CHAPTER 2

UNITS AND MEASUREMENTS

2.1 NEED FOR THE MEASUREMENT

1. *Explain the need for measurement in physics.*

Need for the measurement. Physics is an exact science which gives an accurate knowledge about the nature and the natural phenomena. It expresses various natural phenomena in terms of the relationships among the quantities involved. The exactness or accuracy of these relationships depends upon the measurements we make. The accuracy of the measurements, in turn, depends on the accuracy of the measuring instruments and techniques. The recent progress in science and technology has been possible only due to the development of high precision instruments. One can measure mass as small as that of an electron ($\sim 10^{-30}$ kg) and as large as that of the universe ($\sim 10^{55}$ kg).

Prof. *William S. Franklin* emphasised the importance of measurement in the following words :

> *"The most important thing for a young man to acquire from his first course in physics is an appreciation for precise details."*

Stressing upon the importance of measurement, *Lord Kelvin*, a great English physicist of 19th century, once remarked that

> *"When you can measure what you are speaking about and express it in numbers, you know something about it, but when you cannot measure it and express it in numbers, your knowledge is of a meagre and unsatisfactory kind, but you have scarcely, in your thoughts, advanced to the stage of science whatever the matter may be."*

2.2 PHYSICAL QUANTITIES : FUNDAMENTAL AND DERIVED

2. *What are physical quantities ? Distinguish between fundamental and derived quantities.*

Physical quantities. *All those quantities which can be measured directly or indirectly and in terms of which the laws of physics can be expressed are called physical quantities.* For example, length, mass, temperature, speed, force, electric current, etc. The physical quantities are the building blocks of physics in terms of which the basic laws of physics can be expressed in mathematical forms.

Physical quantities are of *two* types – fundamental and derived.

(*i*) ***Fundamental quantities.*** *The physical quantities which can be treated as independent of other physical quantities and are not usually defined in terms of other physical quantities are called fundamental quantities.* To give a consistent and unambiguous description to all physical quantities, we need a minimum of **seven fundamental** or **base quantities**. These are mass, length, time, electric current, temperature, luminous intensity and amount of substance.

(*ii*) ***Derived quantities.*** *The physical quantities whose defining operations are based on other physical quantities are called derived quantities.* All physical quantities other than the seven base quantities are derived quantities. For example, velocity, acceleration, force, momentum, etc.

2.3 THE MEASURING PROCESS

3. *What is meant by the term measurement of a physical quantity ? How is the result of measurement of a physical quantity expressed ?*

Measurement. The measurement of a physical quantity is the process of comparing this quantity with a standard amount of the physical quantity of the same kind, called its unit.

To express the measurement of a physical quantity, we need to know *two* things :

(*i*) The *unit* in which the quantity is measured.

(*ii*) The *numerical value* or the *magnitude* of the quantity *i.e.*, the number of times that unit is contained in the given physical quantity.

∴ Measure of a physical quantity

= Numerical value of the physical quantity × size of the unit

or $Q = nu$

Let length of a room = 5 m = 500 cm.

Clearly, the smaller the size of the unit, the larger is the numerical value associated with the physical quantity.

Thus the numerical value (n) is inversely proportional to the size (u) of the unit.

$$n \propto \frac{1}{u} \quad \text{or} \quad nu = \text{constant}.$$

If n_1 and n_2 are numerical values for a physical quantity Q corresponding to the units u_1 and u_2, then

$$Q = n_1 u_1 = n_2 u_2 \qquad ...(2.1)$$

2.4 PHYSICAL UNIT AND ITS DESIRABLE CHARACTERISTICS

4. *What is a physical unit ? Write the essential requirements that a physical unit/standard must meet.*

Physical unit. *The standard amount of a physical quantity chosen to measure the physical quantity of the same kind is called a physical unit.*

Desirable characteristics of a physical unit :

1. It should be *well-defined.*
2. It should be of *convenient size, i.e.,* neither too small nor too large in comparison with the measurable physical quantity.
3. It should not change with time.
4. It should be easily *reproducible.*
5. It should be *imperishable* or *indestructible.*
6. It should not be affected by the change in physical conditions such as pressure, temperature, etc.
7. It should be *internationally acceptable.*
8. It should be easily *accessible.*

2.5 FUNDAMENTAL AND DERIVED UNITS

5. *Although the number of physical quantities which we measure is very large, yet we do not need a very large number of units for this measurement. Why ?*

This is possible because the various physical quantities are related to each other and so their units can be expressed in terms of just seven basic or fundamental units.

6. *What are fundamental and derived units ? Give some examples.*

Fundamental units. *The physical units which can neither be derived from one another, nor they can be further resolved into more simpler units are called fundamental units.* The units of fundamental quantities such as mass, length, etc. are fundamental units.

Derived units. *All the other physical units which can be expressed in terms of the fundamental units are called derived units.* Let us consider the unit of speed.

$$\text{Speed} = \frac{\text{Distance travelled}}{\text{Time taken}}$$

$$\therefore \text{ Unit of speed} = \frac{\text{Unit of distance}}{\text{Unit of time}}$$

$$= \frac{\text{metre}}{\text{second}} = \text{ms}^{-1}$$

Thus the unit of speed (ms^{-1}) is a derived unit as it has been expressed in the fundamental units of length and time.

2.6 SYSTEMS OF UNITS

7. *What is a system of units ? Mention the various types of systems of units.*

System of units. *A complete set of units which is used to measure all kinds of fundamental and derived quantities is called a system of units.*

Some of the commonly used systems of units are as follows :

(*i*) ***cgs system.*** It was set up in France. It is based on *centimetre, gram* and *second* as the fundamental units of length, mass and time respectively.

(*ii*) ***fps system.*** It is a British system based on *foot, pound* and *second* as the fundamental units of length, mass and time respectively.

(*iii*) ***mks system.*** It is also a French system based on *metre, kilogram* and *second* as the fundamental units of length, mass and time respectively.

(*iv*) ***SI : The international system of units.*** SI is the abbreviation for *"Systeme Internationale d' Unites"*, which is French equivalent for *international system of units.* It is a modernised and extended form of the metric systems like cgs and mks systems. This system was adopted by eleventh *General Conference of Weights and Measures in 1960.* It covers all branches of science and technology. It is based on the following seven basic units and two supplementary units.

Table 2.1 *Basic SI quantities and units*

S. No.	Basic physical quantity	Basic unit	Symbol
1.	Length	metre	m
2.	Mass	kilogram	kg
3.	Time	second	s
4.	Temperature	kelvin	K
5.	Electric current	ampere	A
6.	Luminous intensity	candela	cd
7.	Quantity of matter	mole	mol

Table 2.2 *Supplementary SI units*

S. No.	Supplementary quantity	Basic unit	Symbol
1.	Plane angle	radian	rad
2.	Solid angle	steradian	sr

For Your Knowledge

- The cgs, mks and SI are *metric* or *decimal systems* of units. This is because the multiples and sub-multiples of their basic units are related to the practical units by powers of 10.
- The fps system is not a metric system. This system is not in much use these days.

2.7 DEFINITIONS OF BASIC AND SUPPLEMENTARY SI UNITS

8. *Name and define all the basic and supplementary units of SI.*

The *seven* **basic SI units** are defined as follows :

(*i*) **Metre (m).** It is the SI unit of length. *One metre is defined as the length of the path travelled by light in vacuum during a time interval of 1/299, 792, 458 of a second.*

(*ii*) **Kilogram (kg).** It is the SI unit of mass. *One kilogram is the mass of prototype cylinder of platinum-iridium alloy (whose height is equal to its diameter)* preserved at the International Bureau of Weights and Measures, at Sevres, near Paris.

(*iii*) **Second (s).** It is the SI unit of time. *One second is the duration of 9, 192, 631, 770 periods of the radiation corresponding to the transition between two hyperfine levels of the ground state of the cesium – 133 atom.*

(*iv*) **Ampere (A).** It is the SI unit of electric current. *One ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible cross-section, and placed 1 metre apart in vacuum, would produce between these conductors a force equal to* 2×10^{-7} *newton per metre of length.*

(*v*) **Kelvin (K).** It is the SI unit of temperature. *One kelvin is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water.* The triple point of water is the temperature at which ice, water and water vapour co-exist.

(*vi*) **Candela (cd).** It is the SI unit of luminous intensity. *One candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency* 540×10^{12} *hertz and* that has a radiant intensity of 1 / 683 watt per steradian in that direction.

(*vii*) **Mole (mol).** *One mole is that amount of a substance which contains as many elementary entities as there are atoms in 0.012 kg of carbon-12 isotope.* The entities may be atoms, molecules, ions etc.

The *two* **supplementary SI units** are defined as follows :

(*a*) **Radian (rad).** *It is defined as the plane angle subtended at the centre of a circle by an arc equal in length to the radius of the circle.*

$$\theta \text{ (in radians)} = \frac{\text{Arc}}{\text{Radius}} = \frac{l}{r}$$

(*b*) **Steradian (sr).** *It is defined as the solid angle subtended at the centre of a sphere by a surface of the sphere equal in area to that of a square, having each side equal to the radius of the sphere.*

$$\Omega \text{ (in steradian)} = \frac{\text{Surface area}}{\text{Radius}^2}$$

For Your Knowledge

A HISTORICAL VIEW OF BASIC SI UNITS

metre (m)

- In 1791, the Paris Academy of Sciences defined metre as *one ten millionth part of the distance measured along the meridian from the north pole to the equator*.
- In 1889, the General Conference of Weights and Measures defined metre *as the distance between two lines marked on a platinum-iridium rod preserved at a constant temperature of 273.16 K at 1 bar pressure in the* International Bureau of Weights and Measures at Sevres, near Paris.

Fig. 2.1 Prototype metre.

- In 1960, the standard metre was defined in terms of wavelength of light. *One metre is defined as the distance which contains 1650, 763.73 wavelengths of certain orange-red radiation (of wavelength 6057.8021 Å) emitted by krypton-86 source kept at the temperature of triple point of nitrogen*.

kilogram (kg)

- In 1791 in France, originally kilogram was defined as the mass of one cubic decimetre (or 1 litre) of water at 4°C. Water has maximum density at this temperature.
- In 1889, the General Conference of Weights and Measures defined kilogram as the mass of platinum-iridium cylinder preserved at Sevres.

second (s)

- The Paris Academy of Sciences defined second as the time taken by a simple pendulum of one metre length to swing from one extreme position to the other.
- A *solar day* is defined as the time interval between the noons of two successive days. A *mean solar day* is the average of the solar days in one complete year. The $\frac{1}{24 \times 60 \times 60}$ or $\frac{1}{86,400}$ part of a mean solar day is called *mean solar second*.
- In 1956, the second was defined in terms of the tropical year 1900, the duration of which was taken as 365.242 mean solar days. So second was defined as the fraction $\frac{1}{365.242 \times 86,400}$ or $\frac{1}{31,556,925.9747}$ of the tropical year 1900 and was called ephimeris second.

2.8 COHERENT SYSTEM OF UNITS

9. *What is a coherent system of units ? Give examples.*

Coherent system. *It is a system of units based on a certain set of fundamental units from which all derived units can be obtained by simple multiplication or division without introducing any numerical factor.* For example, mks system is a coherent system of units in mechanics. All derived units in mechanics such as those of area, volume, density, acceleration, force, etc., can be obtained by the multiplication or division of the fundamental units of mass, length and time. SI is a coherent system of units for all branches of physics.

2.9 ADVANTAGES OF SI

10. *State the advantages of SI over other systems of units.*

Advantages of SI over other systems of units :

(*i*) ***SI is a coherent system of units.*** All derived units can be obtained by simple multiplication or division of fundamental units without introducing any numerical factor.

(*ii*) ***SI is a rational system of units.*** It uses only one unit for a given physical quantity. For example, all forms of energy are measured in joule. On the other hand, in mks system, the mechanical energy is measured in joule, heat energy in calorie and electrical energy in watt hour.

(*iii*) ***SI is a metric system.*** The multiples and sub-multiples of SI units can be expressed as powers of 10.

(*iv*) ***SI is an absolute system of units.*** It does not use gravitational units. The use of '*g*' is not required.

(*v*) ***SI is an internationally accepted system of units.***

2.10 GUIDELINES FOR WRITING SI UNITS IN SYMBOLS

11. *State the rules that are followed in writing SI units in symbolic form.*

Rules for writing SI units in symbolic form :

(*i*) Small letters are used for symbols of units.

(*ii*) Symbols are not followed by a full stop.

(*iii*) The initial letter of a symbol is capital only when the unit is named after a scientist.

(*iv*) The full name of a unit always begins with a small letter even if it has been named after a scientist.

(*v*) Symbols do not take plural form.

2.11 ABBREVIATIONS IN POWERS OF TEN

12. *Make a list of various prefixes used for powers of 10. Give some examples.*

Abbreviations in powers of ten. When the magnitudes of the physical quantities are very large or very small, it is convenient to express them in the multiples or submultiples of the SI units. The various prefixed used for powers of 10 are listed in table 2.3.

Table 2.3 *Prefixes for powers of ten*

Multi-ple	*Prefix*	*Symbol*	*Sub-multiple*	*Prefix*	*Symbol*
10^1	deca	da	10^{-1}	deci	d
10^2	hecto	h	10^{-2}	centi	c
10^3	kilo	k	10^{-3}	milli	m
10^6	mega	M	10^{-6}	micro	μ
10^9	giga	G	10^{-9}	nano	n
10^{12}	tera	T	10^{-12}	pico	p
10^{15}	peta	P	10^{-15}	femto	f
10^{18}	exa	E	10^{-18}	atto	a

Examples :

1 megaohm = 1 MΩ = 10^6 Ω

1 milliampere or 1 mA = 10^{-3} A

1 kilometre = 1 km = 10^3 m

1 microvolt or 1 μV = 10^{-6} V

1 decagram = 1 da g = 10 g

1 nanosecond or 1 ns = 10^{-9} s

1 centimetre = 1 cm = 10^{-2} m

1 picofarad or 1 pF = 10^{-12} F.

