{1}{T} \quad \text{or} \quad \gamma = \frac{1}{T}$$

Hence for an ideal gas, the coefficient of volume expansion decreases with the increase in temperature.

11.14 RELATION BETWEEN α, β AND γ

25. *Derive the relation between α, β and γ.*

Relation between α, β and γ. Consider a cube of side l. Its original volume is

$$V = l^3$$

Suppose the cube is heated so that its temperature increases by ΔT. Its each side will become

$$l' = l + \Delta l = l + \alpha\, l\, \Delta T = l\,(1 + \alpha\, \Delta T)$$

The new volume of the cube will be

$$V' = l'^3 = l^3\,(1 + \alpha\,\Delta T)^3$$
$$= V\,(1 + 3\alpha\,\Delta T + 3\alpha^2\,\Delta T^2 + \alpha^3\,\Delta T^3)$$

As α is small, so the terms containing α^2 and α^3 can be neglected. Then

$$V' = V\,(1 + 3\alpha\,\Delta T)$$

By the definition of the coefficient of cubical expansion,

$$\gamma = \frac{\Delta V}{V\,\Delta T} = \frac{V' - V}{V\,\Delta T} = \frac{V\,(1 + 3\alpha\,\Delta T) - V}{V\,\Delta T} = 3\alpha$$

Similarly, it can be proved that

$$\beta = 2\alpha$$

Hence $\quad \dfrac{\alpha}{1} = \dfrac{\beta}{2} = \dfrac{\gamma}{3}$.

- The three coefficients of expansion α, β and γ are not constant for a given solid. Their values depend on the temperature range.
- For most of the solids, the value of α lies between 10^{-6} to $10^{-5}\ K^{-1}$ in the temperature range 0 to 100°C. The value of α is more for ionic solids than that for non-ionic solids.
- The coefficient of linear expansion of a solid rod is independent of the geometrical shape of its cross-section.
- The coefficient of volume expansion of solids and liquids is rather small, particularly very small for pyrex glass ($1 \times 10^{-5}\ K^{-1}$) and invar (Fe-Ni alloy with $\gamma = 2 \times 10^{-6}\ K^{-1}$).
- For an ideal gas γ varies inversely with temperature *i.e.*, $\gamma = 1/T$. At 0°C or 273 K, $\gamma = 1/273 = 3.7 \times 10^{-3}\ K^{-1}$, which is much larger than that for solids and liquids.
- Water contracts on heating between 0°C and 4°C. This is called *anomalous expansion of water*. It has the minimum volume and hence the maximum density (1000 kg m^{-3}) at 4°C. Silver iodide also contracts on heating between 80°C to 140°C.

11.15 MOLECULAR EXPLANATION OF THERMAL EXPANSION

26. *Explain thermal expansion of solids on the basis of the potential energy curve.*

Molecular explanation of thermal expansion. As shown in Fig. 11.6, the graph between the potential energy $U\,(r)$ of two neighbouring atoms in a crystalline solid and their interatomic separation r is an *asymmetric parabola*. The potential energy curve is asymmetric about its minimum because the attractive part of the potential energy rises slowly compared to the repulsive part.

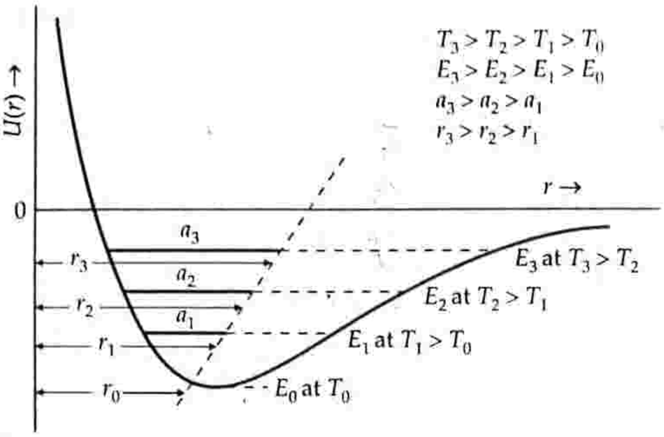


Fig. 11.6 P.E. $U(r)$ of two adjacent atoms in a crystalline solid versus interatomic separation r.

At the temperature $T_0 = 0$ K, the atoms remain at the equilibrium separation r_0 and their oscillation energy E_0 is minimum. As the temperature increases, the energy of the atoms increases and they start vibrating about their equilibrium positions with the interatomic separation oscillating between its minimum and maximum values : r_{min} and r_{max}. The average interatomic separation becomes

$$r = \frac{r_{min} + r_{max}}{2}$$

Clearly, as the temperature increases, the amplitude of vibration of the atoms increases. Due to the asymmetry of the P.E. curve, the equilibrium position shifts to the right on the curve (as shown by the dashed inclined line), *i.e.*, the average interatomic separation increases. It is thus *in consequence of this increase in the average interatomic separation with temperature that a solid expands when heated.*

11.16 PRACTICAL APPLICATIONS OF THERMAL EXPANSION

27. *Mention some applications of thermal expansion in daily life.*

Practical applications of thermal expansion :

(*i*) ***A small gap is left between the iron rails of railway tracks.*** The two rails are joined by fish plates. If no gap is left between the rails, the rails may bend due to expansion in summer and the train may get derailed.

(*ii*) ***Space is left between the girders used for supporting bridges.*** This allows their expansion during summer. Moreover, the ends of the girders are placed on metal rollers to allow the expansion and contraction to take place easily with the change of season.

(*iii*) ***The iron ring to be put on the rim of a cart wheel is always of slightly smaller diameter than that of the wheel.*** When the iron ring is heated to become red hot, it expands and slips on to the wheel easily. When it is cooled, it contracts and grips the wheel firmly.

(*iv*) ***Clock pendulums are made of invar.*** Invar is an alloy. It has extremely small temperature coefficient of expansion. So the length of invar pendulum does not change with the change of season and the clock gives almost correct time.

(*v*) ***A glass stopper jammed in the neck of a glass bottle can be removed by warming the neck of the bottle.*** When the neck of the bottle is slightly warmed, its mouth becomes slightly wider. The stopper becomes loose and comes out easily.

(*vi*) ***Only platinum wire is used for fusing into glass.*** This is because the coefficient of thermal expansion of platinum is almost the same as that of glass.

11.17 EXPANSION OF A LIQUID

28. *What do you mean by coefficients of apparent and real expansion of a liquid ? How are they related ?*

Expansion of a liquid. When a liquid is heated, its container also expands. The observed expansion of the liquid is called ***apparent expansion*** which is different from the ***real expansion*** of the liquid.

Coefficient of apparent expansion. *It is defined as the apparent increase in volume per unit original volume for 1°C rise in temperature.* The coefficient of apparent expansion of the liquid is given by

$$\gamma_a = \frac{\text{Apparent increase in volume}}{\text{Original volume} \times \text{Rise in temperature}}$$

Coefficient of real expansion. *It is defined as the real increase in volume per unit original volume for 1°C rise in temperature.* The coefficient of real expansion of the liquid is given by

$$\gamma_r = \frac{\text{Real increase in volume}}{\text{Original volume} \times \text{Rise in temperature}}$$

It can be proved that $\gamma_r = \gamma_a + \gamma_g$

where γ_g is coefficient of cubical expansion of glass (material) of the container.

11.18 VARIATION OF DENSITY WITH TEMPERATURE

29. *How does the density of a solid or a liquid vary with temperature ? Show that its variation with temperature is given by $\rho' = \rho(1 - \gamma\,\Delta T)$, where γ is the coefficient of cubical expansion.*

Variation of density with temperature. When a given mass of a solid or a liquid is heated, its volume increases and hence density decreases. If V and V' are the volumes and ρ and ρ' are the densities of a given mass M at temperatures T and $T + \Delta T$ respectively, then

$$V' = V(1 + \gamma\,\Delta T)$$

or $$\frac{M}{\rho'} = \frac{M}{\rho}(1 + \gamma\,\Delta T) \quad \text{or} \quad \rho' = \rho\,(1 + \gamma\,\Delta T)^{-1}$$

Expanding by Binomial theorem and neglecting the terms containing higher powers of $\gamma\,\Delta T$, we get

$$\rho' = \rho\,(1 - \gamma\,\Delta T)$$

Clearly, the density of a solid or a liquid decreases with the increase in temperature.

11.19 ANOMALOUS EXPANSION OF WATER

30. *Discuss anomalous expansion of water. Give its practical importance.*

Anomalous expansion of water. Almost all liquids expand on being heated but water behaves in a peculiar manner. When water at 0°C is heated, its volume decreases and, therefore, its density increases, until its temperature reaches 4°C. Above 4°C, the volume increases, and therefore the density decreases. ***Thus water at 4°C has the maximum density.***

Fig. 11.7(*a*) shows the variation of volume of 1 kg of water as the temperature increases from 0°C to 100°C. Fig. 11.7(*b*) shows the variation of density of water with temperature from 0° to 10°C.

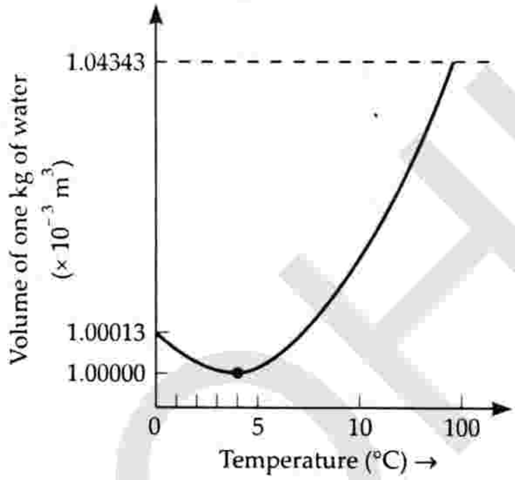


Fig. 11.7 (*a*) Thermal expansion of water.

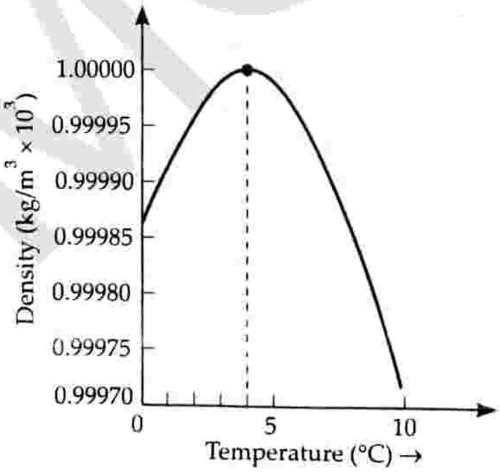


Fig. 11.7 (*b*) Variation of density of water with temperature

Practical importance of anomalous expansion of water. The anomalous expansion of water has a favourable effect on aquatic life. Since the density of water is maximum at 4°C, water at the bottom of the lakes remains at 4°C even if it freezes at the top surface. This allows marine animals to remain alive and move freely near the bottom. If water did not have this property, lakes and ponds would freeze from the bottom up, which would destroy the entire aquatic animal and plant life.

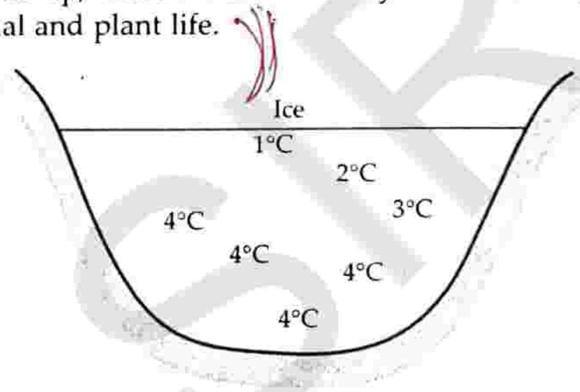


Fig. 11.8 Anomalous expansion of water helps aquatic life.

Examples based on Thermal Expansion

FORMULAE USED

1. Change in length, $l' - l = l\,\alpha\,(T' - T)$ or $\Delta l = l\,\alpha\,\Delta T$
2. Coefficient of linear expansion, $\alpha = \dfrac{\Delta l}{l \Delta T}$
3. Final length, $l' = l\,(1 + \alpha\,\Delta\,T)$
4. Change in surface area, $S' - S = S\beta\,(t' - t)$ or $\Delta S = S\beta\,\Delta\,T$
5. Coefficient of superficial expansion, $\beta = \dfrac{\Delta S}{S\,\Delta\,T}$
6. Final surface area, $S' = S(1 + \beta\,\Delta\,T)$
7. Change in volume, $V' - V = V\,\gamma\,(t' - t)$ or $\Delta V = V\,\gamma\,\Delta T$
8. Coefficient of cubical expansion, $\gamma = \dfrac{\Delta V}{V\,\Delta\,T}$
9. Final volume $V' = V\,(1 + \gamma\,\Delta\,T)$
10. Relation between α, β and γ

$$\frac{\alpha}{1} = \frac{\beta}{2} = \frac{\gamma}{3} \qquad \therefore \qquad \beta = 2\alpha \text{ and } \gamma = 3\alpha$$

11. Final density, $\rho' = \rho\,(1 - \gamma\,\Delta T)$.

UNITS USED

Lengths l, l' and $\Delta\,l$ are in cm. Surface areas S, S' and ΔS are in cm^2. Volumes V, V' and ΔV are in cm^3. Temperatures t, t' and ΔT are in °C and coefficients α, β and γ are in °C^{-1}.

EXAMPLE 7. *Show that the coefficient of area expansions, $(\Delta A/A)/\Delta T$ of a rectangular sheet of the solid is twice its linear expansivity, α.* **[NCERT]**

Solution. As shown in Fig. 11.9, consider a rectangular sheet of the solid material of length a and breadth b. Suppose its temperature increases by ΔT.

Increase in length a, $\Delta a = \alpha a \Delta T$

Increase in breadth b, $\Delta b = \alpha b \Delta T$

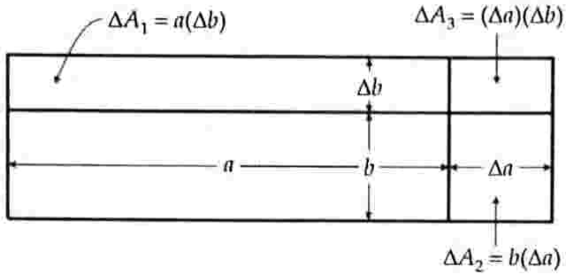


Fig. 11.9

Total increase in area of the sheet is

$$\Delta A = \Delta A_1 + \Delta A_2 + \Delta A_3$$

$$\Delta A = a\Delta b + b\Delta a + (\Delta a)(\Delta b)$$

$$= \alpha ab\,\Delta T + b\alpha\, a\Delta T + (\alpha)^2\, ab\,(\Delta T)^2$$

$$= \alpha ab\,\Delta T\,(2 + \alpha\Delta T) = \alpha A \Delta T\,(2 + \alpha\Delta T)$$

Since $\alpha \simeq 10^{-5}\ K^{-1}$, the product $\alpha\Delta T$ for fractional temperature is small in comparison with 2 and may be neglected. Therefore,

$$\beta = \left(\frac{\Delta A}{A}\right)\frac{1}{\Delta T} \simeq 2\alpha.$$

EXAMPLE 8. *Railway lines are laid with gaps to allow for expansion. If the gap between steel rails 66 m long be 3.63 cm at 10°C, then at what temperature will the lines just touch ? Coefficient of linear expansion of steel $= 11 \times 10^{-6}\ {}^\circ C^{-1}$.*

Solution. As the rails expand in both directions, so the gap between two rails is filled by the expansion of half length of each rail. Equivalently, we can take the expansion of one rail in one direction to fill the gap.

$\therefore \quad l = 66$ m, $\Delta l = 3.63 \times 10^{-2}$ m,

$\alpha = 11 \times 10^{-6}\ {}^\circ C^{-1}$

As $\quad \Delta l = l\,\alpha\,\Delta T$

$$\therefore \quad \Delta T = \frac{\Delta l}{l\,\alpha} = \frac{3.63 \times 10^{-2}}{66 \times 11 \times 10^{-6}} = 50°C$$

Final temperature

= Initial temperature + $\Delta T = 10 + 50 =$ **60°C.**

EXAMPLE 9. *A blacksmith fixes iron ring on the rim of the wooden wheel of a bullock cart. The diameters of the rim and the iron ring are 5.243 m and 5.231 m respectively at 27°C. To what temperature should the ring be heated so as to fit the rim of the wheel ?* **[NCERT]**

Solution. Here $T_1 = 27°C$, $l_1 = 5.231$ m, $l_2 = 5.243$ m

As $\quad l_2 - l_1 = \alpha l_1 (T_2 - T_1)$

$$\therefore \quad T_2 - T_1 = \frac{l_2 - l_1}{\alpha l_1} = \frac{5.243 - 5.231}{1.20 \times 10^{-5} \times 5.231} \approx 191°C$$

or $\quad T_2 = 191 + T_1 = 191 + 27 =$ **218°C.**

EXAMPLE 10. *A clock with an iron pendulum keeps correct time at 20°C. How much will it lose or gain if temperature changes to 40°C ? Coefficient of cubical expansion of iron $= 36 \times 10^{-6}\ {}^\circ C^{-1}$.* **[IIT]**

Solution. Time period of simple pendulum,

$$T_{20} = 2\ \text{s}$$

Let T_{40} be the time period at 40°C. If l_0, l_{20}, l_{40} be the lengths of the pendulum at 0°C, 20°C and 40°C respectively, then

$$l_{20} = l_0(1 + 20\,\alpha)$$

$$l_{40} = l_0(1 + 40\,\alpha)$$

$$T_{20} = 2\pi\sqrt{\frac{l_{20}}{g}} = 2\pi\sqrt{\frac{l_0(1+20\alpha)}{g}}$$

$$T_{40} = 2\pi\sqrt{\frac{l_{40}}{g}} = 2\pi\sqrt{\frac{l_0(1+40\,\alpha)}{g}}$$

$$\therefore \quad \frac{T_{40}}{T_{20}} = \sqrt{\frac{1+40\,\alpha}{1+20\,\alpha}} = (1+40\,\alpha)^{1/2}(1+20\,\alpha)^{-1/2}$$

$$= (1 + \tfrac{1}{2} \times 40\,\alpha)(1 - \tfrac{1}{2} \times 20\,\alpha)$$

[Using Binomial theorem]

$$= (1 + 20\,\alpha)(1 - 10\,\alpha) = 1 + 10\,\alpha$$

Fractional loss in time

$$= \frac{T_{40} - T_{20}}{T_{20}} = 10\,\alpha$$

$$= 10 \times 1.2 \times 10^{-5} = 1.2 \times 10^{-4}$$

As the temperature increases, time period also increases. The clock runs slow.

Time lost in 24 hours

$= 1.2 \times 10^{-4} \times 24 \times 3600 =$ **10.368 s.**

EXAMPLE 11. *A metal ball 0.1 m in radius is heated from 273 to 348 K. Calculate the increase in surface area of the ball. Given coefficient of superficial expansion $= 0.000034\ K^{-1}$.*

Solution. Here $r_{273} = 0.1$ m,

$$\Delta T = 348 - 273 = 75\ K, \quad \beta = 0.000034\ K^{-1}$$

$$S_{273} = 4\pi r_{273}^2 = 4\pi(0.1)^2 = \frac{4\pi}{100}\ m^2$$

Increase in surface area,

$$\Delta S = S_{273}\,\beta\,\Delta T = \frac{4\pi}{100} \times 0.000034 \times 75$$

$$= \mathbf{3.206 \times 10^{-4}\ m^2}.$$

EXAMPLE 12. On heating a glass block of 10,000 cm^3, from 25°C to 40°C, its volume increases by 4 cm^3. Calculate coefficient of linear expansion of glass.

Solution. Here $V = 10{,}000\ cm^3$,

$$\Delta T = (40 - 25) = 15°C,\ \Delta V = 4\ cm^3$$

The coefficient of cubical expansion is given by

$$\gamma = \frac{\Delta V}{V\,.\,\Delta T} = \frac{4}{10{,}000 \times 15} = 26.67 \times 10^{-6}\ °C^{-1}$$

∴ Coefficient of linear expansion,

$$\alpha = \frac{\gamma}{3} = \frac{26.67 \times 10^{-6}}{3} = \mathbf{8.89 \times 10^{-6}\ °C^{-1}}$$

EXAMPLE 13. If the volume of a block of metal changes by 0.12%, when it is heated through 20°C, what is the coefficient of linear expansion of metal ?

Solution. Here $\frac{\Delta V}{V} = 0.12\% = \frac{0.12}{100}$, $\Delta T = 20°C$

Now $\gamma = \frac{\Delta V}{V\,.\,\Delta T} = \frac{0.12}{100 \times 20} = 6.0 \times 10^{-5}\ °C^{-1}$

∴ $\alpha = \gamma / 3 = \mathbf{2.0 \times 10^{-5}\ °C^{-1}}$.

EXAMPLE 14. Density ρ, mass m and volume V are related as $\rho = m/V$. Prove that

$$\gamma = -\frac{1}{\rho}\frac{d\rho}{dT}.$$

Solution. Given $\rho = \frac{m}{V} = mV^{-1}$

Differentiating both sides w.r.t. temperature T, we get

$$\frac{d\rho}{dT} = -mV^{-2}\frac{dV}{dT} \qquad [\because m = \text{constant}]$$

$$= -\frac{m}{V}\cdot\frac{dV}{V\,dT} = -\rho\,\gamma \qquad \left[\because \frac{dV}{V\,.\,dT} = \gamma\right]$$

∴ $\gamma = -\frac{1}{\rho}\frac{d\rho}{dT}$.

✕ PROBLEMS FOR PRACTICE

1. How much the temperature of a brass rod should be increased so as to increase its length by 1% ? Given that α for brass = $0.00002°C^{-1}$. (**Ans.** 500°C)
2. A steel scale measures the length of a copper rod as 80 cm when both are at 20°C, the calibration temperature of the scale. What would the scale read for the length of the rod when both are at 40°C ? Given α for steel = $1.1 \times 10^{-5}\ °C^{-1}$ and α for copper = $1.7 \times 10^{-5}\ °C^{-1}$. (**Ans.** 80.0096 cm)
3. A steel metre scale is to be ruled so that the millimetre intervals are accurate to within about 5×10^{-5} mm at a certain temperature. What is the maximum temperature variation allowable during the ruling ? Given α for steel = $1.1 \times 10^{-5}\ °C^{-1}$. (**Ans.** 4.5°C)
4. A brass rod at 30°C is observed to be 1 metre long when measured by a steel scale which is correct at 0°C. Find the correct length of the rod at 0°C. Given α for steel = 0.00012 per °C and α for brass = 0.000019 per °C. (**Ans.** 99.98 cm)
5. A cylinder of diameter 1.0 cm at 30°C is to be slid into a hole in a steel plate. The hole has a diameter of 0.99970 cm at 30°C. To what temperature must the plate be heated ? For steel, α = $1.1 \times 10^{-5}\ °C^{-1}$. (**Ans.** 57.3°C)
6. What should be the lengths of steel and copper rods at 0°C that the length of steel rod is 5 cm longer than copper at all temperatures ? Given α for copper = $1.7 \times 10^{-5}\ °C^{-1}$ and α for steel = $1.1 \times 10^{-5}\ °C^{-1}$ **[IIT]** (**Ans.** 9.17cm, 14.17 cm)
7. A clock having a brass pendulum beats seconds at 30°C. How many seconds will it lose or gain per day when temperature falls to 10°C ? Given α for brass = $1.9 \times 10^{-5}\ °C^{-1}$. (**Ans.** gain of 16.42 s)
8. A steel wire 2 mm in diameter is stretched between two clamps, when its temperature is 40°C. Calculate the tension in the wire when its temperature falls to 30°C.

   Given α for steel = $1.1 \times 10^{-6}\ °C^{-1}$ and Y for steel = 21×10^{11} dyne cm^{-2}. (**Ans.** 7.26×10^6 dyne)
9. Calculate the force required to prevent a steel wire of 1 mm^2 cross-section from contracting when it cools from 60°C to 15°C, if Young's modulus for steel is $2 \times 10^{11}\ Nm^{-2}$ and its coefficient of linear expansion is $0.000011°C^{-1}$. (**Ans.** 99 N)
10. The design of some physical apparatus requires that there be a constant difference in length at any temperature between iron and copper cylinders laid side by side. What should be the length of the cylinders at 0°C for the difference in length to be 10 cm at all temperatures ? Given α for iron = $1.1 \times 10^{-5}\ °C^{-1}$ and for copper = $1.7 \times 10^{-5}\ °C^{-1}$.

    (**Ans.** Length of iron cylinder = 28.33 cm,
    Length of copper cylinder = 18.33 cm.)
11. An iron sphere has a radius of 10 cm at a temperature of 0°C. Calculate the change in the volume of the sphere, if it is heated to 100°C. Coefficient of linear expansion of iron = $11 \times 10^{-6}\ °C^{-1}$. (**Ans.** 13.8 cm^3)
12. The volume of a metal sphere is increased by 1% of its original volume when it is heated from 320 K to

522 K. Calculate the coefficients of linear, superficial and cubical expansion of the metal.

(**Ans.** $1.67 \times 10^{-5}\,°C^{-1}$, $3.34 \times 10^{-5}\,°C^{-1}$, $5 \times 10^{-5}\,°C^{-1}$)

13. The density of mercury is 13.6 g cm^{-3} at 0°C and its coefficient of cubical expansion is $1.82 \times 10^{-4}\,°C^{-1}$. Calculate the density of mercury at 50°C.

(**Ans.** 13.48 g cm^{-3})

14. Suppose that one early morning when the temperature is 10°C, a driver of an automobile gets his gasoline tank which is made of steel, filled with 75 litre of gasoline, which is also at 10°C. During the day, the temperature rises to 30°C. How much gasoline will overflow ?

Given α for steel $= 1.2 \times 10^{-5}\,°C^{-1}$ and γ for gasoline $= 9.5 \times 10^{-4}\,°C^{-1}$. (**Ans.** 1.37 litre)

15. A one litre flask contains some mercury. It is found that at different temperatures, the volume of air inside the flask remains the same. What is the volume of mercury in this flask ? Given α for glass $= 9 \times 10^{-6}\,°C^{-1}$ and γ for mercury $= 1.8 \times 10^{-4}\,°C^{-1}$.

[IIT] (**Ans.** 150 cm^3)

X. HINTS

2. 1 cm length of steel scale at 40°C

$= 1 + 1.1 \times 10^{-5} \times (40 - 20) = 1.00022$ cm

Length of copper rod at 40°C

$= 80 + 80 \times 1.7 \times 10^{-5} \times (40 - 20) = 80.0272$ cm

Number of divisions on steel scale at 40°

$$= \frac{80.0272}{1.00022} = \mathbf{80.0096\ cm.}$$

3. $$\Delta T = \frac{\Delta l}{\alpha\, l} = \frac{5 \times 10^{-5}\text{ mm}}{1.1 \times 10^{-5}\,°C^{-1} \times 1.0\text{ mm}} = \mathbf{4.5°C.}$$

4. As the scale is correct at 0°C, so each division of scale is 1 cm. At 30°C, each cm division becomes $(1 + 0.000012 \times 30)$ cm $= 1.00036$ cm.

$\therefore$ True length of steel scale at 30°C

$= 100 \times 1.00036 = 100.036$ cm

Length of brass rod at 30°C = 100.036 cm

If l be the length of brass rod at 0°C, then

$l\,(1 + 0.000019 \times 30) = 100.036$

or $$l = \frac{100.036}{1.00057} = \mathbf{99.98\ cm.}$$

6. Let l be the length of copper rod at 0°C, then length of steel rod at 0°C must be $l + 5$

$\therefore \quad l \times 1.7 \times 10^{-5} \times \Delta T = (l + 5) \times 1.1 \times 10^{-5} \times \Delta T$

$1.7\,l = 1.1\,l + 5.5$ or $0.6\,l = 5.5$

$\therefore$ Length of copper rod, $l = \mathbf{9.17\ cm}$

Length of steel rod $= 9.17 + 5 = \mathbf{14.17\ cm.}$

7. Here $T_{30} = 2$ s

$$l_{30} = l_0\,(1 + 30\,\alpha)$$
$$l_{10} = l_0\,(1 + 10\,\alpha)$$
$$T_{10} = 2\pi\sqrt{\frac{l_{10}}{g}} = 2\pi\sqrt{\frac{l_0\,(1 + \alpha \times 10)}{g}}$$
$$T_{30} = 2\pi\sqrt{\frac{l_{30}}{g}} = 2\pi\sqrt{\frac{l_0\,(1 + 30\,\alpha)}{g}}$$
$$\therefore \quad \frac{T_{10}}{T_{30}} = \sqrt{\frac{1 + 10\,\alpha}{1 + 30\,\alpha}} = (1 + 10\,\alpha)^{1/2}\,(1 + 30\,\alpha)^{-1/2}$$
$$= \left(1 + \frac{1}{2}\,10\alpha\right)\left[1 + 30\left(-\frac{1}{2}\right)\alpha\right]$$

[Using Binomial theorem]

$$= (1 + 5\alpha)(1 - 15\alpha) = (1 - 10\alpha)$$
$$\therefore \quad T_{10} = T_{30}\,(1 - 10\alpha)$$
$$\text{or} \quad T_{10} = 2\,(1 - 10 \times 1.9 \times 10^{-5})\text{ s}$$
$$= (2 - 3.8 \times 10^{-4})\text{ s}$$

As $T_{10} < T_{30}$, the clock gains in time when the temperature falls to 10°C.

For 2 seconds, gain in time $= T_{30} - T_{10} = 3.8 \times 10^{-4}$ s

For 1 day, gain in time

$$= \frac{3.8 \times 10^{-4} \times 24 \times 3600}{2}\text{ s} = \mathbf{16.42\ s.}$$

8. $\Delta l = l\,\alpha\,\Delta T$

$= l \times 1.1 \times 10^{-5} \times (40 - 30) = 1.1\,l \times 10^{-4}$ cm

As $$Y = \frac{F \times l}{A \times \Delta l} = \frac{F \times l}{\pi r^2 \times \Delta l}$$

$\therefore$ Tension, $$F = \frac{Y \times \pi r^2 \times \Delta l}{l}$$
$$= \frac{21 \times 10^{11} \times 22 \times (0.1)^2 \times 1.1\,l \times 10^{-4}}{7 \times l}$$
$$= \mathbf{7.26 \times 10^6\ dyne.}$$

10. Let l_1 and l_2 be lengths of iron and copper cylinders respectively and α_1, α_2 be their coefficients of linear expansion.

Increase in length of iron rod $= l_1\,\alpha_1\,t$

Increase in length of copper rod $= l_2\,\alpha_2\,t$

According to the problem,

$$l_1 - l_2 = 10\text{ cm} \qquad \ldots(i)$$

and $(l_1 + l_1\,\alpha_1\,t) - (l_2 + l_2\,\alpha_2\,t) = 10$

or $(l_1 - l_2) + l_1\,\alpha_1\,t - l_2\,\alpha_2\,t = 10 \qquad \ldots(ii)$

From (*i*) and (*ii*), $l_1\,\alpha_1\,t = l_2\,\alpha_2\,t$

or $$\frac{l_1}{l_2} = \frac{\alpha_2}{\alpha_1} = \frac{1.7 \times 10^{-5}}{1.1 \times 10^{-5}} = \frac{17}{11} \quad \text{or} \quad l_1 = \frac{17}{11}\,l_2$$

From (*i*), $$\frac{17}{11}\,l_2 - l_2 = 10 \quad \text{or} \quad \frac{6}{11}\,l_2 = 10$$

or $\quad l_2 = \frac{110}{6} = 18.33$ cm

and $\quad l_1 = 10 + 18.33 = 28.33$ cm

Length of iron cylinder $= \mathbf{28.33\ cm}$

Length of copper cylinder $= \mathbf{18.33\ cm}$.

11. Here $\quad V = \frac{4}{3}\pi r^3 = \frac{4}{3} \times 3.14 \times (10)^3\ \text{cm}^3$

$$\gamma = 3\alpha = 3 \times 11 \times 10^{-6}\,°\text{C}^{-1}$$

$$\Delta T = 100 - 0 = 100°\text{C}$$

Change in volume,

$$\Delta V = V.\gamma.\Delta T = \frac{4}{3} \times 3.14 \times (10)^3 \times 3 \times 11 \times 10^{-6} \times 100$$

$$= \mathbf{13.8\ cm^3}.$$

14. Change in volume of gasoline,

$$\Delta V_g = \gamma_g\, V\, \Delta T = 9.5 \times 10^{-4} \times V \times 20$$
$$= 190 \times 10^{-4}\, V$$

Change in volume of steel tank,

$$\Delta V_s = \gamma_s\, V\, \Delta T = 3\,\alpha_s\, V\, \Delta T$$
$$= 3 \times 1.2 \times 10^{-5} \times V \times 20 = 7.2 \times 10^{-4}\, V$$

∴ Volume of gasoline that overflows,

$$\Delta V_g - \Delta V_s = (190 \times 10^{-4} - 7.2 \times 10^{-4})\, V$$
$$= 182.8 \times 10^{-4} \times 75 = \mathbf{1.37\ litre.}$$

15. Volume of glass flask = Volume of mercury + Volume of air

or $\quad V = V_m + V_a$

As V_a remains constant, so $\Delta V = \Delta V_m$

or $\quad \gamma_g\, V\, \Delta T = \gamma_m\, V_m\, \Delta T$

or $\quad V_m = \frac{\gamma_g V}{\gamma_m} = \frac{3\alpha_g\, V}{\gamma_m}$

$$= \frac{3 \times 9 \times 10^{-6}}{1.8 \times 10^{-4}} \times 1000\ \text{cm}^3 = \mathbf{150\ cm^3}.$$

11.20 SPECIFIC HEAT

31. ***Define the terms specific heat and molar specific heat. Give their CGS and SI units.***

Specific heat. *The specific heat of a substance may be defined as the amount of heat required to raise the temperature of unit mass of the substance through one degree.* It depends on the nature of the substance and its temperature.

If an amount of heat ΔQ is needed to raise the temperature of M mass of a substance through ΔT, then specific heat is given by

$$c = \frac{\Delta Q}{M \times \Delta T}$$

The CGS unit of specific heat is cal $\text{g}^{-1}\,°\text{C}^{-1}$ and the SI unit is $\text{J kg}^{-1}\,\text{K}^{-1}$.

Clearly, the amount of heat required to raise the temperature of M mass of a substance through ΔT is

$$\Delta Q = Mc\,\Delta T.$$

Molar specific heat. *The molar specific heat of a substance is defined as the amount of heat required to raise the temperature of one mole of the substance through one degree.* It depends on the nature of the substance and its temperature.

If an amount of heat ΔQ is required to raise the temperature of n moles of a substance through ΔT, then molar specific heat is given by

$$C = \frac{\Delta Q}{n\,\Delta T}$$

The CGS unit of molar specific heat is cal $\text{mol}^{-1}\,°\text{C}^{-1}$ and SI unit is $\text{J mol}^{-1}\,\text{K}^{-1}$.

Therefore, the amount of heat required to raise the temperature of n moles of a substance through ΔT is

$$\Delta Q = nC\,\Delta T.$$

32. ***Define the terms heat capacity and water equivalent. Give their CGS and SI units.***

Heat capacity or thermal capacity. *The heat capacity of a body is defined as the amount of heat required to raise its temperature through one degree.*

By definition, the amount of heat required to raise the temperature of unit mass of a body is equal to specific heat c. So heat required for m mass is $m \times c$.

∴ Heat capacity = Mass × Specific heat

or $$S = mc$$

The CGS unit of heat capacity is cal $°\text{C}^{-1}$ and the SI unit is JK^{-1}.

Water equivalent. *The water equivalent of a body is defined as the mass of water which requires the same amount heat as is required by the given body for the same rise of temperature.*

Water equivalent = Mass × Specific heat

or $$w = mc$$

The CGS unit of water equivalent is g and the SI unit is kg.

11.21 CALORIMETRY

33. ***What is calorimetry ? State the principle of calorimetry.***

Calorimetry. *The branch of physics that deals with the measurement of heat is called calorimetry.*

Principle of calorimetry or the law of mixtures. Whenever two bodies at different temperatures are placed in contact with one another, heat flows from the body at higher temperature to the body at lower temperature till the two bodies acquire the same temperature.

The principle of calorimetry states that the heat gained by the cold body must be equal to the heat lost by the hot body, provided there is no exchange of heat with the surroundings.

Heat gained = Heat lost

This principle is a consequence of the law of conservation of energy and useful for solving problems relating to calorimetry.

34. *Briefly describe the construction of a calorimeter.*

Calorimeter. *It is a device used for measuring the quantities of heat.* It consists of a cylindrical vessel of copper provided with a stirrer. The vessel is kept inside a wooden jacket. The space between the calorimeter and the jacket is packed with a heat insulating material like glass wool, etc. Thus the calorimeter gets thermally isolated from the surroundings. The loss of heat due to radiation is further reduced by polishing the outer surface of the calorimeter and the inner surface of the jacket. The lid is provided with holes for inserting a thermometer and a stirrer into the calorimeter.