2.12 SOME COMMON PRACTICAL UNITS

13. *Make a list of some commonly used practical units. How these units are related to SI units ?*

A. Practical units for measuring small distances :

(*i*) ***Fermi.*** It is the small practical unit of distance used for measuring nuclear sizes. It is also called femtometre.

$$\textbf{1 fermi = 1 fm} = \mathbf{10^{-15}\ m}$$

The radius of a proton is 1.2 fermi.

(*ii*) ***Angstrom.*** It is used to express wavelength of light.

$$\textbf{1 angstrom = 1 Å} = \mathbf{10^{-10}\ m = 10^{-8}\ cm}$$

(*iii*) ***Nanometre.*** It is also used for expressing wavelength of light.

$$\textbf{1 nanometre = 1 nm} = \mathbf{10^{-9}\ m}$$

(*iv*) ***Micron.*** It is the unit of distance defined as micrometre.

$$\textbf{1 micron} = \mathbf{1\ \mu m = 10^{-6}\ m}$$

B. Practical units used for measuring large distances :

(*i*) ***Light year.*** *It is the distance travelled by light in vacuum in one year.*

1 light year = Speed of light in vacuum × 1 year

$$= 3 \times 10^8\ \text{ms}^{-1} \times 365.25 \times 24 \times 60 \times 60\ \text{s}$$

$$\therefore\ \mathbf{1\ light\ year = 1\ ly = 9.467 \times 10^{15}\ m}$$

Light year is used in astronomy to measure distances of nearby stars. *Alpha centauri*, the nearest star outside the solar system is 4.3 light years away from the earth.

(*ii*) ***Astronomical unit.*** *It is defined as the mean distance of the earth from the sun.* It is used in astronomy to measure distances of planets.

$$\mathbf{1\ astronomical\ unit = 1\ AU = 1.496 \times 10^{11}\ m}$$

(*iii*) ***Parsec (parallactic second).*** It is the largest practical unit of distance used in astronomy. *It is defined as the distance at which an arc of length 1 astronomical unit subtends an angle of 1 second of arc.*

$$\text{As} \quad \theta = \frac{l}{r} \quad \therefore \quad r = \frac{l}{\theta}$$

$$1\ \text{parsec} = \frac{1\ \text{AU}}{1''} = \frac{1.496 \times 10^{11}\ \text{m}}{\dfrac{1}{3600} \times \dfrac{\pi}{180}\ \text{rad}} = 3.08 \times 10^{16}\ \text{m}$$

$$\mathbf{1\ parsec = 3.08 \times 10^{16}\ m = 3.26\ ly}$$

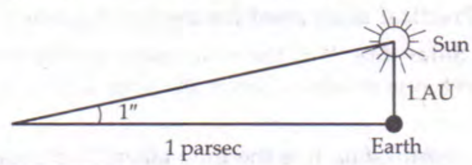


Fig. 2.2 Parsec

Relations between astronomical unit, light year and parsec

$$1\ \text{AU} = 1.5 \times 10^{11}\ \text{m}$$

$$1\ \text{ly} = 9.46 \times 10^{15}\ \text{m}$$

$$1\ \text{parsec} = 3.08 \times 10^{16}\ \text{m}$$

$$\frac{1\ \text{ly}}{1\ \text{AU}} = \frac{9.46 \times 10^{15}}{1.5 \times 10^{11}} = 6.3 \times 10^4$$

$$\therefore \quad \mathbf{1\ ly = 6.3 \times 10^4\ AU}$$

$$\text{Also} \quad \frac{1\ \text{parsec}}{1\ \text{ly}} = \frac{3.08 \times 10^{16}}{9.46 \times 10^{15}} = 3.26$$

$$\therefore \quad \mathbf{1\ parsec = 3.26\ ly}$$

Clearly, **1 parsec > 1 ly > 1 AU**

C. Practical units for measuring areas :

(*i*) ***Barn.*** It is used for very small areas, such as nuclear cross-sections.

$$\mathbf{1\ barn = 10^{-28}\ m^2}$$

(*ii*) ***Acre.*** It is used measuring large areas.

$$\mathbf{1\ acre = 4047\ m^2}$$

(*iii*) ***Hectare.*** It is also used for measuring large areas.

$$\mathbf{1\ hectare = 10^4\ m^2}$$

D. Practical units used for measuring large masses :

1 tonne or 1 metric ton = 1000 kg

1 quintal = 100 kg

1 slug = 14.57 kg

1 pound = 1 lb = 0.4536 kg

1 Chandra Shekher limit = 1 CSL = 1.4 times the mass of the sun.

CSL is the largest practical unit of mass.

E. Practical unit used for measuring very small masses :

Atomic mass unit. It is defined as $\frac{1}{12}$th of the mass of one ${}^{12}_{6}C$ atom.

1 atomic mass unit = 1 amu = 1 u = 1.66×10^{-27} kg

The mass of a proton or a neutron is of the order of one amu.

F. Practical units used for measuring time :

(*i*) ***Solar day.*** It is the time taken by the earth to complete one rotation about its own axis w.r.t. the sun.

(*ii*) ***Sedrial day.*** It is the time taken by the earth to complete one rotation about its own axis w.r.t. a distant star.

(*iii*) ***Solar year.*** It is the time taken by the earth to complete one revolution around the sun in its orbit.

1 solar year = 365.25 average solar days
= 366.25 sedrial days

(*iv*) ***Tropical year.*** The year in which there is total solar eclipse is called tropical year.

(*v*) ***Leap year.*** The year which is divisible by 4 and in which the month of February has 29 days is called a leap year.

(*vi*) ***Lunar month.*** It is the time taken by the moon to complete one revolution around the earth in its orbit.

1 lunar month = 27.3 days

(*vii*) ***Shake.*** It is the smallest practical unit of time.

$$1\ \text{shake} = 10^{-8}\ \text{s}$$

G. Practical units used for measuring pressure :

1 bar = 1 atmospheric pressure
$= 10^5\ \text{Nm}^{-2} = 10^5$ pascal (Pa)

1 millibar = 10^2 Pa

1 torr = 1 mm of Hg column

1 atmospheric pressure
= 1 bar = 760 mm of Hg column
= 760 torr.

Examples based on Simple Conversion of Units

CONVERSIONS USED

$$1\ \text{kg m}^{-3} = 10^{-3}\ \text{g cm}^{-3}$$
$$1\ \text{g cm}^{-3} = 10^3\ \text{kg m}^{-3}$$
$$1\ \text{N} = 10^5\ \text{dyn}$$
$$1\ \text{amu} = 1.66 \times 10^{-27}\ \text{kg}$$
$$1\ \text{AU} = 1.496 \times 10^{11}\ \text{m}$$
$$1\ \text{ly} = 9.46 \times 10^{15}\ \text{m}$$
$$1\ \text{parsec} = 3.08 \times 10^{16}\ \text{m}$$
$$1\ \text{Å} = 10^{-10}\ \text{m} = 0.1\ \text{nm}$$

EXAMPLE 1. *How many light years are there in one metre ?* [Himachal 06C]

Solution. $1\ \text{ly} = 9.46 \times 10^{15}$ m

or 9.46×10^{15} m = 1 ly

$$\therefore\ 1\ \text{m} = \frac{1}{9.46 \times 10^{15}}\ \text{ly} = \mathbf{1.057 \times 10^{-16}\ ly.}$$

EXAMPLE 2. *What is the number of electrons that would weigh 1 kg ? Mass of an electron is* 9.11×10^{-31} *kg.*

Solution. Mass of each electron = 9.11×10^{-31} kg

Total mass = 1 kg

$\therefore$ Number of electrons

$$= \frac{\text{Total mass}}{\text{Mass of each electron}} = \frac{1\ \text{kg}}{9.11 \times 10^{-31}\ \text{kg}}$$

$$= \mathbf{1.1 \times 10^{30}.}$$

EXAMPLE 3. *The radius of gold nucleus is 41.3 fermi. Express its volume in* m^3.

Solution. Here r = 41.3 fermi = 41.3×10^{-15} m

$$\text{Volume,}\quad V = \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times (41.3 \times 10^{-15}\ \text{m})^3$$
$$= \mathbf{2.95 \times 10^{-40}\ m^3.}$$

EXAMPLE 4. *Convert an acceleration of 2* $km\,h^{-2}$ *into* $cm\,s^{-2}$.

$$\textbf{Solution.}\quad a = 2\ \frac{\text{km}}{\text{h}^2} = \frac{2 \times 10^5\ \text{cm}}{\left(\frac{1}{3600}\ \text{s}\right)^2} = \mathbf{0.0154\ cm\ s^{-2}.}$$

EXAMPLE 5. *The Young's modulus of steel is $1.9 \times 10^{11}\ Nm^{-2}$. Express it in dyn cm^{-2}.*

Solution. $Y = 1.9 \times 10^{11}\ \dfrac{\text{N}}{\text{m}^2}$

$$= 1.9 \times 10^{11} \times \frac{10^5 \text{ dyn}}{(100 \text{ cm})^2}$$

$$= \mathbf{1.9 \times 10^{12} \text{ dyn cm}^{-2}}.$$

EXAMPLE 6. *The density of a material is 0.8 g cm^{-3}. Express it in SI units.*

Solution. As 1 g $\text{cm}^{-3} = 10^3$ kg m^{-3}

$\therefore$ $0.8 \text{ g cm}^{-3} = 0.8 \times 10^3 \text{ kg m}^{-3} = \mathbf{800 \text{ kg m}^{-3}}$.

EXAMPLE 7. *How many amu would make up 1 kg ?*

Solution. As 1 amu $= 1.66 \times 10^{-27}$ kg

$$\therefore 1 \text{ kg} = \frac{1}{1.66 \times 10^{-27}} \text{ amu} = \mathbf{6.024 \times 10^{26} \text{ amu}}.$$

EXAMPLE 8. *Express the average distance of the earth from the sun in (i) light year and (ii) parsec.*

Solution. Average distance of the earth from the sun,

$$r = 1 \text{ AU} = 1.496 \times 10^{11} \text{ m}$$

(*i*) As 1 light year $= 9.46 \times 10^{15}$ m

$$\therefore \quad r = \frac{1.496 \times 10^{11}}{9.46 \times 10^{15}} \text{ ly} = \mathbf{1.58 \times 10^{-5} \text{ ly}}.$$

(*ii*) As 1 parsec $= 3.08 \times 10^{16}$ m

$$\therefore \quad r = \frac{1.496 \times 10^{11}}{3.08 \times 10^{16}} \text{ parsec} = \mathbf{4.86 \times 10^{-6} \text{ parsec}}.$$

PROBLEMS FOR PRACTICE

1. Calculate the number of astronomical units in one metre. (**Ans.** 6.67×10^{-12} AU)
2. How many parsec are there in one metre ? (**Ans.** 3.25×10^{-17} parsec)
3. How many parsec are there in one light year ? (**Ans.** 0.3067 parsec)
4. The density of air is 1.293 kg m^{-3}. Express it in cgs units. (**Ans.** 0.001293 g cm^{-3})
5. Convert an acceleration of 10 ms^{-2} in km h^{-2}. (**Ans.** 1.29×10^5 kmh^{-2})
6. The mass of a proton is 1.67×10^{-27} kg. How many protons would make 1 g ? (**Ans.** 5.99×10^{23})

HINTS

1. As 1 AU $= 1.5 \times 10^{11}$ m

$$\therefore \quad 1 \text{ m} = \frac{1}{1.5 \times 10^{11}} \text{ AU} = \mathbf{6.67 \times 10^{-12} \text{ AU}}.$$

2. 1 parsec $= 3.08 \times 10^{16}$ m

$$\therefore \quad 1 \text{ m} = \frac{1}{3.08 \times 10^{16}} \text{ parsec}$$

$$= \mathbf{3.25 \times 10^{-17} \text{ parsec}}.$$

3. As 1 parsec = 3.26 light year

$$\therefore \quad 1 \text{ light year} = \frac{1}{3.26} \text{ parsec} = \mathbf{0.3067 \text{ parsec}}.$$

4. Density $= 1.293 \text{ kg m}^{-3} = 1.293 \times 10^{-3} \text{ g cm}^{-3}$

$$= \mathbf{0.001293 \text{ g cm}^{-3}}.$$

5. Acceleration $= 10 \dfrac{\text{m}}{\text{s}^2} = \dfrac{10 \times 10^{-3} \text{ km}}{\left(\dfrac{1}{3600} \text{ h}\right)^2}$

$$= (3600)^2 \times 10^{-2} \text{ km h}^{-2} = \mathbf{1.29 \times 10^5 \text{ km h}^{-2}}.$$

6. No. of protons $= \dfrac{\text{Total mass}}{\text{Mass of each proton}}$

$$= \frac{10^{-3} \text{ kg}}{1.67 \times 10^{-27} \text{ kg}} = \mathbf{5.99 \times 10^{23}}.$$

2.13 ORDER OF MAGNITUDE

14. *Define order of magnitude of a physical quantity. How is it determined ? Illustrate it with same examples.*

Order of magnitude. *The order of magnitude of a physical quantity is that power of 10 which is closest to its magnitude.* It gives an idea about how big or how small a given physical quantity is.

To determine the order of magnitude of a number *N*, we first express it as

$$N = n \times 10^x.$$

If $0.5 < n \leq 5$, then *x* will be the order of magnitude of *N*.

Few examples are illustrated in table 2.4.

Table 2.4 *Order of magnitude*

Measure number N	*Expressed in nearest power of 10*	*Order of magnitude*
8	0.8×10^1	1
49	4.9×10^1	1
52	0.52×10^2	2
555	0.555×10^3	3
999	0.999×10^3	3
1001	1.001×10^3	3
753000	0.753×10^6	6
0.135	1.35×10^{-1}	− 1
0.05	5×10^{-2}	− 2
0.99	0.99×10^0	0

Examples based on Order of Magnitude

FORMULAE USED

To determine the order of magnitude of a number N, we express it as

$$N = n \times 10^x.$$

If $0.5 < n \leq 5$, then x will be the order of magnitude of N.

CONVERSIONS USED

$$1 \text{ AU} = 1.496 \times 10^{11} \text{ m}, 1 \text{ parsec} = 3.08 \times 10^{16} \text{ m}$$

CONSTANT USED

Speed of light in vacuum, $c = 3 \times 10^8 \text{ ms}^{-1}$

EXAMPLE 9. *Write the order of magnitude of the following measurements :*

(*i*) 25,710,000 *m* (*ii*) 0.00000521 *kg*.

Solution. We first express each number in terms of nearest power of 10.

(*i*) $25{,}710{,}000 \text{ m} = 2.571 \times 10^7 \text{ m}$

As $0.5 < 2.571 < 5$

$\therefore$ Order of magnitude = **7.**

(*ii*) $0.00000521 \text{ kg} = 0.521 \times 10^{-5} \text{ kg}$

As $0.5 < 0.521 < 5$

$\therefore$ Order of magnitude = **– 5.**

EXAMPLE 10. *Express 1 light year in terms of metres. What is its order of magnitude ?*

Solution. One light year is the distance travelled by light in one year.

Speed of light, $c = 3 \times 10^8 \text{ ms}^{-1}$

$t = 1 \text{ year} = 365.25 \text{ days} = 365.25 \times 24 \times 60 \times 60 \text{ s}$

$\therefore$ 1 light year $= ct$

$= 3 \times 10^8 \times 365.25 \times 24 \times 60 \times 60 \text{ m}$

$= \mathbf{9.467 \times 10^{15} \text{ m} = 0.9467 \times 10^{16} \text{ m}}$.

Order of magnitude of light year = **16.**

PROBLEMS FOR PRACTICE

1. Write the order of magnitude of the following :
   (*i*) 8 (*ii*) 49 (*iii*) 52 (*iv*) 999
   (*v*) 1001 (*vi*) 753000 (*vii*) 0.05 (*viii*) 0.99.
   (**Ans.** 1, 1, 2, 3, 3, 6, – 2, 0)
2. What is one astronomical unit ? Express it in metres. Write its order of magnitude.
   (**Ans.** Order of magnitude = 11)
3. What is the order of magnitude of seconds in a day ? (**Ans.** 5)

HINTS

1. (*i*) $8 = 0.8 \times 10^1$ (*ii*) $49 = 4.9 \times 10^1$
   (*iii*) $52 = 0.52 \times 10^2$ (*iv*) $999 = 0.999 \times 10^3$
   (*v*) $1001 = 1.001 \times 10^3$ (*vi*) $753000 = 0.753 \times 10^6$
   (*vii*) $0.05 = 5 \times 10^{-2}$ (*viii*) $0.99 = 0.99 \times 10^0$
2. Astronomical unit is the mean distance of the earth from the sun. $1 \text{ AU} = 1.496 \times 10^{11} \text{ m}$
   $\therefore$ Order of magnitude = 11.
3. $1 \text{ day} = 24 \times 60 \times 60 = 86400 \text{ s} = 0.864 \times 10^5 \text{ s}$
   $\therefore$ Order of magnitude = **5.**

2.14 MEASUREMENT OF LENGTH BY DIRECT METHODS

15. *Define Length. Name the device used for measuring directly the lengths (i) from 10^{-3} m to 10^2 m (ii) to an accuracy of 10^{-4} m and (ii) to an accuracy of 10^{-5} m.*

Length. *Length may be defined as the distance of separation between two points in space.*

(*i*) Metre scale.

(*ii*) Vernier callipers.

(*iii*) Screw gauge or spherometer.

2.15 INDIRECT METHODS FOR MEASURING LARGE DISTANCES

16. *Describe an indirect method to measure the height of an accessible tree or a tower.*

Triangulation method for the height of an accessible object. Let $AB = h$ be the height of the tree or the tower to be measured. Let C be the point of observation at distance x from B. Place a sextant at C and measure the angle of elevation,

$$\angle AOB = \theta.$$

From right ΔABC, we have

$$\tan\theta = \frac{AB}{CB} = \frac{h}{x}$$

or height, $h = x \tan\theta$

Knowing the distance x, the height h can be determined.

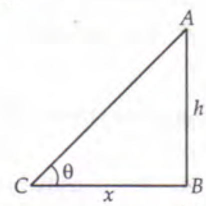


Fig. 2.3 Height of a tree.

17. *Describe a method to measure the height of an inaccessible object like a mountain.*

Triangulation method for the height of an inaccessible object. Let $AB = h$ be the height of the mountain to be measured. By using a sextant, we first measure the angle of elevation of its peak from a point C on the ground. Let it be θ_1.

Move the sextant to another position D such that $CD = d$. Again measure the angle of elevation, $\angle ADB = \theta_2$.

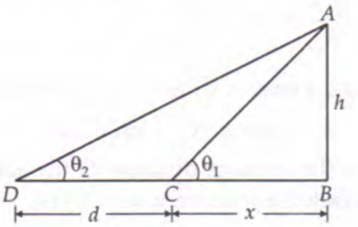


Fig. 2.4 Height of a mountain.

In rt. ΔABC, $\cot\theta_1 = \dfrac{CB}{AB} = \dfrac{x}{h}$

In rt. ΔABD, $\cot\theta_2 = \dfrac{DB}{AB} = \dfrac{d+x}{h}$

$$\therefore \quad \cot\theta_2 - \cot\theta_1 = \frac{d+x}{h} - \frac{x}{h} = \frac{d}{h}$$

or
$$h = \frac{d}{\cot\theta_2 - \cot\theta_1}.$$

Knowing d, the height h can be determined.

18. *What is meant by parallax and parallactic angle ? How can we find the distance of the moon (or any planet) by parallax method ?*

Parallax. *Parallax is the apparent shift in the position of an object with respect to another when we shift our eye sidewise.* The closer object always appears to move in the direction opposite to that of our eye.

To understand parallax, hold a pen O at distance S from the eyes. Look at the pen first by the left eye L (closing the right eye) and then by the right eye R (closing the left eye). The position of the pen appears to change with respect to the background. This is called *parallax*. The distance between the two points of observation is called *basis*. In this case, the distance ($LR = b$) between the two eyes is the basis. $\angle LOR = \theta$ is called *parallax angle* or *parallactic angle*.

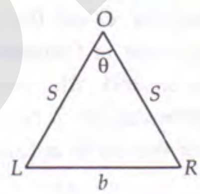


Fig. 2.5 Parallax

Distance of the moon or any planet. To measure the distance S of the moon or a far away planet P, we observe it simultaneously from two different positions (observatories) A and B on the earth, separated by a large distance $AB = b$. We select a distant star S' whose position and direction can be taken approximately same from A and B.

Now $\angle PAS' = \phi_1$ and $\angle PBS' = \phi_2$ are measured from the two observatories at the same time. As $b << S$, so we can take AB as an arc of length b.

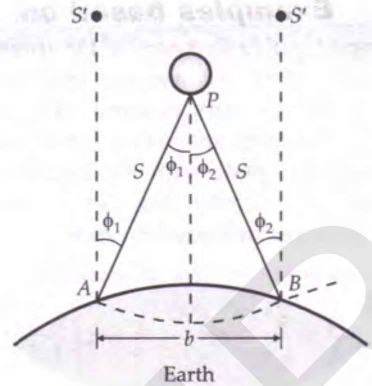


Fig. 2.6 Distance of a planet.

Now $\quad \theta = \dfrac{\text{Arc}}{\text{Radius}} = \dfrac{b}{S}$

$$\therefore \quad S = \frac{b}{\theta}.$$

where $\theta = \angle APB = \phi_1 + \phi_2$, is the parallactic angle.

19. *Describe the parallax method for the determination of the distance of a nearby star from the earth.*

Distance of a nearby star by parallax method.

As shown in Fig. 2.7, suppose N is the nearby star whose distance d from the earth is to be found. F is a far off star whose direction and position is fixed for all positions of the earth in its orbital motion. When the earth is at position A, the parallax angle between distant star F and nearby star N is determined. Let it be θ_1. After six months, the earth is at diametrically opposite position B. The parallax angle, $\angle NBF = \theta_2$ is measured.

Total parallax angle subtended by N on the earth's orbital diameter AB is

$$\theta = \theta_1 + \theta_2$$

As $\quad \theta = \dfrac{\text{Arc}}{\text{Radius}}$

$$\therefore \quad \theta = \frac{AB}{d}$$

or
$$d = \frac{AB}{\theta}.$$

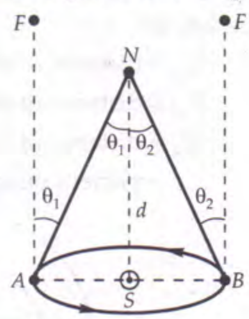


Fig. 2.7 Distance of a nearby star.

The parallax method is useful for measuring distances of the stars which are less than 100 light years away from the earth.

20. *Briefly describe a method for the determination of distance of a far away star.*

Intensity method. This is a spectroscopic method based on **inverse square law of intensity**. According to this law, *the intensity of illumination at a point is inversely proportional to the square of the distance from the source of light*. Here we assume that the intrinsic brightness of all the stars is same. We compare the intensity I_1 of the faint image of a far away star taken on a photographic plate with the intensity I_2 of the bright image of a nearby star. Let r_1 and r_2 be the respective distances of these two stars.

From inverse square law of intensity,

$$\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2} \quad \text{or} \quad r_1 = r_2\left[\frac{I_2}{I_1}\right]^{1/2}$$

knowing the distance r_2 of the nearby star, the distance r_1 of the far away star can be determined. This method is useful for measuring distances of stars which are more than 100 light years away from the earth.

21. *What are inferior and superior planets ? Give the names of these planets.*

Inferior planets. *The planets which are closer to the sun than the earth are called inferior planets.* Mercury and Venus are the inferior planets.

Superior planets. *The planets which are farther from the sun than the earth are called superior planets.* Jupiter, Saturn, Uranus, Neptune and Pluto are the superior planets.

22. *Describe a method for determining the distance of an inferior planet.*

Copernicus method. Copernicus assumed circular orbits for the planets. The angle formed at the earth between the earth-planet direction and earth-sun direction is called the *planet's elongation*. As shown in Fig. 2.8, let

R_{ps} = distance of the planet from the sun

R_{pe} = distance of the planet from the earth

R_{es} = distance of the earth from the sun

ε = planet's elongation

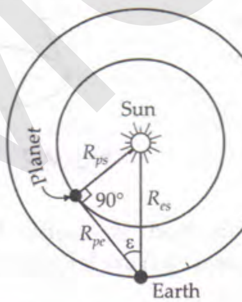


Fig. 2.8 Copernicus method.

When the elongation attains its maximum value and the planet appears farthest from the sun, the angle subtended by the sun and the earth at the planet is 90°. Then from the right angle triangle shown in the figure, we find that

$$\frac{R_{ps}}{R_{es}} = \sin \varepsilon$$

Hence the distance of the planet from the sun is

$$R_{ps} = \sin \varepsilon \,.\, R_{es} = \sin \varepsilon \,.\, \text{AU}$$

where R_{es} is the average distance of the earth from the sun and is called *astronomical unit* (AU).

23. *How can we determine the distance of a superior planet from the earth ?*

Distance of a superior planet. By knowing the distance of any planet from the sun, we can determine the distance of any superior planet. For this purpose we use **Kepler's third law of planetary motion**. This law states *the square of the period (T) of revolution of a planet round the sun is proportional to cube of the semi-major axia (a) of the orbit i.e.,*

$$T^2 \propto a^3$$

If T_1 and T_2 are periods of revolution of two planets, and a_1 and a_2 are their respective semi-major axes, then

$$\frac{a_2^3}{a_1^3} = \frac{T_2^2}{T_1^2} \quad \text{or} \quad a_2 = a_1\left(\frac{T_2}{T_1}\right)^{2/3}$$

Knowing the values of a_1, T_1 and T_2, we can find out a_2.

24. *Describe a method to measure the diameter of the moon or any planet.*

Diameter of the moon. Let $AB = D$ be the diameter of the moon (or planet) which is to be measured by an observer O on the earth. A telescope is focussed on the moon and the angle AOB subtended by it on the point O of the earth is found.

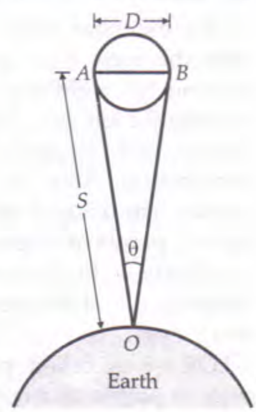


Fig. 2.9 Diameter of the moon.

$$\text{As } \theta = \frac{\text{Arc}}{\text{Radius}} = \frac{AB}{OB} = \frac{D}{S}$$

or $\quad D = S\theta$

Linear diameter = Distance × angular diameter

Knowing S and θ, D can be determined.