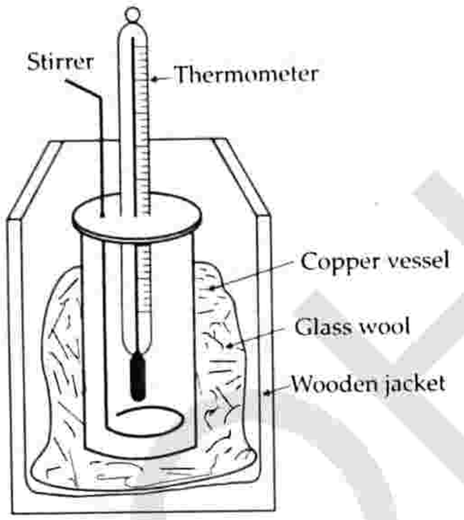


Fig. 11.10 A calorimeter

When bodies at different temperatures are mixed together in the calorimeter, heat is exchanged between the bodies as well as with the calorimeter. If there is no loss of heat to the surroundings, then according to the principle of calorimetry,

Heat gained by cold bodies = Heat lost by hot bodies.

This equation can be used to determine the specific heat and latent heat of different substances.

11.22 CHANGE OF STATE

35. *What do you mean by change of state of a substance ?*

Change of state. Matter exists in *three* states : *solid, liquid* and *gas.* Any state of a substance can be changed into another by heating or cooling it. *The transition of a substance from one state to another is called a change of state.*

The common changes of states are as follows :

1. Melting of a solid :

$$\text{Solid} \xrightarrow{\text{Heat}} \text{Liquid}$$

2. Vaporization of a liquid :

$$\text{Liquid} \xrightarrow{\text{Heat}} \text{Vapour}$$

3. Condensation of vapour :

$$\text{Vapour} \xrightarrow{\text{Cool}} \text{Liquid}$$

4. Freezing of a liquid :

$$\text{Liquid} \xrightarrow{\text{Cool}} \text{Solid}$$

36. *By a simple experiment, show that temperature of a substance remains constant during its change of state.*

Effect of heat on the change of state. Take some ice cubes in a beaker. Note the temperature of ice. It will be 0°C. As shown in Fig. 11.11, start heating it slowly on a constant heat source. Note the temperature after every minute. Continuously stir the mixture of water and ice.

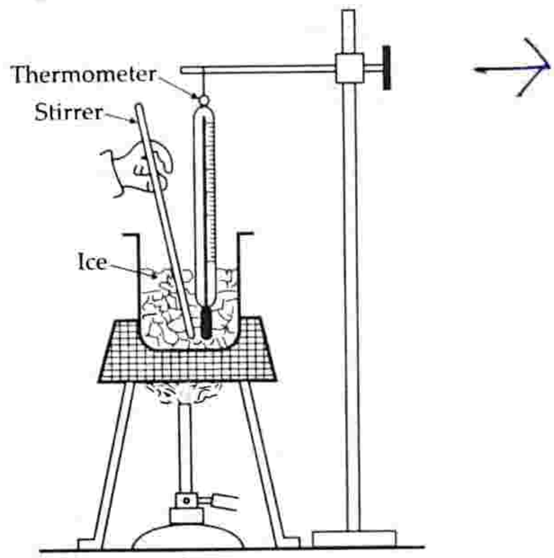


Fig. 11.11 Effect of heat on ice.

Plot a graph between temperature recorded and time. We obtain a curve of the shape shown in Fig. 11.12. It is found that the temperature does not change as long as there is any ice left in the beaker. Here the heat supplied is being used in changing the state from solid (ice) to liquid (water). *The change of state from solid to liquid is called **melting** and from liquid to solid is called **fusion**.* It is seen that the temperature remains constant until the entire amount of the solid substance melts. Thus *both the solid and liquid states of the substance coexist in thermal equilibrium during the change of state from solid to liquid.*

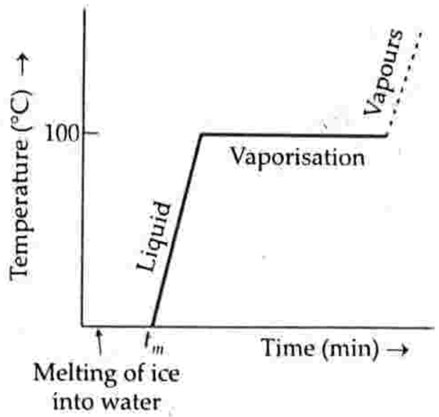


Fig. 11.12 A plot of temperature versus time showing the changes in the state of ice on heating (not to scale).

After the whole of ice gets melted into water and as we continue heating the beaker, we note that the temperature begins to increase till it reaches nearly 100°C when it again becomes steady. The heat supplied is now being used to change state of water from liquid to vapour. *The change of state from liquid to vapour is called **vaporisation**.* It is noted that the temperature remains constant until the entire amount of the liquid is converted into vapour. Thus *both the liquid and the vapour states of the substance coexist in thermal equilibrium.*

Melting point. *The temperature at which the solid and the liquid states of a substance coexist in thermal equilibrium with each other is called its melting point.* It is a characteristic of the substance but also depends on pressure. *The melting point of a substance at standard atmospheric pressure is called its **normal melting point**.*

Boiling point. *The temperature at which the liquid and vapour states of a substance coexist in thermal equilibrium with each other is called its boiling point. The boiling point of a substance at standard atmospheric pressure is called its **normal boiling point**.*

Sublimation. Some substances, on being heated, pass from the solid state to liquid state directly. *The process of transition of a substance from the solid state to the vapour state without passing through the liquid state is called sublimation,* and the substance is said to sublime. Substances like dry ice (solid CO_2), iodine, naphthalene and camphor undergo sublimation when heated. During the sublimation process, the solid and vapour states of a substance coexist in thermal equilibrium with each other.

11.23 EFFECT OF PRESSURE ON MELTING AND BOILING POINTS

37. *What is the effect of pressure on the melting point of a substance ? What is regelation ? Give an experiment to illustrate it.*

Effect of pressure on melting point. Under a given pressure, a pure substance melts at definite temperature. However, the melting point changes with the change in pressure. We know that paraffin wax expands on melting. An increase in pressure will make its expansion difficult. We will have to heat it more to melt it. That is, the increase in pressure increases the melting point of wax. On the other hand, ice contracts on melting. The increase in pressure will help in its contraction. So we expect a decrease in the melting point of ice as the pressure on it is increased. We can generalise these observations as follows :

The melting point of those substances which expand on melting (e.g., paraffin wax, phosphorus, sulphur, etc.) increases with the increase in pressure while the melting point of those substances which contract on melting (e.g., ice, cast iron, bismuth etc.) decreases with increase in pressure.

Effect of pressure on freezing point of ice : Regelation. When two pieces of ice are pressed against one another for few seconds and then released, they get frozen at the surface of contact. As the pressure is increased, the melting point of ice is lowered and ice melts. When pressure is released, the water so formed (at a temperature $< 0°C$) immediately freezes again. This phenomenon of refreezing is called regelation.

*The phenomenon in which ice melts when pressure is increased and again freezes when pressure is removed is called **regelation*** (re = again ; gelare = freeze) :

We can demonstrate the phenomenon of regelation through the following simple experiment.

Take a slab of ice and support it on two wooden blocks. Take a metallic wire and attach two heavy weights, say 5 kg each, at its ends. Put the wire over the slab, as shown in Fig. 11.13. It is seen that the wire

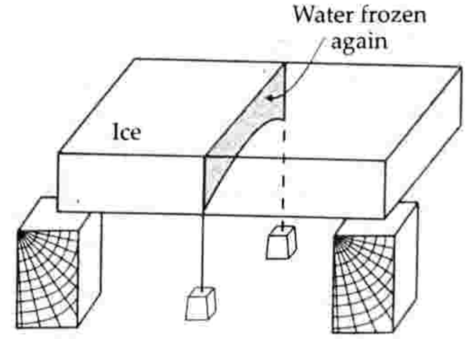


Fig. 11.13 A wire cuts its way through an ice slab without cutting it into two pieces.

gradually cuts its way through the ice without cutting it into two pieces. Just below the wire, ice melts at a lower temperature due to the increase in pressure. When the wire has passed, water above the wire freezes again. Thus the wire passes through the slab and the slab does not split. *This phenomenon of refreezing is called* ***regelation.***

Practical applications of regelation :

1. By pressing snow in our hand, we can transform it into a snow-ball. When snow is pressed, its crystals melt. As the pressure is released, water refreezes forming a snow-ball.
2. When the wheels of cart pass over snow, ice melts due to increase in pressure exerted by the wheels. When pressure is released, water so formed refreezes on the wheels. That is why wheels are covered with ice.
3. Skating is possible due to the formation of water layer below the skates. Water is formed due to the increase of pressure and it acts as a lubricant.
4. The ice of a glacier, pressed against the sides of its valley melts, and in this way adopts itself to the shape of the valley.

38. *What is the effect of pressure on the boiling point of a liquid ? Explain it with the help of a simple experiment.*

Effect of pressure on the boiling point of a liquid. *The boiling point of a liquid increases with the increase in pressure.* At 1 atmospheric pressure, the boiling point of water is 100°C. At 2 atmospheric pressure, the boiling point of water is 128°C.

To study the effect of pressure on boiling point, take a round-bottom flask and fill it more than half with water. Support it over a burner and fix a thermometer and a steam outlet through the cork fitted in the neck of the flask, as shown in Fig. 11.14.

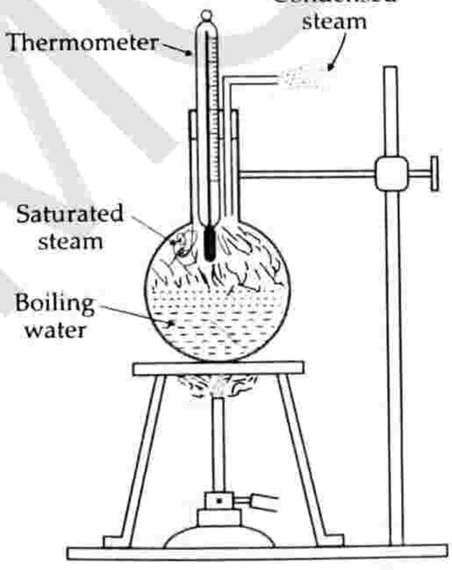


Fig. 11.14 Boiling process

Heat the flask over the burner slowly. As the temperature rises, small bubbles of water begin to form at the bottom which rise through water and escape from the surface. At a temperature of about 70° – 80°C, bubbles of steam begin to form at the bottom. These bubbles of steam rise to the upper cold layers of water, where they condense and disappear producing of a peculiar noise called *singing of the vessel.*

As the temperature of the entire mass of water becomes 100°C, the bubbles of steam begin to escape more rapidly and water gets violently agitated and *boiling is said to occur.* As the steam comes out of the flask, it condenses as tiny droplets of water, giving a foggy appearance.

If the steam outlet is partly closed, the pressure inside the flask increases and boiling stops. Water now has to be heated to a higher temperature to make it boil again. Thus *the boiling point increases with the increase in pressure.*

Remove the burner and allow the water to cool to a temperature below 100°C. If the pressure is reduced by removing the partial covering of the outlet, water begins to boil again. Thus *the boiling point decreases with the decrease in pressure.*

39. *Describe a simple experiment to demonstrate the boiling of water at a temperature much lower than 100°C.*

Franklin's experiment. Take a round-bottom flask and fill it about half with water. Heat it so that water begins to boil. Continue boiling for some time so that air of the flask escapes to the atmosphere and the empty space is filled with steam. Remove the burner. Close the flask with airtight cork. Hold the flask in the clamp of a stand in an inverted position, as shown in Fig. 11.15. Allow the water to cool for some time so that boiling stops altogether. Pour cold water on the flask. Due to the condensation of steam, the pressure over

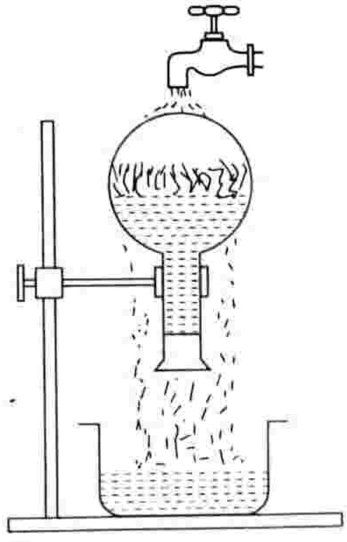


Fig. 11.15 Water boils below 100°C under reduced pressure.

the water surface decreases. Water begins to boil again now at a lower temperature. This shows that the boiling point decreases with the decrease in pressure.

Practical Applications :

1. Cooking is difficult at mountains. The atmospheric pressure at mountains is much lower than that at plains, so water starts boiling at a temperature much lower than 100°C. Hence cooking is difficult.
2. The pressure inside a pressure cooker is increased much above the atmospheric pressure by not allowing the steam to escape. This increases the boiling point. Hence the vegetables are cooked inside a pressure cooker in a shorter time.

11.24 LATENT HEAT

40. *What is latent heat ? Give its units. ~~With the help of a suitable graph~~, explain the terms latent heat of fusion and latent heat of vaporisation.*

Latent Heat. When a solid changes into liquid or a liquid changes into gas, it absorbs heat. But this heat does not show up as an increase in temperature. This heat, used to change the state, is hidden or ***latent*** (lying hidden), and is therefore called latent heat.

The amount of heat required to change the state of unit mass of a substance at constant temperature and pressure is called latent heat of the substance.

If m mass of a substance undergoes a change from one state to another, then the amount of heat required for the process is

$$Q = mL$$

where L is the latent heat of the substance and is a characteristic of the substance. Its value also depends on the pressure. Clearly,

$$L = \frac{Q}{m}$$

∴ SI unit of latent heat = J kg^{-1}

CGS unit of latent heat = cal g^{-1}.

Latent heat of fusion. *The amount of heat required to change the state of unit mass of a substance from solid to liquid at its melting point is called latent heat of fusion or latent heat of melting. It is denoted by L_f.*

Latent heat of vaporisation. *The amount of heat required to change the state of unit mass of a substance from liquid to vapour at its boiling point is called latent heat of vaporisation or latent heat of boiling. It is denoted by L_v.*

Fig. 11.16 shows the plot of the heat supplied *vs.* rise of temperature. Here we take 1 kg of ice at –20°C. As we start heating, the temperature of ice increases until it reaches its melting point (0°C). At this temperature, the addition of more heat does not increase the temperature but causes the ice to melt, or

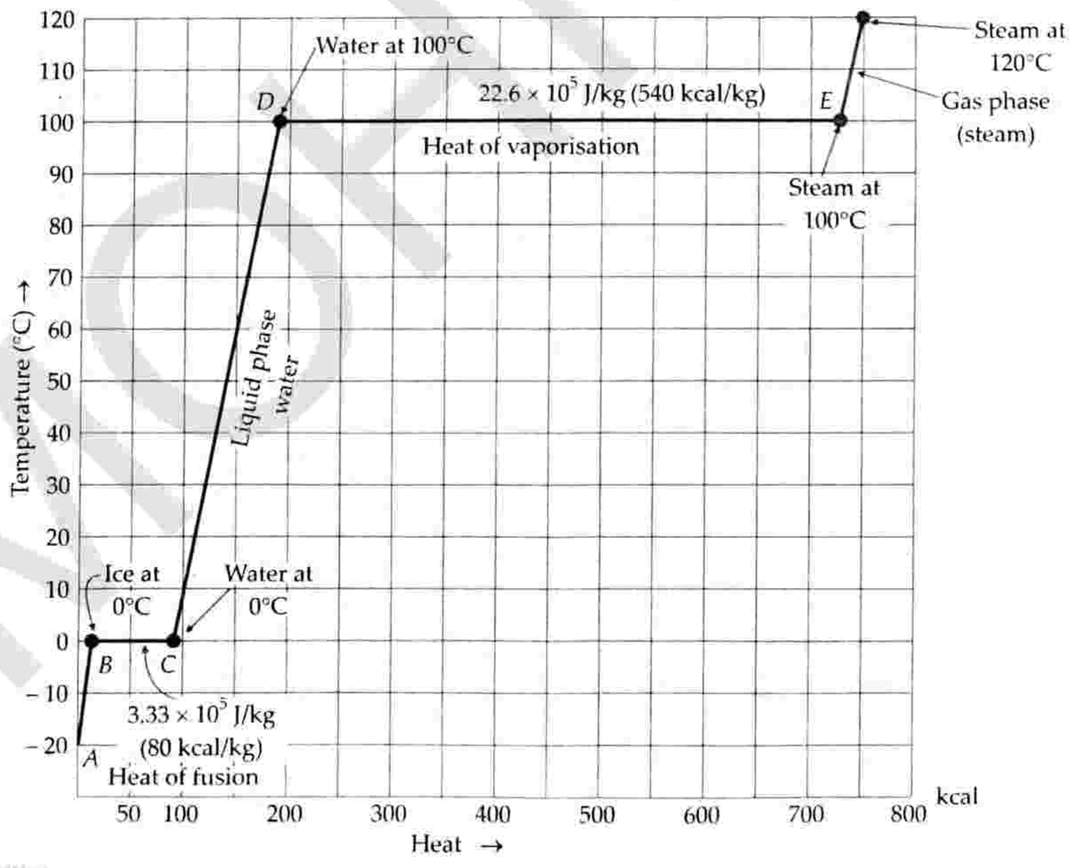


Fig. 11.16 Temperature vs. heat for 1 kg of water at atmospheric pressure.

changes its state. As is obvious from the graph, the latent heat of fusion of ice is,

$$L_f = 3.3 \times 10^5 \text{ J kg}^{-1}$$

That is, 3.3×10^5 J of heat is needed to melt 1 kg of ice into water at 0°C.

Once the entire ice melts, the addition of more heat causes the temperature of water to rise. The temperature keeps on increasing till it reaches 100°C. The water starts boiling. The addition of more heat to the boiling water causes vaporisation, without increase in temperature. Obviously, the latent heat of vaporisation of water is,

$$L_v = 22.6 \times 10^5 \text{ J.}$$

That is, 22.6×10^5 J of heat is needed to convert 1 kg of water to steam at 100°C. Additional heat causes the temperature of the steam to rise.

By conservation of energy, when 1 kg of steam condenses to water, it gives up 22.6×10^5 J of heat. That is why steam burns are more serious than burns from boiling water, even though both are at 100°C.

Table 11.3 *Temperatures of the change of state and latent heats for various substances at 1 atm pressure*

Substance	Melting point (°C)	L_f (10^5 J kg^{-1})	Boiling point (°C)	L_v (10^5 J kg^{-1})
Ethyl alcohol	– 114	1.0	78	8.5
Gold	1063	0.645	2660	15.8
Lead	328	0.25	1744	8.67
Mercury	– 39	0.12	357	2.7
Nitrogen	– 210	0.26	– 196	2.0
Oxygen	– 219	0.14	– 183	2.1
Water	0	3.33	100	22.6

Examples based on Specific Heat and Latent Heat

FORMULAE USED

1. Heat gained or lost, $Q = mc\,\Delta T$
2. According to the principle of calorimetry,
   Heat gained = Heat lost
3. Water equivalent, $w = mc$ (gram)
4. Heat capacity = mc (cal °C^{-1})
5. Latent heat of vaporisation or fusion, $Q = mL$

UNITS USED

In CGS system, heat Q is in cal, mass m in gram, specific heat c in cal g^{-1} °C^{-1} and ΔT in °C. In SI, heat Q is in joule, mass m in kg, specific heat c in J kg^{-1} K^{-1} and ΔT in K.

EXAMPLE 15. *A sphere of aluminium of 0.047 kg is placed for sufficient time in a vessel containing boiling water, so that the sphere is at 100°C. It is then immediately transferred to 0.14 kg copper calorimeter containing 0.25 kg of water at 20°C. The temperature of water rises and attains a steady state at 23°C. Calculate the specific heat capacity of aluminium.* **[NCERT]**

Solution. Mass of aluminium sphere,

$$m = 0.047 \text{ kg}$$

Fall in temperature of aluminium sphere,

$$\Delta T = 100 - 23 = 77°\text{C}$$

Let specific heat of aluminium $= c_{Al}$

Heat lost by the aluminium sphere

$$= mc_{Al}\,\Delta T = 0.047 \text{ kg} \times c_{Al} \times 77°\text{C}$$

Mass of water, $m_1 = 0.25$ kg

Mass of calorimeter, $m_2 = 0.14$ kg

Initial temperature of water and calorimeter = 20°C

Final temperature of the mixture = 23°C

Rise in temperature of mixture,

$$\Delta T' = 23 - 20 = 3°\text{C}$$

Specific heat of water,

$$c_w = 4.18 \times 10^3 \text{ J kg}^{-1} \text{ K}^{-1}$$

Specific heat of copper,

$$c_{Cu} = 0.386 \times 10^3 \text{ J kg}^{-1} \text{ K}^{-1}$$

Heat gained by water and calorimeter

$$= m_1 c_w\,\Delta T' + m_2 c_{Cu}\,\Delta T'$$
$$= (0.25 \times 4.18 \times 10^3 + 0.14 \times 0.386 \times 10^3) \times 3 \text{ J}$$

In the steady state,

Heat lost = Heat gained

or $$0.047 \times c_{Al} \times 77 = (0.25 \times 4.18 \times 10^3 + 0.14 \times 0.386 \times 10^3) \times 3$$

or $$c_{Al} = 0.911 \times 10^3 \text{ J kg}^{-1} \text{ K}^{-1}$$
$$= \mathbf{0.911 \text{ kJ kg}^{-1} \text{ K}^{-1}}.$$

EXAMPLE 16. *A thermally isolated vessel contains 100 g of water at 0°C when air above the water is pumped out, some of the water freezes and some evaporates at 0°C itself. Calculate the mass of ice formed, if no water is left in the vessel. Latent heat of vaporisation of water at 0°C = 2.10×10^6 J kg^{-1} and latent heat of fusion of ice = 3.36×10^5 J kg^{-1}.*

Solution. Latent heat of vaporisation of water at 0°C,

$$L_1 = 2.10 \times 10^6 \text{ J kg}^{-1} = 2.10 \times 10^3 \text{ Jg}^{-1}$$

Latent heat of fusion of ice,

$$L_2 = 3.36 \times 10^5 \text{ J kg}^{-1} = 3.36 \times 10^2 \text{ Jg}^{-1}$$

Let mass of ice formed = m gram

Then mass of water evaporated

$$= (100 - m) \text{ gram}$$

As no water is left in the vessel,

Heat gained by water in evaporation

$$= \text{Heat lost by water in freezing}$$

or $$(100 - m)\, L_1 = mL_2$$

or $$(100 - m) \times 2.10 \times 10^3 = m \times 3.36 \times 10^2$$

or $$m = \mathbf{86.2\ g.}$$

EXAMPLE 17. *When 0.15 kg of ice of 0°C is mixed with 0.30 kg of water at 50°C in a container, the resulting temperature is 6.7°C. Calculate the heat of fusion of ice.* (c_{water} = 4186 J kg^{-1} K^{-1})

Solution. Heat lost by 0.30 kg water when its temp. falls from 50°C to 6.7°C

$$= mc_{\text{water}}\, \Delta T = 0.30 \times 4186 \times (50 - 6.7) \text{ J}$$
$$= 54376.14 \text{ J}$$

Heat required to melt 0.15 kg ice into water at 0°C

$$= mL_f = 0.15 \times L_f \text{ J}$$

Heat required to raise temperature of 0.15 kg water from 0°C to 6.7°C

$$= mc_{\text{water}}\, \Delta T$$
$$= 0.15 \times 4186 \times (6.7 - 0) = 4206.93 \text{ J}$$

By the principle of calorimetry,

Heat gained = Heat lost

$$0.15 \times L_f + 4206.93 = 54376.14$$

or $$L_f = \mathbf{3.34 \times 10^5\ J\ kg^{-1}}.$$

EXAMPLE 18. *Calculate the heat required to convert 3 kg of ice at −12°C kept in a calorimeter to steam at 100°C at atmospheric pressure.*

Given :

specific heat capacity of ice = 2100 J kg K^{-1},

specific heat capacity of water = 4186 J kg^{-1} K^{-1},

latent heat of fusion of ice = 3.35 × 10^5 J kg^{-1}

and *latent heat of steam* = 2.256 × 10^6 J kg^{-1}. **[NCERT]**

Solution. Mass of the ice,

$$m = 3 \text{ kg}$$

Specific heat capacity of ice,

$$c_{\text{ice}} = 2100 \text{ Jkg}^{-1} \text{ K}^{-1}$$

Specific heat capacity of water,

$$c_{\text{water}} = 4186 \text{ J kg}^{-1} \text{ K}^{-1}$$

Latent heat of fusion of ice,

$$L_{f,\text{ ice}} = 3.35 \times 10^5 \text{ J kg}^{-1}$$

Latent heat of steam,

$$L_{\text{steam}} = 2.256 \times 10^6 \text{ J kg}^{-1}$$

Heat required to convert ice at −12°C to ice at 0°C,

$$Q_1 = mc_{ice}\, \Delta T_1$$
$$= (3 \text{ kg})(2100 \text{ J kg}^{-1} \text{ K}^{-1})[0 - (-12)]°\text{C}$$
$$= 75600 \text{ J}$$

Heat required to melt ice at 0°C to water at 0°C

$$Q_2 = mL_{f\text{ ice}} = (3 \text{ kg})(3.35 \times 10^5 \text{ J kg}^{-1})$$
$$= 1005000 \text{ J}$$

Heat required to convert water at 0°C to water at 100°C,

$$Q_3 = mc_w\, \Delta T_2 = (3 \text{ kg})(4186 \text{ J kg}^{-1} \text{ K}^{-1})(100°\text{C})$$
$$= 1255800 \text{ J}$$

Heat required to convert water at 100°C to steam at 100°C,

$$Q_4 = m\, L_{\text{steam}} = (3 \text{ kg})(2.256 \times 10^6 \text{ J kg}^{-1})$$
$$= 6768000 \text{ J}$$

Total heat required to convert 3 kg of ice at −12°C to steam at 100°C,

$$Q = Q_1 + Q_2 + Q_3 + Q_4$$
$$= 75600 \text{ J} + 1005000 \text{ J} + 1255800 \text{ J} + 6768000 \text{ J}$$
$$= \mathbf{9.1 \times 10^6\ J.}$$

✖ PROBLEMS FOR PRACTICE

1. When 10 g of coal is burnt, it raises the temperature of 2 litres of water from 20°C to 55°C. Calculate the heat of combustion of fuel. **[Chandigarh 04]**
   (**Ans.** 7000 cal g^{-1})

2. A normal diet furnishes 2000 kcal to a 60 kg person in a day. If this energy was used to heat the person with no losses to the surroundings, how much would the person's temperature increase ? The specific heat of the human body = 0.83 cal g^{-1}°C^{-1}.
   (**Ans.** 40.16°C)

3. 0.75 gram of petroleum was burnt in a bomb calorimeter which contains 2 kg of water and has a water equivalent 500 gram. The rise in temperature was 3°C. Determine the calorific value of petroleum. (**Ans.** 10^4 cal g^{-1})

4. The heat of combustion of ethane gas is 373 kcal per mole. Assuming that 60% of the heat is useful, how many litres of ethane measured at S.T.P. must be burnt to convert 50 kg of water at 10°C to steam at 100°C ? One mole of a gas occupies 22.4 litre at S.T.P. (**Ans.** 3131.5 litres)

5. A refrigerator converts 50 gram of water at 15°C into ice at 0°C in one hour. Calculate the quantity of heat removed per minute. Take specific heat of water = 1 cal g^{-1}°C^{-1} and latent heat of ice = 80 cal g^{-1}. (**Ans.** 79.2 cal min^{-1})

6. How many grams of ice at – 14°C are needed to cool 200 gram of water from 25°C to 10°C ? Take specific heat of ice = 0.5 cal g^{-1} °C^{-1} and latent heat of ice = 80 cal g^{-1}. **[MNREC 89]**

(**Ans.** 31 g)

7. An electric heater of power 100 W raises the temperature of 5 kg of a liquid from 25°C to 31°C in 2 minutes. Calculate the specific heat of the liquid.

(**Ans.** 400 J kg^{-1} °C^{-1})

8. A piece of iron of mass 100 g is kept inside a furnace for a long time and then put in a calorimeter of water equivalent 10 g containing 240 g of water at 20°C. The mixture attains an equilibrium temperature of 60°C. Find the temperature of ice. Given specific heat of iron = 470 J kg^{-1} °C^{-1}.

(**Ans.** 953.6°C)

9. When 0.45 kg of ice of 0°C mixed with 0.9 kg of water at 55°C in a container, the resulting temperature is 10°C. Calculate the heat of fusion of ice. (c_{water} = 4186 J kg^{-1} K^{-1}) **[Delhi 09]**

(**Ans.** 334400 J kg^{-1})

10. Calculate the heat required to convert 0.6 kg of ice at –20°C, kept in a calorimeter to steam at 100°C at atmospheric pressure. Given the specific heat capacity of ice = 2100 J kg^{-1} K^{-1}, specific heat capacity of water is 4186 J kg^{-1} K^{-1}, latent heat of ice = 3.35×10^5 J kg^{-1}, and latent heat of steam = 2.256×10^6 J kg^{-1}. **[Delhi 08]**

(**Ans.** 1.8×10^8 J)

HINTS

1. Heat produced, $Q = mc\,\Delta T = 2 \times 10^3 \times 1 \times (55 - 20)$

$$= 7 \times 10^4 \text{ cal}$$

$$\text{Heat of combustion} = \frac{Q}{M} = \frac{7 \times 10^4}{10} = \mathbf{7000\ cal\ g^{-1}}.$$

2. Here $m = 60$ kg $= 60 \times 10^3$ g, $c = 0.83$ cal g^{-1} °C^{-1}

$$Q = 2000 \text{ kcal} = 2 \times 10^6 \text{ cal}$$

$$\Delta T = \frac{Q}{mc} = \frac{2 \times 10^6}{60 \times 10^3 \times 0.83} = \mathbf{40.16°C.}$$

3. Total heat gained by water and calorimeter,

$$Q = (m + w)\,c\,\Delta T = (2000 + 500) \times 1 \times 3 = 7500 \text{ cal}$$

$$\text{Calorific value of petroleum} = \frac{7500}{0.75} = \mathbf{10^4\ cal\ g^{-1}}.$$

4. Heat required to convert 50 kg of water at 10°C into steam at 100°C

$$= mc\,\Delta T + mL$$

$$= 50 \times 10^3 \times (100 - 10) + \frac{50 \times 2.25 \times 10^6}{4.2}$$

$$= (4.5 + 26.79) \times 10^6 = 31.29 \times 10^6 \text{ cal}$$

As only 60% of the heat is useful, so total heat produced is

$$Q = \frac{100}{60} \times 31.29 \times 10^6 = 52.15 \times 10^6 \text{ cal}$$

Heat of combustion of ethane

$$= 373 \text{ kcal mole}^{-1} = 373 \times 10^3 \text{ cal mole}^{-1}$$

Number of moles of ethane needed to be burnt

$$= \frac{52.15 \times 10^6}{373 \times 10^3} = 139.8 \text{ mole}$$

Volume of ethane needed to be burnt

$$= 22.4 \times 139.8 = \mathbf{3131.5\ litres.}$$

5. Total heat removed in converting 50 g of water at 15°C into ice at 0°C

$$= mc\,\Delta T + mL = m(c\,\Delta T + L)$$

$$= 50(1 \times 15 + 80) = 4750 \text{ cal}$$

Rate of removal of heat

$$= \frac{4750 \text{ cal}}{60 \text{ min}} = \mathbf{79.2\ cal\ min^{-1}}.$$

6. Heat lost by water in cooling from 25°C to 10°C,

$$Q = mc\,\Delta T = 200 \times 1 \times (25 - 10) = 3000 \text{ cal}$$

Heat gained by ice at – 14°C to change into water at 10°C,

$$Q = (mc\,\Delta T)_{ice} + mL + (mc\,\Delta T)_{water}$$

$$= m \times 0.5 \times 14 + m \times 80 + m \times 1 \times 10$$

$$= 97\,m \text{ cal}$$

By principle of calorimetry, $97\,m = 3000$

∴ Mass of ice = 3000 / 97 = **31 g.**

8. Here $m_1 = 100$ g, $w = 10$ g, $m_2 = 240$ g, $T_1 = 20$°C,

Final temperature, $T = 60$°C,

Temperature of furnace = T_2 (say)

Heat lost by iron piece

= Heat gained by water and calorimeter

$$m_1 c (T_2 - T) = (m_2 + w) \times 1 \times (T - T_1)$$

$$\frac{100 \times 470 \times (T_2 - 60)}{4.2 \times 10^3} = (240 + 10) \times 1 \times (60 - 20)$$

or $$T_2 - 60 = \frac{250 \times 40 \times 4.2 \times 10^3}{100 \times 470} = 893.6$$

or $$T_2 = 893.6 + 60 = \mathbf{953.6°C.}$$

9. Proceed as in Example 17.
10. Proceed as in Example 18.

11.25 MODES OF TRANSFER OF HEAT : INTRODUCTION

41. *What are the three modes of transfer of heat ?*

Modes of transfer of heat. Heat can be transferred from one place to another by three different methods. These are (*i*) *conduction*, (*ii*) *convection and* (*iii*) *radiation.*

Solids are usually heated by the process of conduction. Liquid and gases are heated by the process of convection. The process of radiation requires no intervening medium. We receive heat from the sun by the process of radiation. Conduction and convection are slow processes while radiation is a very fast process.

11.26 THERMAL CONDUCTION

42. *What is thermal conduction ? Briefly explain the molecular mechanism of thermal conduction.*

Thermal conduction. *It is a process in which heat is transmitted from one part of a body to another at a lower temperature through molecular collisions, without any actual flow of matter.*

Molecular mechanism of thermal conduction. Solids are heated through conduction. When one end of a metal rod is heated, the molecules at the hot end vibrate with greater amplitude. So they have greater average kinetic energy. As these molecules collide with the neighbouring molecules of lesser kinetic energy, the energy is shared between them. The kinetic energy of the neighbouring molecules increases. This energy transfer takes place from one layer to the next, without the molecules leaving their average location. This way, heat is passed to the colder end of the rod.

11.27 STEADY STATE AND TEMPERATURE GRADIENT

43. *What do you mean by variable state and steady state in thermal conduction ? Define temperature gradient.*

Variable state, steady state and temperature gradient. Consider a metal rod heated at one end A. Heat flows from the hot end A to the cold end B by conduction.

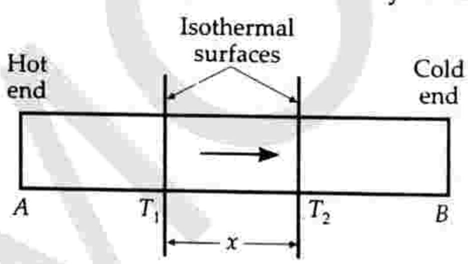


Fig. 11.17 Steady state heat flow by conduction.

In the process of conduction, each cross-section of the rod receives heat from the adjacent cross-section of the hotter side. A part of this heat is absorbed by the cross-section itself whose temperature increases, another part is lost into the atmosphere by convection and radiation from the sides of cross-section and the rest is conducted to the next cross-section. *In this state the temperature of every cross-section of the rod goes on increasing with time.* The rod is said to be in the ***variable state*** of heat conduction.

Suppose the sides of the rod are covered with some insulating material so that no heat is lost from the sides to the surroundings. After some time, a steady state is reached when the temperature of every cross-section of the rod becomes constant. In this state, no heat is absorbed by the rod. *This state of the rod when temperature of every cross-section of the rod becomes constant and there is no further absorption of heat in any part is called* ***steady state***. During steady state,

(*i*) the temperatures of two different parts of the rod are different, but the temperature of each part remains constant.

(*ii*) every transverse section of the rod is an *isothermal surface.*

(*iii*) the temperature decreases as we move away from the hot end.

(*iv*) the quantity of heat flowing per second through every cross-section is constant.

The rate of change of temperature with distance in the direction of flow of heat is called ***temperature gradient***.

If T_1 and T_2 are the temperatures of two isothermal surfaces separated by distance x, then

$$\text{Temperature gradient} = \frac{T_1 - T_2}{x}.$$

11.28 THERMAL CONDUCTIVITY

44. *State the factors on which the conduction of heat through a substance depends. Obtain an expression for the heat conducted and hence define coefficient of thermal conductivity and give its units and dimensions.*

Thermal conductivity. As shown in Fig. 11.18, consider a block of a material of cross-sectional area A and thickness x. Suppose its opposite faces are at temperatures T_1 and T_2, with $T_1 > T_2$.

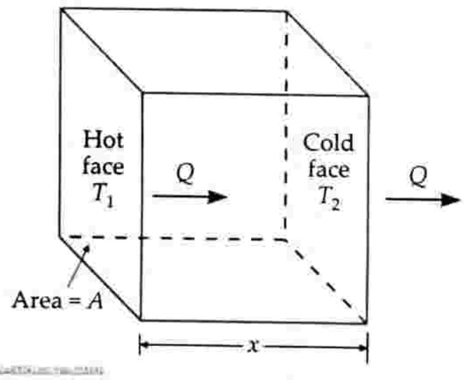


Fig. 11.18 Thermal conductivity.