25. *Describe a method to measure distance of a hill.*

Reflection or echo method. To find the distance of a hill, a gun is fired towards the hill. The time interval between the instant of firing the gun and the instant of hearing the echo of the gun shot is noted. During this time interval, the sound first travels towards the hill and then back from the hill to the place of firing. If v is the speed of sound, S is the distance of the hill and t is the total time taken, then

$$2S = v \times t \quad \text{or} \quad S = \frac{v \times t}{2}$$

26. *Describe a reflection method for measuring the distance of the moon.*

Laser method. The word laser stands for *light amplification by stimulated emission of radiation.* Laser is a source of very intense, highly monochromatic (of one wavelength) and highly directional beam of light. A laser beam is sent towards the moon and its reflected pulse is received. If t is the time elapsed between the instants the laser beam is sent and received back, then the distance of the moon from the earth is given by

$$S = \frac{c \times t}{2}$$

where $c = 3 \times 10^8 \text{ ms}^{-1}$, is the speed of light.

27. *Briefly describe a reflection method for estimating the distance of a nearby planet from the earth.*

Radar method. The word RADAR stands for *radio detection and ranging.* A radar can be used to measure accurately the distance of a nearby planet (such as Venus). Here radiowaves are sent from a transmitter which after reflection from the planet are detected by the receiver. By measuring the time interval (t) between the instants the radiowaves are sent and received, the distance of the planet can be determined as

$$S = \frac{c \times t}{2}$$

where c is the speed of the radiowave. This method can also be used to determine the distance of an aeroplane.

28. *How is a sonar used in finding the depth of the sea-bed ?*

Sonar method. The word SONAR stands for *sound navigation and ranging.* On a sonar, ultrasonic waves (sound waves having frequencies greater than 20,000 Hz) are transmitted through the ocean. They are reflected by the submerged rock or submarines and received by the receiver. By measuring the time delay t of the receipt of the echo, the distance S of the submerged rock or submarine can be determined as

$$S = \frac{v \times t}{2}$$

where v is the speed of the ultrasonic waves in water.

Examples based on Indirect Methods for Long Distances

FORMULAE USED

1. *Reflection or echo method*
$$S = \frac{c \times t}{2} \quad \text{or} \quad \frac{v \times t}{2}$$
2. *Triangulation method*
   (*i*) Height of an accessible object
   $$h = x \tan \theta$$
   where x is the distance of observation point from the foot of the object.
   (*ii*) Height of an inaccessible object
   $$h = \frac{d}{\cot \theta_2 - \cot \theta_1}$$
   where d is the distance between the two observation points.
3. *Parallax method. The distance of an astronomical object*
$$S = \frac{\text{Basis}}{\text{Parallactic angle}} = \frac{b}{\theta}$$
4. Size of an astronomical object,
   Linear diameter = Distance × angular diameter
   or $D = S \times \theta$
5. *Copernicus method*
   (*i*) The distance of a planet from the sun,
   $$r_{ps} = \sin \varepsilon . r_{es} = \sin \varepsilon \text{ AU}$$
   (*ii*) The distance of a planet from the earth,
   $$r_{pe} = \cos \varepsilon . r_{es} = \cos \varepsilon \text{ AU}$$
6. *From Kepler's law of periods*
$$\frac{a_2^3}{a_1^3} = \frac{T_2^2}{T_1^2} \quad \text{or} \quad a_2 = \left[\frac{T_2}{T_1}\right]^{2/3} . a_1$$

UNITS USED

Distances S, x, d, h, a, b and D are in metre, angle θ in radian, velocities v and c in ms^{-1}

CONVERSIONS USED

$$1'' = 4.85 \times 10^{-6} \text{ rad}$$

EXAMPLE 11. *Calculate the angle of (a) 1° (degree) (b) 1′ (minute of arc or arcmin) and (c) 1″ (second of arc or arc second) in radians. Use* $360° = 2\pi$ *rad,* $1° = 60'$ *and* $1' = 60''$. **[NCERT]**

Solution. (*a*) As $360° = 2\pi$ rad

$\therefore \quad 1° = \frac{2\pi}{360}$ rad $= \mathbf{1.745 \times 10^{-2}}$ **rad.**

(*b*) $1° = 60' = 1.745 \times 10^{-2}$ rad

$\therefore \quad 1' = 2.908 \times 10^{-4} \approx \mathbf{2.91 \times 10^{-4}}$ **rad**

(*c*) $1' = 60'' = 2.908 \times 10^{-4}$ rad.

$\therefore \quad 1'' = 4.847 \times 10^{-4}$ rad $\approx \mathbf{4.85 \times 10^{-6}}$ **rad.**

EXAMPLE 12. *The shadow of a tower standing on a level plane is found to be 50 m longer when sun's altitude is 30° than when it is 60°. Find the height of the tower.*

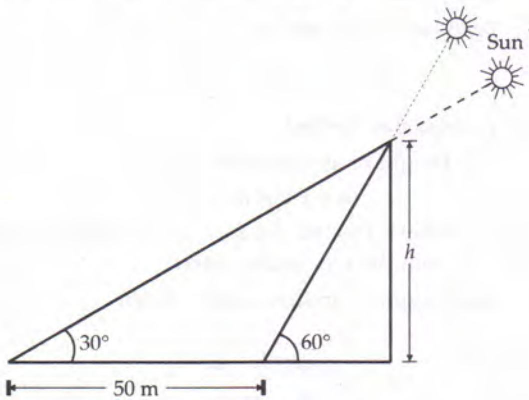


Fig. 2.10

Solution. Here $d = 50$ m, $\theta_1 = 60°$, $\theta_2 = 30°$.

$$h = \frac{d}{\cot\theta_2 - \cot\theta_1} = \frac{50}{\cot 30° - \cot 60°}$$

or
$$h = \frac{50}{\sqrt{3} - 1/\sqrt{3}} = \frac{50\sqrt{3}}{3-1} = 25\sqrt{3}$$

$$= 25 \times 1.732 = \mathbf{43.3\ m.}$$

EXAMPLE 13. *A man wishes to estimate the distance of a nearby tower from him. He stands at a point A in front of the tower C and spots a very distant object O in line with AC. He then walks perpendicular to AC upto B, a distance of 100 m, and looks at O and C again. Since O is very distant, the direction BO is practically the same as AO ; but he finds the line of sight of C shifted from the original line of sight by an angle θ = 40° (θ is known as 'parallax'). Estimate the distance of the tower C from his original position A.* **[NCERT]**

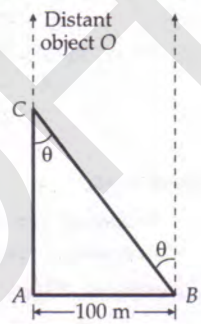


Fig. 11

Solution. Here parallax angle, $\theta = 40°$

Now $\tan\theta = \dfrac{AB}{AC}$

$$\therefore \quad AC = \frac{AB}{\tan\theta} = \frac{100\text{ m}}{\tan 40°} = \frac{100\text{ m}}{0.8391} = \mathbf{119\ m}.$$

EXAMPLE 14. *The moon is observed from two diametrically opposite points A and B on the earth. The angle θ subtended at the moon by the two directions of observation is 1°54′. Given the diameter of the earth to be 1.276×10^7 m, compute the distance of the moon from the earth.* **[NCERT]**

Solution. Here parallactic angle,

$$\theta = 1°54' = 114' = (144 \times 60)''$$
$$= 114 \times 60 \times 4.85 \times 10^{-6}\text{ rad} = 3.32 \times 10^{-2}\text{ rad}$$

Basis, $b = AB = 1.276 \times 10^7$ m

The distance of the moon from the earth,

$$S = \frac{b}{\theta} = \frac{1.276 \times 10^7}{3.32 \times 10^{-2}} = \mathbf{3.84 \times 10^8\ m.}$$

EXAMPLE 15. *The angular diameter of the sun is 1920″. If the distance of the sun from the earth is 1.5×10^{11} m, what is the linear diameter of the sun ?* **[NCERT]**

Solution. Distance of the sun from the earth,

$$S = 1.5 \times 10^{11}\text{ m}$$

Angular diameter of the sun,

$$\theta = 1920'' = 1920 \times 4.85 \times 10^{-6}\text{ rad}$$

Linear diameter of the sun,

$$D = S \times \theta = 1.5 \times 10^{11} \times 1920 \times 4.85 \times 10^{-6}$$
$$= \mathbf{1.4 \times 10^9\ m.}$$

EXAMPLE 16. *Assuming that the orbit of the planet Mercury around the sun to be a circle, Copernicus determined the orbital radius to be 0.38 AU. From this, determine the angle of the maximum elongation for Mercury and its distance from the earth where the elongation is maximum.*

Solution. Here $r_{ps} = 0.38$ AU,

$r_{es} = 1$ AU [By Definition]

The angle of maximum elongation ε for a planet is given by

$$\sin\varepsilon = \frac{r_{ps}}{r_{es}} = \frac{0.38}{1} = 0.38$$

$$\varepsilon = \sin^{-1}(0.38) = \mathbf{22.3°}$$

Distance of mercury from the earth is

$$r_{pe} = \cos\varepsilon \,.\, \text{AU} = \cos(22.3°) \times 1.496 \times 10^{11}\text{ m}$$
$$= 0.9252 \times 1.496 \times 10^{11}\text{ m}$$
$$= 1.384 \times 10^{11}\text{ m} = \mathbf{1.384 \times 10^8\ km.}$$

EXAMPLE 17. *In the case of Venus, the angle of maximum elongation is found to be approximately 47°. Determine the distance between Venus and the Sun r_{vs} and the distance between Venus and the Earth.*

Solution. Here angle of maximum elongation, ε = 47°

Distance between Venus and the sun is

$$r_{vs} = \sin\varepsilon \,.\, \text{AU} = \sin 47° \times 1\text{ AU}$$
$$= 0.73 \times 1.496 \times 10^{11}\text{ m} = \mathbf{1.09 \times 10^{11}\ m.}$$

Distance between Venus and the earth is

$$r_{ve} = \cos\varepsilon \,.\, \text{AU} = \cos 47° \times 1\text{ AU}$$
$$= 0.68 \times 1.496 \times 10^{11}\text{ m} = \mathbf{1.02 \times 10^{11}\ m.}$$

EXAMPLE 18. Suppose there existed a planet that went around the sun twice as fast as the earth. What would be its orbital size as compared to that of the earth ?

Solution. Let the period of revolution of the earth $= T_e$

As the planet goes round the sun twice as fast as the earth, so its period of revolution is

$$T_p = \frac{1}{2} T_e$$

Orbital size of the earth, $a_e = 1$ AU

Orbital size of the planet, $a_p = ?$

From Kepler's law of periods,

$$\frac{T_p^2}{T_e^2} = \frac{a_p^3}{a_e^3}$$

$$\therefore \quad a_p = \left[\frac{T_p}{T_e}\right]^{2/3} \times a_e = \left[\frac{T_e/2}{T_e}\right]^{1/3} \times 1 \text{ AU}$$

$$= (0.5)^{2/3} \text{ AU} = \mathbf{0.63 \ AU.}$$

PROBLEMS FOR PRACTICE

1. In a submarine fitted with a SONAR, the time interval between the generation of an ultrasonic wave and the receipt of its echo is 200 s. What is the distance of the enemy submarine ? The speed of sound in water is 1.450 km s^{-1}. (**Ans.** 145 km)
2. A radar signal is beamed towards a planet and its echo is received 7 minutes later. If the distance between the planet and the earth is 6.3×10^{10} m, calculate the speed of the signal. (**Ans.** 3×10^8 ms^{-1})
3. A rock under water is 1595 m deep. Find the time in which an ultrasonic signal returns after reflection from the rock. Given speed of ultrasonic waves in water = 1450 ms^{-1}. (**Ans.** 2.2 s)
4. The angle of elevation of the top of a hill is 30° from a point on the ground. On walking 1 km towards the hill, angle is found to be 45°. Calculate the height of the hill. (**Ans.** 1.366 km)
5. The moon subtends an angle of 57 minutes at the base-line equal to the radius of the earth. What is the distance of the moon from the earth ? Radius of the earth = 6.4×10^6 m. (**Ans.** 3.86×10^8 **m**)
6. The parallax of a heavenly body measured from two points diametrically opposite on equator of earth is 1.0 minute. If the radius of the earth is 6400 km, find the distance of the heavenly body from the centre of the earth in AU.
   Given 1 AU = 1.5×10^{11} m. (**Ans.** 0.293 AU)
7. Assuming that a planet goes round the sun in a circular orbit of radius 0.5 AU, determine the angle of maximum elongation for the planet and its distance from the earth when elongation is maximum. (**Ans.** 30°, 0.866 AU)
8. Find the period of revolution of planet mars about the sun compared with that of the earth about it. The mean distance of the Mars from the Sun is 1.52 AU. (**Ans.** 1.87 year)

HINTS

1. $S = \frac{v \times t}{2} = \frac{1.450 \times 200}{2} = \mathbf{145 \ km.}$

3. $t = \frac{2S}{v} = \frac{2 \times 1595}{1450} = \mathbf{2.2 \ s.}$

4. $h = \frac{d}{\cot \theta_2 - \cot \theta_1} = \frac{1 \text{ km}}{\cot 30° - \cot 45°} = \mathbf{1.366 \ km.}$

5. Here parallactic angle,

   $$\theta = 57' = \frac{57°}{60} = \frac{57}{60} \times \frac{\pi}{180} \text{ rad}$$

   Basis, $b =$ Radius of earth $= 6.4 \times 10^6$ m

   Distance of the moon from the earth,

   $$S = \frac{b}{\theta} = \frac{6.4 \times 10^6 \times 60 \times 180}{57 \times \pi} = \mathbf{3.86 \times 10^8 \ m.}$$

6. $\theta = 1.0 \text{ min} = \frac{1°}{60} = \frac{1}{60} \times \frac{\pi}{180}$ rad

   $$S = \frac{b}{\theta} = \frac{2 \times 6400 \times 180 \times 60}{\pi}$$

   $$= 4.4 \times 10^7 \text{ km} = 4.4 \times 10^{10} \text{ m}$$

   $$= \frac{4.4 \times 10^{10}}{1.5 \times 10^{11}} \text{ AU} = \mathbf{0.293 \ AU.}$$

7. Here $r_{ps} = 0.5$ AU, $r_{es} = 1$ AU

   $$\therefore \quad \sin \varepsilon = \frac{r_{ps}}{r_{es}} = \frac{0.5}{1} = \sin 30°, \quad \therefore \ \varepsilon = \mathbf{30°}.$$

   Also, $r_{pe} = \cos \varepsilon \,.\, \text{AU} = \cos 30° \text{ AU} = \mathbf{0.866 \ AU.}$

8. For the mars : $a_m = 1.52$ AU

   For the earth : $a_e = 1$ AU, $T_e = 1$ year

   $$T_m = \left[\frac{a_m}{a_e}\right]^{3/2} \times T_e = \left[\frac{1.52}{1}\right]^{3/2} \times 1 = \mathbf{1.87 \ year.}$$

2.16 INDIRECT METHODS FOR MEASURING SMALL DISTANCES

29. *Explain how will you determine the radius of an atom by Avogadro's hypothesis.*

Atomic radius by Avogadro's hypothesis. Atoms are spherical in shape. So when a large number of atoms are packed together, some empty spaces are left between them. According to *Avogadro's hypothesis, the actual volume occupied by the atoms in one gram of a substance is two-third of the volume of one gram of the substance.*

Let M be the molecular mass of a substance. Then M grams of the substance will contain N (Avogadro number) of atoms.

$\therefore$ Number of atoms in 1 gram $= \dfrac{N}{M}$

If r be the radius of each atom, then volume of atoms in one gram $= \dfrac{N}{M} \cdot \dfrac{4}{3}\pi r^3$.

Let V be the actual volume occupied by molecules in 1 gram of the substance. Then by

Avogadro's hypothesis, $\dfrac{N}{M} \cdot \dfrac{4}{3}\pi r^3 = \dfrac{2}{3}V$

If ρ is the density of the substance, then

$$\rho = \frac{\text{Mass}}{\text{Volume}} = \frac{1}{V} \quad \text{or} \quad V = \frac{1}{\rho}$$

$$\therefore \frac{N}{M} \cdot \frac{4}{3}\pi r^3 = \frac{2}{3} \cdot \frac{1}{\rho} \qquad \text{or} \quad r = \left[\frac{M}{2\pi N\rho}\right]^{1/3}$$

Thus, the radius of an atom of the substance can be determined.

30. *Describe a method for measuring the molecular size of oleic acid.*

Size of molecule of oleic acid. Oleic acid is a soapy liquid with large molecular size. We dissolve 1 cm^3 of oleic acid in 20 cm^3 of alcohol and then redissolve 1 cm^3 of this solution in 20 cm^3 of alcohol. Then the concentration of oleic acid is $1/400\ cm^3$ in 1 cm^3 of alcohol. We then determine the approximate volume of each drop (V cm^3). Now pour n drops of the solution on the surface of water taken in a broad vessel. We stretch the film carefully. As the alcohol evaporates, a very thin film of oleic acid is left on water surface. We measure the area A of the film using a graph paper.

Volume of n drops of the solution $= nV\ cm^3$

Amount of oleic acid in this solution $= \dfrac{nV}{400}\ cm^3$

Thickness of the oil film,

$$t = \frac{\text{Volume of the film}}{\text{Area of the film}} = \frac{nV}{400\ A}\ \text{cm}$$

Assuming that the film has one molecular thickness, then t will be approximately the size or diameter of a molecule of oleic acid. The value of t is found to be of the order of 10^{-9} m.

Table 2.5 Orders of Magnitude of length

S. No.	Size of object or distance	Length (m)
1.	Radius of a proton	10^{-15}
2.	Size of atomic nucleus	10^{-14}
3.	Size of hydrogen atom	10^{-10}
4.	Size of typical virus	10^{-8}
5.	Wavelength of light	10^{-7}
6.	Size of red blood corpuscle	10^{-5}
7.	Thickness of a paper	10^{-4}
8.	Height of a person	10^{0}
9.	Height of Mount Everest	10^{4}
10.	Radius of the earth	10^{7}
11.	Distance of the moon from the earth	10^{8}
12.	Radius of the sun	10^{9}
13.	Distance of the sun from the earth	10^{11}
14.	Distance of Pluto from the earth	10^{13}
15.	Size of Milkway	10^{21}
16.	Distance to Andromeda galaxy	10^{22}
17.	Distance to the boundary of observable universe	10^{26}

Examples based on Indirect Methods for Small Distances

FORMULAE USED

1. Molar volume = Volume of 1 mole of a gas at S.T.P. = 22.4 L
2. Volume of a sphere $= \dfrac{4}{3}\pi r^3$
3. Thickness of an oil film $= \dfrac{\text{Volume of oil drop}}{\text{Area of the film}}$

UNITS USED

Volume V is in m^3 or litre, radius r in metre.

CONSTANT USED

Avogadro's number,
$N = 6.023 \times 10^{23}$

CONVERSIONS USED

$1\ \text{Å} = 10^{-10}$ m, $1\text{L} = 10^{-3}\ m^3$

EXAMPLE 19. *The radius of a muonic hydrogen atom is* 2.5×10^{-13} m. *What is the total atomic volume in* m^3 *of a mole of such hydrogen atoms ?*

Solution. Radius, $r = 2.5 \times 10^{-13}$ m

Volume of one atom $= \dfrac{4}{3}\pi r^3$

Number of atoms in 1 mole $= 6.023 \times 10^{23}$

Volume of 1 mole of H-atoms

$$= N \times \frac{4}{3}\pi r^3$$

$$= 6.023 \times 10^{23} \times \frac{4}{3} \times 3.14 \times (2.5 \times 10^{-13})^3$$

$$= \mathbf{3.94 \times 10^{-14}\ m^3}.$$

EXAMPLE 20. A drop of olive oil of radius 0.25 mm spreads into a circular film of radius 10 cm on the water surface. Estimate the molecular size of olive oil.

Solution. Thickness of oil film

$$= \frac{\text{Volume of oil drop}}{\text{Area of the film}}$$

$$= \frac{\frac{4}{3}\pi \times (0.025)^3 \text{ cm}^3}{\pi \times (10)^2 \text{ cm}^2} = \frac{4}{3} \times (25)^3 \times 10^{-11} \text{ cm}$$

$$= 2.08 \times 10^{-7} \text{ cm}$$

Assuming that the film has one molecular thickness, then molecular size of olive oil

$$= \mathbf{2.08 \times 10^{-7} \text{ cm}.}$$

✖ PROBLEM FOR PRACTICE

1. A drop of olive oil of radius 0.30 mm spreads into a rectangular film of 30 cm × 15 cm on the water surface. Estimate the molecular size of olive oil. (**Ans.** 7.5×10^{-7} cm)

Examples based on Magnification of Sizes

FORMULAE USED

1. Linear magnification $= \dfrac{\text{Final size}}{\text{Initial size}} = \dfrac{\text{Size of image}}{\text{Size of object}}$
2. Linear magnification $= \sqrt{\text{Areal magnification}}$

UNITS USED

Magnification has no units.

EXAMPLE 21. If the size of a nucleus ($\simeq 10^{-15}$ m) is scaled upto the tip of a sharp pin ($\simeq 10^{-5}$ m), what roughly is the size of the atom ? **[NCERT]**

Solution. Magnification in size

$$= \frac{\text{Size of the tip of sharp pin}}{\text{Size of nucleus}} = \frac{10^{-5}}{10^{-15}} = 10^{10}$$

Actual size of an atom $= 1 \text{ Å} = 10^{-10}$ m, which is scaled up by a factor 10^{10}.

$\therefore$ Apparent size of atom $= 10^{-10} \times 10^{10} = \mathbf{1 m}.$

Thus a nucleus in an atom is as small in size as the tip of a sharp pin placed at the centre of a sphere of radius about one metre.

Example 22. A 35 mm wide slide with a 24 mm × 36 mm picture is projected on a screen placed 12 cm from the slide. The image of the slide picture on the screen measures 1.0 m × 1.5 m. What is the linear magnification of the projector-screen arrangement ?

Solution. Areal magnification

$$= \frac{1.0 \text{ m} \times 1.5 \text{ m}}{24 \times 10^{-3} \text{ m} \times 36 \times 10^{-3} \text{ m}}$$

$$\simeq 1736$$

Linear magnification $= \sqrt{1736} = \mathbf{41.67.}$

✖ PROBLEMS FOR PRACTICE

1. If the size of an atom ($\simeq$ 1Å) were enlarged to the tip of a sharp pin ($\simeq 10^{-5}$ m), how large would the height of Mount Everest be ? (**Ans.** 10^9 m)
2. If the size of an atom ($\simeq$ 1Å) were enlarged to the size of the earth ($\simeq 10^7$ m), how large would its nucleus be ? Take size of nucleus $= 10^{-15}$ m. (**Ans.** 100 m)
3. If the universe were shrunk to the size of the earth, how large would the earth be on this scale ? (**Ans.** 10^{-11} m *i.e.* size of the atom)

✖ HINTS

1. Magnification $= \dfrac{10^{-5} \text{ m}}{10^{-10} \text{ m}} = 10^5$

   Apparent height of Mount Everest

   $= $ Actual height × magnification

   $= 10^4 \text{ m} \times 10^5 = \mathbf{10^9 \text{ m}}.$

2. Magnification $= \dfrac{\text{Size of earth}}{\text{Size of atom}} = \dfrac{10^7 \text{ m}}{10^{-10} \text{ m}} = 10^{17}$

   Apparent size of nucleus

   $=$ Actual size × magnification

   $= 10^{-15} \times 10^{17} = \mathbf{100 \text{ m}}.$

3. Magnification $= \dfrac{\text{Size of earth}}{\text{Size of universe}} = \dfrac{10^7 \text{ m}}{10^{25} \text{ m}} = 10^{-18}$

   Apparent size of earth $= 10^7 \text{ m} \times 10^{-18} = \mathbf{10^{-11} \text{ m}}.$

2.17 MASS AND WEIGHT

31. *Define the terms mass and weight. Give their SI units.*

Mass. *The mass of a body is the quantity of matter contained in it.* It is a basic property of matter. It does not depend on the temperature, pressure or location of the body in space. The SI unit of mass is *kilogram* (kg).

Weight. *The weight of a body is the force with which a body is pulled towards the centre of the earth.* It is equal to the product of the mass (m) of the body and the acceleration due to gravity (g) of the earth on body.

Thus $\quad W = mg$

As the value of 'g' changes from place to place, so the weight of a body is different at different places.

The SI unit of weight is *newton* (N).

32. *Distinguish between mass and weight.*

Mass	***Weight***
1. Mass is the measure of inertia.	Weight is the measure of gravity
2. It is a scalar quantity.	It is a vector quantity.
3. It is a constant quantity.	It varies from place to place.
4. It cannot be zero for a body.	Weight of a body is zero at the centre of the earth.
5. It is an essential property of material bodies.	It is not an essential property.
6. It is not affected by the presence of other bodies.	It is affected by the presence of other bodies.
7. Its units are gram, kilogram, etc.	Its units are dyne, newton, etc.

2.18 INERTIAL AND GRAVITATIONAL MASSES

33. *Distinguish between inertial mass and gravitational mass.*

Inertial mass. *The mass of a body which determines its inertia in translatory motion is called its inertial mass.* It is defined by Newton's second law of motion and is equal to the ratio of the external force applied on the body to the acceleration produced in it. By Newton's second law,

$$F = m_i a \quad \text{or} \quad m_i = \frac{F}{a}$$

Here m_i is the inertial mass of the body which can be measured by using an inertial balance.

Gravitational mass. *The mass of a body which determines the gravitational pull acting upon it due to the earth is called its gravitational mass.* It is defined by Newton's law of gravitation. According to this law, the force of gravitation of the earth on a body of mass m_g is given by

$$F = \frac{GMm_g}{R^2} \quad \text{or} \quad m_g = \frac{FR^2}{GM}$$

Here m_g is the gravitational mass of the body which can be measured by using a physical balance.

34. *Briefly explain how can we measure the inertial mass of a body.*

Measurement of inertial mass. The inertial mass of a body is measured by using a device called *inertial balance*. As shown in Fig. 2.12, it consists of a long strip of metal.

One end of the strip is clamped to a table. The other end of the strip carries a pan in which body whose inertial mass is to be measured is kept. When the strip vibrates horizontally, its inertia comes into play and not the force of gravity of the earth. It is found that *period of vibration T is directly proportional to the square root of the inertial mass m of the body.*

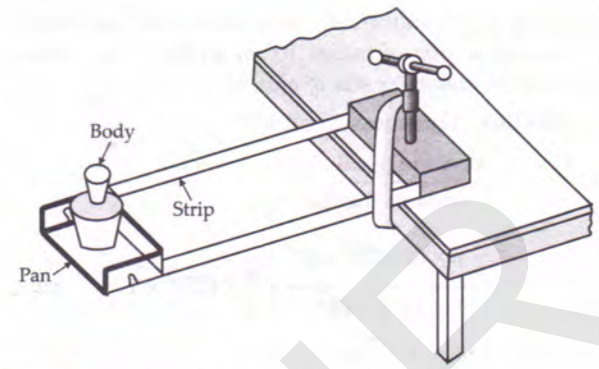


Fig. 2.12 Inertial balance.