It is found that the amount of heat Q that flows from hot to cold face during the steady state

(*i*) is directly proportional to the cross-sectional area A, (*ii*) is directly proportional to the temperature difference $(T_1 - T_2)$ between the opposite faces,

(*iii*) is directly proportional to time t for which the heat flows,

(*iv*) is inversely proportional to thickness x of the block, and

(*v*) depends on the nature of the material of the block.

$$\therefore \quad Q \propto \frac{A(T_1 - T_2)\,t}{x}$$

or
$$Q = \frac{KA(T_1 - T_2)\,t}{x}$$

The proportionality constant K is called ***coefficient of thermal conductivity*** of the given material. Its value depends on the nature of the material.

If $A = 1,\ T_1 - T_2 = 1,\ t = 1,\ x = 1$, then $Q = K$

Hence, *the coefficient of thermal conductivity of a material may be defined as the quantity of heat that flows per unit time through a unit cube of the material when its opposite faces are kept at a temperature difference of one degree.*

If the area of cross-section is not uniform or if the steady state condition is not reached, then we consider a thin layer of the material normal to the direction of heat flow. If A be the area of the cross-section at a place, dx be a small thickness along the direction of heat flow and dT be the temperature difference across this thickness dx, then the rate of flow of heat or heat current H will be

$$H = \frac{dQ}{dt} = -\,KA\,\frac{dT}{dx}$$

The quantity dT/dx is called the *temperature gradient*. The negative sign indicates that dT/dx is negative in the direction of flow of heat *i.e.*, temperature decreases along the positive x-direction. Thus the negative sign in the above equation ensures that K is positive.

Units and dimensions of K. The numerical value of K is

$$K = \frac{Q\,.\,x}{A(T_1 - T_2)\,t}$$

$\therefore$ SI unit of K

$$= \frac{\text{J . m}}{\text{m}^2\text{ . K . s}}$$

$$= \mathbf{J\,s^{-1}\,m^{-1}\,K^{-1}\ \text{or}\ W\,m^{-1}\,K^{-1}}.$$

CGS unit of K = $\mathbf{cal\ s^{-1}\ cm^{-1}\ {}^\circ C^{-1}}$.

Dimensions of K

$$= \frac{[\text{ML}^2\text{T}^{-2}]\,.\,[\text{L}]}{[\text{L}^2]\,.\,[\text{K}]\,[\text{T}]} = [\text{MLT}^{-3}\text{K}^{-1}]$$

Table 11.4 gives the thermal conductivities of some common materials. We can note that solids are better conductors than liquids and liquids are better conductors than gases. Moreover, metals are much better thermal conductors than the non-metals. This is because metals have a large number of free electrons which carry heat from hotter to colder regions very fastly. For most of the materials the value of K increases slightly with temperature.

Table 11.4 Thermal conductivities of some common materials

Material	*Thermal conductivity in $J\,s^{-1}\,m^{-1}\,K^{-1}$*
METALS	
Silver	406
Copper	385
Aluminum	205
Brass	109
Steel	50.2
Lead	34.7
Mercury	8.3
NON-METALS	
Insulating brick	0.15
Concrete	0.8
Body fat	0.20
Felt	0.04
Glass	0.8
Ice	1.6
Rock Wool	0.04
Wood	0.12 – 0.04
Water	0.8
GASES	
Air	0.024
Argon	0.016
Hydrogen	0.14

11.29 HEAT CURRENT AND THERMAL RESISTANCE

45. *Define heat current and thermal resistance. Write mathematical expressions for them in terms of thermal conductivity K.*

Heat current and thermal resistance. We know that charge flows in a circuit due to potential difference between its two points. The flow of charge per unit time is called electric current. Similarly, heat flows in a conductor due to temperature difference between its two points. *The flow of heat per unit time is called heat current,* denoted by H. Thus

$$H = \frac{Q}{t}$$

Its SI unit is Js^{-1} or watt (W).

From Ohm's law, electric resistance is given by

$$R = \frac{V}{I}$$

That is electric resistance is the ratio of the potential difference and the electric current. Similarly, *the ratio of the temperature difference between the ends of a conductors to the heat current through it is called thermal resistance,* denoted by R_H. Thus

$$R_H = \frac{\Delta T}{H}$$

As $$Q = KA\frac{\Delta T}{\Delta x}t$$

$\therefore$ $$H = \frac{Q}{t} = KA\frac{\Delta T}{\Delta x}$$

and $$R_H = \frac{\Delta T}{H} = \frac{\Delta x}{KA}$$

Hence *greater the coefficient of thermal conductivity of a material, smaller is the thermal resistance of rod of that material.*

Units and dimensions of R_H. As $R_H = \Delta T / H$, so

SI unit of $R_H = \frac{\text{K}}{\text{J s}^{-1}} = \frac{\text{K}}{\text{W}} = \mathbf{KW^{-1}}$.

Dimensions of R_H

$$= \frac{[\text{K}]}{[\text{ML}^2\text{T}^{-2}].[\text{T}^{-1}]} = [\text{M}^{-1}\text{L}^{-2}\text{T}^{3}\text{K}].$$

11.30 APPLICATIONS OF CONDUCTIVITY IN DAILY LIFE

46. *Describe some applications of conductivity in daily life.*

Some applications of conductivity in daily life :

(*i*) ***In winter, a metallic handle appears colder than the wooden door.*** In winter, the human body is at a higher temperature than the surrounding objects. As we touch the metallic handle (good conductor), heat flows from our body to the handle and feels cold. But no heat flows from our body to the wooden door (bad conductor), so it does not feel that cold as the metallic handle.

(*ii*) ***Cooking utensils are provided with wooden handles.*** Wood is a bad conductor of heat. A wooden handle does not allow heat to be conducted from the hot utensil to the hand. So we can easily hold the hot utensils with the help of wooden handles.

(*iii*) ***A new quilt is warmer than an old quilt.*** A new quilt contains more air in its pores as compared to the old quilt. As air is bad conductor of heat, it does not allow heat to be conducted away from our body to the surroundings and we feel warmer in it.

(*iv*) ***Birds swell their feathers in winter.*** By doing so the birds enclose air between their feathers and the body. Air is poor conductor of heat. It prevents the loss of heat from the bodies of the birds to the surroundings and as such they do not feel cold in winter.

(*v*) ***Ice is packed in saw dust.*** Saw dust and air trapped inside it are poor conductors of heat. This prevents the conduction of heat from the surroundings to the ice which may otherwise melt the ice.

(*vi*) ***Eskimos make double wall houses of the blocks of ice.*** Air trapped between the two walls of ice does not allow the heat to be conducted away from the inside of the house to the colder surroundings.

(*vii*) ***When a wire gauge is placed over the burning Bunsen's burner, the flame does not go beyond the gauge.*** Copper is a very good conductor of heat. The copper gauge absorbs most of the heat. Therefore, the temperature of the gas above the gauge is not high enough to ignite the gas.

(*viii*) ***A refrigerator is provided with insulated walls.*** Generally, fibre glass is used as an insulating material. This is done to minimise the chances of heat flowing into the refrigerator from outside.

Examples based on Thermal Conductivity

FORMULAE USED

1. The amount of heat that flows in time t across the opposite faces of a slab of thickness x and cross-section A,

$$Q = \frac{KA(T_1 - T_2)t}{x}$$

where T_1 and T_2 are the temperatures of hot and cold faces and K is the coefficient of thermal conductivity of the material of the slab.

2. Rate of flow of heat,

$$\frac{dQ}{dt} = -KA\frac{dT}{dx}$$

Here dT/dx is the rate of fall of temperature with distance and is called temperature gradient.

UNITS USED

SI unit of K is $\text{Js}^{-1}\ \text{m}^{-1}\ \text{K}^{-1}$ and CGS unit is $\text{cal s}^{-1}\ \text{cm}^{-1}\ {}^\circ\text{C}^{-1}$.

EXAMPLE 19. *Calculate the rate of loss of heat through a glass window of area 1000 cm^2 and thickness 0.4 cm when temperature inside is 37°C and outside is – 5°C.*

Coefficient of thermal conductivity of glass is 2.2×10^{-3} cal s^{-1} cm^{-1} K^{-1}. **[Delhi 02]**

Solution. Here $A = 1000 \text{ cm}^2$, $x = 0.4$ cm,

$$T_1 - T_2 = 37 - (-5) = 42°\text{C},$$
$$K = 2.2 \times 10^{-3} \text{cal s}^{-1} \text{ cm}^{-1} \text{ K}^{-1}$$

Rate of loss of heat,

$$H = \frac{Q}{t} = \frac{KA(T_1 - T_2)}{x}$$
$$= \frac{2.2 \times 10^{-3} \times 1000 \times 42}{0.4} = \mathbf{231 \text{ cal s}^{-1}}.$$

EXAMPLE 20. *Steam at 100°C is passed into a copper cylinder 10 mm thick and of 200 cm^2 area. Water at 100°C collects at the rate of 150 g min^{-1}. Find the temperature of the outer surface, if the conductivity of copper is 0.8 cal s^{-1} cm^{-1} °C^{-1} and the latent heat of steam is 540 cal g^{-1}.*

Solution. Here $T_1 = 100°\text{C}$, $x = 10$ mm $= 1$ cm,

$$A = 200 \text{ cm}^2, \quad m = 150 \text{ g}, \quad t = 1 \text{ min} = 60 \text{ s},$$
$$K = 0.8 \text{ cal s}^{-1} \text{ cm}^{-1} \text{ °C}^{-1}, \quad L = 540 \text{ cal g}^{-1}$$

Heat drawn from steam,

$$Q = mL = 150 \times 540 \text{ cal}$$

As $$Q = \frac{KA(T_1 - T_2)\,t}{x}$$

$$\therefore \quad 150 \times 540 = \frac{0.8 \times 200\,(100 - T_2) \times 60}{1}$$

or $$100 - T_2 = \frac{150 \times 540}{0.8 \times 200 \times 60} = 8.44°\text{C}$$

$$\therefore \quad T_2 = 100 - 8.44 = \mathbf{91.56°C}.$$

EXAMPLE 21. *A metal rod of length 20 cm and diameter 2 cm is covered with non-conducting substance. One of its ends is maintained at 100°C, while the other end is put in ice at 0°C. It is found that 25 g of ice melts in 5 minutes. Calculate the coefficient of thermal conductivity of the metal. Given latent heat of ice = 80 cal g^{-1}.*

Solution. Here length of rod, $x = 20$ cm,

$T_1 = 100°\text{C}$, $T_2 = 0°\text{C}$, $t = 5$ min $= 300$ s,

Diameter of rod, $d = 2$ cm

Area of cross-section, $A = \dfrac{\pi d^2}{4} = \dfrac{\pi \times (2)^2}{4} = \pi \text{ cm}^2$

Mass of ice melted, $m = 25$ g

Latent heat of ice, $L = 80 \text{ cal g}^{-1}$

Heat used in melting ice,

$$Q = mL = 25 \times 80 = 2000 \text{ cal}$$

As $$Q = \frac{KA(T_1 - T_2)\,t}{x}$$

$$\therefore \quad K = \frac{Qx}{A(T_1 - T_2)\,t} = \frac{2000 \times 20}{\pi \times (100 - 0) \times 300}$$
$$= \frac{2000 \times 20}{3.142 \times 100 \times 300}$$
$$= \mathbf{0.424 \text{ cal s}^{-1} \text{ cm}^{-1} \text{ °C}^{-1}}.$$

EXAMPLE 22. *A layer of ice 2 cm thick is formed on a pond. The temperature of air is – 20°C. Calculate how long it will take for the thickness of ice to increase by 1 mm. Density of ice = 1 g cm^{-3}. Latent heat of ice = 80 cal g^{-1}. Conductivity of ice = 0.008 cal s^{-1} cm^{-1} °C^{-1}.* **[Delhi 98]**

Solution. Let surface area of the ice layer

$$= A \text{ cm}^2$$

Thickness of the ice layer to be formed

$$= 1 \text{ mm} = 0.1 \text{ cm}$$

$\therefore$ Volume of the ice to be formed,

$$V = A \times 0.1 = 0.1\,A \text{ cm}^3$$

Mass of the ice to be formed,

$$m = \text{Volume} \times \text{Density} = 0.1\,A \times 1 = 0.1\,A \text{ gram}$$

In order to increase the thickness of ice layer, heat has to be conducted from the pond to the air through ice layer already formed. If Q is the amount of heat conducted from the pond to the air, to form $0.1\,A$ gram of ice, then

$$Q = mL = 0.1\,A \text{ g} \times 80 \text{ cal g}^{-1} = 8\,A \text{ cal}$$

As thickness of layer through which heat is being conducted increases continuously, so we calculate mean thickness x of the layer.

$$x = \frac{\text{Initial thickness} + \text{Final thickness}}{2}$$
$$= \frac{2 \text{ cm} + 2.1 \text{ cm}}{2} = 2.05 \text{ cm}$$

Temperature difference on two sides of ice layer,

$$T_1 - T_2 = 0 - (-20) = 20°\text{C}$$

Let t be the time taken by the ice layer to increase thickness by 1 mm. Then

$$Q = \frac{KA(T_1 - T_2)\,t}{x}$$

or $$t = \frac{Qx}{KA(T_1 - T_2)} = \frac{8\,A \times 2.05}{0.008 \times A \times 20} = \mathbf{102.5 \text{ s}}.$$

EXAMPLE 23. *Two metal cubes A and B of same size are arranged as shown in Fig. 11.19. The extreme ends of the combination are maintained at the indicated temperatures. The arrangement is thermally insulated. The co-efficients of thermal conductivity of A and B are 300W/m°C and 200 W/m°C, respectively. After steady state is reached, what will be the temperature T of the interface ?* **[IIT 96]**

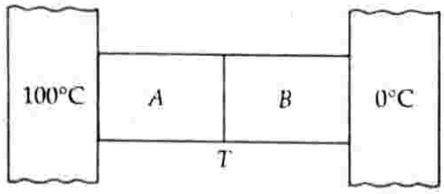


Fig. 11.19

Solution. In the steady-state,

Rate of flow of heat through cube A

= Rate of flow of heat through cube B.

or $$\frac{K_1 A(100-T)}{x} = \frac{K_2 A(T-0)}{x}$$

or $$\frac{300\ A(100-T)}{x} = \frac{200\ A(T-0)}{x}$$

or $$300 - 3T = 2T \quad \text{or} \quad 5T = 300$$

$\therefore$ $$T = \mathbf{60°C}.$$

EXAMPLE 24. *Three bars of equal lengths and equal area of cross-section are connected in series. Their thermal conductivities are in the ratio of 2 : 4 : 3. If the open ends of the first and the last bars are at temperatures 200°C and 18°C respectively in the steady state, calculate the temperatures of both the junctions.*

Solution. Let θ_1 and θ_2 be the temperatures of junctions B and C respectively.

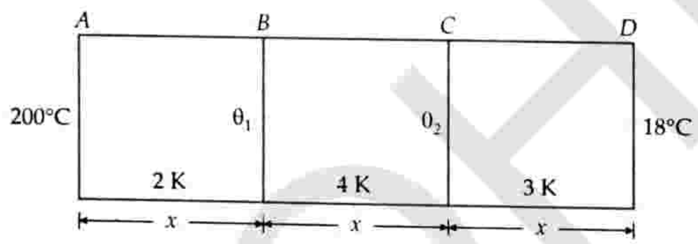


Fig. 11.20

In the steady state, the rate of flow of heat through each bar will be same.

$\therefore$ $$\frac{Q}{t} = \frac{2\ K \times A(200-\theta_1)}{x}$$

$$= \frac{4\ K \times A(\theta_1 - \theta_2)}{x} = \frac{3\ K \times A(\theta_2 - 18)}{x}$$

or $\quad 2\ (200-\theta_1) = 4\ (\theta_1 - \theta_2) = 3\ (\theta_2 - 18)$

On solving, we get, $\theta_1 = \mathbf{116°C}$ and $\theta_2 = \mathbf{74°C}$.

EXAMPLE 25. *One end of a copper rod of uniform cross-section and of length 1.5 m is kept in contact with ice and the other end with water at 100°C. At what point along its length should a temperature of 200°C be maintained so that in steady state, the mass of ice melted be equal to that of the steam produced in the same interval of time ? Assume that the whole system is insulated from the surroundings. Latent heat of fusion of ice = 80 cal g^{-1}. Latent heat of vaporisation of water = 540 cal g^{-1}.* **[REC 92]**

Solution. Let the temperature of 200°C be maintained at a distance x from the end at 0°C. Heat will flow from this point towards both ice and water.

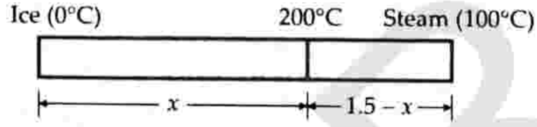


Fig. 11.21

If m is the mass of ice melted = mass of steam produced, then

$$m \times L_{ice} = \frac{KA\ (200-0)}{x} \qquad ...(i)$$

$$m \times L_{steam} = \frac{KA\ (200-100)}{1.5-x} \qquad ...(ii)$$

Dividing (*i*) by (*ii*), we get

$$\frac{L_{ice}}{L_{steam}} = \frac{200}{x} \times \frac{1.5-x}{100}$$

or $$\frac{80}{540} = \frac{2\ (1.5-x)}{x}$$

On solving, $\quad x = \mathbf{1.396\ m}.$

EXAMPLE 26. *What is the temperature of the steel-copper junction in the steady state of the system shown in Fig. 11.22 ? Length of the steel rod = 15.0 cm, length of the copper rod = 10.0 cm, temperature of the furnace = 300°C, temperature of the other end = 0°C. The area of cross-section of the steel rod is twice that of the copper rod. Thermal conductivity of steel = 50.2 $Js^{-1}\ m^{-1}\ °C^{-1}$ and of copper = 385 $Js^{-1}\ m^{-1}\ °C^{-1}$.* **[NCERT ; Delhi 2010]**

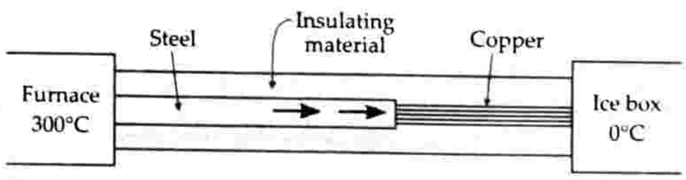


Fig. 11.22

Solution. Here $K_1 = 50.2\ Js^{-1}\ m^{-1}\ °C^{-1}$,

$K_2 = 385\ Js^{-1}\ m^{-1}\ °C^{-1}$, $A_1 = 2\ A_2$,

$x_1 = 15.0$ cm, $\quad x_2 = 10.0$ cm

Let T be the temperature of the steel-copper junction in the steady state of the system. Then in the steady state,

Rate of heat flowing into the system

= Rate of heat flowing out of the system.

or $$\frac{K_1\ A_1\ (300-T)}{x_1} = \frac{K_2\ A_2\ (T-0)}{x_2}$$

or $$\frac{300-T}{T}=\frac{K_2}{K_1}\times\frac{A_2}{A_1}\times\frac{x_1}{x_2}$$

$$=\frac{385}{50.2}\times\frac{1}{2}\times\frac{15}{10}=5.75$$

or $T = \mathbf{44.4°C}$.

EXAMPLE 27. *An electric heater is used in a room of total wall area 137 m^2 to maintain a temperature of $+20°C$ inside it, when the outside temperature is $-10°C$. The walls have three different layers materials. The innermost layer is of wood of thickness 2.5 cm, the middle layer is of cement of thickness 1.0 cm and the outermost layer is of brick of thickness 25.0 cm. Find the power of the electric heater. Assume that there is no heat loss through the floor and the ceiling. The thermal conductivities of wood, cement and brick are 0.125, 1.5 and 1.0 Watt/m/°C respectively.* **[IIT 86]**

Solution. Equivalent thermal conductivity of the series combination of three walls is

$$K=\frac{d_1+d_2+d_3}{\frac{d_1}{K_1}+\frac{d_2}{K_2}+\frac{d_3}{K_3}}=\frac{0.025+0.01+0.25}{\frac{0.025}{0.125}+\frac{0.01}{1.5}+\frac{0.25}{1.0}}$$

$$=\frac{0.285\times 300}{137}\ \text{Wm}^{-1}\,°\text{C}^{-1}$$

Rate of flow of heat is

$$\frac{Q}{t}=KA\frac{\theta_1-\theta_2}{(d_1+d_2+d_3)}$$

$$=\frac{0.285\times 300}{137}\times 137\times\frac{20-(-10)}{0.285}=\mathbf{9000\ W.}$$

This should be equal to the power of the heater.

EXAMPLE 28. *An iron bar ($L_1=0.1\,m$, $A_1=0.02\,m^2$, $K_1=79\,W\,m^{-1}\,K^{-1}$) and a brass bar ($L_2=0.1\,m$, $A_2=0.02\,m^2$, $K_2=109\,W\,m^{-1}\,K^{-1}$) are soldered end to end as shown in Fig. 11.23. The free ends of the iron bar and brass bar are maintained at 373 K and 273 K respectively. Obtain expressions for and hence compute (i) the temperature of the junction of the two bars, (ii) the equivalent thermal conductivity of the compound bar, and (iii) the heat current through the compound bar.* **[NCERT]**

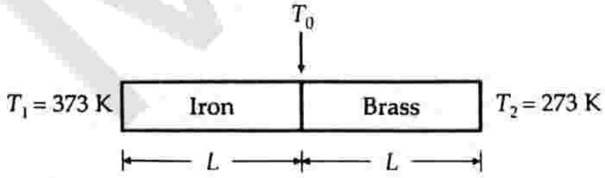


Fig. 11.23

Solution. Here

$L_1=L_2=L=0.1$ m, $A_1=A_2=A=0.02$ m^2,
$K_1=79$ W m^{-1} K^{-1}, $K_2=109$ W m^{-1} K^{-1},
$T_1=373$ K, and $T_2=273$ K

In the steady state,

Heat current through iron bar = Heat current through brass bar

or $$H_1=H_2=H \quad \text{(say)}$$

or $$\frac{K_1A_1(T_1-T_0)}{L_1}=\frac{K_2A_2(T_0-T_2)}{L_2}$$

or $$\frac{K_1A(T_1-T_0)}{L}=\frac{K_2A(T_0-T_2)}{L}$$

or $$K_1(T_1-T_0)=K_2(T_0-T_2)$$

Thus the junction temperature T_0 of the two bars is

$$T_0=\frac{(K_1T_1+K_2T_2)}{(K_1+K_2)}$$

Using this value of T_0, the heat current through either bar will be

$$H=\frac{K_1A(T_1-T_0)}{L}=\frac{K_1A}{L}\left(T_1-\frac{K_1T_1+K_2T_2}{K_1+K_2}\right)$$

$$=\frac{K_1K_2A}{K_1+K_2}\cdot\frac{T_1-T_2}{L}$$

Thus, the heat current H' through the compound bar of length $L_1+L_2=2L$ and the equivalent thermal conductivity K', of the compound bar are given by

$$H'=H$$

or $$\frac{K'A(T_1-T_2)}{2L}=\frac{K_1K_2A}{K_1+K_2}\cdot\frac{T_1-T_2}{L}$$

or $$K'=\frac{2K_1K_2}{K_1+K_2}.$$

(*i*) $$T_0=\frac{(K_1T_1+K_2T_2)}{(K_1+K_2)}$$

$$=\frac{(79\ \text{Wm}^{-1}\text{K}^{-1})(373\ \text{K})+(109\ \text{Wm}^{-1}\text{K}^{-1})(273\ \text{K})}{79\ \text{Wm}^{-1}\text{K}^{-1}+109\ \text{Wm}^{-1}\text{K}^{-1}}$$

$= \mathbf{315\ K.}$

(*ii*) $$K'=\frac{2K_1K_2}{K_1+K_2}$$

$$=\frac{2\times(79\ \text{Wm}^{-1}\text{K}^{-1})\times(109\ \text{Wm}^{-1}\text{K}^{-1})}{79\ \text{Wm}^{-1}\text{K}^{-1}+109\ \text{Wm}^{-1}\text{K}^{-1}}$$

$= \mathbf{91.6\ m^{-1}\ K^{-1}}$.

(*iii*) $$H'=H=\frac{K'A(T_1-T_2)}{2L}$$

$$=\frac{(91.6\ \text{Wm}^{-1}\text{K}^{-1})\times(0.02\ \text{m}^2)\times(373\ \text{K}-273\ \text{K})}{2\times(0.1\ \text{m})}$$

$= \mathbf{916.1\ W}$

✖ PROBLEMS FOR PRACTICE

1. Heat is flowing through a rod of length 25.0 cm having cross-sectional area $8.80\,cm^2$. The coefficient of thermal conductivity for the material of the rod is $K = 9.2 \times 10^{-2}$ kcal s^{-1} m^{-1} $°C^{-1}$. The temperatures of the ends of the rod are 125°C and 0°C in the steady state. Calculate (*i*) temperature gradient in the rod (*ii*) temperature of a point at a distance of 10.0 cm from the hot end and (*iii*) rate of flow of heat. [**Ans.** (*i*) $-5°C\,cm^{-1}$ (*ii*) 75°C (*iii*) 4.048×10^{-2} kcal s^{-1}]

2. Calculate the difference in temperatures between two sides of an iron plate 20 mm thick, when heat is conducted at the rate of 6×10^5 cal / min / m^2. K for metal is 0.2 cal s^{-1} cm^{-1} $°C^{-1}$ (**Ans.** 10°C)

3. A flat-bottom kettle placed on a stove is being used to boil water and the thermal conductivity of the material is 0.5 cal s^{-1} cm^{-1} $°C^{-1}$. If the amount of steam being produced in the kettle is at the rate 10 g min^{-1}, calculate the difference of temperature between the inner and outer surfaces of the bottom. The latent heat of steam is 540 cal g^{-1}. (**Ans.** 0.2°C)

4. An iron boiler is 1 cm thick and has a heating area of 2 m^2. The two surfaces of the boiler are at 234°C and 100°C respectively. If the latent heat of steam is 536 kcal kg^{-1} and thermal conductivity of iron is 1.6×10^{-2} kcal s^{-1} m^{-1} $°C^{-1}$, then how much water will be evaporated into steam per minute ? (**Ans.** 48 kg)

5. One end of a 0.25 m long metal bar is in steam and the other is in contact with ice. If 12 g of ice melts per minute, what is the thermal conductivity of the metal ? Given cross-section of the bar = 5×10^{-4} m^2 and latent heat of ice is 80 cal g^{-1}. (**Ans.** 80 cal s^{-1} m^{-1} $°C^{-1}$)

6. A layer of ice 0.15 m thick has formed on the surface of a deep pond. If the temperature of upper surface of ice is constant and equal to that of the air which is – 12°C, determine the time it will take to increase the thickness of ice layer by 0.2 mm. Take latent heat of ice = 80 cal g^{-1}, density of ice = 0.91 g cm^{-3} and thermal conductivity of ice = 0.5 cal s^{-1} m^{-1} K^{-1}. (**Ans.** 364.2 s)

7. Water is boiled in a rectangular steel tank of thickness 2 cm by a constant temperature furnace. Due to vaporisation, water level falls at a steady rate of 1 cm in 9 minutes. Calculate the temperature of the furnace. Given K for steel = 0.2 cal s^{-1} m^{-1} $°C^{-1}$. (**Ans.** 110 °C)

8. Estimate the rate at which ice would melt in a wooden box 2.0 cm thick and of inside measurements 200 cm × 120 cm × 120 cm assuming that the external temperature is 30°C and coefficient of thermal conductivity of wood is 0.0004 cal s^{-1} cm^{-1} °C. (**Ans.** 9.36 gs^{-1})

9. Steam at 373 K is passed through a tube of radius 50 cm and length 3 m. If thickness of the tube be 2 mm and conductivity of its material be 2×10^4 cal s^{-1} cm^{-1} K^{-1}, calculate the rate of loss of heat in Js^{-1}. The outside temperature is 282 K. (**Ans.** 36026 Js^{-1})

10. The thermal conductivity of copper is four times that of brass. Two rods of copper and brass of same length and cross-section are joined end to end. The free end of copper rod is at 0°C and that of brass rod at 100°C. Calculate the temperature of junction at equilibrium. Neglect radiation losses. (**Ans.** 20°C)

11. The temperature difference between the two ends of a bar 1.0 m long is 50°C and that for the other bar 1.25 m long 75°C. Both the bars have same area of cross-section. If the rates of conduction of heat in the two bars are the same, find the ratio of the coefficients of thermal conductivity of the materials of the two bars. (**Ans.** 6 : 5)

12. The ratio of the areas of cross-section of two rods of different materials is 1 : 2, and the ratio of the thermal conductivities of their materials is 4 : 3. On keeping equal temperature-difference between the ends of these rods, the rates of conduction of heat are equal. Determine the ratio of the lengths of the rods. (**Ans.** 2 : 3)

13. In Fig. 11.24, two bars of the same metal are connected. The length of the first bar is half of that of the second, but the cross-sectional area is double. What is the temperature of the junction of the bars ? (**Ans.** 20°C)

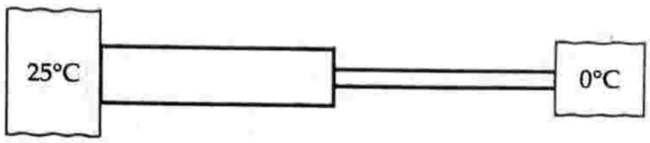


Fig. 11.24

14. A room at 20°C is heated by a heater of resistance 20 ohm connected to 200 V mains. The temperature is uniform throughout the room and heat is transmitted through a glass window of area 1 m^2 and thickness 0.2 cm. Calculate the temperature outside. Thermal conductivity of glass is 0.2 cal s^{-1} m^{-1} $°C^{-1}$ and $J = 4.2$ J cal^{-1}. (**Ans.** 15.2°C)

✖ HINTS

1. (*i*) Temperature gradient,

$$\frac{dT}{dx} = \frac{T_2 - T_1}{x} = \frac{0 - 125}{25.0} = -5°C\ cm^{-1}.$$

(*ii*) Change in temperature at a point distant 10.0 cm from the hot end

$= \text{Temp. gradient} \times \text{distance} = -5 \times 10.0 = -50°C$

Temperature at this point = 125 – 50 = **75° C.**

Rate of flow of heat, $\frac{Q}{t} = \frac{KA(T_1 - T_2)}{x}$

$$= \frac{9.2 \times 10^{-2} \times 8.80 \times 10^{-4} \times 125}{0.25}$$

$$= \mathbf{4.048 \times 10^{-2}\ kcal\ s^{-1}}.$$

2. Here $x = 20$ mm $= 2$ cm, $Q = 6 \times 10^5$ cal,

$t = 1$ min $= 60$ s, $A = 1\ m^2 = 10^4\ cm^2$,

$K = 0.2\ cal\ s^{-1}\ cm^{-1}\ K^{-1}$

As $Q = \frac{KA(T_1 - T_2)t}{x}$

$$\therefore\ T_1 - T_2 = \frac{Qx}{KAt} = \frac{6 \times 10^5 \times 2}{0.2 \times 10^4 \times 60} = \mathbf{10°\,C}.$$

3. Rate of flow of heat from the stove into the kettle is

$$\frac{Q}{t} = \frac{KA(T_1 - T_2)}{x} = \left(\frac{m}{t}\right) L$$

$$\therefore\ T_1 - T_2 = \frac{m}{t} L \times \frac{x}{KA} = \frac{10}{60} \times 540 \times \frac{0.3}{0.5 \times 270} = \mathbf{0.2°\,C}.$$

4. Here $x = 1$ cm $= 0.01$ m, $A = 2\ m^2$,

$T_1 - T_2 = 234 - 100 = 134°C$

$K = 1.6 \times 10^{-2}\ kcal\ s^{-1}\ m^{-1}\ °C^{-1}$, $t = 1$ min $= 60$ s

$$m = \frac{KA(T_1 - T_2)t}{xL} = \frac{1.6 \times 10^{-2} \times 2 \times 134 \times 60}{0.01}$$

$= \mathbf{48\ kg.}$

5. Here $x = 0.25$ m, $T_1 - T_2 = 100 - 0 = 100°C$,

$t = 1$ min $= 60$ s, $A = 5 \times 10^{-4}\ m^2$

$Q = mL = 12 \times 80 = 960$ cal

But $Q = \frac{KA(T_1 - T_2)t}{x}$

or $960 = \frac{K \times 5 \times 10^{-4} \times 100 \times 60}{0.25}$

$$\therefore\ K = \frac{960 \times 0.25}{5 \times 10^{-2} \times 60} = \mathbf{80\ cal\ s^{-1}\ m^{-1}\ °C^{-1}}.$$

6. Let A be the area of the upper face of ice layer. Increase in thickness = 0.2 mm = 0.02 cm.

Mass of ice of to be frozen, $m = A \times 0.02 \times 0.91$ g

As latent heat of ice, $L = 80\ cal\ g^{-1}$

$\therefore\ Q = mL = A \times 0.02 \times 0.91 \times 80$ cal

Average thickness through which heat is to pass,

$$x = \frac{15 + (15 + 0.02)}{2} = 15.01\ cm$$

and $K = 0.5\ cal\ s^{-1}\ m^{-1}\ K^{-1}$

$= 0.5 \times 10^{-2}\ cal\ s^{-1}\ cm^{-1}\ K^{-1}$

As $Q = \frac{KA(T_1 - T_2)t}{x}$

$$t = \frac{Qx}{KA(T_1 - T_2)}$$

$$= \frac{A \times 0.02 \times 0.91 \times 80 \times 15.01}{0.5 \times 10^{-2} \times A \times [0 - (-12)]} = \mathbf{364.2\ s.}$$

7. Let area of the bottom of the tank = $A\ cm^2$

Volume of water that vaporises in 9 min (or 540 s)

$= A \times 1\ cm^3$

Mass of water that vaporises in 540 s

$= A\ cm^2 \times 1\ g\ cm^{-3} = A$ g

$Q = mL = A \times 540$ cal

But $Q = \frac{KA(T_1 - T_2)t}{x}$.

or $T_1 - T_2 = \frac{Qx}{KAt} = \frac{A \times 540 \times 2}{0.2 \times A \times 540} = 10$

$\therefore\ T_1 = T_2 + 10 = 100 + 10 = \mathbf{110°\,C.}$

8. A = Area of all the six faces

$= (2 \times 120 \times 120 + 4 \times 200 \times 120) = 124800\ cm^2$

$x = 2$ cm, $T_1 - T_2 = 30 - 0 = 30°C$,

$K = 0.004\ cal\ s^{-1}\ cm^{-1}\ °C^{-1}$

$Q = mL = \frac{KA(T_1 - T_2)t}{x}$

Rate of melting of ice,

$$\frac{m}{t} = \frac{KA(T_1 - T_2)}{xL} = \frac{0.0004 \times 124800 \times 30}{2 \times 80}$$

$= \mathbf{9.36\ gs^{-1}}.$

9. Here $T_1 - T_2 = 373 - 282 = 91$ K, $x = 2$ mm $= 0.2$ cm

$K = 2 \times 10^{-4}\ cal\ s^{-1}\ cm^{-1}\ K^{-1}$

$A = 2\pi rl = 2\pi \times 50 \times 300 = 3\pi \times 10^4\ cm^2$

[$\because l = 3$ m $= 300$ cm]

$$\frac{Q}{t} = \frac{KA(T_1 - T_2)}{x} = \frac{2 \times 10^{-4} \times 10^4 \times 91}{0.2}\ cal\ s^{-1}$$

$= 8577.7\ cal\ s^{-1} = \mathbf{36026\ Js^{-1}}.$ [$\because$ 1 cal = 4.2 J]

14. Rate of production of heat by the heater is,

$$\frac{V^2}{R} = \frac{(200\ V)^2}{20\ \Omega} = 2000\ Js^{-1} = \frac{2000}{4.2}\ cal\ s^{-1}$$

Rate of loss of heat through the window,

$$\frac{Q}{t} = \frac{KA(T_1 - T_2)}{x} = \frac{0.2 \times 1 \times (20 - T_2)}{0.2 \times 10^{-2}}$$

As the temperature of the room is uniform and constant, so

$$\frac{0.2 \times 1 \times (20 - T_2)}{0.2 \times 10^{-2}} = \frac{2000}{0.2} \quad \text{or} \quad 20 - T_2 = \frac{20}{4.2} = 4.8$$

$\therefore\ T_2 = 20 - 4.8 = \mathbf{15.2°\,C.}$

11.31 CONVECTION

47. *What is thermal convection ? Briefly explain how are convection currents set up in water ? Distinguish between natural and forced convections.*

Convection. *It is the process by which heat flows from the region of higher temperature to the region of lower temperature by the actual movement of the material particles.*

Fluids (liquids and gases) are heated mainly by the process of convection in which buoyancy and gravity play an important role. As shown in Fig. 11.25, when a fluid is heated from below, the hot portion at the bottom expands and becomes less dense. Because of buoyancy, this lighter portion rises up. The denser colder fluid takes its place by moving downwards. Thus *convection current* is set up in the fluid. The actual movement of a liquid can be seen by colouring the liquid with potassium permanganate crystals placed at the bottom of the vessel.