Thus $T \propto \sqrt{m}$ or $T^2 \propto m$

Let m_1 and m_2 be the inertial masses of two objects and T_1 and T_2 be their corresponding periods of vibration, then

$$\frac{m_2}{m_1} = \frac{T_2^2}{T_1^2} \quad \text{or} \quad m_2 = m_1 \cdot \frac{T_2^2}{T_1^2}$$

If m_1 is a standard mass, then unknown mass m_2 can be determined.

35. *How is gravitational mass of a body measured by a common (or physical) balance ?*

Gravitational mass by a physical balance. A physical balance works on the *principle of moments i.e.,* when an object is balanced under the action of several forces acting in the same plane, the sum of the clockwise moments is equal to the sum of anticlockwise moments. The essential parts of a physical balance are shown in Fig. 2.13.

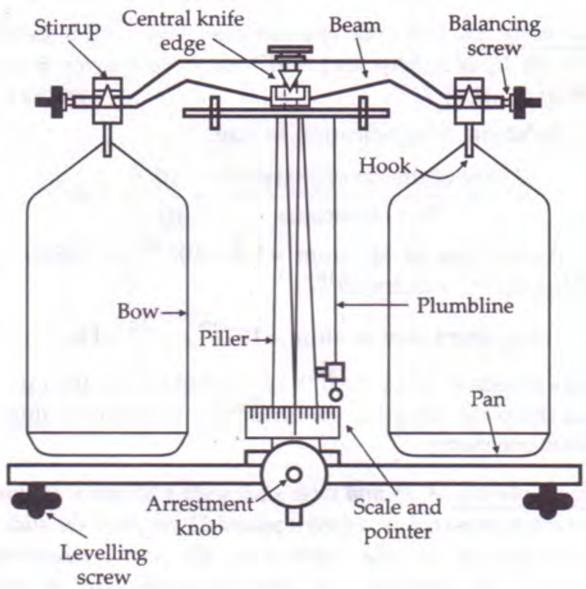


Fig. 2.13 Physical balance.

The object to be weighed is placed in the left pan and the standard weights are placed in the right pan. The weights are adjusted till the beam becomes horizontal. In this condition, the gravitational force on the object is equal to the gravitational force on the standard weights. Hence the gravitational mass of the object is equal to that of the standard weights (cancelling the effect of g).

36. *How can a spring balance be used to measure the gravitational mass of a body ?*

Gravitational mass by a spring balance. In a spring balance, the gravitational force on a body stretches the spring and we measure the elongation of the spring. The elongation depends on gravitational force which, in turn, is proportional to the gravitational mass of the body. Thus the elongation of the spring gives a measure of the gravitational mass. For this we first calibrate the spring balance by using standard masses and measuring the elongations. The calibration of spring balance and the mass measurement should be done at same place because the value of g varies from place to place.

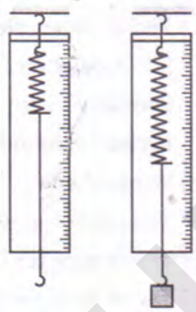


Fig. 2.14 Spring balance.

Table 2.6 Orders of magnitude of mass

S.No.	Object	Mass (kg)
1.	Electron	10^{-30}
2.	Proton	10^{-27}
3.	Uranium atom	10^{-25}
4.	Cell	10^{-10}
5.	Dust particle	10^{-9}
6.	Rain drop	10^{-6}
7.	Mosquito	10^{-5}
8.	Grape	10^{-3}
9.	Man	10^{2}
10.	Automobile	10^{3}
11.	Ship	10^{5}
12.	Moon	10^{23}
13.	Earth	10^{25}
14.	Milkyway	10^{41}
15.	Observable universe	10^{55}

Examples based on Mass Density

FORMULAE USED

1. Density $= \dfrac{\text{Mass}}{\text{Volume}}$ or $\rho = \dfrac{M}{V}$
2. Volume of a sphere, $V = \dfrac{4}{3}\pi r^3$

UNITS USED

Radius r is in metre, volume V in m^3 and density ρ in kg m^{-3}.

EXAMPLE 23. *Consider a white dwarf and a neutron star each of one solar mass. The radius of the white dwarf is same as that of the earth ($\simeq$ 6400 km) and the radius of the neutron star is 10 km. Determine the densities of the two types of the stars. Take mass of the sun $= 2.0 \times 10^{30}$ kg.*

Solution. *For white dwarf* : $M = 2 \times 10^{30}$ kg, $r = 6400\text{ km} = 64 \times 10^5$ m.

Density of white dwarf

$$= \frac{M}{\frac{4}{3}\pi r^3} = \frac{2 \times 10^{30}}{\frac{4}{3}\pi \times (64 \times 10^5)^3} = \mathbf{1.822 \times 10^9\ kg\ m^{-3}}$$

For neutron star : $M = 2 \times 10^{30}$ kg, $r = 10\text{ km} = 10^4$ m

Density of neutron star

$$= \frac{2 \times 10^{30}}{\frac{4}{3}\pi \times (10^4)^3} = \mathbf{4.77 \times 10^{17}\ kgm^{-3}}.$$

EXAMPLE 24. *Assume that the mass of a nucleus is given by $M = A\,m_p$, where A is the mass number and radius of a nucleus $r = r_0\,A^{1/3}$, where $r_0 = 1.2$ f. Estimate the density of nuclear matter in kg m^{-3}. Given $m_p = 1.67 \times 10^{-27}$ kg.*

Solution. Here $m_p = 1.67 \times 10^{-27}$ kg,

$$r_0 = 1.2\ f = 1.2 \times 10^{-15}\text{ m}$$

Nuclear density $$= \frac{Am_p}{\frac{4}{3}\pi r^3} = \frac{Am_p}{\frac{4}{3}\pi (r_0\,A^{1/3})^3} = \frac{3m_p}{4\pi r_0^3}$$

$$= \frac{3 \times 1.67 \times 10^{-27}}{4 \times 3.14 \times (1.2 \times 10^{-15})^3}\text{ kg m}^{-3}$$

$$= \mathbf{2.3 \times 10^{17}\ kg\ m^{-3}}.$$

PROBLEM FOR PRACTICE

1. A neutron star has a density equal to that of nuclear matter ($\simeq 2.8 \times 10^{17}$ kg m^{-3}). Assume the star to be spherical, find the radius of the neutron star whose mass is 4.0×10^{10} kg (twice the mass of the sun).

(Ans. 15 km)

2.19 MEASUREMENT OF TIME

37. *What type of phenomena can be used as a time standard ? Give some examples. Which one do you select as the most suitable standard for measuring time ?*

Measurement of time. According to Einstein, *"Time is simply what a clock reads. Any phenomenon that repeats itself after equal intervals of time can be used as a time standard"*. Examples of such a phenomenon are

1. Beating of human heart.
2. Oscillations of a pendulum.
3. Rotation of the earth about its own-axis.
4. Revolution of the earth around the sun.
5. Vibrations of a quartz crystal in quartz wristwatch.
6. Period of vibration of cesium – 133 atom.

Of the above examples, the period of vibration of cesium – 133 atom serves as the most accurate standard of time.

38. *What are the advantages of defining second in terms of period of radiation from cesium – 133 atom ?*

Advantages of defining second in terms of period of radiation from cesium – 133 atom :

1. Cesium atomic clock has high accuracy of 1 part in 10^{13}.
2. It can be easily reproduced in any good laboratory.
3. It is not affected by physical conditions of pressure, temperature, etc.
4. It is imperishable.

39. *Give a brief account of the various techniques used for measuring small and large time intervals.*

Some techniques used for measuring time intervals over different ranges :

1. ***Electrical oscillators.*** In India, a.c. is supplied at a frequency of 50 Hz. A motor running synchronously with this a.c. can be used to provide a time scale.

2. ***Electronic oscillators.*** A junction transistor can be used to produce oscillations of very high frequency which can be used to measure small time intervals.

3. ***Quartz clocks.*** A quartz crystal shows piezo-electric effect. If fluctuating pressure is applied across a pair of its parallel faces, an oscillatory emf is developed across another pair of perpendicular faces and vice versa. The oscillations produced can be used to measure time intervals.

4. ***Atomic clocks.*** These clocks are based on periodic vibrations taking place within the atoms. The first cesium atomic clock was set up in 1964.

5. ***Decay of elementary particles.*** The life of many elementary particles varies from 10^{-16} to 10^{-24} s. By making use of their decay times, very small time intervals can be measured.

6. ***Radioactive dating.*** This technique is used to measure long time intervals by finding the ratio of the number of radioactive atoms that have undergone decay to the number of atoms left undecayed. Carbon dating is used to estimate the age of fossils, whereas uranium dating is used to estimate the age of the rocks.

Table 2.7 Orders of magnitude of time intervals

S. No.	Event	Time (s)
1.	Life of most unstable particle	10^{-24}
2.	Time taken by light to cross a nuclear distance	10^{-22}
3.	Time period of electron in hydrogen atom	10^{-15}
4.	Period of visible light waves	10^{-15}
5.	Life time of an excited state of atom	10^{-8}
6.	Period of radio wave	10^{-6}
7.	Period of sound wave	10^{-3}
8.	Wink of eye	10^{-1}
9.	Time between two successive heart beats	10^{0}
10.	Travel time for light from moon to earth	10^{0}
11.	Travel time for light from sun to earth	10^{2}
12.	Time period of a satellite	10^{4}
13.	Period of rotation of the earth (one day)	10^{5}
14.	Period of revolution of the earth (1 year)	10^{7}
15.	Travel time for light from nearest star	10^{8}
16.	Average human life-span	10^{9}
17.	Age of the universe	10^{17}

For Your Knowledge

- In India, the National Physical Laboratory (New Delhi) has the responsibility of maintenance and improvement of physical standards of length, mass, time, etc.
- The iodine stabilized helium neon laser has been used to realize the latest definition of standard metre.
- Italian physicist Galileo was the first to set up pendulum clocks.
- The cesium atomic clocks have an high accuracy of 1 part in 10^{13}. They lose or gain not more than 3 μs in one year.
- Table 2.5 shows that the ratio of the largest and shortest lengths of objects in the universe is about 10^{41}.
- Table 2.7 shows that the ratio of the largest and the shortest time intervals associated with the events in the universe is also about 10^{41}.
- Table 2.6 shows that the ratio of the largest and the smallest masses of the objects in the universe is about $(10^{41})^2$.

Examples based on Measurement of Time

FORMULAE USED

$$\text{Fractional error in time} = \frac{\text{Difference in time}}{\text{Time internal}} = \frac{\Delta t}{t}$$

UNITS USED

Times t and Δt are in second.

EXAMPLE 25. *The average life of an Indian is 56 years. Find the number of times the human heart beats in the life of an Indian, if the heart beats once in 0.8 s.*

Solution. Average life of an Indian = 56 years

$$= 56 \times 365.25 \times 24 \times 60 \times 60 \text{ s}$$

Period of heart beat = 0.8 s

Total number of heart beats in 56 years

$$= \frac{56 \times 365.25 \times 24 \times 60 \times 60}{0.8} = \mathbf{2.2 \times 10^9 \text{ times.}}$$

EXAMPLE 26. *The mean life of an elementary particle pion is 2×10^{-7} nanosecond. The age of the universe is about 4×10^9 years. Identify a physically meaning time interval that is approximately half way between these two on a logarithmic scale.*

Solution. Mean life of pion,

$$t_1 = 2 \times 10^{-7} \text{ ns} = 2 \times 10^{-7} \times 10^{-9} \text{ s}$$
$$= 2 \times 10^{-16} \text{ s} \simeq 10^{-16} \text{ s}$$

Mean life of universe,

$$t_2 = 4 \times 10^9 \text{ years} = 4 \times 10^9 \times 3 \times 10^7 \text{ s}$$
$$= 1.2 \times 10^{17} \text{ s} \simeq 10^{17} \text{ s}$$

Let t be the time interval that is half way between t_1 and t_2 on the logarithmic scale. Then

$$\log t = \frac{1}{2}[\log t_1 + \log t_2]$$
$$= \frac{1}{2}[\log 10^{-16} + \log 10^{17}]$$
$$= \frac{1}{2}[-16 + 17] = 0.5 \simeq 1$$

$\therefore \quad t = 10$ s

which is the time taken by an athlete to run a 100 m track.

PROBLEMS FOR PRACTICE

1. Find the number of seconds in 1 year. Express them in order of magnitude. (**Ans.** 3.158×10^7 s, 10^7)
2. Find the number of times the human heart beats in the life of 60 years of a man, assuming that the heart beats once in 0.8 s. (**Ans.** 2.3668×10^9 s)
3. Two atomic clocks allowed to run for average life of an Indian (say, 70 years) differ by 0.2 s only. Calculate the accuracy of standard atomic clock in measuring a time interval of 1s. (**Ans.** 1 s in 10^{10} s)
4. Age of the universe is about 10^{10} years whereas the mankind has existed for 10^6 years. For how many seconds would the man have existed if age of universe were 1 day ? (**Ans.** 8.64 s)

HINTS

3. Here t = 70 years = $70 \times 365 + 17$

[17 days added due to 17 leap years in 70 years]

or $\quad t = 25567 \text{ days} = 25567 \times 86400 \text{ s}$

$\Delta t = 0.2$ s

Fractional error in time

$$= \frac{\Delta t}{t} = \frac{0.2}{25567 \times 86400} = 0.904 \times 10^{-10} \simeq 10^{-10}.$$

So the accuracy shown by atomic clock is 10^{-10} part in 1 s or 1 s in 10^{10} s.