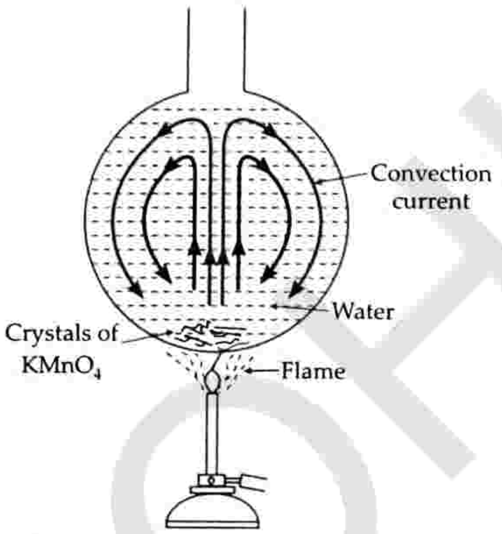


Fig. 11.25 Convection currents in water.

Natural convection. *If the material moves due to difference in density, the process of heat transfer is called natural or free convection.* Natural convection arises due to unequal heating (of fluid) and gravity. Here the more heated and less dense parts of the fluid rise and are replaced by the cooler parts. Natural convection is responsible for the origin of different types of winds in the atmosphere.

Forced convection. *If the heated material is forced to move by an agency like a pump or a blower, the process of heat transfer is called forced convection.* Air-conditioning, central heating systems and heating a liquid by brisk stirring are examples of forced convection.

11.32 PHENOMENA BASED ON THERMAL CONVECTION

48. *Describe some of the phenomena which are based on thermal convection.*

Phenomena based on thermal convection :

(*i*) ***In regulating the temperature of human body.*** In the human body, the heart acts as the pump that circulates blood through different parts of the body, transferring heat by forced convection. This maintains a remarkably constant body temperature.

(*ii*) ***In maintaining comfortable room temperature in cold countries.*** In cold countries during winter, the outside temperature is much below 0°C while the room temperature is comfortably maintained around 20°C. However, the inside air close to the glass window is cooler than 20°C while the outside close to the window is warmer than the chilling temperature of the atmosphere. Thus heat is continuously transferred from the room to the outside by convection of air inside the room, conduction across the glass pane and again convection of air outside. But this heat loss is compensated by the heating system provided in the room.

(*iii*) ***In the formation of trade winds.*** Natural convection plays an important role in the formation of trade winds. The surface of the earth and hence the air above it near the equator gets strongly heated by the sun. The heated air expands and rises upwards. The colder air from polar region rushes in towards the equator. This produces northward wind in northern hemisphere and southward in southern hemisphere. Due to rotation of the earth about its axis from west to east, the air close to the equator has an eastward speed of 1600 kmh^{-1}, while it is zero close to the poles. As a result, the actual direction of the wind in the northern hemisphere is north east and in the southern hemisphere, south west. These winds are called *trade winds*. In ancient times, the traders used these winds to find the direction of motion of their sailing vessels.

(*iv*) ***Land and sea breezes.*** These are local convection currents. Specific heat of water is higher than that of soil. So land and hence air above it is heated faster in summer during day time than air above the sea. The air above land expands and rises and its place is taken up by the colder air from sea to land and is called sea breeze. At night the land gets cooled faster than water. So colder air flows from land to sea and is called land breeze.

(*v*) ***Monsoons.*** Water has much more specific heat than soil or rock. In summer, the land mass of the Indian subcontinent gets much hotter than the Indian Ocean. This sets up convention current with hot air from the land rising and moving towards the Indian Ocean, while the moisture-laden air from the Ocean moves towards the land. When obstructed by mountains, the moist air rushes upwards to great height and gets cooled. The moisture condenses and causes wide-spread rains in India. In winter, the landmass is cooler than the ocean. Winds blow from the land to ocean. These winds take up moisture as they pass the Bay of Bengal and cause rainfall in Tamilnadu and Srilanka.

11.33 RADIATION

49. *What is radiation ? Give examples.*

Radiation. *It is the process by which heat is transmitted from one place to another without heating the intervening medium.*

When we stand near a fire, we feel warmth because of the heat we receive by the process of radiation. The heat from the sun reaches the earth by the process of radiation, covering millions of kilometers of the empty space or vacuum.

For Your Knowledge

▲ The word *radiation* is used in two meanings. It refers to the process by which the energy is emitted by a body, is transmitted in space and falls on another body. It also refers to the energy itself which is being transmitted in space.

11.34 PREVOST'S THEORY OF HEAT EXCHANGE*

50. *Write the main features of Prevost's theory of heat exchange. How does this theory lead to the fact that good absorbers are good radiators ?*

Prevost's theory of heat exchange. Much before the nature of heat radiation was understood, the Swiss physicist *Pierre Prevost* in 1792 aptly described radiation as a mode of heat transfer. This theory is known as the *theory of heat exchange.* The salient features of this theory are

(*i*) *All bodies at temperatures above 0 K emit heat to the surroundings and gain heat from the surroundings at all times.*

(*ii*) *The amount of heat radiated per second depends on the nature of the emitting surface, its surface area and its temperature.*

(*iii*) *The rate of heat radiated by a body increases with the increase of its temperature and is unaffected by the presence of surrounding bodies.*

(*iv*) *There is a continuous exchange of heat between a body and its surroundings.*

(*v*) *The rise or fall in temperature of a body is the net result of the exchange of heat between the body and the surroundings.*

(*vi*) *The exchange of heat between a body and its surrounding continues till a dynamic thermal equilibrium is established between them and their temperatures become equal.*

When we stand near a fire, we feel the sensation of warmth, because our body is receiving more energy from the fire than it is losing by its own radiation. Similarly, when we stand near a huge block of ice, we feel a sensation of cold because our body loses more energy by radiation than it receives from the ice which is at a temperature lower than that of our body.

When the temperature of a body is equal to that of its surroundings, it radiates heat to the surroundings at the same rate at which it absorbs. The body is then in the state of *dynamic equilibrium.* In this state, if a body absorbs a large fraction of the total heat falling upon it, it must radiate the same amount of heat back to the surroundings, otherwise its temperature will change. This shows that *a body which is a good absorber is also a good radiator of heat and vice-versa.*

11.35 NATURE AND PROPERTIES OF THERMAL RADIATION

51. *What are electromagnetic waves ? In what respect is the thermal radiation different from light ?*

Electromagnetic waves. *These are the waves constituted by oscillating electric and magnetic fields. The oscillations of the two fields are mutually perpendicular to each other as well as to the direction of propagation of the waves.*

Every body at any temperature emits electromagnetic waves. These waves can have different wavelengths. Light is an electromagnetic wave. Visible light wavelengths extend from 4000 Å (violet) to 7500 Å (red). Beyond the red of the electromagnetic spectrum are the *infrared waves* having wavelengths from 1 μm to 100 μm which produce heating effect. These radiations emitted by hot bodies are also called *thermal radiations.* Much beyond the red end are the *medium wavelength radiowaves* (200 m to 500 m) and *short wavelength radiowaves* (20 m to 50 m). All electromagnetic waves travel through vacuum with a speed of 3×10^8 ms^{-1}.

The atoms or molecules of a substance can be excited to higher energy states by thermal collisions or by some other means. When such atoms or molecules de-excite to lower energy states, electromagnetic radiations are emitted.

52. *What is thermal radiation ? Give its important properties.*

Thermal radiation. *The electromagnetic radiation emitted by a body by virtue of its temperature is called thermal radiation or radiant energy.* All bodies having temperature above 0 K emit thermal radiation continuously. For example, the radiation emitted by red-hot iron or light from a filament lamp is thermal radiation.

Properties of thermal radiation :

(*i*) These are electromagnetic waves having wavelength range from 1 μm to 100 μm. These are also called infrared waves.

(*ii*) Like light, thermal radiations travel in straight lines.

(*iii*) These radiations obey the laws of reflection and refraction like light does.

(*iv*) They show the phenomena of interference, diffraction and polarisation.

(*v*) Thermal radiations produce heat when they are absorbed by a body.

11.36 NEWTON'S LAW OF COOLING

53. *State Newton's law of cooling. Express it mathematically. How can this law be verified experimentally ?*

Newton's law of cooling. The rate at which a body loses heat by radiation depends on (*i*) the temperature of the body, (*ii*) the temperature of the surrounding medium, and (*iii*) the nature and extent of the exposed surface.

Newton's law of cooling *states that the rate of cooling (or rate of loss of heat) of a body is directly proportional to the temperature difference between the body and its surroundings, provided the temperature difference is small.*

This is in accordance with Newton's law of cooling that a hot water bucket cools fast initially until it gets lukewarm after which it stays so for a longer time.

Mathematical expressions for Newton's law of cooling. Consider a hot body at temperature T. Let T_0 be the temperature of its surroundings. According to Newton's law of cooling,

Rate of loss of heat ∝ Temperature difference between the body and its surroundings

or $$-\frac{dQ}{dt} \propto (T - T_0)$$

or $$-\frac{dQ}{dt} = k\,(T - T_0) \qquad ...(1)$$

where k is a proportionality constant depending upon the area and nature of the surface of the body.

Let m be the mass and c the specific heat of the body at temperature T. If the temperature of the body falls by small amount dT in time dt, then the amount of heat lost is

$$dQ = mc\,dT$$

∴ Rate of loss of heat is given by

$$\frac{dQ}{dt} = mc\frac{dT}{dt}$$

Combining the above equations, we get

$$-mc\frac{dT}{dt} = k\,(T - T_0)$$

or $$\frac{dT}{dt} = -\frac{k}{mc}(T - T_0) = -K\,(T - T_0) \qquad ...(2)$$

where $K = k/mc$ is another constant. The negative sign indicates that as the time passes, the temperature of the body decreases. The above equation can be written as

$$\frac{dT}{T - T_0} = -K\,dt$$

On integrating both sides, we get

$$\int \frac{1}{T - T_0}\,dT = -K\int dt$$

or $$\log_e (T - T_0) = -Kt + c \qquad ...(3)$$

or $$T - T_0 = e^{-Kt + c}$$

or $$T = T_0 + e^c e^{-Kt}$$

or $$T = T_0 + C\,e^{-Kt} \qquad ...(4)$$

where c is a constant of integration and $C = e^c$. Equations (1), (2), (3) and (4) are the different mathematical representations for Newton's law of cooling. Using equation (4), one can calculate the time of cooling of a body through a particular range of temperature.

If we plot a graph by taking different values of temperature difference $\Delta T = T - T_0$ along y-axis and the corresponding values of t along x-axis, we get a curve of the form shown in Fig. 11.26. It clearly shows that the rate of cooling is higher initially and then decreases as the temperature of the body falls.

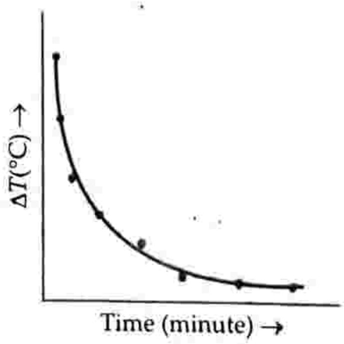


Fig. 11.26 Curve showing cooling of hot water with time.

Moreover, the equation (3) is of the form $y = mx + c$. So if we plot a graph, by taking $\log_e (T - T_0)$ along y-axis and time t along x-axis, we must get a straight line, as shown in Fig. 11.27. It has a negative slope equal to $-K$ and intercept on y-axis equal to c.

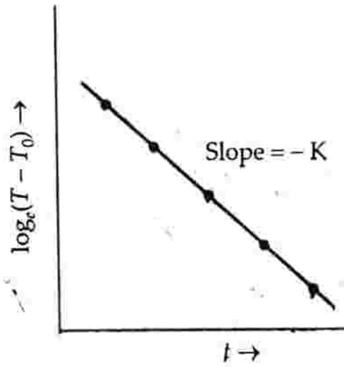


Fig. 11.27 Straight line graph between $\log_e (T - T_0)$ and time t.

In both of the above situations, Newton's law of cooling stands verified.

Experimental verification of Newton's law of cooling. The experimental set-up used for verifying Newton's law of cooling is shown in Fig. 11.28. The set-up consists of a double walled vessel (V) containing water in between the two walls. A copper calorimeter (C) containing hot water is placed inside the double walled vessel. Two thermometers through the corks are used to note the temperatures T of hot water in calorimeter and T_0 of water in between the double walls respectively.

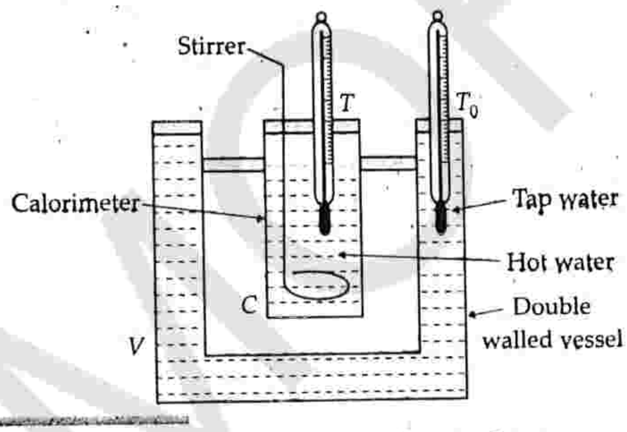


Fig. 11.28 Experimental set-up for verifying Newton's law of cooling.

Temperature of hot water in the calorimeter is noted after fixed intervals of time, say after every one minute of stirring the water gently with a stirrer. Continue noting its temperature till it attains a temperature about 5°C above that of surroundings. Plot a graph between $\log_e (T - T_0)$ and time (t). The nature of the graph is observed to be a straight line, having a negative slope, as shown in Fig. 11.27. This verifies newton's law of cooling.

Examples based on Newton's Law of Cooling

FORMULAE USED

Newton's law of cooling. If the temperature difference between body and surroundings is small, then

Rate of loss of heat $\propto$ Temperature difference from the body

$\therefore$ Rate of loss of heat from the body is

$$mc\frac{(T_1 - T_2)}{t} = k(T - T_0) = k\left(\frac{T_1 + T_2}{2} - T_0\right)$$

Here temperature of the body falls from T_1 to T_2 in time-interval t.

UNITS USED

Temperatures T_1, T_2 and T_0 are in °C or K.

EXAMPLE 29. *A body cools in 7 minutes from 60°C to 40°C. What will be its temperature after the next 7 minutes ? The temperature of the surroundings is 10°C. Assume that Newton's law of cooling holds good throughout the process.* **[REC 92]**

Solution. *In first case.* $T_1 = 60°$, $T_2 = 40°C$,

$T_0 = 10°C$, $t = 7$ min $= 420$ s

According to Newton's law of cooling,

$$mc\frac{T_1 - T_2}{t} = K\left(\frac{T_1 + T_2}{2} - T_0\right)$$

$$\therefore \quad mc\frac{60 - 40}{420} = K\left(\frac{60 + 40}{2} - 10\right)$$

or

$$mc\frac{20}{420} = K \times 40 \qquad ...(i)$$

In second case. $T_1 = 40°C$, $T_2 = ?$, $T_0 = 10°C$, $t = 7$ min $= 420$ s

$$\therefore \quad mc\frac{40 - T_2}{420} = K\left(\frac{40 + T_2}{2} - 10\right) \qquad ...(ii)$$

Dividing equation (ii) by (i), we get :

$$\frac{20}{40 - T_2} = \frac{40}{\frac{40 + T_2}{2} - 10}$$

On solving, we get : $T_2 = 28°C$.

✖ PROBLEM FOR PRACTICE

1. A pan filled with hot food cools from 94°C to 86°C in 2 minutes when the room temperature is at 20°C. How long will it take to cool from 71°C to 69°C ? **[NCERT]** **(Ans.** 42 s)

✖ HINT

1. Proceed as in NCERT Exercise 11.22 on page 11.62.

11.37 REFLECTANCE, ABSORPTANCE AND TRANSMITTANCE

54. *Define the terms reflectance, absorptance and transmittance. How are they related ?*

Reflectance, absorptance and transmittance. When thermal radiations fall on a body, they are partly reflected, absorbed and transmitted. Let Q be the amount of radiant energy incident on a body. Suppose the part R is reflected, A is absorbed and T is transmitted. Then

$$R + A + T = Q$$

On dividing both sides by Q, we get

$$\frac{R}{Q} + \frac{A}{Q} + \frac{T}{Q} = 1$$

or $$r + a + t = 1$$

i.e. Reflectance + Absorptance + Transmittance = 1.

Reflectance. *It is defined as the ratio of the amount of thermal energy reflected by a body in a certain time to the total amount of thermal energy falling upon the body in the same time.*

$\therefore$ Reflectance, $r = \frac{R}{Q}$.

Absorptance. *It is defined as the ratio of the amount of thermal energy absorbed by a body in a certain time to the total amount of thermal energy incident upon the body in the same time.*

$\therefore$ Absorptance, $a = \frac{A}{Q}$.

Transmittance. *It is defined as the ratio of the amount of thermal energy transmitted by a body in a certain time to the total amount of thermal radiation incident on it in the same time.*

$\therefore$ Transmittance, $t = \frac{T}{Q}$.

The reflectance, absorptance and transmittance of a body depend upon

(*i*) Nature of the surface of the body.

(*ii*) Wavelength of incident radiation.

However, these quantities do not depend on the nature of the material of the body. Hence for radiations of different wavelength, a given body may have different values of reflectance, absorptance and transmittance. It is more useful to define these quantities for a given wavelength λ. Then these quantities are called ***monochromatic reflectance*** (r_λ), ***monochromatic absorptance*** (a_λ) *and* ***monochromic transmittance*** (t_λ) which are related as $r_\lambda + a_\lambda + t_\lambda = 1$.

Special cases. (*i*) If a body does not transmit radiation, $t = 0$, then $r + a = 1$.

Clearly, if r is more, a is less and vice versa. That is *good reflectors are bad absorbers and bad reflectors are good absorbers of heat.*

(*ii*) If a body neither reflects nor transmits any radiation, $r = 0$ and $t = 0$, then $a = 1$.

Such *a body which neither reflects nor transmits, but absorbs whole of the heat radiation incident on it is called a black body.*

11.38 ABSORPTIVE AND EMISSIVE POWERS

55. *Define the terms absorptive power, emissive power and emissivity.*

Absorptive power. *The absorptive power of a body for a given wavelength λ is defined as the ratio of amount of heat energy absorbed in a certain time to the total heat energy incident on it in the same time within a unit wavelength range around the wavelength λ.* It is denoted by a_λ. A perfect black body absorbs all the heat radiations incident upon it. So its absorptive power is unity.

If the radiant energy dQ in wavelength range λ and $\lambda + d\lambda$ is incident on a body of absorptive power a_λ, then amount of radiant energy absorbed by the body $= a_\lambda \, dQ$.

The absorptive power is a dimensionless quantity.

Emissive power. The amount of heat energy radiated by a body per second depends upon (*i*) the area of its surface, (*ii*) the temperature of its surface and (*iii*) the nature of its surface. The strength of emission is measured by a quantity called emissive power. The *emissive power of a body at a given temperature and for a given wavelength λ is defined as the amount of radiant energy emitted per unit time per unit surface area of the body within a unit wavelength range around the wavelength λ.*

If a heat radiation of wavelength range λ to $\lambda + d\lambda$ is incident on the surface of a body of emissive power e_λ, then the amount of radiant energy emitted per second per unit area $= e_\lambda \, d\lambda$.

The SI unit of emissive power is $\mathrm{J\,s^{-1}\,m^{-2}}$ or $\mathrm{Wm^{-2}}$.

Emissivity. *The emissivity of a body is defined as the ratio of the heat energy radiated per unit time per unit area by the given body to the amount of heat energy radiated per unit time per unit area by a perfect black body of the same temperature i.e.,* it is the ratio of the emissive power (e) of a body to the emissive power (E) of a black body at the same temperature. It is denoted by ε.

Thus $$\varepsilon = \frac{e}{E}$$

It is dimensionless quantity. Its value lies between 0 and 1. The emissivity of a perfect black body is 1. The emissivities of polished copper, polished aluminium and lamp black are 0.018, 0.05 and 0.95 respectively.

11.39 BLACK BODY

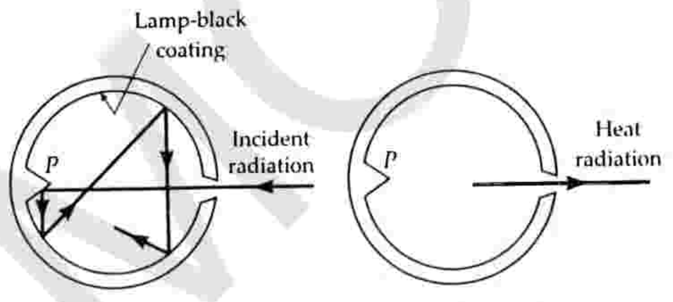
56. *What is a black body ? How can it be realised in practice ?*

Black body. *A black body is one which neither reflects nor transmits but absorbs whole of the heat radiation incident on it.* The absorptive power of a perfect black body is unity.

When a black body is heated to a high temperature, it emits radiations of all possible wavelengths within a certain wavelength range. The radiations emitted by a block body are called ***full*** or ***black body radiations***.

In practice, a surface coated with lamp black or platinum black absorbs 95 to 97% of the incident radiation. But on heating, it does not emit full radiation spectrum. So it acts as a black body only for absorption of heat radiation. It is observed that if a hollow cavity is heated, the radiation coming out from its inner surface through a small opening is a full radiation spectrum. Such a radiation is called **cavity radiation**. Hence the small opening of a heated hollow cavity acts as a perfect black body both for absorption and emission of heat radiation.

Fery's black body. Fery's black body consists of a hollow double walled metal sphere coated inside with lamp black and nickel polished from outside. Heat radiations entering the sphere through the small opening are completely absorbed due to multiple reflections. The conical projection opposite the opening prevents direct reflection.



(*a*) A black body absorber. (*b*) A black body emitter.

Fig. 11.29

To use it as a source of heat radiation, the enclosure is heated in a suitable bath to maintain its temperature constant. The radiations coming out from the small hole are black body radiations. The *wavelength range of emitted radiation is independent of the material of the body and depends only on the temperature of the black body.*

11.40 KIRCHHOFF'S LAW

57. *State and explain the Kirchhoff's law of heat radiation.*

Kirchhoff's law. *Kirchhoff's law of heat radiation states that at any given temperature, the ratio of the emissive power to the absorptive power corresponding to the certain wavelength is constant for all bodies and this constant is equal to emissive power of the perfect black body at the same temperature and corresponding to the same wavelength.*

If e_λ and a_λ are the emissive and the absorptive powers of a body corresponding to wavelength λ,

then
$$\frac{e_\lambda}{a_\lambda} = E_\lambda \text{ (constant)} \qquad ...(i)$$

where E_λ is the emissive power of perfect black body at the same temperature and corresponding to the same wavelength. Thus, if a_λ is large, then e_λ will also be large *i.e.*, if a body absorbs a radiation of certain wavelength strongly, then it will also strongly emit the radiation of that wavelength.

As the emissivity ε of a body is defined as the ratio of its emissive power to that of the emissive power of a black body at the same temperature, so

$$\frac{e_\lambda}{E_\lambda} = \varepsilon \qquad ...(ii)$$

From equations (*i*) and (*ii*), we get

$$a_\lambda = \varepsilon$$

Thus, *the absorptive power of a body is equal to its emissivity.* This is another form of ***Kirchhoff's law***.

Hence *a good absorber is a good emitter.* Since a good absorber is a poor reflector, so the ability of a body to emit radiation is related oppositely to its ability to reflect. That is, *a good emitter is a poor reflector.*

11.41 APPLICATIONS OF KIRCHHOFF'S LAW

58. *Describe some experimental observations to which Kirchhoff's law of heat radiation is applicable.*

Applications of Kirchhoff's law. According to Kirchhoff's law, a body strongly absorbs a radiation of certain wavelength, it must emit strongly the radiation of same wavelength. It is clear from the following observations :

(*i*) Take a piece of china having some dark paintings engraved on it. Heat it in a furnace to about 1000°C and then examine in a dark room immediately. The dark paintings will appear much brighter than white china. This is because the dark paintings are better absorbers and, therefore, also better emitters.

(*ii*) A green glass heated in a furnace when taken out in dark glows with red light. Green glass (when cold) is a good absorber of red light and a good reflector of green light. When heated, it becomes a good emitter of red light in accordance with Kirchhoff's law.

(*iii*) If a polished metal ball with a spot of platinum black on it is heated in a furnace to about 1200 K and then taken out into a dark room, the black spot appears brighter than the polished surface. This is because the black spot is a better absorber and hence, by Kirchhoff's law, a better emitter of radiation.

(*iv*) A Dewar flask or thermos bottle consists of a double-walled glass vessel with its inner and outer walls coated with silver. Radiation from the inner walls is reflected back into the contents of the bottle. Similarly, the outer wall reflects back any incoming radiation. The space between the walls is evacuated to reduce losses due to conduction and convection. The device helps in keeping hot contents hot and cold contents cold for a long time.

11.42 FRAUNHOFFER LINES*

59. *What are Fraunhoffer lines ? Explain their origin. What is their importance ?*

Fraunhoffer lines. *When light from the sun is seen through a spectrometer, it is found to be crossed by several dark lines which are called Fraunhoffer lines.*

Origin. The existence of Fraunhoffer lines in the solar spectrum can be satisfactorily explained on the basis of Kirchhoff's law.

The sun has a hot central core which emits continuous spectrum. The central hot core is surrounded by various elements in the vaporised state and comparatively cooler than the core. When white light from the central core passes through the elements in the vapour state, they absorb their characteristic wavelengths, thus producing dark lines in the sun's spectrum.

Importance. By comparing the wavelengths of Fraunhoffer's lines with those emitted by elements on the earth, we have identified various elements like H_2, He, N_2, O_2, Na, Fe, Cu, etc. in the atmosphere of the sun.

11.43 STEFAN-BOLTZMANN LAW

60. *State and explain Stefan-Boltzmann law of black body radiation.*

Stefan-Boltzmann law. *This law states that the total heat energy emitted by a perfect black body per second per unit area is directly proportional to the fourth power of the absolute temperature of its surface.* Thus

$$E \propto T^4 \quad \text{or} \quad E = \sigma T^4.$$

If H is the rate of radiant energy emitted by a black body of surface area A, then the Stefan-Boltzmann law takes the form

$$H = EA = \sigma T^4 A$$

Here σ is a universal constant called Stefan-Boltzmann constant. The above relation was first deduced experimentally by Stefan and later proved theoretically by Boltzmann and hence is known as Stefan-Boltzmann law.

In SI units, $\sigma = 5.67 \times 10^{-8}\ \text{J s}^{-1}\ \text{m}^{-2}\ \text{K}^{-4}$

$= 5.67 \times 10^{-8}\ \text{W m}^{-2}\ \text{K}^{-4}$

In CGS units, $\sigma = 5.67 \times 10^{-5}\ \text{erg s}^{-1}\ \text{cm}^{-2}\ \text{K}^{-4}$.

If a black body is in an enclosure at temperature T_0, then the rate at which the black body absorbs radiation from the enclosure is σT_0^4. Therefore, the net loss of energy by the black body per unit time per unit area is

$$E = \sigma (T^4 - T_0^4)$$

If the body is not a perfect black body and has emissivity ε, then above relations get modified as follows :

$$E = \varepsilon \sigma T^4$$
$$E = \varepsilon \sigma (T^4 - T_0^4).$$

61. *Derive Newton's law of cooling from Stefan's law.*

Derivation of Newton's law of cooling from Stefan's law. Suppose a body of surface area A at an absolute temperature T is kept in an enclosure at lower temperature T_0. According to Stefan-Boltzmann law, the net rate of loss of heat from the body due to radiation is

$$H = \varepsilon\sigma A(T^4 - T_0^4)$$
$$= \varepsilon\sigma A(T - T_0)(T + T_0)(T^2 + T_0^2)$$
$$= \varepsilon\sigma A(T - T_0)(T^3 + T^2T_0 + TT_0^2 + T_0^3)$$

As $T - T_0$ is small, T can be taken approximately equal to T_0.

$$\therefore \quad H = \varepsilon\sigma A(T - T_0)(T_0^3 + T_0^3 + T_0^3 + T_0^3)$$
$$= 4\varepsilon\sigma A T_0^3 (T - T_0)$$

If c is the specific heat of the body and m its mass, then the rate of fall of temperature will be

$$-\frac{dT}{dt} = \frac{H}{mc} = \frac{4\varepsilon\sigma A T_0^3}{mc}(T - T_0)$$

or $$\frac{dT}{dt} = -kA(T - T_0)$$

This prove the Newton's law of cooling.

Examples based on Stefan's Law

FORMULAE USED

1. *Stefan's law.* Energy emitted per second per unit area by a black body at absolute temperature T,
$$E = \sigma T^4, \quad \text{where } \sigma = \text{Stefan's constant.}$$
2. *Stefan-Boltzmann law.* When a black body at temperature T is placed in an enclosure at temperature T_0, the net heat energy radiated per second per unit area, $E = \sigma(T^4 - T_0^4)$
3. Energy radiated by a surface of emissivity ε, area A in time t,

(i) $E = \varepsilon \sigma T^4 \times A \times t$ (*Stefan's law*)

(ii) $E = \varepsilon \sigma (T^4 - T_0^4) \times A \times t$ (*Stefan-Boltzmann law*)

UNITS USED

Here E is in $Jm^{-2} s^{-1}$ or Wm^{-2} and T in kelvin.

CONSTANT USED

Stefan's constant,

(i) $\sigma = 5.67 \times 10^{-8}\ Js^{-1}\ m^{-2}\ K^{-4}$
$= 5.67 \times 10^{-8}\ Wm^{-2}\ K^4$

(ii) $\sigma = 5.67 \times 10^{-5}\ erg\ s^{-1}\ cm^{-2}\ K^{-4}$

EXAMPLE 30. *Calculate the temperature (in K) at which a perfect black body radiates energy at the rate of 5.67 W cm^{-2}. Given $\sigma = 5.67 \times 10^{-8}\ Wm^{-2}\ K^{-4}$.* **[Delhi 05]**

Solution. Here $E = 5.67\ W\ cm^{-2} = 5.67 \times 10^4\ Wm^{-2}$
$$\sigma = 5.67\ Wm^{-2}\ K^{-4}$$

According to Stefan's law, $E = \sigma T^4$

$$\therefore \quad T = \left(\frac{E}{\sigma}\right)^{1/4} = \left(\frac{5.67 \times 10^4}{5.67 \times 10^{-8}}\right)^{1/4}$$
$$= (10^{12})^{1/4} = 10^3 = \mathbf{1000\ K.}$$

EXAMPLE 31. *Luminosity of Rigel star in Orion constellation is 17,000 times that of our sun. If the surface temperature of the sun is 6000 K, calculate the temperature of the star.*

Solution. Let E_1 and E_2 be the luminosities and T_1 and T_2 be the absolute temperatures of the star and sun respectively. According to Stefan's law,
$$E = \sigma T^4$$
$$\therefore \quad \frac{E_1}{E_2} = \frac{\sigma T_1^4}{\sigma T_2^4} = \frac{T_1^4}{T_2^4} \quad \text{or} \quad T_1 = \left(\frac{E_1}{E_2}\right)^{1/4} T_2$$

But $\quad \frac{E_1}{E_2} = 17{,}000, \quad T_2 = 6000\ K$

$$\therefore \quad T_1 = (17000)^{1/4} \times 6000$$
$$= 11.42 \times 6000 = \mathbf{68520\ K.}$$

EXAMPLE 32. *Due to the change in mains voltage, the temperature of an electric bulb rises from 3000 K to 4000 K. What is the percentage rise in electric power consumed ?*

Solution. Electric power consumed in first case,
$$P_1 = \sigma T_1^4 = \sigma (3000)^4$$
Electric power consumed in second case,
$$P_2 = \sigma T_2^4 = \sigma (4000)^4$$
$$\therefore \quad \frac{P_2}{P_1} = \frac{(4000)^4}{(3000)^4} = \frac{256}{81}$$
$$\frac{P_2 - P_1}{P_1} = \frac{P_2}{P_1} - 1 = \frac{256}{81} - 1 = \frac{175}{81}$$
$\therefore$ Percentage rise in power
$$= \left(\frac{P_2 - P_1}{P_1}\right) \times 100 = \frac{175}{81} \times 100 = \mathbf{216.}$$

EXAMPLE 33. *Consider the sun to be a perfect sphere of radius 6.8×10^8 m. Calculate the energy radiated by the sun in one minute. Surface temperature of the sun = 6200 K. Stefan's constant = $5.67 \times 10^{-8}\ Jm^{-2}\ s^{-1}\ K^{-4}$.*

Solution. Radius of the sun, $r = 6.8 \times 10^8$ m

Surface area of the sun,
$$A = 4\pi r^2 = 4 \times 3.142 \times (6.8 \times 10^8)^2$$
$$= 5.81 \times 10^{18}\ m^2$$
Temperature, $T = 6200$ K; Time $t = 1$ min $= 60$ s

Total energy radiated by the sun in 1 min,
$$E = \sigma T^4 \times A \times t$$
$$= 5.6 \times 10^{-8} \times (6200)^4 \times 5.81 \times 10^{18} \times 60$$
$$= \mathbf{2.92 \times 10^{28}\ J.}$$

EXAMPLE 34. *At what temperature will the filament of 100 W lamp operate if it is supposed to be perfectly black body of area 1 cm^2 ?*

Given $\sigma = 5.67 \times 10^{-5}\ erg\ cm^{-2}\ s^{-1}\ K^{-4}$.

Solution. Power of lamp = 100 W = 100 Js^{-1}

$\therefore$ Rate of emission of energy,
$$E = 100 \times 10^7\ erg\ s^{-1}$$
Area, $A = 1\ cm^2$, Temperature, $T = ?$

As $\quad E = \sigma T^4 \times A$
$$\therefore \quad T^4 = \frac{E}{\sigma A} = \frac{100 \times 10^7}{5.67 \times 10^{-5} \times 1} = \frac{100 \times 10^{12}}{5.67}$$
or $$T = \left(\frac{100}{5.67}\right)^{1/4} \times 10^3 = 2.049 \times 10^3 = \mathbf{2049\ K.}$$

EXAMPLE 35. *Suppose the surface area of a person's body is 1.8 m^2 and the room temperature is 20°C. The skin temperature is 27°C and the emissivity of the skin is about 0.97 for the relevant region of electromagnetic radiation. Estimate the rate of heat radiation from the body of the person.*

Solution. Here $A = 1.8\ m^2$, $\varepsilon = 0.97$,

$T = 27 + 273 = 300\ K$, $T_0 = 20 + 273 = 293\ K$,

$\sigma = 5.67 \times 10^{-8}\ Jm^{-2}\ s^{-1}\ K^{-4}$

The rate of heat loss

$$H = Q/t = \varepsilon \sigma A (T^4 - T_0^4)$$

$$= 0.97 \times 5.67 \times 10^{-8} [(300)^4 - (293)^4]$$

$$= 72.3\ Js^{-1} = \mathbf{72.3\ W.}$$

EXAMPLE 36. A thin brass rectangular sheet of sides 15.0 cm and 12.0 cm is heated in a furnace to 600°C and taken out. How much electric power is needed to maintain the sheet at this temperature, given that its emissivity is 0.0250 ? Negletct heat loss due to convection. (Stefan-Boltzmann constant, $\sigma = 5.67 \times 10^{-8}\ W\ m^{-2}\ K^{-4}$).

[NCERT]

Solution. As the energy is radiated from both surfaces of the sheet, so

$$A = 2 \times 15.0 \times 12.0 \times 10^{-4} m^2 = 3.60 \times 10^{-2}\ m^2$$

$T = 600 + 273 = 873\ K$, $\varepsilon = 0.250$,

$\sigma = 5.67 \times 10^{-8}\ Wm^{-2}\ K^{-4}$.

The rate of heat loss by the sheet,

$$H = Q/t = \varepsilon \sigma A T^4$$

$$= 0.250 \times 5.67 \times 10^{-8} \times 3.60 \times 10^{-2} \times (873)^4$$

$$= 296\ Js^{-1} = \mathbf{296\ W.}$$

EXAMPLE 37. A spherical body with radius 12 cm radiates 450 W power at 500 K. If the radius were halved and the temperature doubled, what would be the power radiated ?

[IIT 97]

Solution. Power radiated,

$$E = A \sigma T^4 = 4\pi r^2 \sigma T^4$$

When radius is halved and temperature is doubled, power radiated becomes

$$E' = 4\pi (r/2)^2 \sigma (2T)^4 = 4 \times 4\pi r^2 \sigma T^4 = 4E$$

$$= 4 \times 450 = \mathbf{1800\ W.}$$

EXAMPLE 38. Calculate the maximum amount of heat which may be lost per second by radiation by a sphere 14 cm in diameter at a temperature of 227°C, when placed in an enclosure at 27°C.
Given Stefan's constant $= 5.7 \times 10^{-8}\ Wm^{-2}\ K^{-4}$.

Solution. Temperature of sphere,

$$T = 227 + 273 = 500\ K$$

Temperature of surroundings,

$$T_0 = 27 + 273 = 300\ K$$

Radius of sphere, $r = 7\ cm = 0.07\ m$

Area of sphere,

$$A = 4\pi r^2 = 4 \times \frac{22}{7} \times (0.07)^2 = 6.16 \times 10^{-2}\ m^2$$

According to Stefan-Boltzmann law, the net heat lost per second by the sphere,

$$E = \sigma (T^4 - T_0^4) \times A$$

$$= 5.7 \times 10^{-8} (500^4 - 300^4) \times 6.16 \times 10^{-2}\ Js^{-1}$$

$$= 5.7 \times 6.16 \times 10^{-10} \times 100^4 (5^4 - 3^4)\ Js^{-1}$$

$$= 5.7 \times 6.16 \times 10^{-2} \times 544\ Js^{-1}$$

$$= \frac{5.7 \times 6.16 \times 10^{-2} \times 544}{4.2} = \mathbf{45.48\ cal\ s^{-1}}.$$

EXAMPLE 39. Two bodies A and B are kept in evacuated vessels maintained at a temperature of 27°C. The temperature of A is 527°C and that of B is 127°. Compare the rates at which heat is lost from A and B.

Solution. Here $T_0 = 27 + 273 = 300\ K$

$$T_1 = 527 + 273 = 800\ K$$

$$T_2 = 127 + 273 = 400\ K$$

According to Stefan-Boltzmann law, the net rates of loss heat by the bodies A and B will be

$$E_A = \sigma (T_1^4 - T_0^4)$$

and $$E_B = \sigma (T_2^4 - T_0^4)$$

$$\therefore \quad \frac{E_A}{E_B} = \frac{T_1^4 - T_0^4}{T_2^4 - T_0^4} = \frac{(800)^4 - (300)^4}{(400)^4 - (300)^4} = \mathbf{23.}$$

EXAMPLE 40. How much faster does a cup of coffee cool off by one degree from 100°C than from 30°C in a room at 20°C ? Assume the coffee to act as a black body.

Solution. According to Stefan-Boltzmann law, the rate of loss of heat by the body at temperature T is given by

$$E = \sigma (T^4 - T_0^4)$$

In first case : $T = 10 + 273 = 373\ K$,

$$T_0 = 20 + 273 = 293\ K$$

$$\therefore \quad E_1 = \sigma [(373)^4 - (293)^4]. \quad ...(i)$$

In second case : $T = 30 + 273 = 303\ K$,

$$T_0 = 20 + 273 = 293\ K$$

$$\therefore \quad E_2 = \sigma [(303)^4 - (293)^4] \quad ...(ii)$$

Dividing equation (*i*) by (*ii*), we get

$$\frac{E_1}{E_2} = \frac{(373)^4 - (293)^4}{(303)^4 - (293)^4}$$

$$= \frac{[1.93568 - 0.73701] \times 10^{10}}{[0.84289 - 0.73701] \times 10^{10}}$$

$$= \frac{1.19867}{0.10588} = \mathbf{11.32.}$$

Thus the coffee cools off by one degree from 100°C about 11.32 times faster than from 30°C.

✖ Problems For Practice

1. A small hole is made in a hollow sphere whose walls are at 273°C. Find the total energy radiated per second per cm^2. Given Stefan's constant $= 5.7 \times 10^{-5}$ erg cm^{-2} s^{-1} K^{-4}. (**Ans.** 5.61×10^7 erg)

2. How much energy is radiated per minute from the filament of an incandescent lamp at 3000 K, if the surface area is 10^{-4} m^2 and its emissivity is 0.4 ? Stefan's constant $\sigma = 5.67 \times 10^{-8}$ Wm^{-2} K^{-4}.
(**Ans.** 11022.5 J)

3. A full radiator at 0°C radiates energy at the rate of 3.2×10^4 erg cm^{-2} s^{-1}. Find (*i*) Stefan's constant and (*ii*) the amount of heat radiated per second by a sphere of radius 4 cm and at a temperature of 1000°C. [**Ans.** (*i*) 5.76×10^{-5} erg cm^{-2} s^{-1} K^{-4} (*ii*) 3.042×10^{10} erg s^{-1})]

4. To what temperature must a black body be raised in order to double total radiation, if original temperature is 727°C ? (**Ans.** 916°C)

5. The temperature of a body is increased from 27°C to 127°C. By what factor would the radiation emitted by it increase ? **[IIT 90]** (**Ans.** 256/81)

6. A black body initially at 27°C is heated to 327°C. How many times is total heat emitted at the higher temperature than that emitted at lower temperature ? What is the wavelength of the maximum energy radiation at the higher temperature ? Wien's constant $= 2.898 \times 10^{-3}$ mK. (**Ans.** 16, 48300 Å)

7. An electric bulb with tungsten filament having an area of 0.25 cm^2 is raised to a temperature of 3000 K, when a current passes through it. Calculate the electrical energy being consumed in watt, if the emissivity of the filament is 0.35. Stefan's constant, $\sigma = 5.67 \times 10^{-5}$ erg s^{-1} cm^{-2} K^{-4}. If due to fall in main voltage the filament temperature falls to 2500 K, what will be wattage of the bulb ?
(**Ans.** 49.19 W, 19.38 W)

8. A sphere of radius 10 cm is hung inside an oven whose walls are at a temperature of 1000 K. Calculate total heat energy incident per second (in Js^{-1}) on the sphere. Given $\sigma = 5.67 \times 10^{-8}$ SI units.
(**Ans.** 7128 Js^{-1})

9. A body which has surface area of 5.0 cm^2 and a temperature of 727°C radiates 300 J of energy each minute. What is its emissivity ? Stefan's constant $\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4}. (**Ans.** 0.18)

10. An iron ball having a surface area of 200 cm^2 and at a temperature of 527°C is placed in an enclosure at 27°C. If the surface emissivity of iron be 0.4, at what rate is heat being lost by radiation by the ball ?
(**Ans.** 45.59 cal s^{-1})

✖ Hints

1. Here $T = 723 + 273 = 996$ K,
$$\sigma = 5.7 \times 10^{-5} \text{ erg cm}^{-2}\text{ s}^{-1}\text{ K}^{-4}$$
Total energy radiated per second per cm^2,
$$E = \sigma T^4 = 5.7 \times 10^{-5} \times (996)^4 = \mathbf{5.61 \times 10^7 \text{ erg.}}$$

2. Here $t = 1$ min $= 60$ s, $T = 3000$ K, $A = 10^{-4}$ m^2, $\varepsilon = 0.4$
Total energy radiated from the filament per minute,
$$E = \varepsilon \times \sigma T^4 \times A \times t$$
$$= 0.4 \times 5.67 \times 10^{-8} \times (3000)^4 \times 10^{-4} \times 60 \text{ J}$$
$$= \mathbf{11022.5 \text{ J.}}$$

3. Here $T = 0 + 273 = 273$ K
Energy radiated per second per unit area,
$$E = 3.2 \times 10^5 \text{ erg cm}^{-2}\text{ s}^{-1}$$
(*i*) $\sigma = \dfrac{E}{T^4} = \dfrac{3.2 \times 10^5}{(273)^4}$
$$= \mathbf{5.76 \times 10^{-5} \text{ erg cm}^{-2}\text{ s}^{-1}\text{ K}^{-4}}.$$
(*ii*) Radius of sphere, $r = 4$ cm
$\therefore$ Surface area of sphere,
$$A = 4\pi r^2 = 4 \times 3.142 \times (4)^2 \text{ cm}^2$$
Also, $T = 1000 + 273 = 1273$ K
Energy radiated per second, $E = \sigma T^4 \times A$
$$= 5.76 \times 10^{-5} \times (1273)^4 \times 4 \times 3.142 \times 4^2$$
$$= \mathbf{3.042 \times 10^{10} \text{ erg s}^{-1}}.$$

4. Here $T_1 = 727 + 273 = 1000$ K, $E_2 / E_1 = 2$
As $\dfrac{E_2}{E_1} = \dfrac{T_2^4}{T_1^4}$
$\therefore \quad 2 = \dfrac{T_2^4}{(1000)^4}$
or $\quad T_2 = (2)^{1/4} \times 1000 = 1.189 \times 1000$
$$= 1189 \text{ K} = \mathbf{916°C}.$$

5. $$\frac{E_2}{E_1} = \frac{T_2^4}{T_1^4} = \frac{(127 + 273)^4}{(27 + 273)^4} = \frac{(400)^4}{(300)^4} = \mathbf{\frac{256}{81}}.$$

6. $T_1 = 273 + 27 = 300$ K, $T_2 = 327 + 273 = 600$ K,
$b = 2.889 \times 10^{-3}$ mK
$$\frac{E_2}{E_1} = \left[\frac{T_2}{T_1}\right]^4 = \left[\frac{600}{300}\right]^4 = \mathbf{16.}$$
Also $\lambda_m = \dfrac{b}{T} = \dfrac{2.898 \times 10^{-3}}{600}$
$$= 0.483 \times 10^{-5} \text{ m} = \mathbf{48300 \text{ Å}}$$

7. (*i*) Here $A = 0.25$ cm^2 $= 0.25 \times 10^{-4}$ m^2, $T = 3000$ K, $\varepsilon = 0.35$
Energy consumed per second $= \varepsilon \times \sigma T^4 \times A$
$$= 0.35 \times 5.67 \times 10^{-8} \times (3000)^4 = 0.25 \times 10^{-4}$$
$$= 40.19 \text{ Js}^{-1} = \mathbf{40.19 \text{ W.}}$$

(*ii*) Here $T = 2500$ K,

Energy consumed per second $= \varepsilon \times \sigma T^4 \times A$

$= 0.35 \times 5.67 \times 10^{-8} \times (2500)^4 \times 0.25 \times 10^{-4}$

$= 19.38 \text{ Js}^{-1} = \mathbf{19.38\ W.}$

8. Here, $r = 10$ cm $= 0.10$ m, $T = 1000$ K

$\sigma = 5.67 \times 10^{-8}$ SI units

Heat energy incident/sec/area $= \sigma T^4$

Total heat energy incident/sec

$= \sigma T^4 \times$ area of sphere

or $E = \sigma T^4 \times 4\pi r^2$

$= 5.67 \times 10^{-8} \times (10^3)^4 \times 4 \times \frac{22}{7} (0.01)^2$

$= \mathbf{7128\ J\ s^{-1}}$.

9. Here $A = 5.0 \text{ cm}^2 = 5.0 \times 10^{-4} \text{ m}^2$,

$T = 727 + 273 = 1000$ K,

Energy radiated per sec, $E = \frac{300}{60} = 5 \text{ Js}^{-1}$

As $E = \varepsilon(\sigma T^4) \times A$

$\therefore \varepsilon = \frac{E}{\sigma T^4 \times A}$

$= \frac{5}{5.67 \times 10^{-8} \times (1000)^4 \times 5.0 \times 10^{-4}} = \mathbf{0.18}.$

10. $E = \varepsilon \times \sigma(T^4 - T_0^4)\ A$

$= 0.4 \times 5.7 \times 10^{-5} \times (800^4 - 300^4) \times 200 \text{ erg s}^{-1}$

$= \frac{0.4 \times 5.7 \times (4096 - 81) \times 2 \times 10^5}{4.2 \times 10^7} = \mathbf{45.59\ cal\ s^{-1}}.$

11.44 WIEN'S DISPLACEMENT LAW

62. *State and illustrate Wien's displacement law. Give its importance.*

Wien's displacement law. The total energy radiated by a black body is not uniformly distributed over all the wavelengths but is maximum for a particular wavelength λ_m. The value of λ_m decreases with the increase of temperature.

Wien's displacement law states that the wavelength (λ_m) corresponding to which the energy emitted by a black body is maximum is inversely proportional to its absolute temperature (T). Mathematically,

$$\lambda_m \propto \frac{1}{T} \quad \text{or} \quad \lambda_m T = b$$

where b is Wien's constant. Its value is 2.9×10^{-3} mK.

Illustration. When an iron piece is heated in a hot flame, its colour first becomes dull red, then reddish yellow and finally white. This observation is an accordance with Wien's law because with increasing temperature, the emission of energy is maximum corresponding to smaller wavelength.

Importance. Wien's law can be used to estimate the surface temperatures of the moon, sun and other stars. Light from the moon shows a maximum of intensity at $\lambda_m = 14\ \mu$m. By applying Wien's law, the temperature of the surface of the moon turns out to be 200 K. Similarly, solar radiation shows a maximum at $\lambda_m = 4753$ Å. This corresponds to a surface temperature of 6060 K.

Examples based on Wien's Displacement Law

FORMULAE USED

1. *Wien's displacement law* : The wavelength λ_m corresponding to maximum energy emission by a black body at absolute temperature T is given by

$$\lambda_m = \frac{b}{T}$$

where b = Wien's constant = 0.002898 mK

UNITS USED

Wavelength λ_m is in metre, temperature T in kelvin.

EXAMPLE 41. *Wavelength corresponding to E_{max} for the moon is 14 microns. Estimate the surface temperature of the moon, if $b = 2.884 \times 10^{-3}$ mK.*

Solution. Here $\lambda_m = 14$ microns $= 14 \times 10^{-6}$ m,

$b = 2.884 \times 10^{-3}$ mK

By Wien's law,

$$T = \frac{b}{\lambda_m} = \frac{2.884 \times 10^{-3}}{14 \times 10^{-6}} = \mathbf{206\ K.}$$

EXAMPLE 42. *The surface temperature of a hot body is 1227°C. Find the wavelength at which it radiates maximum energy. Given Wien's constant = 0.2898 cm K.*

Solution. Here $T = 1227 + 273 = 1500$ K,

$b = 0.2898$ cm K

By Wien's law,

$$\lambda_m = \frac{b}{T} = \frac{0.2898}{1500}$$

$= 19320 \times 10^{-8}$ cm $= \mathbf{19320}$ **Å**.

EXAMPLE 43. *The spectral energy distribution of the sun has a maximum at 4753 Å. If the temperature of the sun is 6050 K, what is the temperature of a star for which this maximum is at 9506 Å ?*

Solution. Here $\lambda_m = 4753$ Å, $T = 6050$ K,

$\lambda'_m = 9506$ Å, $T' = ?$

By Wien's law, $\lambda_m T = \lambda'_m T'$

$$\therefore \quad T' = \frac{\lambda_m T}{\lambda'_m} = \frac{4753 \times 6050}{9506} = \mathbf{3025\ K.}$$

EXAMPLE 44. *An indirectly heated filament is radiating maximum energy of wavelength 2.16×10^{-5} cm. Find the net amount of heat energy lost per second per unit area, the temperature of surrounding air is 13°C. Given $b = 0.288$ cm K, $\sigma = 5.77 \times 10^{-5}$ erg s^{-1} cm^{-2} K^{-4}.*

Solution. Here $b = 0.288$ cm K, $\lambda_m = 2.16 \times 10^{-5}$ cm

By Wien's law, $\lambda_m T = b$

or $$T = \frac{b}{\lambda_m} = \frac{0.288}{2.16 \times 10^{-5}} = 13333.3 \text{ K}$$

Also, $\sigma = 5.77 \times 10^{-5}$ erg s^{-1} cm^{-2} K^{-4}

Temperature of surrounding,

$$T_0 = 13 + 273 = 286 \text{ K}$$

$\therefore$ The net amount of heat energy lost per second per unit area,

$$E = \sigma (T^4 - T_0^4) = 5.77 \times 10^{-5} [(13333.3)^4 - (286)^4]$$
$$= 5.77 \times 10^{-5} [3.161 \times 10^{16} - 6.69 \times 10^9]$$
$$= 5.77 \times 10^{-5} \times 3.161 \times 10^{16}$$
$$= \mathbf{1.824 \times 10^{12} \text{ erg s}^{-1} \text{ cm}^{-2}}.$$

PROBLEMS FOR PRACTICE

1. The sun radiates maximum energy at wavelength 4753 Å. Estimate the surface temperature of the sun, if $b = 2.888 \times 10^{-3}$ mK. (**Ans.** 6076 K)
2. The temperature of an ordinary electric bulb is around 3000 K. At what wavelength will it radiate maximum energy ? Will this wavelength be within visible region ? Given $b = 0.288$ cm K. (**Ans.** 9600 Å, No)
3. A furnace is at a temperature of 2000 K. At what wavelength will it radiate maximum intensity ? Is it in the visible region ? (**Ans.** 14400 Å, No)

11.45 DISTRIBUTION OF ENERGY IN THE BLACK BODY SPECTRUM

63. *Explain the distribution of energy in the spectrum of a black body. What conclusions can be drawn from it ?*

Energy distribution in a black body spectrum. When a black body is heated, it emits heat radiations of different wavelengths. When these wavelengths are arranged in the increasing order, we get black body spectrum. Radiation of a particular wavelength has a definite energy at a particular temperature. Fig. 11.30 shows the experimental curves drawn between the wavelength λ and intensity of radiation E_λ (energy per second per unit area) emitted by a black body maintained at different constant temperatures.

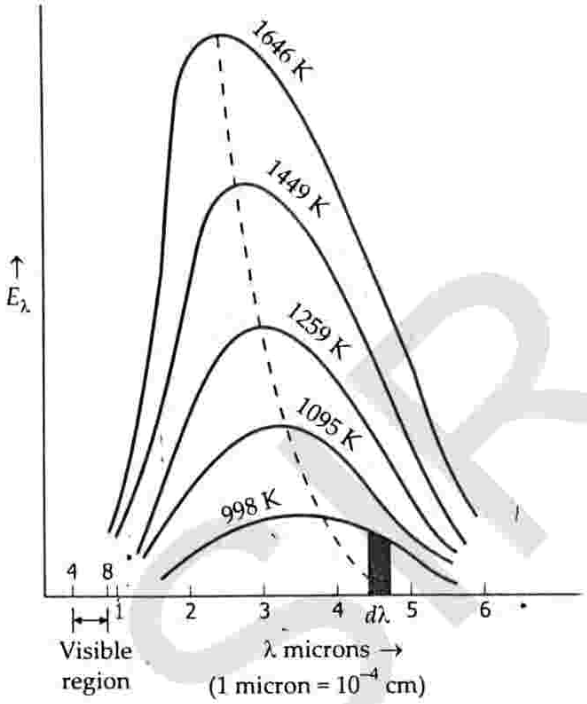


Fig. 11.30 Energy distribution in a black body spectrum

Conclusions drawn from the black body spectrum :

(*i*) At each temperature, a black body emits continuous heat radiation spectrum. The energy is not distributed equally amongst all wavelengths.

(*ii*) The energy associated with a radiation of particular wavelength increases with the increase in temperature.

(*iii*) As wavelength increases, the energy emitted increases, reaches a maximum for a particular wavelength λ_m and then decreases.

(*iv*) The wavelength (λ_m) of maximum emission shifts towards the lower wavelength side as the temperature of the black body increases.

$$\lambda_m T = \text{a constant}$$

This is ***Wien's displacement law.***

(*v*) Area under a curve represents the total energy (E) emitted by a perfect black body per second per unit area over the complete wavelength range at that temperature. This area is found to increase with fourth power of absolute temperature. Thus $E \propto T^4$.

This is ***Stefan-Boltzmann law*** of heat radiation.

The black body radiation curves are *universal*. They depend only on the temperature and not on the size, shape or material of the black body.

11.46 PHASES AND PHASE DIAGRAMS*

64. *Draw isotherms for water at different temperatures both above and below its critical temperature. What important conclusions can be drawn from them? Define the three critical constants.*

Isoterms of water. *A graph drawn between the pressure and volume of a system at constant temperature is called an isotherm.* Fig. 11.31 shows few isotherms for water-steam system in the temperature range 350° – 390°.

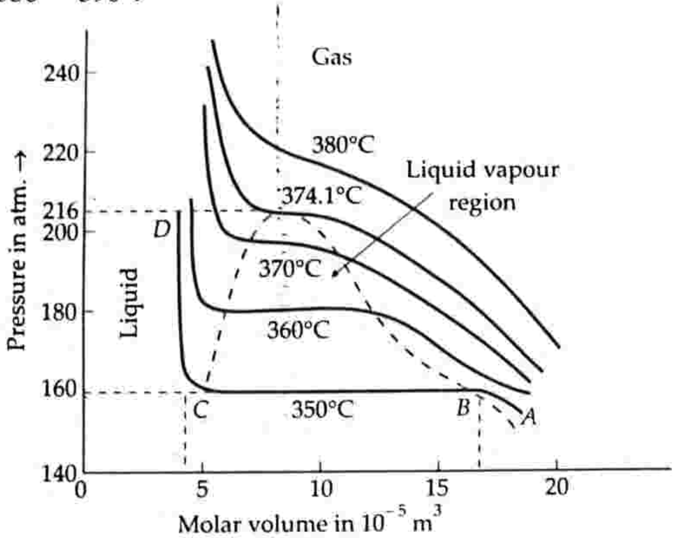


Fig. 11.31 Water-steam phase diagram for one mole of H_2O near the critical point C.

Consider the isotherm $ABCD$ at 350°C. AB represents the vapour phase (steam) which is compressible. This means when pressure is increased from A to B, the volume is decreased. Here steam is at 350°C and this is possible only at high pressure of about 160 atmosphere. From B to C, the pressure remains constant. At B the substance is in vapour state and at C it is in liquid state. Thus along BC liquid and vapour coexist in equilibrium.

Let V_l and V_g be the molar volumes of water in liquid and gaseous phases respectively. If V is the total volume of the system, then the fractions of the volume in liquid and gaseous phases will be

$$x_l = \frac{V_g - V}{V_g - V_l} \quad \text{and} \quad x_g = 1 - x_l$$

When pressure becomes more than 163 atm, the substance is in the liquid state (water). Along CD the pressure is increased. There is almost no change in volume. This shows that the liquids are incompressible.

As the temperature is increased, the volume difference $V_l - V_g$ decreases and at a temperature 374.1°C and pressure 216 atm, $V_l - V_g = 0$. For this isotherm there is no horizontal portion. This temperature is called the critical temperature (T_c) of water. This means that water can exist as liquid till 374.1°C only, there is only one phase *i.e.* vapour. This means if a gas is above critical temperature whatever pressure is applied we cannot liquify it.

Critical constants. *The critical temperature, T_c is the temperature below which a gas can be liquified by the application of pressure. The pressure required is called critical pressure P_c and the volume occupied by unit mass of the gas at critical temperature and critical pressure is called critical volume V_c.*

65. *Draw a labelled P-T diagram of water. Explain its behaviour, when both pressure P and temperature T are above and below the triple point. Give the importance of triple point.*

Pressure-temperature phase diagram for water. Fig. 11.32 shows the *P-T* phase diagram for water. It consists of the following *three* curves :

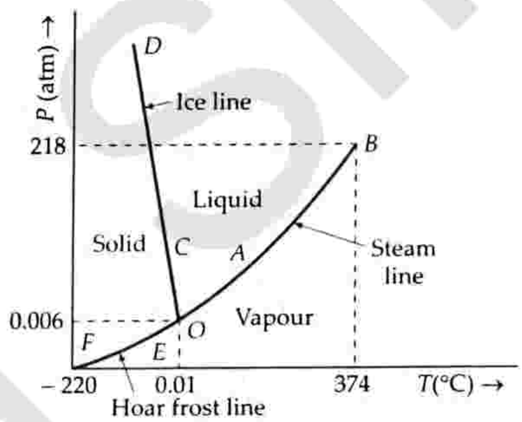


Fig. 11.32 Pressure-temperature phase diagram for water.

(*i*) **Vaporisation curve (Steam line *AB*).** It is a graph between pressure and the boiling point of the substance in the liquid state. Each point on this curve fixes a set of pressure and temperature at which the liquid and the gaseous phases can co-exist. If the pressure is increased, the vapour will at once condense into liquid but if the pressure is decreased, the liquid will evaporate. So, all points above the vaporisation curve correspond to liquid phase and below it to vapour phase.

(*ii*) **Fusion curve (Ice line *CD*).** *It is a graph between the pressure and the melting point of the substance in the solid state.* Each point on this curve gives the value of the pressure and the temperature at which the solid and liquid phases can co-exist. If pressure is increased, the solid would melt into liquid but if the pressure is decreased liquid will turn into solid. So all the points above the fusion curve correspond to liquid phase and those below it to solid phase.

(*iii*) **Sublimation curve (Hoar frost line *EF*).** *It is a graph between pressure and temperature at which a solid directly changes to vapour state.* Each point on this curve gives the values of pressure and temperature at which the solid and vapour phases can co-exist. If pressure is increased, the vapour changes to solid phase and if the pressure is decreased, the solid changes to vapour state. So all the points above this curve correspond to solid phase while those below it correspond to vapour state.

Conclusions. (*i*) In the space above the steam line and on the right of ice-line, water exists in liquid phase as water.

(*ii*) In the space below the steam line and on the right of hoar frost line, water exists in gaseous phase as steam.

(*iii*) In the space above the hoar-frost line and on the left of ice-line, water exists in solid phase as ice.

Triple point. *It is a unique point on P-T diagram at which all the three phases of a substance can co-exist in equilibrium with each other.* The three curves *AB*, *CD* and *EF* on being extended meet at point *O* which represents the triple point. The values of pressure and temperature corresponding to this point for water are 0.46 cm of Hg and 273.16 K.

The negative slope of ice line for water indicates that melting point of ice decreases with the increase in pressure. The triple point of such substances is above its melting point at normal pressure.

66. *Draw a labelled P-T diagram for CO_2. Explain, its behaviour, when both pressure and temperature are above and below the triple point. Give importance of triple point.*

Pressure-temperature phase diagram for CO_2.

Figure 11.33 shows the *P-T* phase diagram for CO_2. It consists of the following *three* curves :

(*i*) **Vaporisation curve (*AB*).** Above *AB* the substance is in liquid phase and below it in vapour phase. Along *AB*, we get a set of values for *P* and *T* for which the liquid and the vapour co-exist. This graph shows the variation of boiling point with pressure. For all substances boiling point increases with pressure.

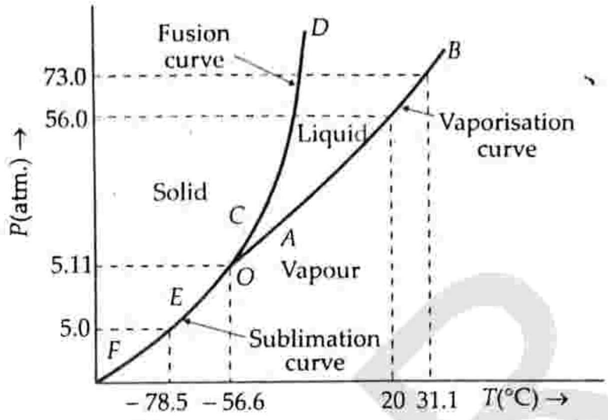


Fig. 11.33 Pressure-temperature diagram for CO_2.

(*ii*) **Fusion curve (*CD*).** Above *CD* the substance is in the solid state and below it in the liquid state. Along *CD* solid and liquid co-exist *i.e.* the substance melts and the temperature is called melting point. The graph shows that the melting point of the substance increases with increase in pressure.

(*iii*) **Sublimation curve (*EF*).** Above *EF* the substance is in solid state and below in vapour state. Along *EF*, the solid and the vapour co-exist. Here the solid changes directly to vapour state and process is called sublimation.

The three curves meet at *O*. This is called the *triple point* of CO_2. The values of pressure and temperature corresponding to triple point for CO_2 are 5.11 atm and 216.4 K.

Here all the three curves have positive slopes. So the triple point of CO_2 is below its melting point at the normal pressure.

Very Short Answer Conceptual Problems

Problem 1. Is temperature a macroscopic or microscopic concept ?

Solution. Temperature is macroscopic concept. It is related to the average kinetic energy of a large number of molecules forming a system. It is not possible to define the temperature for a single molecule.

Problem 2. What is dynamical theory of heat ?

Solution. According to dynamical theory, the amount of heat possessed by a body is equal to the total kinetic energy of its molecules and its temperature is proportional to the average kinetic energy of its molecules.

Problem 3. Two thermometers are constructed in the same way except that one has a spherical bulb and the other an elongated cylindrical bulb. Which of the two will respond quickly to temperature changes ?

Solution. A cylindrical bulb has a greater surface area than a spherical bulb of the same volume. Hence the thermometer with elongated bulb will respond to temperature changes more quickly than the one with a spherical bulb.

Problem 4. Why a clinical thermometer should not be sterilized by boiling ?

Solution. The range of clinical thermometer is usually from 95° F to 110° F and the boiling point of water is 212° F. So on sterilization by boiling, the capillary of thermometer will burst due to thermal expansion of mercury in the capillary.

Problem 5. Why should a thermometer bulb have a small heat capacity ?

Solution. The thermometer bulb having small heat capacity will absorb less heat from the body whose temperature is to be measured. Hence the temperature of that body will practically remain unchanged.

Problem 6. What do you mean by triple point of water ? Why it is unique ?

Solution. It is the temperature at which the three phases of water : ice, liquid water and water vapour are equally stable and exist simultaneously. It is unique because it occurs at specific temperature (= 273.16 K) and a specific pressure of about 0.46 cm of Hg column.

Problem 7. Why are gas thermometers are more sensitive than mercury thermometers ?

Solution. The coefficient of expansion of a gas is very large as compared to the coefficient of expansion of mercury. For the same temperature range, a gas would undergo a much larger change in volume as compared to mercury.

Problem 8. Can the temperature of a body be negative on the kelvin scale ?

Solution. No. This is because the absolute zero on the kelvin scale is the minimum possible temperature.

Problem 9. Mercury boils at 357°C. How can then a mercury thermometer be used to measure temperature upto 500°C ?

Solution. The space above the mercury is filled with nitrogen. This increases the boiling point of mercury and raises the upper limit of the mercury thermometer.

Problem 10. Why the temperatures above 1200°C cannot be measured accurately by a platinum resistance thermometer ?

Solution. This is because platinum begins to evaporate above 1200°C.

Problem 11. Why is a constant volume gas thermometer preferred as a standard thermometer than a constant pressure gas thermometer ?

Solution. This is because the changes in pressure can be measured with greater accuracy than changes in volume.

Problem 12. A mercury thermometer is transferred from melting ice to a hot liquid. The mercury rises 0.9 of the distance between lower and upper fixed points. What is the temperature of the liquid in °C ? What in °F ?

Solution. $t_C = \frac{l_t - l_0}{l_{100} - l_0} \times 100°\text{C} = \frac{0.9 - 0}{1.0 - 0} \times 100 = \mathbf{90°C}.$

$$t_F = \frac{9}{5} t_C + 32 = \frac{9}{5} \times 90 + 32 = \mathbf{194°F}.$$

Problem 13. The readings of air thermometer at 0°C and 100°C are 50 cm and 75 cm of mercury column respectively. What is the temperature at which its reading is 80 cm of Hg column ?

Solution. $t_C = \frac{P_t - P_0}{P_{100} - P_0} \times 100 = \frac{80 - 50}{75 - 50} \times 100 = \mathbf{120°C}.$

Problem 14. Two bodies at different temperatures T_1 and T_2, if brought in thermal contact do not necessarily settle at the mean temperature $(T_1 + T_2)/2$. Why ?

Solution. The two bodies may have different masses and different materials *i.e.*, they may have different thermal capacities. In case the two bodies have equal thermal capacities they would settle at the mean temperature $(T_1 + T_2)/2$.

Problem 15. A body at higher temperature contains more heat, comment.

Solution. The statement is not always true. The heat content of a body depends upon its mass, specific heat and temperature.

Problem 16. Two hollow glass balls are connected by a tube, which has a pellet of mercury in the middle. Can the temperature of the surrounding air be determined from the position of the drop ?

Solution. Yes. If the tube is held vertically, the position of the pellet will change with any change in the temperature of the surrounding air. This can be used as a thermometer.

Problem 17. Do all solids expand on heating ? If not, give an example.

Solution. No. Camphor contracts on heating.

Problem 18. Why does a solid expand on heating ?

Solution. The average distance between the positions of equilibrium of the atoms of a solid increases with an increase in temperature which results in the thermal expansion of a solid.

Problem 19. Is the temperature coefficient always positive ?

Solution. No. Temperature coefficient α is positive for metals and alloys and negative for semiconductors and insulators.

Problem 20. The diameters of steel rods *A* and *B* having the same length are 2 cm and 4 cm respectively. They are heated through 100°C. What is the ratio of increase of length of *A* to that of *B* ?

Solution. 1 : 1. This is because the increase in length does not depend on the diameter of the steel rod.

Problem 21. The difference between lengths of a certain brass rod and that of a steel rod is claimed to be constant at all temperatures. Is this possible ?

Solution. Yes. This is possible when the lengths of the rods are in inverse ratio of their coefficients of linear expansion. For the difference in the lengths of the two rods to remain same,

$$\Delta l_1 = \Delta l_2 \quad \text{or} \quad l_1 \alpha_1 \Delta T = l_2 \alpha_2 \Delta T \quad \text{or} \quad \frac{l_1}{l_2} = \frac{\alpha_2}{\alpha_1}.$$

Problem 22. Why a small gap is left between the iron rails of railway tracks ?

Solution. If no gap is left between the iron rails, the rails may bend due to expansion in summer and the train may get derailed.

Problem 23. Why are loops provided in long metal pipes used for carrying oil and any other liquid over long distances ?

Solution. The loops in metal pipes are provided in order to avoid the strain that would develop in the pipes when the temperature changes and hence the pressure of liquids in them changes.

Problem 24. Pendulum clocks generally run fast in winter and slow in summer. Why ?

Solution. The time period of a simple pendulum is given by

$$T = 2\pi\sqrt{\frac{l}{g}} \quad i.e., \quad T \propto \sqrt{l}.$$

In winter, l decreases with the fall in temperature, so T decreases and clocks run fast. In summer l increases with the increase in temperature, so T increases and clock runs slow.

Problem 25. Why is invar used in making a clock pendulum ?

Solution. Invar, which is an alloy of nickel and steel, has extremely small temperature coefficient of expansion and so the length of a pendulum made of invar does not change appreciably during summer and winter seasons and hence the clock gives almost correct time.

Problem 26. Why must telephone or power lines necessarily slag a little ?

Solution. The sag is allowed for contraction in winter. If no sag is allowed, the wire may snap in extremely cold weather.

Problem 27. A brass disc fits snugly in a hole in a steel plate. Should we heat or cool the system to loosen the disc from the hole ?

Solution. α for brass is greater than that for steel. On cooling, the disc shrinks to a greater extent than the hole, and hence the disc would get loosened.

Problem 28. A tightened glass stopper can be taken out easily by pouring hot water around the neck of the bottle. Why ?

Solution. The neck expands but not the stopper due to poor conductivity of glass. Thus the stopper can be taken out easily.

Problem 29. A long cylindrical vessel having linear coefficient of expansion α is filled with a liquid up to a certain level. On heating, it is observed that the length of the liquid in the cylinder remains the same. What is the volume coefficient of expansion of the liquid ?

Solution. Since the level of liquid remains the same, therefore, the volume coefficient of expansion of the liquid is the same as the volume coefficient of expansion of the cylinder. So, the volume coefficient of expansion of the liquid is 3α.

Problem 30. Thick bottomed drinking glasses frequently crack if hot water is poured into them. Why ?

Solution. Glass is a bad conductor of heat. It does not pass down the heat quickly to the lower surface. Different layers of the bottom are at different temperatures and expand differently. This causes breakage of the glass at the bottom.

Problem 31. Two identical rectangular strips of copper, and the other of steel are riveted to form a bimetallic strip. What will happen on heating ?

Solution. Since α for copper is more than α for steel, hence on heating, the bi-metallic strip will bend in such a way that the copper strip remains on outer or convex side.

Problem 32. Why iron rims are heated red hot before being put on the cart wheels ?

Solution. The iron ring to be put on the rim of a cart wheel is always of slightly smaller diameter than that of the wheel. When the iron ring is heated to become red hot, it expands and slips on to the wheel easily. When it is cooled, it contracts and grips the wheel firmly.

Problem 33. In riveting boiler plates, red hot rivets are used. Why ?

Solution. Red hot rivets are used so that on cooling, the grip becomes firm and steam-tight.

Problem 34. How does the diameter of the opening in the cast iron plate of a kitchen stove change, when the stove is heated ?

Solution. The opening in the stove is circular. But its diameter undergoes linear expansion on heating.

The increase in diameter is given by : $\Delta D = \alpha \ D \Delta T$.

Problem 35. A metal ball is heated through a certain temperature. Out of mass, radius, surface area and volume, which will undergo largest percentage increase and which one the least ?

Solution. The mass of the ball will not change. Its volume ($\frac{4}{3}\pi r^3$) will undergo largest percentage increase while the percentage increase in radius will be minimum.

Problem 36. Explain why a beaker filled with water at 4°C overflows if the temperature is decreased or increased ?