4. Magnification in time

$$= \frac{\text{Age of mankind}}{\text{Age of universe}} = \frac{10^6}{10^{10}} = 10^{-4}$$

Apparent age of mankind

$$= 10^{-4} \times 1 \text{ day} = 10^{-4} \times 86400 \text{ s} = \mathbf{8.64 \text{ s.}}$$

2.20 DIMENSIONS OF A PHYSICAL QUANTITY

40. *What do you mean by seven dimensions of the world ?*

Seven dimensions of the world. All the derived physical quantities can be expressed in terms of some combination of the seven fundamental or base quantities. We call these fundamental quantities as the *seven dimensions of the world,* which are denoted with square brackets [ ].

Dimension of length = [L]
Dimension of mass = [M]
Dimension of time = [T]
Dimension of electric current = [A]
Dimension of thermodynamic temperature = [K]
Dimension of luminous intensity = [cd]
Dimension of amount of substance = [mol]

41. *What do you mean by dimensions of a physical quantity ? Explain with the help of an example.*

Dimensions of a physical quantity. *The dimensions of a physical quantity are the powers (or exponents) to which the fundamental quantities must be raised to represent that quantity completely.*

For example,

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} = \frac{\text{Mass}}{\text{Length} \times \text{breadth} \times \text{height}}$$

∴ Dimensions of density

$$= \frac{[M]}{[L][L][L]} = [ML^{-3}] = [M^1L^{-3}T^0]$$

Hence the dimensions of density are '1' in mass, '–3' in length and '0' in time.

2.21 DIMENSIONAL FORMULAE AND DIMENSIONAL EQUATIONS

42. *What is meant by dimensional formula and dimensional equation ? Give examples.*

Dimensional formula. *The expression which shows how and which of the fundamental quantities represent the dimensions of a physical quantity is called the dimensional formula of the given physical quantity.*

Examples : The dimensional formula of the volume is $[M^\circ L^3 T^\circ]$ and that of momentum is $[MLT^{-1}]$.

Dimensional equation. *The equation obtained by equating a physical quantity with its dimensional formula is called the dimensional equation of the given physical quantity.*

Examples : The dimensional equation of force is

$$[\text{Force}] = [MLT^{-2}]$$

The dimensional equation for pressure is

$$[\text{Pressure}] = [ML^{-1}T^{-2}]$$

Dimensional formuale and SI units of some physical quantities are given in table 2.8.

Table 2.8 Dimensional Formulae and SI units of some physical quantities

S. No.	*Physical Quantity*	*Relation with other quantities*	*Dimensional formula*	*SI unit*
		A. Mechanical Quantities		
1.	Area	Length × breadth	$L \times L = L^2 = [M^0L^2T^0]$	m^2
2.	Volume	Length × breadth × height	$L \times L \times L = [M^0L^3T^0]$	m^3
3.	Density	$\frac{\text{Mass}}{\text{Volume}}$	$\frac{M}{L^3} = [ML^{-3}T^0]$	$kg\,m^{-3}$
4.	Speed or velocity	$\frac{\text{Distance}}{\text{Time}}$	$\frac{L}{T} = [M^0LT^{-1}]$	ms^{-1}
5.	Acceleration	$\frac{\text{Change in velocity}}{\text{Time}}$	$\frac{LT^{-1}}{T} = LT^{-2} = [M^0LT^{-2}]$	ms^{-2}
6.	Momentum	Mass × velocity	$M \times LT^{-1} = [MLT^{-1}]$	$kg\,ms^{-1}$
7.	Force	Mass × acceleration	$M \times LT^{-2} = [MLT^{-2}]$	N
8.	Work	Force × distance	$MLT^{-2} \times L = [ML^2T^{-2}]$	J
9.	Energy	Amount of work	$[ML^2T^{-2}]$	J
10.	Power	$\frac{\text{Work}}{\text{Time}}$	$\frac{ML^2T^{-2}}{T} = [ML^2T^{-3}]$	W
11.	Pressure	$\frac{\text{Force}}{\text{Area}}$	$\frac{ML^1T^{-2}}{L^2} = [ML^{-1}T^{-2}]$	Pa or Nm^{-2}
12.	Moment of force or torque	Force × $\perp_r$ distance	$MLT^{-2} \times L = [ML^2T^{-2}]$	Nm
13.	Gravitational constant 'G'	$\frac{\text{Force} \times (\text{distance})^2}{\text{Mass} \times \text{mass}}$	$\frac{MLT^{-2}L^2}{M \times M} = [M^{-1}L^3T^{-2}]$	Nm^2kg^{-2}
14.	Impulse of a force	Force × time	$MLT^{-2} \times T = [MLT^{-1}]$	Ns
15.	Stress	$\frac{\text{Force}}{\text{Area}}$	$\frac{MLT^{-2}}{L^2} = [ML^{-1}T^{-2}]$	Nm^{-2}
16.	Strain	$\frac{\text{Change in dimension}}{\text{Original dimension}}$	$[M^0L^0T^0]$ (dimensionless)	–
17.	Coefficient of elasticity	$\frac{\text{Stress}}{\text{Strain}}$	$\frac{ML^{-1}T^{-2}}{1} = [ML^{-1}T^{-2}]$	Nm^{-2}

S. No.	Physical Quantity	Relation with other quantities	Dimensional formula	SI unit
18.	Surface tension	$\frac{\text{Force}}{\text{Length}}$	$\frac{MLT^{-2}}{L} = MT^{-2} = [ML^0T^{-2}]$	Nm^{-1}
19.	Surface energy	$\frac{\text{Work}}{\text{Area}}$	$\frac{ML^2T^{-2}}{L^2} = MT^{-2} = [ML^0T^{-2}]$	Jm^{-2}
20.	Coefficient of viscosity	$\frac{\text{Force} \times \text{distance}}{\text{Area} \times \text{velocity}}$	$\frac{MLT^{-2} \times L}{L^2 \times LT^{-1}} = [ML^{-1}T^{-1}]$	Nm^{-2} s or Pa s
21.	Angle	$\frac{\text{Arc}}{\text{Radius}}$	$\frac{L}{L} = 1 = [M^0L^0T^0]$ (dimensionless)	rad
22.	Angular velocity	$\frac{\text{Angle}}{\text{Time}}$	$\frac{1}{T} = T^{-1} = [M^0L^0T^{-1}]$	rad s^{-1}
23.	Angular acceleration	$\frac{\text{Angular velocity}}{\text{Time}}$	$\frac{T^{-1}}{T} = T^{-2} = [M^0L^0T^{-2}]$	rad s^{-2}
24.	Moment of inertia	Mass × $(\text{distance})^2$	$ML^2 = [ML^2T^0]$	kg m^2
25.	Radius of gyration	Distance	$L = [M^0LT^0]$	m
26.	Angular momentum	Mass × velocity × radius	$M \times LT^{-1} \times L = [ML^2T^{-1}]$	kg m^2s^{-1}
27.	T-ratios (sin θ, cos θ, tan θ)	$\frac{\text{Length}}{\text{Length}}$	$\frac{L}{L} = 1 = [M^0L^0T^0]$ (dimensionless)	—
28.	Time period	Time	$T = [M^0L^0T]$	s
29.	Frequency	$\frac{1}{\text{Time period}}$	$\frac{1}{T} = T^{-1} = [M^0L^0T^{-1}]$	s^{-1} or Hz
30.	Planck's constant '*h*'	$\frac{E}{\nu} = \frac{\text{Energy}}{\text{Frequency}}$	$\frac{ML^2T^{-2}}{T^{-1}} = [ML^2T^{-1}]$	Js
31.	Relative density	$\frac{\text{Density of substance}}{\text{Density of water at 4°C}}$	$\frac{ML^{-3}}{ML^{-3}} = 1 = [M^0L^0T^0]$ (dimensionless)	—
32.	Velocity gradient	$\frac{\text{Velocity}}{\text{Distance}}$	$\frac{LT^{-1}}{L} = T^{-1} = [M^0L^0T^{-1}]$	s^{-1}
33.	Pressure gradient	$\frac{\text{Pressure}}{\text{Distance}}$	$\frac{ML^{-1}T^{-2}}{L} = [ML^{-2}T^{-2}]$	Pa m^{-1}
34.	Force constant	$\frac{\text{Force}}{\text{Displacement}}$	$\frac{MLT^{-2}}{L} = MT^{-2} = [ML^0T^{-2}]$	Nm^{-1}
		B. Thermal Quantities		
35.	Heat or enthalpy	Energy	$[ML^2T^{-2}]$	J
36.	Specific heat	$\frac{\text{Heat}}{\text{Mass} \times \text{Temperature}}$	$\frac{ML^2T^{-2}}{M \cdot K} = [M^0L^2T^{-2}K^{-1}]$	J kg^{-1} K^{-1}
37.	Latent heat	$\frac{\text{Heat}}{\text{Mass}}$	$\frac{ML^2T^{-2}}{M} = [M^0L^2T^{-2}]$	J kg^{-1}
38.	Thermal conductivity	$\frac{\text{Heat} \times \text{distance}}{\text{Area} \times \text{temp.} \times \text{time}}$	$\frac{ML^2T^{-2} \cdot L}{L^2 \cdot K \cdot T} = [MLT^{-3}K^{-1}]$	$Js^{-1}m^{-1}K^{-1}$
39.	Entropy	$\frac{\text{Heat}}{\text{Temperature}}$	$\frac{ML^2T^{-2}}{K} = [ML^2T^{-2}K^{-1}]$	JK^{-1}
40.	Universal gas constant	$\frac{PV}{nT}$	$\frac{ML^{-1}T^{-2}L^3}{mol \cdot K} = [ML^2T^{-2}K^{-1}\ mol^{-1}]$	J mol^{-1} K^{-1}

S. No.	Physical Quantity	Relation with other quantities	Dimensional formula	SI unit
41.	Boltzmann's constant	$\frac{\text{Energy}}{\text{Temperature}}$	$\frac{ML^2T^{-2}}{K} = [ML^2T^{-2}K^{-1}]$	JK^{-1}
42.	Stefan's constant	$\frac{\text{Energy}}{\text{Area} \times \text{time} \times (\text{temp.})^4}$	$\frac{ML^2T^{-2}}{L^2 \cdot T \cdot K^4} = [ML^0T^{-3}K^{-4}]$	$Js^{-1}m^{-2}K^{-4}$
43.	Solar constant	$\frac{\text{Energy}}{\text{Area} \times \text{time}}$	$\frac{ML^2T^{-2}}{L^2 \cdot T} = [ML^0T^{-3}]$	$Js^{-1}m^{-2}$
44.	Mechanical equivalent of heat	$J = \frac{W}{H}$	$\frac{ML^2T^{-2}}{ML^2T^{-2}} = [M^0L^0T^0]$ (dimensionless)	—
		C. Electrical Quantities		
45.	Electric charge	Time × Current	$T.A = [M^0L^0TA]$	C (coulomb)
46.	Electrical potential	$\frac{\text{Work}}{\text{Charge}}$	$\frac{ML^2T^{-2}}{TA} = [ML^2T^{-3}A^{-1}]$	V (volt)
47.	Resistance	$\frac{\text{Potential difference}}{\text{Current}}$	$\frac{ML^2T^{-3}A^{-1}}{A} = [ML^2T^{-3}A^{-2}]$	Ω (ohm)
48.	Capacitance	$\frac{\text{Charge}}{\text{Potential difference}}$	$\frac{TA}{ML^2T^{-3}A^{-1}} = [M^{-1}L^{-2}T^4A^2]$	F (farad)
49.	Inductance	$\frac{\text{EMF}}{\text{Current / time}}$	$\frac{ML^2T^{-3}A^{-1}}{AT^{-1}} = [ML^2T^{-2}A^{-2}]$	H (henry)
50.	Permittivity of free space	$\varepsilon_0 = \frac{q_1 q_2}{Fr^2}$	$\frac{AT \cdot AT}{MLT^{-2}L^2} = [M^{-1}L^{-3}T^4A^2]$	$A^2C^2N^{-1}m^{-2}$
51.	Relative permittivity or dielectric constant	ε_r or $\kappa = \frac{\varepsilon_0}{\varepsilon}$	a pure ratio $= [M^0L^0T^0]$ (dimensionless)	—
52.	Intensity of electric field	$E = \frac{F}{q} = \frac{\text{Force}}{\text{Charge}}$	$\frac{MLT^{-2}}{AT} = [MLT^{-3}A^{-1}]$	NC^{-1} or Vm^{-1}
53.	Conductance	$C = \frac{1}{R}$	$\frac{1}{ML^2T^{-3}A^{-2}} = [M^{-1}L^{-2}T^3A^2]$	Ω^{-1} or mho
54.	Specific resistance or resistivity	$\rho = \frac{RA}{l}$	$\frac{ML^2T^{-3}A^{-2} \cdot L^2}{L} = [ML^3T^{-3}A^{-2}]$	Ωm
55.	Specific conductance or conductivity	$\sigma = \frac{1}{\rho}$	$[M^{-1}L^{-3}T^3A^2]$	$\Omega^{-1}m^{-1}$
56.	Electric dipole moment	$q \times 2l$	$AT \cdot L = [M^0LTA]$	Cm
		D. Magnetic Quantities		
57.	Magnetic field	$B = \frac{F}{qv \sin\theta}$	$\frac{MLT^{-2}}{AT \cdot LT^{-1} \cdot 1} = [ML^0T^{-2}A^{-1}]$	T (tesla)
58.	Magnetic flux	$\phi = BA$	$MT^{-2}A^{-1}.L^2 = [ML^2T^{-2}A^{-1}]$	Wb (weber)
59.	Permeability of free space	$\mu_0 = \frac{4\pi r . F}{I_1 I_2 l}$	$\frac{L \cdot MLT^{-2}}{A^2 \cdot L} = [MLT^{-2}A^{-2}]$	
60.	Magnetic moment	Current × area	$A \cdot L^2 = [M^0L^2T^0A]$	Am^2
61.	Pole strength	$\frac{\text{Magnetic moment}}{\text{Magnetic length}}$	$\frac{AL^2}{L} = [M^0LT^0A]$	Am

Examples based on Derivation of Dimensional Formulae

CONCEPT USED

The dimensional formula of a physical quantity can be obtained by defining its relation with other quantities, whose dimensions in M, L and T are known.