Solution. It is because of the anomalous expansion of water. Water has a maximum density at 4°C. Therefore, water expands whether it is heated above 4° C or cooled below 4°C.

Problem 37. A block of wood is floating on water at 0°C with a certain volume V above the level of water. The temperature of water is gradually increased from 0° C to 8°C. How does the volume V change with the change of temperature ?

Solution. The density of water increases from 0° C to 4°C and decreases from 4°C to 8°C. So, V will increase till the temperature of water reaches 4°C and then it will go on decreasing.

Problem 38. Is J a physical quantity ?

Solution. No. J is not a physical quantity. It is a conversion factor.

Problem 39. Why is J called a conversion factor ?

Solution. Because it helps us to convert work measured in terms of joules into heat expressed in calories or *vice-versa.*

Problem 40. A thermos bottle containing water is vigorously shaken. What will be the effect on the temperature of water ?

Solution. The temperature of the water will increase slightly. This is because some of the work done against opposing viscous force will be converted into heat.

Problem 41. When we rub our hands, they are warmed but only to a certain maximum temperature. Why ?

Solution. The work done in rubbing is converted into heat. But after some time when the temperature of the hands is raised, the total heat produced goes into the atmosphere.

Problem 42. There is a slight temperature difference between the water fall at the top and the bottom. Why ?

Solution. The potential energy of water at the top of the fall gets converted into heat kinetic energy at the bottom of the fall. When water hits the ground, a part of its kinetic energy gets converted into heat which increases its temperature slightly.

Problem 43. Why do the brake drums of a car get heated, when the car moves down a hill at a constant speed ?

Solution. As the car moves down a hill at constant speed, its kinetic energy does not change. But its gravitational potential energy constantly decreases, which gets converted into heat and so the brake drums get heated.

Problem 44. Can a given amount of mechanical energy be completely converted into heat ?

Solution. Yes, a given amount of mechanical work can be completely converted into heat. This is because whole of the mechanical energy can be absorbed by the molecules of a system in the form of their kinetic energy which gets converted into heat.

Problem 45. A match stick can be lighted by rubbing it against a rough surface. Why ?

Solution. When the match stick is rubbed against a rough surface, work is done against friction. This work done appears as heat and lights up the match stick.

Problem 46. If an electric fan be switched in a closed room, will the air of the room be cooled ? If not, why do we feel cold ?

Solution. The air will not be cooled. In fact, it will get heated up due to the increase in the speed of its molecules. We feel cold due to faster evaporation of sweat.

Problem 47. Give an example of a system in which no heat is transferred to or from a system but the temperature of the system changes.

Solution. When a paddle-wheel arrangement is worked while dipping in water, the temperature of water increases without any addition of heat.

Problem 48. What is the difference between the specific heat and the molar specific heat ?

Solution. The specific heat is the heat capacity per unit mass whereas the molar specific heat is the heat capacity per mole.

Problem 49. Why water is preferred to any other liquid in the hot water bottles ?

Solution. Water is preferred to any other liquid in the hot water bottles because due to high specific heat it does not cool fast. Also for the given mass of water, the amount of heat contained is higher and it can provide more warmth as compared to any other liquid.

Problem 50. The coolant used in a nuclear reactor should have high specific heat. Why ? *[Delhi 2011]*

Solution. The purpose of a coolant is to absorb maximum heat with least rise in its own temperature. This is possible only if specific heat is high because $Q = mc\,\Delta T$. For a given value of m and Q, the rise in temperature ΔT will be small if c is large. This will prevent different parts of the nuclear reactor from getting too hot.

Problem 51. Why juice bottles are placed under water in the cold countries ?

Solution. This is done so to prevent the freezing of juice. Water has to release comparatively large amount of heat to lower its temperature to the same extent than juice and hence the chances of freezing are reduced.

Problem 52. Why is water used as an effective coolant ?

Solution. The specific heat of water is very high. When it runs over hot parts of an engine or machinery, it absorbs a large amount of heat. This helps in maintaining the temperature of the engine low.

Problem 53. What kind of thermal conductivity and specific heat requirements would you specify for cooking utensils ? *[Central Schools 2004]*

Solution. A cooking utensil should have (*i*) *high conductivity* so that it can conduct heat through itself and transfer it to the contents quickly. (*ii*) *low specific heat* so that it immediately attains the temperature of the source.

Problem 54. Why do the metal utensils have wooden handles ?

Solution. Wood is a bad conductor of heat. Wooden handle does not allow heat to be conducted from the hot utensil to the hand. So we can easily hold the hot utensil with its help.

Problem 55. Why birds are often seen to swell their feathers in winter ? *[Himachal 07]*

Solution. When the birds swell their feathers, they are able to enclose air in the feathers. Air, being a poor conductor of heat, prevents the loss of heat from the bodies of the birds to the surroundings and as such they do not feel cold in winter.

Problem 56. Why an ice box is constructed with a double wall ?

Solution. An ice-box is made of double wall, and the space between the walls is filled with some non-conducting material to provide heat insulation, so that the loss of heat can be minimised.

Problem 57. Why are two thin blankets are warmer than a single blanket of double the thickness ?

Solution. The air enclosed between two blankets prevents the transfer of heat from our body to outside. Thus it provides a better insulation than a single blanket of double thickness.

Problem 58. A squirred wraps its bushy tail round its body during its winter sleep. Why ?

Solution. The bushy tail provides its body a non- conducting blanket. So the loss of heat by conduction is minimised.

Problem 59. Calorimeters are made of metals not glass. Why ?

Solution. This is because metals are good conductors of heat and have low specific heat capacity.

Problem 60. When we step barefoot into an office with a marble floor, we feel cold. Why ?

Solution. This is because marble is a better conductor of heat than concrete. When we walk barefooted on a marble floor, heat flows our body through the feet and we feel cold.

Problem 61. Why we can easily boil water in a paper cup ?

Solution. This is because heat is easily conducted through the paper to the water, and as such the temperature attained is not sufficient for the paper to be charred.

Problem 62. A piece of paper wrapped tightly on a wooden rod is observed to get charred quickly when held over a flame as compared to a similar piece of paper when wrapped on a brass rod. Explain why.

Solution. Brass is a good conductor of heat. It quickly conducts away the heat. So, the paper does not alter its ignition point easily. On the other hand, wood is a bad conductor of heat and is unable to conduct away the heat. So, the paper quickly reaches its ignition point and is charred.

Problem 63. A piece of wire gauze is placed over the Bunsen burner. If the gas is turned on below the gauge, will the flame go above the gauge ?

Solution. Copper is a very good conductor of heat. The copper gauze absorbs most of the heat. So the temperature of the gas above the gauze does not reach its ignition temperature.

Problem 64. Woolen clothes are worn in winter. Why ?

Solution. Woolen fibres enclose a large amount of air in them. Both wool and air are bad conductors of heat. The small coefficient of thermal conductivity prevents the loss of heat from our body due to conduction. So we feel warm in woolen clothes.

Problem 65. Why do we use copper gauze in Davy's safety lamp ?

Solution. In Davy's safety lamp used in mines, a copper gauze is placed around the flame of the lamp. Since the copper gauze is good conductor of heat, it absorbs heat of the flame. This keeps the temperature outside the copper gauze less than the ignition temperature and so the marsh gas does not catch fire.

Problem 66. Place a safety pin on a sheet of paper. Hold the sheet over a burning candle, until the paper becomes yellow and charr. On removing the pin, its white trace is observed on the paper. Why ?

Solution. The safety pin is made of steel which is good conductor of heat. So the safety pin takes heat from the paper under it and transfers it away to the surroundings. The portion of the paper under the safety pin remains comparatively colder than the remaining part.

Problem 67. Stainless steel cooking pans are preferred with extra copper bottom. Why ? *[Himachal 07]*

Solution. The thermal conductivity of copper is much larger than that of steel. The copper bottom allows more heat to flow into the pan and hence helps in cooking the food faster.

Problem 68. If a drop of water falls on a very hot iron, it does not evaporate for a long time. Give reason.

Solution. When a drop of water falls on a very hot iron, it gets insulated from the iron by a layer of poor conducting water vapour. As the heat is conducted very slowly through this layer, it takes quite long for the drop to evaporate. But if the drop of water falls on iron which is not very hot, then it comes in direct contact with iron and evaporates immediately.

Problem 69. Pieces of copper and glass are heated to the same temperature. Why does the piece of copper feel hotter on touching ?

Solution. Copper is much better conductor of heat than glass. When we touch the hot copper piece, heat readily flows to our hand. But this is not the case when the hot glass piece is touched.

Problem 70. Usually a good conductor of heat is a good conductor of electricity also. Give reason.

Solution. Electrons contribute largely both towards the flow of electricity and the flow of heat. A good conductor contains a large number of free electrons. So it is both a good conductor of heat and electricity.

Problem 71. Why do electrons in insulators not contribute towards its thermal conductivity ?

Solution. Insulators do not have free electrons inside them. So electrons have no contribution towards their thermal conductivity.

Problem 72. Why felt rather than air is employed for thermal insulation ?

Solution. Though air is a bad conductor of it, it transfers heat easily by convection. Felt traps air between its fibres and convection currents cannot be set up in it. This makes felt a better thermal isosulator than air.

Problem 73. If air is poor conductor of heat, why do we not feel warm without clothes ?

Solution. Although air is poor conductor of heat, it carries away heat from body due to convection when we are without clothes. Hence we feel cold.

Problem 74. Why small holes are provided at the bottom of the chimney of the lamp ?

Solution. The hot air and burnt gases rise upwards through the chimney. Fresh air enters through the holes provided at the bottom. In the absence of these holes, convection currents will not be set up and the lamp would go off.

Problem 75. Why rooms are provided with the ventilators near the roof ?

Solution. It is done so to remove the harmful impure air, and to replace it by the cool fresh air. The air we breath out is warm and so it is lighter. It rises upwards and can go out through the ventilator provided near the roof. The cold fresh air from outside enters the room though the doors and windows. Thus the convection current is set up in the air.

Problem 76. Why it is much hotter above a fire than by its side ?

Solution. Heat carried away from a fire sideways mainly by radiation. Above the fire, heat is carried by both radiation and convection of air. But convection carries much more heat than radiation. So it is much hotter above a fire than by its sides.

Problem 77. Why snow is a better heat insulator than ice ?

Solution. When the temperature of the atmosphere reaches below 0°C, the water vapours present in air freeze directly in the form of minute particles of ice. Many particles combine and take cotton-like shape which is called snow. Snow contains a large number of air pockets which prevent the formation of convection currents. Hence snow acts as a good heat insulator than ice.

Problem 78. Can we boil water inside an earth satellite ?

Solution. No. The process of transfer of heat by convection is based on the fact that a liquid becomes lighter on becoming hot and rises up. In condition of weightlessness, this is not possible. So transfer of heat by convection is not possible in a satellite.

Problem 79. Water is heated from below. Why ?

Solution. When water is heated, its density decreases and it rises up. Cooler liquid of the upper part takes its place and so convection currents are set up and water gets heated up. If heated from the top, it will conduct very small amount of heat to the bottom because water is poor conductor of heat.

Problem 80. Suppose you want to cool your drink. Should you keep ice cubes floating on the top or should you arrange to keep the ice cubes at the bottom ?

Solution. Ice cubes should be kept floating in the drink. The liquid will then cool by convection. If the ice cubes are placed at the bottom, no convection currents are set up and liquid is not cooled. Also it cannot be cooled by conduction because liquid is a poor conductor of heat.

Problem 81. Why are the cooling coils fitted near the ceiling of a refrigerator ?

Solution. As the air gets cooled in the upper part of the refrigerator, it becomes denser and goes down. The warmer air of the lower part moves up. Thus convection currents are set up. This quickly cools up the entire inside of the refrigerator.

Problem 82. After some time of the switching on an electric heater, the temperature of the heater becomes constant although current remains continuously flowing in it, why so ?

Solution. When the steady state is reached, the rate of loss of heat by conduction, convection and radiation becomes equal to the rate of production of heat in the heater due to the flow of current.

Problem 83. The earth constantly receives heat radiation from the sun and gets warmed up. Why does the earth not get as hot as the sun ?

Solution. Because the earth is located at a very large distance from the sun, hence it receives only a small fraction of the heat radiation emitted by the sun. Further, due to loss of heat from the surface of earth due to convection and radiation also, the earth does not become as hot as the sun.

Problem 84. Why do animals curl into a ball, when they feel very cold ? *[Himachal 04, 07]*

Solution. The total energy radiated by a body depends on its surface area. Thus when the animals feel very cold, they curl their bodies into a ball so as to decrease the surface area of their bodies which in turn helps to reduce the amount of heat lost by them.

Problem 85. Two thermos flasks are of the same height and same capacity. One has a circular cross-section while the other has a square cross-section. Which of the two is better ?

Solution. As both flasks have same height and capacity, the area of the cylindrical wall will be less than that of the square wall. Hence the thermos flask of circular cross-section will transmit less heat as compared to the thermos flask of square cross-section and will be better.

Problem 86. Why a body with large reflectivity is a poor emitter ?

Solution. A body whose reflectivity is large would naturally absorb less heat. So, a body with large reflectivity is a poor emitter.

Problem 87. Why does a piece of red glass when heated and taken out glow with green light ?

Solution. At low temperature, the red glass absorbs green colour strongly. But at higher temperatures, it emits green colour strongly.

Problem 88. Two stars radiate maximum energy at wavelengths 3.6×10^{-7} m and 4.8×10^{-7} m respectively. What is the ratio of their temperatures ?

Solution. Here $\lambda_m = 3.6 \times 10^{-7}$ m,

$$\lambda'_m = 4.8 \times 10^{-7} \text{ m}$$

By Wien's law, $\lambda_m T = \lambda'_m T'$

$$\therefore \quad \frac{T}{T'} = \frac{\lambda'_m}{\lambda_m} = \frac{4.8 \times 10^{-7}}{3.6 \times 10^{-7}} = \mathbf{4:3}.$$

Problem 89. If all the objects radiate electromagnetic energy, why do not the objects around us in everyday life become colder and colder ?

Solution. According to the Prevost theory of heat exchanges, all the objects (above 0 K) not only radiate electromagnetic energy but also absorb at the same rate from their surroundings. Thus they do not become colder.

Problem 90. Is it necessary that all black coloured objects should be considered black bodies ?

Solution. No. A polished black surface is not a black body because it reflects radiation incident on it. On the other hand, the sun, which is a dazzling white body, is a black body.

Problem 91. Why are clear nights colder than cloudy nights ? *[Himachal 07C]*

Solution. Clouds are opaque to heat radiations. So on a cloudy night, radiations from the earth's surface fail to escape. But on a clear night, the surface of the earth is cooled due to excessive radiation. So a clear night is colder than a cloudy night.

Problem 92. White clothes are more comfortable in summer while colourful clothes are more comfortable in winter. Why ? *[Himachal 07]*

Solution. White clothes absorb very little heat radiation and hence they are comfortable in summer. Coloured clothes absorb almost whole of the incident radiation and keep the body warm in winter.

Problem 93. Explain why cooking utensils are often blackened at the bottom and polished at the top.

Solution. Black surfaces are good absorbers of heat radiations. The bottom of the cooking utensils is blackened so that it absorbs maximum heat radiations. Polished white surfaces are bad absorbers and hence bad emitters of heat radiations. By polishing the upper parts of the cooking utensils, the loss of heat by radiation is minimised.

Problem 94. Gasoline tanks are generally painted with aluminium paint. Why ?

Solution. The shining aluminium paint is a bad absorber of heat. So the tank painted with aluminium paint on the outside is prevented from getting excessively heated in the sun.

Problem 95. A hole in the cavity of a radiator is a black body. Why ?

Solution. A hole in the cavity of a radiator does not reflect any radiation and absorbs all the radiation incident on it. So it is a black body.

Problem 96. Why is there the word displacement in Wien's displacement law ?

Solution. As the temperature is increased, the wavelength having maximum intensity is displaced towards the shorter wavelength region. Hence the word displacement is used.

Problem 97. Black body radiation is white. Comment.

Solution. True. A black body absorbs radiations of all wavelengths. When heated to a suitable temperature, it emits radiations of all wavelengths. Hence a black body radiation is white.

Problem 98. Which object will cool faster when kept in open air, the one at 300° C or the one at 100° C? Why ? *[Central Schools 03]*

Solution. The object at 300°C will cool faster than the object at 100°C. This is in accordance with Newton's law of cooling,

Rate cooling of an object $\propto$ Temperature between the object and its surroundings

Problem 99. In what respect is the thermal radiation different from light ?

Solution. Thermal radiations are electromagnetic waves having wavelength range from 1 μm to 100 μm. When they are absorbed by a body, they produce heat. Light radiations are electromagnetic waves having wavelength range from 4000 Å to 7500 Å. They produce sensation of vision.

Problem 100. What is critical temperature ?

Solution. It is the temperature of a substance in the gaseous state below which the gas can be liquified by pressure only, and above which the gas cannot be liquified. This implies that gas is simply a vapour below its critical temperature.

Problem 101. Can a gas be liquified at any temperature by the increase of pressure alone ?

Solution. No. A gas can be liquified by pressure alone, only when its temperature is below its critical temperature.

Problem 102. What is the effect of pressure on melting point of a solid ?

Solution. The melting point of a solid may increase or decrease depending on the nature of solid. For solids such as ice which contracts on melting, it is lowered while for solids such as sulphur and wax which expand on melting it increases.

Problem 103. How does the boiling point of water change with pressure ?

Solution. The boiling point of water increases with the increase in pressure.

Problem 104. What is the temperature above which steam will not condense to water even if it is compressed (isothermally) to very large pressure ?

Solution. Above the critical temperature (374.1°C for water), steam will not condense to water.

Problem 105. What is the significance of negative slope of ice line of water ?

Solution. It indicates that the melting point of ice decreases with increase in pressure (on ice line). This is because volume of water formed on melting is less than the volume of ice (before melting).

Problem 106. Are the relative amounts of ice, water and vapour fixed at the triple point water ?

Solution. At the triple-point of water, the temperature and pressure are fixed. However, the relative amounts of the three phases are not unique. The relative amounts of the three phases can be varied by adding or taking out heat from the system.

Problem 107. What happens if water vapour at a pressure of 0.004 atm is cooled to 0°C ?

Solution. The pressure corresponding to triple point is 610 Pa which is equal to nearly 0.006 atm. It follows from the *P-T* phase diagram of water that at a pressure lower than this pressure, water vapour condenses directly to ice without passing through the liquid phase.

Problem 108. Water exists in liquid phase at 30°C at 1 atmospheric pressure. How would you convert this water to vapour form without increasing its temperature ?

Solution. Water at 30°C can be converted into vapour by reducing its pressure until it equals the vapour pressure of water at 30°C. This route is along a vertical line on the *P–T* diagram, with $T = 30°C$.

Problem 109. What are the critical temperature and pressure for CO_2 ? What is their significance ?

Solution. The critical temperature and pressure of CO_2 are 31.1°C and 73.0 atm respectively. Above this temperature, CO_2 will not liquify even if compressed to high pressures.

Problem 110. Ice of 0° is converted into steam at 100° C. State the isothermal changes in the process.

Solution. The isothermal changes are

(*i*) Conversion of ice at 0°C into water at 0°C.

(*ii*) Conversion of water at 100°C into steam at 100°C.

Problem 111. Explain why a new quilt is warmer than an old one. *[Himachal 05]*

Solution. Refer answer to Q. 46(*iii*) on page 11.26.

Short Answer Conceptual Problems

Problem 1. Give reasons why water is considered unsuitable for use in thermometers.

Solution. Water is considered unsuitable for use in thermometers due to following reasons :

(*i*) The expansion of water with temperature is non-uniform.

(*ii*) Due to its large specific heat and low thermal conductivity, a water thermometer does not respond to changes in temperature quickly.

(*iii*) Water is invisible, sticks to glass and has high rate of evaporation.

(*iv*) Its temperature range is small from 0°C to 100°C

Problem 2. Give four reasons why is mercury used in thermometers.

Solution. The reasons for using mercury in a thermometer are

(*i*) Mercury has a uniform expansion over a wide range of temperature.

(*ii*) Mercury is opaque and bright, so it can be easily seen in a glass tube.

(*iii*) It does not stick to the walls of the glass tube.

(*iv*) It is a good conductor of heat and has low thermal capacity.

Problem 3. Give reasons why is a platinum wire used in a resistance thermometer.

Solution. The reasons for using a platinum wire in a resistance thermometer are : (*i*) The resistance of a platinum wire increases *uniformly* with the rise in temperature (from 200°C to 1200°C). (*ii*) It does not react chemically with other substances. (*iii*) Its melting point is quite high (1800°C).

Problem 4. Give some merits of gas thermometers over those of mercury thermometers.

Solution. Some merits of gas thermometers are (*i*) A gas thermometer is more sensitive than a mercury thermometer. (*ii*) The working of a gas thermometer is independent of the nature of the gas used. (*iii*) A gas thermometer can measure very low and very high temperatures.

Problem 5. Name the suitable thermometers to measure the following temperatures : – 80° C, 60° C, 250° C, 780° C, 2000° C.

Solution. Gas thermometer for – 80°C.

Mercury thermometer for 60°C.

Platinum resistance thermometer for 250°C and 780°C.

Total radiation pyrometer for 2000°C.

Problem 6. Suggest suitable methods for measuring the temperature of

(*i*) surface of the sun, (*ii*) surface of the earth,

(*iii*) an insect, and (*iv*) liquid helium.

Solution. (*i*) By using total radiation pyrometer.

(*ii*) By using a thermoelectric thermometer by embedding its hot junction in the earth.

(*iii*) By using a thermoelectric thermometer by touching its hot junction with the insect.

(*iv*) By using a magnetic thermometer which is based on Curie's law : the susceptibility of a peramagnetic material varies inversely with its absolute temperature.

Problem 7. Two large holes are cut in a metal sheet. If the sheet is heated, how will the diameters of the holes change ?

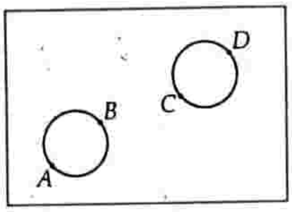


Fig. 11.34

Solution. When a body is heated, the distance between any of its two points increases. Hence the diameters AB and CD of the two holes will increase.

Problem 8. In problem 7, will the distance between the two holes increase or decrease on heating ?

Solution. When the metal sheet is heated, it expands a whole. Therefore, the holes will increase in diameter as well as move outwards. The distance BC between the two holes increases.

Problem 9. There are two spheres of same radius and material at same temperature but one being solid while the other hollow. Which sphere will expand more if (*i*) they are heated to the same temperature (*ii*) same amount of heat is given to each of them ?

Solution. (*i*) As thermal expansion of isotropic solids is similar to true photographic enlargement, the expansion of a cavity is same as if it were a solid body of the same material *i.e.*, $\Delta V = \gamma V \, \Delta T$. As here V, γ and ΔT are same for both solid and hollow spheres, so the expansions of both will be equal.

(*ii*) If same amount of heat is given to the two spheres, then due to lesser mass, rise in temperature of hollow sphere will be more (as $\Delta T = Q/Mc$) and hence the expansion will be more as $\Delta V = \gamma V \, \Delta T$.

Problem 10. Two bodies of specific heats c_1 and c_2 having same heat capacities are combined to form a single composite body. What is the specific heat of the composite body ?

Solution. As the heat capacities are equal, so $m_1 c_1 = m_2 c_2$.

Let c be the specific heat of the composite body. Then

$$(m_1 + m_2)\, c = m_1 c_1 + m_2 c_2 = m_1 c_1 + m_1 c_1 = 2 m_1 c_1$$

or
$$c = \frac{2 m_1 c_1}{m_1 + m_2} = \frac{2 m_1 c_1}{m_1 + m_1 \frac{c_1}{c_2}} = \frac{2 c_1 c_2}{c_1 + c_2}.$$

Problem 11. Two rods A and B are of equal length. Each rod has its ends at temperatures T_1 and T_2. What is the condition that will ensure equal rates of flow of heat through the rods A and B ? *[IIT]*

Solution. Let x be the length of each rod. The rates of flow of heat through the rods A and B will be equal if

$$\frac{K_1 A_1 (T_1 - T_2)}{x} = \frac{K_2 A_2 (T_1 - T_2)}{x}$$

or
$$K_1 A_1 = K_2 A_2 \quad \text{or} \quad \frac{A_1}{A_2} = \frac{K_2}{K_1}$$

Hence for equal rates of flow of heat, the areas of cross-section of the two rods should be inversely proportional to their coefficients of thermal conductivity.

Problem 12. Two vessels of different materials are identical in size and wall-thickness. They are filled with equal quantities of ice at $0°C$. If the ice melts completely in 10 and 25 minutes respectively, compare the coefficients of thermal conductivity of the materials of the vessels.

Solution. Let K_1 and K_2 be the coefficients of thermal conductivity of the materials and t_1 and t_2 be the times in which ice melts in the two vessels.

As the same quantity of ice melts in the two vessels, the quantity of heat flowed into the vessels must be same.

$$\therefore \quad Q = \frac{K_1 A (T_1 - T_2) t_1}{x} = \frac{K_2 A (T_1 - T_2) t_2}{x}$$

or
$$K_1 t_1 = K_2 t_2 \quad \therefore \quad \frac{K_1}{K_2} = \frac{t_2}{t_1} = \frac{25 \text{ min}}{10 \text{ min}} = 5 : 2.$$

Problem 13. Two vessels A and B of different materials but having identical shape, size and wall-thickness are filled with ice and kept at the same place. Ice melts at the rate of 100 g min^{-1} and 150 g min^{-1} in A and B respectively. Assuming that heat enters the vessels through the walls only, calculate the ratio of thermal conductivities of their materials.

Solution. Let m_1 and m_2 be the masses of ice melted in same time t ($= 1$ min) in vessels A and B respectively. Then the amounts of heat flowed into the two vessels will be

$$Q_1 = \frac{K_1 A (T_1 - T_2) t}{x} = m_1 L \quad ...(i)$$

$$Q_2 = \frac{K_2 A (T_1 - T_2) t}{x} = m_2 L \quad ...(ii)$$

where L is latent heat of ice. Dividing (*i*) by (*ii*), we get

$$\frac{K_1}{K_2} = \frac{m_1}{m_2} = \frac{100 \text{ g}}{150 \text{ g}} = \frac{2}{3} = 2 : 3.$$

Problem 14. Water in a closed tube is heated with one arm placed vertically above an arc lamp. Water will begin to circulate along the tube in a counterclockwise direction. Is this true or false ?

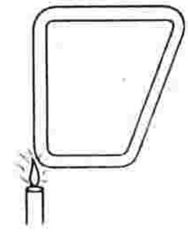


Fig. 11.35

Solution. *False.* Water will circulate in the clockwise direction. The molecules immediately above the arc receive heat by conduction. They rise up and get replaced by cold molecules from the right side. This will make the water circulate in clockwise direction.

Problem 15. A sphere, a cube and a thin circular plate, all made of the same material and having the same mass are initially heated to a temperature of 200°C. Which of these objects will cool fastest and which one slowest when left in air at room temperature ? Give reasons.

Solution. The thin circular plate has the largest surface area. The sphere has the smallest surface area. Thus the plate will radiate maximum heat while the sphere will radiate minimum heat. Hence the plate will cool fastest and the sphere will cool slowest.

Problem 16. There are two rods of the same metal, same length, same area of cross-section, but one of square cross-section and the other of circular cross-section. One end of each is kept immersed in steam. After the steady state is reached, the other ends of the rods are touched. Which one will be hotter ? Give reason.

Solution. The surface area of the rod of circular cross-section will be smaller, because for a given area a circle has least perimeter. So the loss of heat by radiation will be small. Hence the other end of the circular rod will be hotter than the other end of the square rod.

Problem 17. A solid sphere of copper of radius R and a hollow sphere of the same material of inner radius r and outer radius R are heated to the same temperature and allowed to cool in the same environment. Which of them starts cooling faster ?

Solution. Rate of loss of heat by any sphere,

$$m \times c \times \left(-\frac{dT}{dt}\right) = \sigma A (T^4 - T_0^4)$$

Now σ, A, $(T^4 - T_0^4)$ are same for both the spheres, so the rate of cooling,

$$-\frac{dT}{dt} \propto \frac{1}{m}$$

Since the hollow sphere has less mass, its rate of cooling will be faster.

Problem 18. On a hot day, a car is left in sunlight with all the windows closed. After some time, it is found that the inside of the car is considerably warmer than the air outside. Explain, why.

Solution. Glass transmits about 50% of heat radiation coming from a hot source like the sun but does not allow the radiation from moderately hot bodies to pass through it. Due to this, when a car is left in the sun, heat radiation from the sun gets into the car but as the temperature inside the car is moderate, they do not pass back through its windows. Hence, inside of the car becomes considerably warmer.

Problem 19. How does tea in a thermos flask remain hot for a long time ?

Solution. The air between the two walls of the thermos flask is evacuated. This prevents heat loss due to conduction and convection. The loss of heat due to radiation is minimised by silvering the inside surface of the double wall. As the loss of heat due to the three processes is minimised, the tea remains hot for a long time.

Problem 20. Distinguish between conduction, convection and radiation.

Solution.

	Conduction	*Convection*	*Radiation*
1.	Material medium is required.	Material medium is required.	No material medium is required.
2.	It is due to temperature difference. Heat flows from high temperature region to low temperature region.	It is due to difference in density. Heat flows from low density region to high density region.	It occurs from all bodies at temperatures above 0 K.
3.	It occurs in solids through molecular collisions, without actual flow of matter.	It occurs in fluids by actual flow of matter.	It can take place at large distances and does not heat the intervening medium.
4.	It is a slow process.	It is also a slow process.	It propagates at the speed of light.
5.	It does not obey the laws of reflection and refraction.	It does not obey the laws of reflection and refraction.	It obeys the laws of reflection and refraction.

Problem 21. A blackened platinum wire, when gradually heated, first appears dull red, then blue and finally white. Explain why.

Solution. According to Wien's displacement law, when blackened platinum wire is gradually heated, it first emits radiations of longer wavelengths, so it appears red. At higher temperatures, it emits blue radiations more strongly than red and appears blue. At very high temperatures, it emits all radiations strongly and appears white.

Problem 22. In a coal fire, the pockets formed by coals appear brighter than the coals themselves. Is the temperature of such a pocket higher than the surface temperature of a glowing coal ?

Solution. The temperature of pockets formed by coals are not appreciably different from the surface temperatures of glowing coals. However, the pockets formed by coals act as cavities. The radiations from these cavities are black body radiations and so have maximum intensity. Hence the pockets appear brighter than the glowing coals.

Problem 23. Answer the following questions :

(a) A vessel with a movable piston maintained at a constant temperature by a thermostat contains a certain amount of liquid in equilibrium with its vapour. Does this vapour obey Boyle's law ? In other words, what happens when the volume of vapour is decreased ? Does the vapour pressure increase ?

(b) What is meant by 'superheated water' and 'supercooled vapour' ? Do these states of water lie on its P-V-T surface ? Give some practical applications of these states of water.

Solution. (a) No, the vapour in equilibrium with its liquid does not obey Boyle's law. When the volume of the vapour is decreased by applying pressure, some of the vapours condense into liquid, maintaining the same pressure of the vapour at the given temperature *i.e.*, vapour pressure does not increase when the volume of vapours is decreased.

(b) **Superheated water.** Water in liquid phase having temperature above the boiling point of water at the given pressure is called superheated water. It is highly unstable stage.

Supercooled vapour. Water in vapour phase having temperature below its boiling point at the given pressure is called supercooled vapour. It is also highly unstable stage.

As the above states of water are not equilibrium states, so they do not lie on P-V-T surface of water.

Applications. These unstable states of water are used in bubble chamber and cloud chamber for detecting high speed charged particles.

Problem 24. A fat man is used to consuming about 3000 kcal worth of food everyday. His food contains 50 g of butter plus a plate of sweets everyday, besides items which provide him with other nutrients (proteins, vitamins, minerals, etc.) in addition to fats and carbohydrates. The caloric value of 10 g of butter is 60 kcal and that of a plate of sweets is of average 700 kcal. What dietary strategy should he adopt to cut down his calories to about 2100 kcal per day ? Assume the man cannot resist eating the full plate of sweets once it is offered to him !

Solution. The man intends to cut down $3000 - 2100 = 900$ kcal. But avoiding sweets completely, he will cut down 700 kcal. To cut down another 200 kcal, he should cut down butter by $\frac{10}{60} \times 200 \simeq 33$ g per day. He should not cut down consumption of food, that provides him with vitamins and other vital nutrients.

HOTS

Problems on Higher Order Thinking Skills

Problem 1. *2 kg of ice at $-20°C$ is mixed with 5 kg of water at $20°C$ in an insulating vessel having a negligible heat capacity. Calculate the final mass of water remaining in the container. It is given that the specific heats of water and ice are 1 kcal / kg/°C and 0.5 kcal / kg/°C, while the latent heat of fusion of ice is 80 kcal / kg.* **[IIT Screening 03]**

Solution. Suppose m kg of ice melts into water. As the net heat change is zero, so

$$m_{ice}\, C_{ice}\,[0-(-20)] + mL + m_{water}\, C_{water}\,[0-20] = 0$$

$$2 \times 1 \times 20 + m \times 80 + 5 \times 1 \times (-20) = 0$$

or $$m = 1 \text{ kg}$$

$\therefore$ Final mass of water remaining in the beaker

$$= 5 + 1 = \mathbf{6\ kg}.$$

Problem 2. *Two rods, one of aluminum and the other made of steel, having initial lengths l_1 and l_2 are connected together to form a single rod of length $l_1 + l_2$. The coefficients of linear expansion for aluminum and steel are α_a and α_s respectively. If the length of each rod increases by the same amount when their temperatures are raised by $t°C$, then find the ratio $l_1/(l_1 + l_2)$.* **[IIT Screening 03]**

Solution. As the lengths of the two rods increase by the same amount, so

$$l_1\,\alpha_a t = l_2\,\alpha_s t \quad \text{or} \quad \frac{l_2}{l_1} = \frac{\alpha_a}{\alpha_s} \quad \text{or} \quad \frac{l_2 + l_1}{l_1} = \frac{\alpha_a + \alpha_s}{\alpha_s}$$

$$\therefore \quad \frac{l_1}{l_1 + l_2} = \frac{\alpha_s}{\alpha_a + \alpha_s}.$$

Problem 3. *Three rods made of the same material and having the same cross-section have been joined as shown in Fig. 11.35. Each rod is of the same length. The left and right ends are kept at $0°C$ and $90°C$ respectively. What will be the temperature of the junction of the three rods ?* **[IIT Screening 01]**

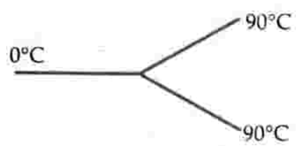


Fig. 11.36

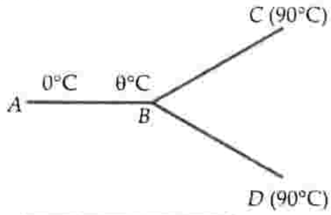


Fig. 11.37

Solution. Let R be the thermal resistance of each rod and θ be the temperature of the junction as shown in Fig. 11.36.

Then

Heat current in CB + Heat current in DB = Heat current in BA

or $$\frac{90-\theta}{R}+\frac{90-\theta}{R}=\frac{\theta-0}{R}$$

or $$180-2\theta=\theta \quad \text{or} \quad \theta = \mathbf{60°C}$$

Problem 4. *When a block of iron floats in mercury at 0°C, a fraction k_1 of its volume is submerged, while at the temperature 60°C, a fraction k_2 is seen to be submerged. If the coefficient of volume expansion of iron is γ_{Fe} and that of mercury is γ_{Hg}, then find the ratio k_1/k_2.* **[IIT Screening 01]**

Solution. Let ρ_{Fe} and ρ_{Hg} denote the densities of iron and mercury at 0°C, and m = mass of the block.

$\therefore$ Volume of the block $=\dfrac{m}{\rho_{Fe}}$,

Volume of displaced mercury $=\dfrac{k_1 m}{\rho_{Fe}}$

Now,

Weight of mercury displaced = Weight of the block

or $$\left(\frac{k_1 m}{\rho_{Fe}}\right)\rho_{Hg}\, g = mg \quad \text{or} \quad k_1=\frac{\rho_{Fe}}{\rho_{Hg}}.$$

Also, $$\rho_t=\frac{\rho_0}{1+\gamma t}$$

$$\therefore \quad k_2=\frac{\rho_{Fe}}{1+\gamma_{Fe}\times 60}\times\frac{1+\gamma_{Hg}\times 60}{\rho_{Hg}}=k_1\,\frac{1+60\gamma_{Hg}}{1+60\gamma_{Fe}}$$

or $$\frac{k_1}{k_2}=\frac{\mathbf{1+60\gamma_{Fe}}}{\mathbf{1+60\gamma_{Hg}}}.$$

Problem 5. *An ice cube of mass 0.1 kg at 0°C is placed in an isolated container which is at 227°C. The specific heat S of the container varies with temperature T according to the empirical relation $S = A + BT$, where $A = 100$ cal / kg-K and $B = 2\times10^{-2}$ cal / kg-K^2. If the final temperature of the container is 27°C, determine the mass of the container. (Latent heat of fusion of water $= 8\times10^4$ cal / kg. Specific heat of water $= 10^3$ cal / kg - K).* **[IIT Mains 01]**

Solution. Heat gained by the ice cube,

$$Q_1 = mL + mC\,dT = 0.1\times8\times10^4+0.1\times10^3\times27$$
$$=10700 \text{ cal}$$

Heat lost by the container,

$$Q_2=-\int_{500}^{300} m_c(A+BT)\,dT=-m_c\left[AT+\frac{BT^2}{2}\right]_{500}^{300}$$

$$=-m_c\left[100\,(300-500)+\frac{2\times10^{-2}}{2}(300^2-500^2)\right]$$

$$=21600\; m_c$$

But $Q_2 = Q_1$

$$\therefore \quad m_c=\frac{10700}{21600}=0.495 \text{ kg}\approx \mathbf{0.5\ kg.}$$

Problem 6. *Hot oil is circulated through an insulated container with a wooden lid at the top whose conductivity $K = 0.149$ J / (m-°C-sec), thickness $t = 5$ mm, emissivity = 0.6. Temperature of the top of the lid is maintained at $T_l = 127$°C. If the ambient temperature $T_a = 27$°C, calculate*

(*a*) *rate of heat loss per unit area due to radiation from the lid.*

(*b*) *temperature of the oil.*

(Given $\sigma=\frac{17}{3}\times10^{-8}$) **[IIT Mains 03]**

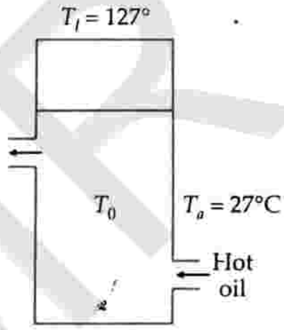


Fig. 11.38

Solution. (*a*) The rate of loss of heat per unit area from the lid,

$$\frac{dQ}{dt}=\sigma\,\varepsilon\,[T_l^{\,4}-T_a^{\,4}]$$
$$=\frac{17}{3}\times10^{-8}\times0.6\,[400^4-300^4]=\mathbf{595\ Wm^{-2}}.$$

(*b*) Let T_0 be the temperature of hot oil.

Then $$\frac{KA\,(T_0-T_l)}{t}=595\text{ A}$$

or $$\frac{0.149\text{ A}\,(T_0-400)}{5\times10^{-3}}=595\text{ A}$$

or $$T_0-400=\frac{595\times5\times10^{-3}}{0.149}=19.97\approx20$$

$$\therefore \quad T_0=\mathbf{420\ K}$$

Problem 7. *The temperature of the two outer surfaces of a composite slab, consisting of two materials having coefficients of thermal conductivity K and 2 K and thickness x and 4x, respectively are T_2 and T_1 ($T_2 > T_1$). What is the rate of flow of heat through the slab in a steady state ?* **[AIEEE 04]**

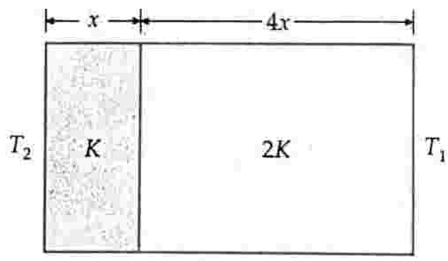


Fig. 11.39

Solution. Let T be the temperature of the interface. In the steady state, the rates of flow of heat through the two slabs will be equal, *i.e.*,

$$H_1 = H_2$$

or $$\frac{KA(T_2 - T)}{x} = \frac{2KA(T - T_1)}{4x}$$

or $$T_2 - T = \frac{T - T_1}{2} \quad \text{or} \quad T = \frac{T_1 + 2T_2}{3}$$

$$\therefore \quad Q_1 = \frac{KA}{x}\left[T_2 - \frac{T_1 + 2T_2}{3}\right] = \frac{KA(T_2 - T_1)}{3x}.$$

Problem 8. *Fig. 11.40 shows a system of two concentric spherical shells of radii r_1 and r_2 and kept at temperatures T_1 and T_2. Find the radial rate of flow of heat through a substance of thermal conductivity κ filled in the space between the two shells.* **[AIEEE 05]**

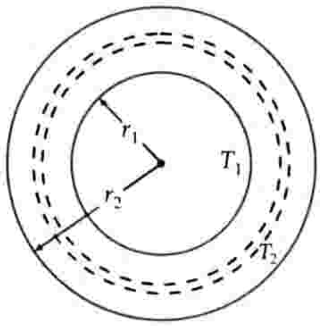


Fig. 11.40

Solution. Consider a thin concentric shell of radius r and thickness dr. The radial rate of flow of heat through this elementary shell will be

$$H = -\kappa A \frac{dT}{dr} = -\kappa\, 4\pi r^2 \frac{dT}{dr}$$

or $$H\frac{dr}{r^2} = -4\pi\kappa\, dT$$

Integrating both sides between the limits of radii and temperatures of the two shells, we get

$$H\int_{r_1}^{r_2} r^{-2}\, dr = -4\pi\kappa \int_{T_1}^{T_2} dT$$

or $$H\left[\frac{r^{-1}}{-1}\right]_{r_1}^{r_2} = -4\pi\kappa\,[T]_{T_1}^{T_2}$$

or $$H\left[-\frac{1}{r}\right]_{r_1}^{r_2} = 4\pi\kappa\,[-T]_{T_1}^{T_2}$$

or $$H\left[\frac{1}{r_1} - \frac{1}{r_2}\right] = 4\pi\kappa\,(T_1 - T_2)$$

or $$H = \frac{4\pi\kappa\, r_1 r_2 (T_1 - T_2)}{(r_2 - r_1)}.$$

Problem 9. *A 5 m long cylindrical steel wire with radius 2×10^{-3} m is suspended vertically from a rigid support and carries a bob of mass 100 kg at the other end. If the bob gets snapped, calculate the change in temperature of the wire ignoring radiation losses. (For the steel wire : Young's modulus $= 2.1 \times 10^{11}$ Pa ; Density $= 7860$ kg / m^3 ; Specific heat $= 420$ J / kg - K).* **[IIT Mains 01]**

Solution. The elongated wire has only elastic potential energy which ultimately gets converted into heat energy and brings the change in the temperature of the wire.

Elastic potential energy

$$= \frac{1}{2}\frac{(\text{Stress})^2}{Y} \times \text{Volume}$$

$$= \frac{1}{2Y}\left(\frac{Mg}{A}\right)^2 . AL = \frac{(Mg)^2 L}{2YA}$$

Heat produced $= ms\Delta T = (AL\rho)\, s\, \Delta T$

$$\therefore \quad (AL\rho)\, s\, \Delta T = \frac{(Mg)^2 L}{2Y A}$$

or $$\Delta T = \frac{(Mg)^2}{2\rho Y s A^2} = \frac{(Mg)^2}{2\rho Y s (\pi r^2)^2}$$

$$= \frac{(100)^2 \times (10)^2}{2 \times 7860 \times 2.1 \times 10^{11} \times 420 \times (3.14 \times 4 \times 10^{-6})^2}$$

$= \mathbf{0.00457\ K}$

Problem 10. *A sphere of diameter 7 cm and mass 266.5 g floats in a bath of a liquid. As the temperature is raised, the sphere just begins to sink at a temperature of 35°C. If the density of the liquid at 0°C is 1.527 g cm^{-3}, find the coefficient of cubical expansion of the liquid. Neglect the expansion of the sphere.* **[IIT]**

Solution. Volume of sphere

$$= \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 = \frac{539}{3}\ \text{cm}^3$$

Mass of sphere $= 266.5$ g

Density of sphere

$$= \frac{\text{Mass}}{\text{Volume}} = \frac{266.5 \times 3}{539} = 1.483\ \text{g cm}^{-3}$$

Density of liquid at $0°\text{C} = 1.527$ g cm^{-3}

As the temperature of the bath is raised, the liquid expands and its density decreases. When the sphere just floats,

Density of liquid at 35°C = Density of the body

$$\therefore \rho_t = 1.483\ \text{g cm}^{-3},\ \rho_0 = 1.527\ \text{g cm}^{-3},\ \Delta T = 35°\text{C}$$

$$\gamma = \frac{\rho_0 - \rho_t}{\rho_0\, \Delta T} = \frac{1.527 - 1.483}{1.527 \times 35}$$

$= \mathbf{0.000084°C^{-1}}.$

Guidelines to NCERT Exercises

11.1. *The triple points of neon and carbon dioxide are 24.57 K and 216.55 K respectively. Express these temperatures on the Celsius and Fahrenheit scales.* **[Delhi 2011]**

Ans. *For neon* : Triple point, $T = 24.57$ K

$\therefore \quad T_C = T\,(\text{K}) - 273.15$

$= 24.57 - 273.15 = -\mathbf{248.58°C.}$

$T_F = \frac{9}{5} T_C + 32 = \frac{9}{5} \times (-248.58) + 32 = -\mathbf{415.44°F.}$

For carbon dioxide : Triple point, $T = 216.55$ K

$\therefore \quad T_C = T\,(K) - 273.15$

$= 216.55 - 273.15 = -\mathbf{56.6°C.}$

$T_F = \frac{9}{5} T_C + 32 = \frac{9}{5} \times (-56.6) + 32 = -\mathbf{69.88°C.}$

11.2. *Two absolute scales A and B have triple points of water defined to be 200 A and 350 B. What is relation between T_A and T_B ?*

Ans. $\frac{T_A}{T_B} = \frac{200}{350} = \frac{4}{7}$ or $\quad \boldsymbol{T_A = \frac{4}{7} T_B}$.

11.3. *The electrical resistance in ohms of a certain thermometer varies with temperature according to the approximate law :*

$$R = R_0\,[1 + 5 \times 10^{-3}\,(T - T_0)]$$

The resistance is 101.6 Ω at the triple point of water, and 165.5 Ω at the normal melting point of lead (600.5 K). What is the temperature when the resistance is 123.4 Ω ?

Ans. When $\quad T = 273$ K, $\quad R = 101.6\,\Omega$

$$\therefore \quad 101.6 = R_0\,[1 + 5 \times 10^{-3}\,(273 - T_0)] \quad ...(i)$$

When $\quad T = 600.5$ K, $\quad R = 165.5\,\Omega$

$$\therefore \quad 165.5 = R_0\,[1 + 5 \times 10^{-3}\,(600.5 - T_0)] \quad ...(ii)$$

Dividing (*ii*) by (*i*), we get

$$\frac{165.5}{101.0} = \frac{1 + 5 \times 10^{-3}\,(600.5 - T_0)}{1 + 5 \times 10^{-5}\,(273 - T_0)}$$

On solving, $T_0 = -49.3$ K

Substituting in (*i*), we get

$$101.6 = R_0\,[1 + 5 \times 10^{-3}\,(273 + 49.3)]$$

or
$$R_0 = \frac{101.6}{1 + 5 \times 10^{-3} \times 322.3} = 38.9\,\Omega$$

For $R = 123.4\,\Omega$, we have

$$123.4 = 38.9\,[1 + 5 \times 10^{-3}\,(T + 49.3)]$$

On solving, $T = \mathbf{384.8\ K.}$

11.4. *Answer the following :*

(*a*) *The triple-point of water is a standard fixed point in modern thermometry. Why ? What is wrong in taking the melting point of ice and the boiling point of water as standard fixed points (as was originally done in the Celsius scale) ?*

(*b*) *There were two fixed points in the original Celsius scale as mentioned above which were assigned the number 0° C and 100° C respectively. On the absolute scale, one of the fixed points is the triple-point of water, which on the Kelvin absolute scale is assigned the number 273.16 K. What is the other fixed point on this (Kelvin) scale ?*

(*c*) *The absolute temperature (Kelvin scale) T is related to the temperature t_c on the Celsius scale by $t_c = T - 273.15$. Why do we have 273.15 in this relation, and not 273.16 ?*

(*d*) *What is the temperature of the triple-point of water on an absolute scale whose unit interval size is equal to that of the Fahrenheit scale ?*

Ans. (*a*) The melting point of ice as well as the boiling point of water change with change in pressure. The presence of impurities also changes the melting and boiling points. However, the triple point of water has a unique temperature and is independent of external factors.

(*b*) The other fixed point on Kelvin scale is absolute zero, which is the temperature at which the volume and pressure of any gas become zero.

(*c*) As the triple point of water on Celsius is 0.01°C (and not 0°C) and on Kelvin scale 273.16 and the size of degree on the two scales is same, so

$$t_c - 0.01 = T - 273.16 \quad \therefore \quad t_c = T - 273.15.$$

(*d*) One degree on Fahrenheit scale

$$= \frac{180}{100} = \frac{9}{5} \text{ divisions on Celsius scale.}$$

But one Celcius scale division is equal to one division on Kelvin scale.

$\therefore$ Triple point on Kelvin scale (whose size of a degree is equal to that of the Fahrenheit scale)

$$= 273.16 \times \frac{9}{5} = \mathbf{491.69.}$$

11.5. *Two ideal gas thermometers A and B use oxygen and hydrogen respectively. The following observations are made :*

Temperature	Pressure thermometer A	Pressure thermometer B
Triple-point of water	1.250×10^5 Pa	0.200×10^5 Pa
Normal melting point of sulphur	1.797×10^5 Pa	0.287×10^5 Pa

(a) What is the absolute temperature of normal melting point of sulphur as read by thermometers A and B ?

(b) What do you think is the reason for slightly different answers from A and B ?

Ans. (*i*) *For pressure thermometer A :*

$$T_{tr} = 273 \text{ K}, \; P_{tr} = 1.250 \times 10^3 \text{ Pa}, \quad P = 1.797 \times 10^5 \text{ Pa}$$

Normal freezing point of sulphur,

$$T = \frac{P}{P_{tr}} \times T_{tr} = \frac{1.795 \times 10^5 \times 273}{1.250 \times 10^5} = \mathbf{392.46 \text{ K}}$$

(*ii*) *For pressure thermometer B :*

$$T_{tr} = 273 \text{ K}, \; P_{tr} = 0.200 \times 10^5 \text{ Pa}, \quad P = 0.287 \times 10^5 \text{ Pa}$$

$$\therefore \quad T = \frac{0.287 \times 10^5 \times 273}{0.200 \times 10^5} = \mathbf{391.75 \text{ K}}$$

(*b*) The slight difference is due to the fact that oxygen and hydrogen do not behave strictly as ideal gases.

11.6. *A steel tape 1 m long is correctly calibrated for a temperature of 27.0° C. The length of a steel rod measured by this tape is found to be 63.0 cm on a hot day when the temperature is 45.0° C. What is the actual length of the steel rod on that day ? What is the length of the same steel rod on a day when the temperature is 27.0°C ? Coefficient of linear-expansion of steel = 1.20 × 10^{-5} °C^{-1} ?*

Ans. Here $t_1 = 27°\text{C}, \; l_1 = 63 \text{ cm},$

$$t_2 = 45°\text{C}, \; \alpha = 1.20 \times 10^{-5} \; °\text{C}^{-1}$$

Length of the rod on the hot day is

$$l_2 = l_1 [1 + \alpha (t_2 - t_1)]$$
$$= 63 [1 + 1.20 \times 10^{-5} (45 - 27)]$$
$$= \mathbf{63.0136 \text{ cm}}$$

As the steel tape has been calibrated for a temperature of 27° C, so length of the steel rod at 27° C = **63 cm.**

11.7. *A large steel wheel is to be fitted on to a shaft of the same material. At 27° C, the outer diameter of the shaft is 8.70 cm and the diameter of the central hole in the wheel is 8.69 cm. The shaft is cooled using 'dry ice' (solid carbon dioxide). At what temperature of the shaft does the wheel slip on the shaft? Assume coefficient of linear expansion of the steel to be constant over the required temperature range.*
$\alpha_{steel} = 1.20 \times 10^{-5} \; K^{-1}$.

Ans. Here $l_1 = 8.70 \text{ cm}, \; l_2 = 8.69 \text{ cm},$

$$T_1 = 27 + 273 = 300 \text{ K}, \quad T_2 = ?$$

As $\quad l_2 - l_1 = \alpha \, l_1 (T_2 - T_1)$

$$\therefore \quad T_2 - T_1 = \frac{l_2 - l_1}{\alpha \, l_1}$$

or $\quad T_2 - 300 = \dfrac{8.69 - 8.70}{1.20 \times 10^{-5} \times 8.70} = -95.8$

or $\quad T_2 = 300 - 95.8 = 204.2 \text{ K} = \mathbf{-68.8° \text{C}}.$

11.8. *A hole is drilled in a copper sheet. The diameter of the hole is 4.24 cm at 27.0° C. What is the change in the diameter of the hole when the sheet is heated to 227° C ? Coefficient of linear expansion of copper = 1.70 × 10^{-5} °C^{-1}.*

Ans. When the copper sheet is heated, the diameter of its hole increases in the same manner as the length of a rod.

$$\therefore \quad l = 4.24 \text{ cm}, \; \alpha = 1.70 \times 10^{-5} \; °\text{C}^{-1},$$
$$\Delta T = 227 - 27 = 200° \text{C}$$

Increase in diameter,

$$\Delta l = l \, \alpha \, \Delta T = 4.24 \times 1.70 \times 10^{-5} \times 200 \text{ cm}$$
$$= \mathbf{1.44 \times 10^{-2} \text{ cm.}}$$

11.9. *A brass wire 1.8 m long at 27° C is held taut with little tension between two rigid supports. If the wire is cooled to a temperature of – 39° C, what is the tension developed in the wire, if its diameter is 2.0 mm ? Coefficient of linear expansion of brass = 2.0 × 10^{-5} °C^{-1}, Young's modulus of brass = 0.91 × 10^{11} Pa.*

Ans. Here $l = 1.8 \text{ m}, \; t_1 = 27°\text{C}, \quad t_2 = -39°\text{C}$

$$r = \frac{2.0}{2} = 1.0 \text{ mm} = 1.0 \times 10^{-3} \text{ m}$$

$$\alpha = 2.0 \times 10^{-5} \; °\text{C}^{-1}, \; Y = 0.91 \times 10^{11} \text{ Pa}$$

As $\quad \Delta l = l \, \alpha \, (t_2 - t_1)$

$$\therefore \text{ Strain}, \; \frac{\Delta l}{l} = \alpha \, (t_2 - t_1)$$

Stress = Strain × Young's modulus

$$= \alpha \, (t_2 - t_1) \times Y$$
$$= 2.0 \times 10^{-5} \times (-39 - 27) \times 0.91 \times 10^{11}$$
$$= 1.2 \times 10^8 \text{ Nm}^{-2} \qquad \text{[Numerically]}$$

∴ Tension developed in the wire

= Stress × Area of cross-section

$$= \text{Stress} \times \pi r^2$$
$$= 1.2 \times 10^8 \times 3.14 \times (1.0 \times 10^{-3})^2$$
$$= \mathbf{3.77 \times 10^2 \text{ N.}}$$

11.10. *A brass rod of length 50 cm and diameter 3.0 mm is joined to a steel rod of the same length and diameter. What is the change in length of the combined rod at 250° C, if the original lengths are at 40.0° C ? Is there a 'thermal stress' developed at the junction ? The ends of the rod are free to expand. Coefficient of linear expansion of brass = 2.0 × 10^{-5} °C^{-1} and that of steel = 1.2 × 10^{-5} °C^{-1}.*

Ans. *For brass rod :*

$$l = 50 \text{ cm}, \; t_1 = 40°\text{C}, \; t_2 = 250°\text{C},$$
$$\alpha = 2.0 \times 10^{-5} \; °\text{C}^{-1}$$

Change in length of brass rod is

$$\Delta l = \alpha l \, (t_2 - t_1)$$
$$= 2.0 \times 10^{-5} \times 50 \times (250 - 40) = 0.21 \text{ cm}$$

For steel rod :

$$l = 50\text{ cm},\ t_1 = 40°\text{C},\ t_2 = 250°\text{C},$$

$$\alpha = 1.2 \times 10^{-5}\ °\text{C}^{-1}$$

Change in length of steel rod is

$$\Delta l' = \alpha l\,(t_2 - t_1)$$

$$= 1.2 \times 10^{-5} \times 50 \times (250 - 40) = 0.13\text{ cm}$$

Change in length of the combined rod at 250°C

$$= \Delta l + \Delta l' = 0.21 + 0.13 = \mathbf{0.34\ cm.}$$

As the rods expand freely, so no thermal stress is developed at the junction.

11.11. *The coefficient of volume expansion of glycerine is $49 \times 10^{-5}\ °C^{-1}$. What is the fractional change in its density for a 30°C rise in temperature ?*

Ans. Let M be the mass of glycerine, ρ_0 its density at 0°C, ρ_t its density at t°C. Then

$$\gamma = \frac{V_t - V_0}{V_0\,\Delta T} = \frac{\dfrac{M}{\rho_t} - \dfrac{M}{\rho_0}}{(M/\rho_0)\,\Delta T} = \frac{\dfrac{1}{\rho_t} - \dfrac{1}{\rho_0}}{(1/\rho_0)\,\Delta T} = \frac{\rho_0 - \rho_t}{\rho_t\,\Delta T}$$

$\therefore$ Fractional change in density,

$$\frac{\rho_0 - \rho_t}{\rho_0} = \gamma\,\Delta T = 49 \times 10^{-5} \times 30 = \mathbf{0.0147.}$$

11.12. *A 10 kW drilling machine is used to drill a bore in a small aluminium block of mass 8.0 kg. How much is the rise in temperature of the block in 2.5 minutes, assuming 50% of power is used up in heating the machine itself or lost to the surroundings. Specific heat of aluminium = 0.91 $Jg^{-1}\ °C^{-1}$.*

Ans.

Power $P = 10\text{ kW} = 10 \times 10^3\text{ W}$

Time $t = 2.5\text{ min} = 2.5 \times 60\text{ s}$

Total energy used

$$= Pt = 10 \times 10^3 \times 2.5 \times 60 = 1.5 \times 10^6\text{ J}$$

Energy absorbed by aluminium block,

$$Q = 50\%\text{ of the total energy}$$

$$= \frac{1.5 \times 10^6}{2} = 0.75 \times 10^6\text{ J}$$

Also, $m = 8.0\text{ kg} = 8.0 \times 10^3\text{ g},\ c = 0.91\text{ Jg}^{-1}\,°\text{C}^{-1}$

As $Q = mc\,\Delta T$

$$\therefore\quad \Delta T = \frac{Q}{mc} = \frac{0.75 \times 10^6}{8.0 \times 10^3 \times 0.91} = \mathbf{103.02°C.}$$

11.13. *A copper block of mass 2.5 kg is heated in a furnace to a temperature of 500°C and then placed on a large ice block. What is the maximum amount of ice that can melt? (Specific heat of copper = 0.39 $Jg^{-1}\,°C^{-1}$, and heat of fusion of water = 335 Jg^{-1}).*

Ans. Mass of copper block,

$$M = 2.5\text{ kg} = 2.5 \times 10^3\text{ g}$$

Specific heat of copper, $c = 0.39\text{ J g}^{-1}\,°\text{C}^{-1}$

Fall in temperature, $\Delta T = 500 - 0 = 500°\text{C}$

Heat lost by copper block

$$= mc\,\Delta T = 2.5 \times 10^3 \times 0.39 \times 500\text{ J}$$

Let mass of ice melted = M gram

Heat of fusion of ice, $L = 335\text{ Jg}^{-1}$

Heat gained by ice = $ML = M \times 335$ J

$\therefore$ Heat gained = Heat lost

$$\therefore\quad M \times 335 = 2.5 \times 10^3 \times 0.39 \times 500$$

or
$$M = \frac{2.5 \times 10^3 \times 0.39 \times 500}{335}$$

$$= 1455.2\text{ g} = \mathbf{1.455\ kg.}$$

11.14. *In an experiment on the specific heat of a metal, a 0.20 kg block of the metal at 150°C is dropped in a copper calorimeter (of water equivalent 0.025 kg) containing 150 cm³ of water at 27°C. The final temperature is 40°C. Compute the specific heat of the metal.*

Ans. Mass of metal block,

$$m = 0.20\text{ kg} = 200\text{ g}$$

Fall in temperature of metal block,

$$\Delta T = 150 - 40 = 110°\text{C}$$

Let specific heat of metal block = c cal $\text{g}^{-1}\,°\text{C}^{-1}$

$\therefore$ Heat lost by metal block

$$= mc\,\Delta T = 200 \times c \times 110\text{ cal}$$

Volume of water in calorimeter = 150 cm³

Mass of water, $m' = 150$ g

Water equivalent of calorimeter,

$$w = 0.025\text{ kg} = 25\text{ g}$$

Specific heat of water,

$$c' = 1\text{ cal g}^{-1}\,°\text{C}^{-1}$$

$\therefore$ Heat gained by water and calorimeter

$$= (m' + w)\,c'\,\Delta T'$$

$$= (150 + 25) \times 1 \times (40 - 27)\text{ cal} = 175 \times 13\text{ cal}$$

By principle of calorimetry,

Heat lost = Heat gained

$$\therefore\quad 200 \times c \times 110 = 175 \times 13$$

or
$$c = \frac{175 \times 13}{200 \times 110} = \mathbf{0.1\ cal\ g^{-1}\,°C^{-1}.}$$

11.15. *Given below are observations on molar specific heats at room temperature of some common gases.*

Gas	*Molar specific heat (C_V) (cal mol^{-1} K^{-1})*
Hydrogen	4.87
Nitrogen	4.97
Oxygen	5.02
Nitric oxide	4.99
Carbon monoxide	5.01
Chlorine	6.17

The measured molar specific heats of these gases are markedly different from those for monoatomic gases. [Typically, molar specific heat of a monoatomic gas is 2.92 cal/mol K]. Explain this difference. What can you infer from the somewhat larger (than the rest) value for chlorine ?

Ans. A monoatomic gas has three degrees of freedom, while a diatomic gas possesses five degrees of freedom. Therefore, molar specific heat of a diatomic gas (at constant volume),

$$C_V = \frac{f}{2} R = \frac{5}{2} R = \frac{5}{2} \times \frac{8.31}{4.2} = 5 \text{ cal mol}^{-1} \text{ K}^{-1}$$

In the given table, all the gases are diatomic gases and for all of them (except chlorine), the value of C_V is about 5 cal mol^{-1} K^{-1}.

The slightly higher value of C_V for chlorine is due to the fact that even at room temperature, a chlorine gas molecule possesses the vibrational mode of motion also.

11.16. *Answer the following questions based on the P-T phase diagram of carbon dioxide as shown in Fig. 11.41.*

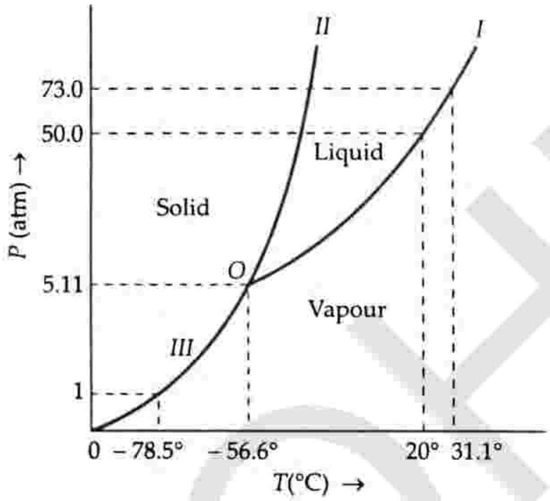


Fig. 11.41

(*i*) *At what temperature and pressure can the solid, liquid and vapour phases of CO_2 co-exist in equilibrium ?*

(*ii*) *What is the effect of decrease of pressure on the fusion and boiling point of CO_2 ?*

(*iii*) *What are the critical temperature and pressure for CO_2 ? What is their significance ?*

(*iv*) *Is CO_2 solid, liquid or gas at (a) $-70°C$ under 1 atm., (b) $-60°C$ under 10 atm., (c) $15°C$ under 56 atm ?*

Ans. (*i*) The solid, liquid and vapour phases of CO_2 co-exist in equilibrium at its triple point O for which

$$P_{tr} = \mathbf{5.11\ atm} \quad \text{and} \quad T_{tr} = \mathbf{-56.6°C}.$$

(*ii*) The vaporisation curve *I* and fusion curve *II* show that both the boiling point and fusion point of CO_2 decrease with decrease of pressure.

(*iii*) For CO_2, $P_c = \mathbf{73.0\ atm}$ and $T_c = \mathbf{31.1°C}$.

Above its critical temperature, CO_2 gas cannot be liquefied, however large pressure may be applied.

(*iv*)

(*a*) *– 70° C under 1 atm*. This point lies in ***vapour*** region. Therefore, at – 70°C under 1 atm, CO_2 is ***vapour***.

(*b*) *– 60° C under 10 atm*. This point lies in ***solid*** region. Therefore, CO_2 is ***solid*** at – 60°C under 10 atm.

(*c*) *15° C under 56 atm*. This point lies in ***liquid*** region. Therefore, CO_2 is ***liquid*** at 15°C under 56 atm.

11.17. *Answer the following questions based on the P-T phase diagram of CO_2 :*

(*i*) *CO_2 at 1 atm pressure and temperature $-60°C$ is compressed isothermally. Does it go through the liquid phase ?*

(*ii*) *What happens when CO_2 at 4 atm pressure is cooled from room temperature at constant pressure ?*

(*iii*) *Describe qualitatively the changes in a given mass of solid CO_2 at 10 atm. pressure and temperature $-65°C$ as it is heated up to room temperature at constant pressure.*

(*iv*) *CO_2 is heated to a temperature 70° C and compressed isothermally. What changes in its properties do you expect to observe ?*

Ans. (*i*) No. When CO_2 at 1 atm pressure and at – 60°C is compressed isothermally, it changes directly from vapour phase to solid phase without going through the liquid phase. This can be checked by drawing a vertical line at – 60°C which intersects the sublimation curve III.

(*ii*) CO_2 at 4 atm pressure and at temperature (say 25°C) is vapour. If it is cooled at constant temperature, it condenses directly into solid without going through liquid phase. This can be checked by drawing a horizontal line at $P = 4$ atm which intersects the sublimation curve III.

(*iii*) CO_2 at 10 atm pressure and at – 65°C is solid. As CO_2 is heated at constant pressure, it will go to ***liquid phase and then to the vapour phase***. It is because, the horizontal line through the initial point intersects both the fusion and the vaporisation curves. The fusion and boiling points can be known from the points, where the horizontal line at 10 atm (initial point) intersects the respective curves.

(*iv*) When the carbon dioxide is heated to 70°C (which is greater than its critical temperature), it will not exhibit any clear phase transition to the liquid phase. At this state, it will deviate more and more from ideal gas behaviour, as its pressure increases.

11.18. *A child running a temperature of 101° F is given an antipyrin (i.e. a medicine that lowers fever) which causes an increase in the rate of evaporation of sweat from his body. If the* *fever is brought down to 98° F in 20 min, what is the average rate of extra evaporation caused by the drug ? Assume the evaporation mechanism to be the only way by which heat is lost. The mass of the child is 30 kg. The specific heat of human body is approximately the same as that of water, and latent heat of evaporation of water at that temperature is about 580 cal g^{-1}.*

Ans. Mass of child, $M = 30\text{ kg} = 30 \times 10^3\text{ g}$

Fall in temperature,

$$\Delta T = 101 - 98 = 3°\text{F} = 3 \times \frac{5}{9} = \frac{5}{3}\text{ °C}$$

Specific heat of human body,

$$c = \text{specific heat of water} = 1\text{ cal g}^{-1}\text{°C}^{-1}$$

Heat lost by child in the form of evaporation of sweat,

$$Q = Mc\,\Delta T = 30 \times 10^3 \times 1 \times \frac{5}{3} = 50{,}000\text{ cal}$$

If M' gram of sweat evaporates from the body of the child, then heat gained by sweat

$$Q = M'L = M' \times 580\text{ cal} \qquad [\because L = 580\text{ cal g}^{-1}]$$

$\therefore$ Heat gained = Heat lost

$\therefore$ $M' \times 580 = 50000$

or $$M' = \frac{50000}{580} = 86.2\text{ g}$$

Time taken by sweat to evaporate = 20 min

$\therefore$ Rate of evaporation of sweat $= \frac{86.2}{20} = \mathbf{4.31\ g\ min^{-1}}$.

11.19. *A 'thermocole' cubical icebox of side 30 cm has a thickness of 5.0 cm. If 4.0 kg of ice are put in the box, estimate the amount of ice remaining after 6 h. The outside temperature is 45°C and coefficient of thermal conductivity of thermocole = 0.01 $Js^{-1}\ m^{-1}\ °C^{-1}$. Given heat of fusion of water = $335 \times 10^3\ J\ kg^{-1}$.*

Ans. Here $A = 6 \times \text{side}^2 = 6 \times 30 \times 30$,

$$= 5400\text{ cm}^2 = 0.54\text{ m}^2,$$
$$x = 5\text{ cm} = 0.05\text{ m}$$
$$t = 6\text{ h} = 6 \times 3600\text{ s},$$
$$T_1 - T_2 = 45 - 0 = 45°\text{C},$$
$$K = 0.01\text{ Js}^{-1}\text{ m}^{-1}\text{ °C}^{-1},$$
$$L = 335 \times 10^3\text{ J kg}^{-1}.$$

Total heat entering the box through all the six faces,

$$Q = \frac{KA(T_1 - T_2)t}{x} = \frac{0.01 \times 0.54 \times 45 \times 6 \times 3600}{0.05}$$
$$= 104976\text{ J}$$

Let m kg of ice melt due to this heat. Then

$$Q = mL$$

or $$m = \frac{Q}{L} = \frac{104976\text{ J}}{335 \times 10^3\text{ J kg}^{-1}} = 0.313\text{ kg}$$

Mass of ice left after six hours

$$= 4 - 0.313 = \mathbf{3.687\ kg.}$$

11.20. *A brass boiler has a base area of 0.15 m^2 and thickness 1.0 cm. It boils water at the rate of 6.0 kg min^{-1}, when placed on a gas stove. Estimate the temperature of the part of the flame in contact with the boiler. Thermal conductivity of brass = 109 $Js^{-1}\ m^{-1}\ °C^{-1}$ and heat of vaporisation of water = 2256 Jg^{-1}.*

Ans. Here $A = 0.15\text{ m}^2$, $x = 1.0\text{ cm} = 0.01\text{ m}$,

$$K = 109\text{ Js}^{-1}\text{ m}^{-1}\text{ °C}^{-1}, \quad L = 2256\text{ Jg}^{-1},$$
$$T_2 = 100°\text{C}, \quad t = 1\text{ min} = 60\text{ s}$$

Let T_1 be the temperature of the part of the flame in contact with boiler. Then amount of heat that flows into water in 1 min,

$$Q = \frac{KA(T_1 - T_2)t}{x} = \frac{109 \times 0.15 \times (T_1 - 100) \times 60}{0.01}\text{ J}$$

Mass of water boiled per min = 6 kg = 6000 g

Heat used to boil water,

$$Q = mL = 6000\text{ g} \times 2256\text{ J g}^{-1} = 6000 \times 2256\text{ J}$$

$$\therefore \quad \frac{109 \times 0.15 \times (T_1 - 100) \times 60}{0.01} = 6000 \times 2256$$

or $$T_1 - 100 = \frac{6000 \times 2256 \times 0.01}{109 \times 0.15 \times 60} = 138°\text{C}$$

or $$T_1 = 138 + 100 = \mathbf{238°C.}$$

11.21. *Explain why :*

(*a*) *a body with large reflectivity is a poor emitter.*

(*b*) *a brass tumbler feels much colder than a wooden tray on a chilly day.*

(*c*) *an optical pyrometer (for measuring high temperatures) calibrated for an ideal black body radiation gives too low a value for the temperature of a red hot iron piece in the open, but gives a correct value for the temperature when the same piece is in the furnace.*

(*d*) *the earth without its atmosphere would be inhospitably cold.*

(*e*) *heating systems based on circulation of steam are more efficient in warming a building than those based on circulation of hot water.*

Ans. (*a*) A body with large reflectivity is a poor absorber of heat. According to Kirchhoff's law, a poor absorber of heat is a poor emitter. Hence a body with large reflectivity is a poor emitter.

(*b*) Brass is a good conductor of heat. When a brass tumbler is touched, heat quickly flows from human body to tumbler. Consequently, the tumbler appears colder. Wood is a bad conductor. So, heat does not flow from the human body to the tray in this case. Thus, it appears comparatively hotter.

(*c*) Let T be the temperature of the hot iron in the furnace.

Heat radiated per second per unit area,

$$E = \sigma T^4$$

When the body is placed in the open at temperature T_0,

the heat radiated/second/area, $E' = \sigma(T_4 - T_0^4)$.

Clearly $E' < E$. So the optical pyrometer gives too low a value for the temperature in the open.

(*d*) Heat radiated out by earth is reflected back by the atmosphere. In the absence of atmosphere, at night all heat would escape from the earth's surface and thereby earth's surface would be inhospitably cold. Also atmosphere helps in maintaining the temperature through convection current.

(*e*) Though steam and boiling water are at the same temperature but each unit mass of steam contains a larger amount of additional heat called the latent heat. For example, each gram of steam has 540 calories of more heat than each gram of boiling water. Hence steam loses more heat than boiling water.

11.22. *A body cools from 80°C to 50°C in 5 minutes. Calculate the time it takes to cool from 60°C to 30°C, the temperature of the surrounding is 20°C.*

Ans. According to Newton's law of cooling, when the temperature difference is not large, rate of loss of heat is proportional to the temperature difference between the body and the surroundings.

$$mc\frac{T_1 - T_2}{t} = K(T - T_0)$$

where $T = \frac{T_1 + T_2}{2}$ = average of the initial and final temperatures of the body and T_0 is the temperature of the surroundings.

Here $T_1 = 80°C$, $T_2 = 50°$, $T_0 = 20°C$,

$$t = 5 \text{ min} = 300 \text{ s}$$

$$T = \frac{T_1 + T_2}{2} = \frac{80 + 50}{2} = 65°C$$

$$\therefore \quad mc\frac{80 - 50}{300} = K(65 - 20) \quad ...(i)$$

If the liquid takes t seconds to cool from 60°C to 30°C, then

$$T = \frac{60 + 30}{2} = 45°C$$

$$\therefore \quad mc\frac{60 - 30}{t} = K(45 - 20) \quad ...(ii)$$

Dividing equation (*i*) by (*ii*), we get :

$$\frac{30}{300} \times \frac{t}{30} = \frac{45}{25}$$

or $$t = \frac{45}{25} \times 300 = 540 \text{ s} = \mathbf{9 \text{ min.}}$$

Text Based Exercises

Type A : Very Short Answer Questions **1 Mark Each**

1. What is heat ?
2. What are the SI and CGS units of heat ? How are they related ?
3. What physical changes may be observed, if an object is heated ?
4. Define temperature.
5. What is thermometry ?
6. State the principle of a thermometer.
7. Name the various thermometric properties.
8. What is the normal temperature of a human body ?
9. The temperature of a gas is increased by 10°C. What is the corresponding change on the Kelvin scale ?
10. At what temperature will wood and iron appear equally hot or equally cold ?
11. What is the value of 0°F on the Kelvin scale ?
12. What is the temperature at which Celsius and Fahrenheit scales give the same reading ?
13. Which thermometer should be used to measure the temperature of – 150°C or 750°C ?
14. What are the values of the triple point of water on the Centigrade, Fahrenheit and absolute scales of temperature ?
15. Taking absolute zero as the basis of the scale of temperature, what are the values of melting point and boiling point of water ?
16. Name the gas law that forms the principle of a constant volume air thermometer.
17. Which one is more sensitive — a mercury thermometer or a gas thermometer ?
18. What is the minimum possible temperature ? Is there also a maximum possible temperature ?
19. What is pyrometry ?
20. When two bodies are said to be in thermal equilibrium ?
21. Of metals and alloys, which have a greater value of temperature coefficient of expansion ?
22. What are the units of α, β and γ ?
23. How are the three coefficients of expansion related to each other ?

24. A liquid of cubical expansivity γ is heated in a vessel having linear expansivity $\gamma / 3$. What would be the effect on the level of the liquid ?
25. A long metal rod is bent to form a ring with a small gap. If this rod is heated, will the gap increase or decrease ?
26. Is the coefficient of expansion constant for a given solid ?
27. A metal disc has a hole in it. What happens to the size of the hole, when the disc is heated ?
28. Define Joule's mechanical equivalent of heat. Give its value.
29. Why does a rifle bullet get heated on striking a target ?
30. If a piece of iron is hammered, it becomes warmer. Why ?
31. A spark is produced when two stones are struck against each other. Why ?
32. Tea gets cooled, when sugar is added to it. Why ?
33. Write the SI unit of specific heat.
34. Which substance has the highest specific heat ?
35. What is the specific heat of water in the SI units ?
36. Write the values of the specific heats of water at 20°C and ice.
37. Which has the maximum and which has the minimum specific heat amongst the following :
    Carbon, Silver, Aluminium, Tungsten ?
38. What is the relation between heat capacity and water equivalent of a body ?
39. Name three modes of transmission of heat energy from one place to another.
40. Which is the only way of heat transfer through a solid ?
41. How does the heat energy from the sun reach the earth ?
42. Out of the three modes of transmission of heat, which one is fastest ?
43. The temperature gradient in a rod 0.5 m long is 40°C per metre. The temperature of the hotter end is 30°C. What is the temperature of its colder end ?
44. Write the SI unit of the coefficient of thermal conductivity.
45. Define coefficient of thermal conductivity. **[Delhi 98, 02]**
46. What is the dimensional formula of the coefficient of thermal conductivity ?
47. Arrange the metals Cu, Al and Ag in the order of their increasing thermal conductivity.
48. Why does the temperature of every part of a metallic rod become constant in steady state ?
49. What is thermal conductivity of a perfect heat conductor and a perfect heat insulator ?
50. Can you imagine a condition that on heating a rod at one end the temperature of the whole rod becomes the same ?
51. Write the unit and dimensional formula of thermal resistance.
52. In which methods of heat transfer, gravity does not play any part ?
53. In which methods of heat transfer, mass of the substance heated does not play any part ?
54. Do the thermal radiations obey the laws of reflection and refraction ?
55. Can thermal radiations pass through vacuum ?
56. What is the nature of heat radiations ?
57. What is the absorptivity of a perfect black body ?
58. If a hot body is kept in surroundings which are cooler than the body, only the hot body radiates heat, while the cooler surroundings do not ? Is it true or false ?
59. What is meant by saying that thermal radiations inside a constant temperature enclosure are isotropic ?
60. If the temperature of a black body is increased from 300 K to 900 K, by what factor the rate of emission will increase ?
61. The temperature of a body is 0°C. Is it radiating ?
62. At what temperature will a body stop radiating ?
63. Name the electrical analogies of temperature and temperature gradient.
64. What is sublimation ?
65. How much is the critical temperature of water ?
66. Why do the fusion line, vaporisation line and sublimation line of a substance, meet necessarily at a single point ?
67. At what temperature and pressure, can the solid, liquid and vapour phases of water co-exist in equilibrium ?
68. At what temperature and pressure, can the solid, liquid and vapour phases of CO_2 co-exist in equilibrium ?
69. What is the effect of decrease of pressure on the fusion and boiling points of CO_2 ?
70. Is CO_2 solid, liquid or gas at (*a*) – 70°C under 1 atm (*b*) – 60°C under 10 atm (*c*) – 15°C under 56 atm ?
71. CO_2 at 1 atm pressure and temperature – 60°C is compressed isothermally. Does it go through liquid phase ?
72. What happens when CO_2 at 4 atm pressure is cooled from room temperature at constant pressure ?
73. Draw the graph showing cooling of hot water with time. **[Delhi 09]**

Answers

1. Heat is a form of energy which produces in us the sensation of warmth.
2. SI unit of heat is joule and CGS unit of heat is calorie.

$$1 \text{ calorie} = 4.18 \text{ joule}$$

3. When an object is heated, the physical changes such as expansion, contraction, change of states, change of electrical properties, etc. are observed.
4. Temperature is the degree of hotness of a body. It is a condition which determines the direction of flow of heat, when the two bodies are placed in contact with each other.
5. The branch of physics that deals with the measurement of temperature is called the thermometery.
6. The working of a thermometer is based on the fact that some physical property of substance changes linearly with temperature.
7. Some of the thermometric properties are length, volume, pressure, electrical resistance, thermo-electric e.m.f., radiated power, etc.
8. 98.4°F or 37°C.
9. 10 K.
10. When both wood and iron are at the temperature of the human body, they appear equally hot or equally cold.
11. 0°F = 255.23 K.
12. – 40°C = – 40°F.
13. Radiation pyrometer.
14. 0.01°C, 32.018°F, 273.16 K.
15. 273.15 K and 373.15 K.
16. Charle's law of pressure.
17. A gas thermometer.
18. 0 K is the minimum possible temperature. There is no limit to maximum temperature.
19. The branch of physics that deals with the measurement of high temperature is called pyrometry.
20. Two bodies are said to be in thermal equilibrium if they have the same temperature.
21. The value of coefficient of linear expansion (α) is greater for metals than for alloys.
22. All have same units, K^{-1} or $°C^{-1}$.
23. $\frac{\alpha}{1} = \frac{\beta}{2} = \frac{\gamma}{3}$.
24. There will be no change in the level of the liquid.
25. The gap will increase on heating.
26. No. The coefficient of expansion of a solid changes with temperature.
27. The size of the hole increases.
28. Joule's mechanical equivalent of heat may be defined as the amount of work that must be done to produce a unit quantity of heat. Its value is 4.18 J cal^{-1}.
29. The kinetic energy of the bullet gets converted into heat energy, which in turn heats up the bullet.
30. When the hammer strikes the piece of iron, its kinetic energy gets converted into heat.
31. The work done in striking the two stones against each other is converted into heat which produces spark.
32. When sugar is added to tea, its heat gets shared by sugar. So the temperature of the tea decreases.
33. SI unit of specific heat = $\text{J kg}^{-1} \text{ K}^{-1}$.
34. Water.
35. $4186 \text{ J kg}^{-1} \text{ K}^{-1}$.
36. Specific heat of water at 20°C = $1 \text{ cal g}^{-1} \text{ °C}^{-1}$ and specific heat of ice = $0.5 \text{ cal g}^{-1} \text{ °C}^{-1}$.
37. Aluminium has the maximum and tungsten the minimum specific heat.
38. The heat capacity and water equivalent of a body are numerically equal.
39. Conduction, convection and radiation.
40. Conduction.
41. By radiation.
42. Radiation.
43. Temperature of colder end

$$= 30 - 0.5 \times 40 = \mathbf{10°C}.$$

44. SI unit of K = $\text{J s}^{-1} \text{ m}^{-1} \text{ K}^{-1}$ or $\text{W m}^{-1} \text{ K}^{-1}$.
45. The coefficient of thermal conductivity of a material may be defined as the quantity of heat that flows per unit time through a unit cube of the material when its opposite faces are kept at a temperature difference of 1 degree.
46. $[MLT^{-3}K^{-1}]$.
47. Al < Cu < Ag.
48. In steady state, no part of the rod absorbs heat.
49. (*i*) For a perfect heat conductor, $K = \infty$.
    (*ii*) For a perfect heat insulator, $K = 0$.
50. Yes, this is possible if the rod is a perfect conductor *i.e.*, $K = \infty$.
51. SI unit of thermal resistance = KW^{-1}.
    Dimensional formula of thermal resistance is $[M^{-1}L^{-2}T^{-3} K]$.

52. Conduction and radiation.
53. Conduction and radiation.
54. Yes, thermal radiations obey the laws of reflection and refraction.
55. Yes, thermal radiations can pass through vacuum.
56. Heat radiations are electromagnetic waves having wavelength range from 1 μm to 100 μm. These are also called infrared waves.
57. One.
58. False. According to Prevost theorem of heat exchanges, both the hot body and the surroundings emit and absorb radiations.
59. This means that the intensities of different wavelengths inside a hollow enclosure are same in all directions.
60. 81 times, because $E \propto T^4$.
61. Yes, a body radiates heat at 0°C.
62. A body stops radiating at 0 K.
63. Potential and potential gradient respectively.
64. When a substance changes from solid state into gaseous state directly without undergoing the liquid state, the process is called sublimation.
65. 374.1°C.
66. This is because if these curves do not meet at a single point, they would enclose some finite area. In this region, solid, liquid and vapour phases of the substance would exist simultaneously, which is not possible.
67. At the temperature of 263.16 K and a pressure of 0.46 cm of Hg.
68. At the temperature of – 50.6°C and under the pressure of 5.11 atmosphere.
69. Both the boiling point and freezing point of CO_2 decrease if pressure decreases.
70. (*i*) Vapour (*ii*) Solid (*iii*) Liquid.
71. No. Vapour condenses directly into solid.
72. It condenses to solid directly without passing through the liquid phase.
73. See Fig. 11.26 on page 11.34.

Type B : Short Answer Questions — 2 or 3 Marks Each

1. Distinguish clearly between heat and temperature.
2. What is meant by the statement that heat is an energy in transit ?
3. Why is mercury used in a thermometer ?
4. State Joule's law of equivalence between work and heat. Hence define mechanical equivalent of heat. Give its value.
5. Starting from Charle's law, develop the concept of absolute zero and absolute scale of temperature.
6. What is meant by triple point of water ? What is the advantage of taking triple point of water as the fixed point for a temperature scale ?
7. Define ideal gas temperature. Does it depend on the nature of the gas ?
8. What is a liquid thermometer ? Briefly describe its working principle.
9. Describe the working principle of a platinum resistance thermometer.
10. How does the coefficient of cubical expansion of a substance vary with temperature ? Draw γ versus T curve for copper.
11. Prove that the coefficient of cubical expansion of an ideal gas at constant pressure is equal to the reciprocal of its absolute temperature.
12. What do you mean by coefficients of apparent and real expansion of a liquid ? How are they related ?
13. How does the density of a solid or liquid change with temperature ? Show that its variation with temperature is given by $\rho' = \rho\,(1 - \gamma\,\Delta T)$, where γ is the coefficient of cubical expansion.
14. Define the terms specific heat and molar specific heat. Give their SI units.
15. Define the terms heat capacity and water equivalent. Give their SI units.
16. Define latent heat of fusion of ice and latent heat of vaporisation of steam. **[Meghalaya 98]**
17. Define thermal conduction. Briefly explain its molecular mechanism.
18. Briefly explain, how does a metallic rod heated at one end attain steady state ? Write the important features of steady state.
19. Explain the following in reference to thermal conduction in a rod :
    (*i*) Steady state,
    (*ii*) Isothermal surface,
    (*iii*) Temperature gradient.
20. Define terms heat current and thermal resistance. Write their SI units.
21. Define thermal resistance. On what factors does it depend ? Deduce its dimensions.

22. Define thermal convection. Briefly explain how convection currents are set up in water.
23. Distinguish between natural and forced convections. Give one example of each.
24. Briefly explain the formation of land and see breezes.
25. Write the main features of Prevost theory of heat exchange. How does this theory lead to the fact that good absorbers are good radiators ?
26. What are thermal radiations ? Give their two important properties.
27. Define the terms
    (*i*) absorptive power,
    (*ii*) emissive power and
    (*iii*) emissivity. Write their SI units, if any.
28. What is a black body ? How can it be realised in practice ?
29. What are Fraunhoffer lines ? Explain their origin. How do they help in identifying the atmosphere of the sun ?
30. State Stefan-Boltzmann law. Write the CGS and SI units of Stefan-Boltzmann constant.
31. Define thermal conduction and convection. How are these modes of transfer different from thermal radiation ? **[Delhi 03C]**
32. Define thermal radiation. State Prevost's theory of heat exchange. **[Delhi 03C]**
33. What is meant by coefficient of linear expansion and coefficient of cubical expansion ? Derive relationship between them. **[Himachal 03]**
34. Define coefficient of thermal conductivity. Write its SI unit. **[Himachal 03, Delhi 02]**
35. Name the three modes of transfer of heat from one object to other. Also cite one example for each one of them. **[Himachal 07 ; Central Schools 08]**
36. Define Newton's law of cooling, write the expression. **[Central Schools 08, 12]**
37. State and prove Kirchoff's law of radiation. Hence show that a good absorber is also a good emitter of radiation. **[Delhi 2006]**
38. State Stefan's Law and Newton's Law of cooling. How do you deduce the later from the former ? **[Central School 05 ; Chandigarh 04]**
39. Discuss briefly energy distribution of black body radiation. Hence deduce Wien's displacement law and Stefan's law. **[Chandigarh 03]**
40. Draw energy distribution curves for a black body at two different temperature T_1 and $T_2 (T_1 > T_2)$. Write any two conclusions that can be drawn from these curves. **[Delhi 2004]**

Answers

1. Refer answer to Q. 6 on page 11.2.
2. Refer answer to Q. 2 on page 11.1.
3. Refer answer to Q. 15 on page 11.5.
4. Refer answer to Q. 4 on page 11.2.
5. Refer answer to Q. 11 on page 11.3.
6. Refer answer to Q. 12 on page 11.4.
7. Refer answer to Q. 14 on page 11.5.
8. Refer answer to Q. 15 on page 11.5.
9. Refer answer to Q. 16 on page 11.6.
10. Refer answer to Q. 23 on page 11.9.
11. Refer answer to Q. 24 on page 11.10.
12. Refer answer to Q. 28 on page 11.11.
13. Refer answer to Q. 29 on page 11.11.
14. Refer answer to Q. 31 on page 11.16.
15. Refer answer to Q. 32 on page 11.16.
16. Refer to points 26 and 27 of Glimpses.
17. Refer answer to Q. 2 on page 11.1.
18. Refer answer to Q. 3 on page 11.2.
19. Refer answer to Q. 42 on page 11.24.
20. Refer answer to Q. 45 on page 11.25.
21. Refer answer to Q. 45 on page 11.25.
22. Refer answer to Q. 47 on page 11.32.
23. Refer answer to Q. 47 on page 11.32.
24. Refer answer to Q. 48(*iv*) on page 11.32.
25. Refer answer to Q. 34 on page 11.17.
26. Refer answer to Q. 52 on page 11.34.
27. Refer answer to Q. 55 on page 11.36.
28. Refer answer to Q. 56 on page 11.37.
29. Refer answer to Q. 59 on page 11.38.
30. Refer answer to Q. 60 on page 11.38.
31. Refer to points 30, 35 and 36 of Glimpses.
32. Refer to points 33 and 37 of Glimpses.
33. Refer answer to Q. 22 on page 11.9.
34. Refer to point 33 of Glimpses.
35. Refer to points 30, 35 and 36 of Glimpses.
36. Refer to point 45 of Glimpses.
37. Refer answer to Q. 57 on page 11.37.
38. Refer answer to Q. 61 on page 11.38.
39. Refer answer to Q. 63 on page 11.43.
40. Refer answer to Q. 63 on page 11.43.

Type C : Long Answer Questions

5 Marks Each

1. Describe the principle, construction and working of a constant volume gas thermometer. Give its two advantages over mercury thermometer.
2. What is meant by coefficient of linear expansion, superficial expansion and cubical expansion ? Derive the relationship between them. **[Himachal 04, 05C, 07C]**
3. Thermal expansion is a consequence of atomic vibrations and asymmetry of potential energy function. Explain.
4. Draw a labelled *P-T* diagram of water. Explain its behaviour, when both pressure *P* and temperature *T* are above and below the triple point. Give the importance of triple point.
5. Draw a *P-T* diagram for CO_2. Explain its behaviour, when both pressure and temperature are above and below the triple point. Give the importance of triple point.
6. On what factors does the rate of heat conduction in a metallic rod in the steady state depend ? Write the necessary expression and hence define the coefficient of thermal conductivity. Write its units and dimensions also.
7. What is meant by Steady state heat flow by conduction in case of a thick copper bar with its two ends maintained at two different temperatures ? On what factors does the amount of heat flowing through the bar depend ? **[Delhi 03]**
8. State and explain the three modes of transference of heat. Explain how the loss of heat due to these three modes is minimised in a thermos flask. **[Himachal 05 ; Delhi 02]**
9. What are convection currents ? What role do they play in relation to trade winds and monsoons ?
10. State and prove the Kirchhoff's law of heat radiation. Why does a glass heated in a furnace when taken out in dark glow with red light ?
11. State : (*i*) Stefan's law and (*ii*) Wein's displacement law.

    How you will derive Newton's law of cooling from Stefan's law ? **[Himachal 01C]**
12. State Newton's law of cooling. Deduce the relations :

    $$\log_e (T - T_0) = -kt + c$$

    and $$\Delta t = T - T_0 = Ce^{-kt}$$

    where the symbols have their usual meanings. Represent Newton's law of cooling graphically by using each of the above equations.
13. (*a*) Explain the terms specific heat and heat capacity.

    (*b*) State Newton's law of cooling. Derive mathematical expression for it. **[Delhi 10]**
14. State Newton's law of cooling. Express it mathematically. How can this law be verified experimentally ? **[Delhi 12]**

Answers

1. Refer answer to Q. 13 on page 11.4.
2. Refer to points 14, 15 and 16 of Glimpses and answer to Q. 25 on page 11.10.
3. Refer answer to Q. 26 on page 11.10.
4. Refer answer to Q. 65 on page 11.44.
5. Refer answer to Q. 66 on page 11.45.
6. Refer answer to Q. 44 on page 11.24.
7. Refer answer to Q. 43 and Q. 44 on page 11.24.
8. Refer to points 30, 35 and 36 of Glimpses. Refer answer to Q. 58(*iv*) on page 11.38.
9. Refer answer to Q. 48(*iii*) and (*v*) on page 11.32.
10. Refer answer to Q. 57 on page 11.37 and Q. 58(*ii*) on page 11.37.
11. Refer answer to Q. 60 on page 11.38 and Q. 61 on page 11.38.
12. Refer answer to Q. 53 on page 11.34.
13. (*a*) Refer to points 21 and 23 of Glimpses.

    (*b*) Refer answer to Q. 61 on page 11.38.
14. Refer answer to Q. 53 on page 11.34.

Competition Section

Thermal Properties of Matter

GLIMPSES

1. **Heat.** Heat is a form of energy which produces in us the sensation of hotness or coldness. According to *dynamic theory*, heat may be regarded as the energy of molecular motion which is equal to the sum total of the kinetic energy possessed by the molecules by virtue of their translational, vibrational and rotational motions.
2. **Units of heat.** *Calorie* (cal) is the C.G.S. unit of heat. One calorie is defined as the heat energy required to raise the temperature of one gram of water from 14.5°C to 15.5°C. Like all other forms of energy, the S.I. unit of heat is joule.

   1 calorie = 4.186 joule.
3. **Joule's mechanical equivalent of heat.** Whenever a given amount of work (W) is converted into heat, always the same amount of heat Q is produced.

   Thus $W \propto Q$

   or $W = JQ$

   or $J = \frac{W}{Q}$.

   The proportionality constant J is called Joule's mechanical equivalent of heat. It may be defined as the amount of work that must be done to produce a unit quantity of heat.

   $$J = 4.2 \times 10^7 \text{ erg cal}^{-1} = 4.2 \text{ J cal}^{-1}.$$
4. **Temperature.** It is the degree of hotness of a body. The temperature of a body gives a measure of the average kinetic energy of its molecules.
5. **Thermometer.** It is a device used to measure the temperature of a body. It makes use of some measurable property (called thermometric property) of a substance which changes linearly with temperature.
6. **Different temperature scales.**

Temperature scale	Lower fixed point (Melting point of ice)	Upper fixed point (Boiling point of water)
1. Celsius	0°C	100°C
2. Fahrenheit	32°F	212°F
3. Reaumer	0°R	80°R
4. Kelvin	273.15 K	373.15 K

7. **Relations between different temperature scales.** If T_C, T_F, T_R and T are the temperatures of a body on Celsius, Fahrenheit, Reaumer and Kelvin scales respectively, then

   (i) $$\frac{T_C - 0}{100 - 0} = \frac{T_F - 32}{212 - 32} = \frac{T_R - 0}{80 - 0} = \frac{T - 273.15}{373.15 - 273.15}$$

   or $$\frac{T_C}{5} = \frac{T_F - 32}{9} = \frac{T_R}{4} = \frac{T - 273.15}{5}$$

   (ii) $T_C = \frac{5}{9}(T_F - 32)$, $\quad T_F = \frac{9}{5} T_C + 32$

   (iii) $T = T_C + 273.15$, $\quad T_C = T - 273.15$

   (iv) $T_F = \frac{9}{5}(T - 273.15) + 32 = \frac{9}{5} T - 459.67$,

   $$T = \frac{5}{9} T_F + 255.37.$$
8. **Absolute scale of temperature.** The lowest possible temperature of -273.15°C at which a gas is supposed to have zero volume (and zero pressure) and at which entire molecular motion stops is called absolute zero of temperature. The temperature scale which starts with -273.15°C as its zero is called Kelvin scale or absolute scale. The size of degree on Kelvin scale is same as that on Celcius scale.

   $$T\,(\text{K}) = t\,°(\text{C}) + 273.15.$$

9. **Triple point of water.** The triple point of water is the state at which the three phases of water namely ice, liquid water and water vapour are equally stable and co-exist in equilibrium. It is unique because it occurs at a specific temperature of 273.16 K and a specific pressure of 0.46 cm of Hg column.

10. **Constant volume air thermometer.** It is used to measure the pressure of a definite mass of air at different temperatures, the volume of air remaining constant. It is based on the pressure law,

$$\frac{P}{T} = \frac{P_0}{T_0} \quad \text{or} \quad T = T_0 \times \frac{P}{P_0}$$

In terms of triple point of water, $T = T_{tr} \times \frac{P}{P_{tr}}$.

11. **Ideal gas temperature scale.** The ideal gas temperature on the Kelvin scale is defined by the equation

$$T = \lim_{P_{tr} \to 0} 273.16 \left(\frac{P}{P_{tr}} \right)$$

It is independent of the nature of the gas.

12. **Platinum resistance thermometer.** It is based on the fact that resistance of a platinum wire varies with temperature. If R_0 and R are the resistances at 0°C and t °C respectively, then

$$R = R_0 (1 + \alpha t)$$

Here α is the temperature coefficient of resistance of platinum and is defined as the increase in resistance per unit resistance at 0°C for 1°C rise in temperature.

$$\alpha = \frac{R - R_0}{R_0 \times t}$$

Unit of α is °C^{-1}.

If R_0 and R_{100} are the resistances of a platinum wire at ice point and steam point respectively, then the temperature t_R of a body for which the corresponding resistance is R_t is given by

$$t_R = \frac{R_t - R_0}{R_{100} - R_0} \times 100°\text{C}.$$

13. **Thermoelectric thermometer.** It uses thermoelectric e.m.f. (ε) as thermometric property. For the linear part of the thermo e.m.f., the unknown temperature is given by

$$t_\varepsilon = \frac{\varepsilon_t - \varepsilon_0}{\varepsilon_{100} - \varepsilon_0} \times 100 \text{ degrees}$$

14. **Linear expansion.** When a solid rod of initial length l is heated through a temperature ΔT, its final (increased) length is given by

$$l' = l(1 + \alpha \Delta T)$$

where α is coefficient of linear expansion. It is given by

$$\alpha = \frac{l' - l}{l \times \Delta T}$$

The coefficient of linear expansion of the material of a solid rod is defined as the increase in length per unit length per degree rise in temperature.

15. **Superficial expansion.** When a solid sheet of initial surface area S is heated through a temperature ΔT, its final (increased) surface area is given by

$$S' = S(1 + \beta \Delta T)$$

where β is coefficient of superficial expansion. It is given by

$$\beta = \frac{S' - S}{S \times \Delta T}$$

It is defined as the increase in surface area per unit surface area per degree rise in temperature.

16. **Cubical expansion.** When a solid of initial volume V is heated through a temperature ΔT, its final (increased) volume is given by $V' = V(1 + \gamma \Delta T)$ where γ is coefficient of cubical expansion. It is given by

$$\gamma = \frac{V' - V}{V \times \Delta T}$$

It is defined as the increase in volume per unit volume per degree rise in temperature.

The coefficient of cubical expansion of an ideal gas is equal to the reciprocal of its absolute temperature.

$$\gamma = \frac{1}{T}.$$

17. **Relation between α, β and γ.** The three coefficients of thermal expansion are related as

$$\frac{\alpha}{1} = \frac{\beta}{2} = \frac{\gamma}{3} \quad \text{or} \quad \beta = 2\alpha \quad \text{and} \quad \gamma = 3\alpha.$$

The units of α, β and γ are same *viz.* °C^{-1} or K^{-1}.

18. **Coefficient of apparent expansion of a liquid.** It is defined as the apparent increase in volume per unit original volume per degree rise in temperature.

$$\gamma_a = \frac{\text{Apparent increase in volume}}{\text{Original volume} \times \text{Rise in temperature}}.$$

19. **Coefficient of real expansion of a liquid.** It is defined as the real increase in volume per unit original volume per degree rise in temperature.

$$\gamma_r = \frac{\text{Real increase in volume}}{\text{Original volume} \times \text{Rise in temperature}}$$

If γ_g is the coefficient of cubical expansion of glass (material of the container), then

$$\gamma_r = \gamma_a + \gamma_g.$$

20. **Variation of density with temperature.** The density of a solid or liquid decreases with the increase of temperature.

$$\rho' = \rho(1 - \gamma \Delta T)$$

21. **Specific heat.** It may be defined as the amount of heat required to raise the temperature of unit mass of a substance through one degree. If Q heat is needed to raise the temperature of m mass of a substance through ΔT, then specific heat is

$$c = \frac{Q}{m\Delta T}$$

The cgs unit of specific heat is cal $g^{-1}\,{}^{\circ}C^{-1}$ and the SI unit is $Jkg^{-1}\,K^{-1}$. Clearly,

Heat gained or lost, $Q = mc\,\Delta T$.

22. **Molar specific heat.** *It is defined as the amount of heat required to raise the temperature of 1 mole of the substance through one degree.* If Q heat is needed to raise the temperature of n moles of a substance through ΔT, then molar specific heat is

$$C = \frac{Q}{n\Delta T}$$

The cgs unit of molar specific heat is cal $mol^{-1}\,{}^{\circ}C^{-1}$ and the SI unit is $J\,mol^{-1}\,K^{-1}$.

23. **Heat capacity or thermal capacity.** *It is defined as the amount of heat required to raise the temperature of a body through one degree.*

Heat capacity = Mass × Specific heat = mc

The cgs unit of heat capacity is cal$^{\circ}C^{-1}$ and the S.I. unit is JK^{-1}.

24. **Water equivalent.** The water equivalent of a body is defined as the mass of water which requires the same amount of heat as is required by the given body for the same rise of temperature.

w = Mass × specific heat = mc

The cgs unit of water equivalent is g and the SI unit is kg.

25. **Latent heat.** The amount of heat required to change the state of unit mass of a substance at a constant temperature is called its latent heat. It is denoted by L.

26. **Latent heat of fusion.** The amount of heat required to change the state of unit mass of a substance from solid to liquid at its melting point is called latent heat of fusion.

27. **Latent heat of vaporisation.** The amount of heat required to change the state of unit mass of a substance from liquid to vapour at its boiling point is called latent heat of vaporisation.

28. **Principle of calorimetry.** When two bodies at different temperatures are placed in contact with each other, the heat lost by the hot body is equal to the heat gained by the cold body. This is the principle of calorimetry or the principle of mixtures.

Heat gained = Heat lost

29. **Transfer of heat.** The three modes of transfer of heat are conduction, convection and radiation.

30. **Conduction.** It is a process in which heat is transmitted from one part of a body to another at a lower temperature through molecular collisions, without any actual flow of matter.

31. **Steady state.** The state of the rod when temperature of every cross-section of the rod becomes constant and there is no further absorption of heat in any part is called steady state.

32. **Factors on which conduction of heat depends.** When two opposite faces of a slab each of area of cross-section A and separated by a distance x are maintained at temperatures T_1 and T_2 ($T_1 > T_2$), then amount of heat that flows in time t,

$$Q = \frac{KA\,(T_1 - T_2)\,t}{x}$$

where K is coefficient of thermal conductivity of the material of the slab between its two faces. The rate of flow of heat through the slab is

$$\frac{dQ}{dt} = -KA\frac{dT}{dx}$$

Here dT/dx is the rate of fall of temperature with distance and is called temperature gradient.

33. **Coefficient of thermal conductivity.** *It may be defined as the quantity of heat energy that flows in unit time between the opposite faces of a cube of unit side, the faces being kept at one degree difference of temperature.* Its cgs unit is cal $s^{-1}cm^{-1}\,{}^{\circ}C^{-1}$ and SI unit is $Js^{-1}m^{-1}K^{-1}$ or $Wm^{-1}K^{-1}$.

Dimensional formula of K is $[MLT^{-3}K^{-1}]$.

Thermal conductivities of metals are much greater than those for metals. Gases are poor thermal conductors.

34. **Heat current and resistance.** The flow of heat per unit time in conduction is called *heat current*. The ratio of the temperature difference between the ends of a conductor to the heat current through it is called *thermal resistance*.

Heat current, $H = \frac{Q}{t} = KA\frac{\Delta T}{\Delta x}$.

Thermal resistance, $R_H = \frac{\Delta T}{H} = \frac{\Delta x}{KA}$.

SI unit of $R_H = KW^{-1}$.

Dimensions of $R_H = [M^{-1}L^{-2}T^{-3}K]$.

35. **Convection.** It is the process by which heat is transmitted through a substance from one point to another due to the bodily motion of the heated particles of the substance. Fluids are mainly heated through convection. *Natural convection* arises due to unequal heating and gravity, when more heated and less dense parts rise and are replaced by the cooler parts of the fluid. In *forced convection*, a material is forced to move by an agency like a pump or a blower.

36. **Radiation.** It is the process by which heat is transmitted from one place to another without heating the intervening medium.

37. **Prevost's theory of heat exchanges.** All bodies emit radiations irrespective of their temperatures. They emit radiations to the surroundings and receive radiations from the surroundings. In the equilibrium state, the exchange of energy between a body and its surroundings occurs in equal amounts.

38. **Thermal radiation.** *The electromagnetic radiation emitted by a body by virtue of its temperature is called thermal radiation or radiant energy.* Thermal radiations are electromagnetic waves of long wavelength ranging from 1 μm to 100 μm.

39. **Absorptive power or absorptivity** (a_λ). The absorptive power of a body for a given wavelength λ is defined as the ratio of amount of heat energy absorbed in a certain time to the total heat energy incident on it in the same time within a unit wavelength range around the wavelength λ. It is a dimensionless quantity.

40. **Emissive power** (e_λ). The emissive power of a body at a given temperature and for a given wavelength λ is defined as the amount of radiant energy emitted per unit time per unit surface area of the body within a unit wavelength range around the wavelength λ. Its SI unit is J s^{-1} m^{-2} or W m^{-2}.

41. **Emissivity** (ε). It is the ratio of the emissive power of a body (e) to the emissive power (E) of a black body at the same temperature. It is given by

$$\varepsilon = \frac{e}{E}$$

It is a dimensionless quantity having value between 0 and 1. The emissivity of a black body is 1.

42. **Black body.** *A black body is one which neither reflects nor transmits but absorbs whole of the heat radiation incident on it. The absorptive power of a black body is unity.*

43. **Kirchhoff's law.** It states that at any given temperature, the ratio of the emissive power (e_λ) to the absorptive power (a_λ) corresponding to certain wavelength is constant for all bodies and this constant is equal to the emissive power of the perfect black body (E_λ) at the same temperature and corresponding to the same wavelength. That is

$$\frac{e_\lambda}{a_\lambda} = E_\lambda \text{ (constant)}$$

Hence a good absorber is a good emitter.

44. **Stefan Boltzmann Law.** It states that the total amount of energy radiated per second per unit area of a perfect black body is directly proportional to the fourth power of the absolute temperature of the body.

Thus $E \propto T^4$ or $E = \sigma T^4$

where σ is called *Stefan's constant*. Its value is

$$\sigma = 5.67 \times 10^{-5} \text{ erg s}^{-1} \text{ cm}^{-2} \text{ K}^{-4}$$

(in CGS system)

or $\sigma = 5.67 \times 10^{-8} \text{ Js}^{-1}\text{m}^{-2}\text{K}^{-4}$

$= 5.67 \times 10^{-8} \text{ Wm}^{-2} \text{ K}^{-4}$ (in SI)

If a perfect black body at temperature T is placed in an enclosure at temperature T_0, then net amount of energy radiated per second per unit area by the black body is

$$E = \sigma(T^4 - T_0^4)$$

If the body and the enclosure are not perfect black bodies and have relative emissivity ε, then

$$E = \varepsilon\sigma(T^4 - T_0^4)$$

45. **Newton's law of cooling.** For small temperature difference between a body and its surroundings, the rate of cooling of the body is directly proportional to the temperature difference and the surface area exposed. This is known as Newton's law of cooling.

$$\frac{dT}{dt} = -kA(T - T_0).$$

46. **Wien's displacement law.** It states that the wavelength (λ_m) corresponding to which the energy emitted by a perfect black body is maximum is inversely proportional to the absolute temperature (T) of the black body *i.e.,*

$$\lambda_m \propto \frac{1}{T} \quad \text{or} \quad \lambda_m = \frac{b}{T}$$

where b is a constant of proportionality and is called *Wien's constant*. Its value is

$$b = 2.9 \times 10^{-3} \text{ mK}$$

47. **Solar constant.** It is defined as the amount of solar radiant energy that a unit area of a perfect black body on the earth would receive per second in the absence of the atmosphere, with its surface perpendicular to the direction of the sun rays. Its value is 1340 W m^{-2}.

48. **Surface temperature of the sun.** If S_0 is solar constant, R_s the solar radius and R_0 is the mean distance of the earth from the sun, then surface temperature of the sun will be

$$T = \left[\frac{R_0^2 S_0}{R_s^2 \sigma}\right]^{1/4}.$$