EXAMPLE 27. *Name the physical quantities whose dimensional formulae are as follows :*

(i) ML^2T^{-2} (ii) ML^2T^{-3} (iii) MT^{-2}
(iv) $ML^{-1}T^{-1}$ (v) $ML^{-1}T^{-2}$.

Solution. (i) $\mathrm{ML^2T^{-2} = MLT^{-2} \cdot L}$ = Force × distance = Work.

(ii) $\mathrm{ML^2T^{-3} = \dfrac{ML^2T^{-2}}{T}} = \dfrac{\text{Work}}{\text{Time}}$ = Power.

(iii) $\mathrm{MT^{-2} = \dfrac{MLT^{-2}}{L}} = \dfrac{\text{Force}}{\text{Length}}$

= Surface tension or force constant.

(iv) $\mathrm{ML^{-1}T^{-1} = \dfrac{MLT^{-2}}{L^2T^{-1}}} = \dfrac{\text{Force}}{\text{Area} \times \text{velocity gradient}}$

= Coefficient of viscosity.

(v) $\mathrm{ML^{-1}T^{-2} = \dfrac{MLT^{-2}}{L^2}}$.

$= \dfrac{\text{Force}}{\text{Area}}$ = Pressure or stress.

EXAMPLE 28. *Deduce the dimensional formulae for the following physical quantities :*

(i) *Gravitational constant* (ii) *Power*
(iii) *Young's modulus* (iv) *Coefficient of viscosity*
(v) *Surface tension* (vi) *Planck's constant.*

Solution. (i) According to Newton's law of gravitation,

$$F = G\frac{m_1 m_2}{r^2}$$

$$\therefore \quad [G] = \frac{[F][r^2]}{[m_1][m_2]} = \frac{\mathrm{MLT^{-2}.\,L^2}}{\mathrm{MM}} = \mathbf{[M^{-1}L^3T^{-2}]}.$$

(ii) $\text{Power} = \dfrac{\text{Work}}{\text{Time}} = \dfrac{\text{Force} \times \text{Distance}}{\text{Time}}$

$$\therefore \quad [\text{Power}] = \frac{\mathrm{MLT^{-2}.\,L}}{\mathrm{T}} = \mathbf{[ML^2T^{-3}]}.$$

(iii) Young's modulus,

$$Y = \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}} = \frac{F/A}{\Delta l/l} = \frac{F}{A}.\frac{l}{\Delta l}$$

$$\therefore \quad [Y] = \frac{\mathrm{MLT^{-2}.\,L}}{\mathrm{L^2.\,L}} = \mathbf{[ML^{-1}T^{-2}]}.$$

(iv) Coefficient of viscosity,

$$\eta = \frac{\text{Force}}{\text{Area} \times \text{velocity gradient}} = \frac{\text{Force}}{\text{Area}} \times \frac{\text{Distance}}{\text{Velocity}}$$

$$[\eta] = \frac{\mathrm{MLT^{-2}.L}}{\mathrm{L^2.\,LT^{-1}}} = \mathbf{[ML^{-1}T^{-1}]}.$$

(v) Surface tension,

$$\sigma = \frac{\text{Force}}{\text{Length}}$$

$$[\sigma] = \frac{\mathrm{MLT^{-2}}}{\mathrm{L}} = \mathbf{[MT^{-2}]}.$$

(vi) Planck's constant,

$$h = \frac{\text{Energy}}{\text{Frequency}}$$

$$[h] = \frac{\mathrm{ML^2T^{-2}}}{\mathrm{T^{-1}}} = \mathbf{[ML^2T^{-1}]}.$$

EXAMPLE 29. *Deduce the dimensional formulae of the following physical quantities :*

(i) *Heat* (ii) *Specific heat* (iii) *Latent heat*
(iv) *Gas constant*
(v) *Boltzmann's constant*
(vi) *Coefficient of thermal conductivity*
(vii) *Mechanical equivalent of heat.*

Solution. (i) Heat = Energy

$$\therefore \quad [\text{Heat}] = \mathbf{[ML^2T^{-2}]}.$$

(ii) $\text{Specific heat} = \dfrac{\text{Heat}}{\text{Mass} \times \text{temperature}}$

$$\therefore \quad [\text{Specific heat}] = \frac{\mathrm{ML^2T^{-2}}}{\mathrm{M\,.\,K}} = \mathbf{[L^2T^{-2}K^{-1}]}.$$

(iii) $\text{Latent heat} = \dfrac{\text{Heat}}{\text{Mass}}$

$$\therefore \quad [\text{Latent heat}] = \frac{\mathrm{ML^2T^{-2}}}{\mathrm{M}} = \mathbf{[L^2T^{-2}]}.$$

(iv) $PV = nRT$

or $R = \dfrac{PV}{nT} = \dfrac{\text{Force} \times \text{volume}}{\text{mol} \times \text{area} \times \text{temp.}}$

$$\therefore \quad [R] = \frac{\mathrm{MLT^{-2}.\,L^3}}{\mathrm{mol\,.\,L^2\,.\,K}} = \mathbf{[ML^2T^{-2}K^{-1}mol^{-1}]}.$$

(v) Boltzmann's constant,

$$k = \frac{\text{Heat}}{\text{Temperature}}$$

$$[k] = \frac{\mathrm{ML^2T^{-2}}}{\mathrm{K}} = \mathbf{[ML^2T^{-2}K^{-1}]}.$$

(*vi*) Coefficient of thermal conductivity,

$$\kappa = \frac{\text{Heat} \times \text{distance}}{\text{Area} \times \text{time} \times \text{temperature difference}}$$

$$[\kappa] = \frac{ML^2T^{-2}.\,L}{L^2\,.\,T\,.\,K} = [\mathbf{MLT^{-3}K^{-1}}]$$

(*vii*) Joule's mechanical equivalent of heat,

$$J = \frac{\text{Work}}{\text{Heat}}$$

$$[J] = \frac{ML^2T^{-2}}{ML^2\,T^{-2}} = 1 = [\mathbf{M^0L^0T^0}].$$

EXAMPLE 30. *Find the dimensional formulae of (i) charge (ii) potential (iii) resistance (iv) capacitance.*

Solution. (*i*) Charge, $q = \text{Current} \times \text{time}$

$\therefore$ $$[q] = [\mathbf{AT}].$$

(*ii*) Potential,

$$V = \frac{\text{Work}}{\text{Charge}}$$

$$[V] = \frac{ML^2T^{-2}}{AT} = [\mathbf{ML^2A^{-1}T^{-3}}].$$

(*iii*) Resistance,

$$R = \frac{\text{Potential difference}}{\text{Current}}$$

$$= \frac{ML^2A^{-1}T^{-3}}{A} = [\mathbf{ML^2A^{-2}T^{-3}}].$$

(*iv*) Capacitance,

$$C = \frac{\text{Charge}}{\text{Potential}}$$

$$[C] = \frac{AT}{ML^2A^{-1}T^{-3}} = [\mathbf{M^{-1}L^{-2}A^2T^4}]$$

EXAMPLE 31. *Taking velocity, time and force as the fundamental quantities, find the dimensions of mass.*

Solution. Force = Mass × Acceleration

$$= \text{Mass} \times \frac{\text{Velocity}}{\text{Time}}$$

or $$\text{Mass} = \frac{\text{Force} \times \text{Time}}{\text{Velocity}}$$

$\therefore$ $$[\text{Mass}] = [\mathbf{FTV^{-1}}].$$

EXAMPLE 32. *If density ρ, acceleration due to gravity g and frequency ν are the basic quantities, find the dimensions of force.*

Solution. We have $\rho = ML^{-3},\ g = LT^{-2},\ \nu = T^{-1}$

Solving for M, L and T in terms of ρ, g and ν, we get

$$M = \rho g^3 \nu^{-6},\ L = g\nu^{-2},\ T = \nu^{-1}$$

$\therefore$ $$[\text{Force}] = MLT^{-2} = \rho g^3\nu^{-6}\,.\,g\nu^{-2}\,.\,\nu^2$$

$$= [\boldsymbol{\rho g^4\,\nu^{-6}}].$$

EXAMPLE 33. *If the velocity of light c, acceleration due to gravity g and atmospheric pressure p are the fundamental quantities, find the dimensions of length.*

Solution. We have,

$$c = LT^{-1},\ g = LT^{-2},\ p = ML^{-1}\,T^{-2}$$

$$[\text{Length}] = L = \frac{L^2\,T^{-2}}{LT^{-2}} = \frac{(LT^{-1})^2}{LT^{-2}} = \left[\frac{c^2}{g}\right].$$

EXAMPLE 34. *The number of particles crossing a unit area perpendicular to X-axis in unit time is given by*

$$n = -D\,\frac{n_2 - n_1}{x_2 - x_1}$$

where n_1 and n_2 are number of particles per unit volume for the values of x meant to be x_1 and x_2. Find the dimensions of the diffusion constant D.

Solution. As $n = -D\,\dfrac{n_2 - n_1}{x_2 - x_1}$

$\therefore$ $$D = \frac{n\,(x_2 - x_1)}{(n_2 - n_1)} \qquad \text{(numerically)}$$

Now n = number of particles per unit area per second,

$\therefore$ $[n] = L^{-2}T^{-1}$

$n_2 - n_1$ = number of particles per unit volume

$\therefore$ $[n_2 - n_1] = L^{-3}$

$x_2 - x_1$ = position

$\therefore$ $[x_2 - x_1] = L$

Hence $$[D] = \frac{L^{-2}T^{-1}.\,L}{L^{-3}} = [\mathbf{L^2T^{-1}}].$$

PROBLEMS FOR PRACTICE

1. Deduce dimensional formulae for (*i*) angle (*ii*) angular velocity (*iii*) angular acceleration (*iv*) torque (*v*) angular momentum and (*vi*) moment of inertia.
[**Ans.** (*i*) Dimensionless (*ii*) T^{-1} (*iii*) T^{-2} (*iv*) $ML^2\,T^{-2}$ (*v*) $ML^2\,T^{-1}$ (*vi*) ML^2]
2. Obtain dimensions of (*i*) impulse (*ii*) power (*iii*) surface energy (*iv*) coefficient of viscosity (*v*) bulk modulus (*vi*) force constant.
[**Ans.** (*i*) MLT^{-1} (*ii*) ML^2T^{-3} (*iii*) ML^0T^{-2} (*iv*) $ML^{-1}\,T^{-1}$] (*v*) $ML^{-1}\,T^{-2}$ (*vi*) $ML^0\,T^{-2}$]
3. By the use of dimensions, show that energy per unit volume is equal to the pressure.
4. Show that angular momentum has the same physical units as the Planck's constant h which is given by the relation $E = h\nu$.
5. If force (F), length (L) and time (T) are chosen as the fundamental quantities, then what would be the dimensional formula for density ? (**Ans.** $FL^{-4}\,T^{-1}$)

6. Calculate the dimensions of force and impulse taking velocity, density and frequency as basic quantities. **(Ans.** $\rho v^4 \nu^{-2}, \rho v^4 \nu^{-3}$)

7. Find the dimensions of linear momentum and surface tension in terms of velocity v, density ρ and frequency ν as fundamental quantities. **(Ans.** $\rho v^4 \nu^{-3}, \rho v^3 \nu^{-1}$)

8. In the expression $P = El^2 m^{-5} G^{-2}$; E, m, l and G denote energy, mass angular momentum and gravitational constant, respectively. Show that P is a dimensionless quantity. **[Exemplar Problem]**

HINTS

3. Energy per unit volume

$$= \frac{\text{Energy}}{\text{Volume}} = \frac{\text{ML}^2 \,.\, \text{T}^{-2}}{\text{L}^3} = \frac{\text{MLT}^{-2}}{\text{L}^2}$$

$$= \frac{\text{Force}}{\text{Area}} = \text{Pressure.}$$

4. Angular momentum,

$$L = mvr$$

$$[\text{L}] = \text{MLT}^{-1} . \text{L} = \text{ML}^2\text{T}^{-1}$$

As $E = h\nu$

$$\therefore \quad h = \frac{E}{\nu}$$

$$[h] = \frac{\text{ML}^2\ \text{T}^{-2}}{\text{T}^{-1}} = \text{ML}^2\ \text{T}^{-1} = [\text{L}]$$

6. $v = \text{LT}^{-1}$, $\rho = \text{ML}^{-3}$, $\nu = \text{T}^{-1}$

Solving for M, L and T in terms of ν, ρ and v, we get

$$\text{T} = \nu^{-1}, \quad \text{L} = v\nu^{-1}, \quad \text{M} = \rho v^3 \nu^{-3}$$

$$[\text{Force}] = \text{MLT}^{-2} = \rho v^3 \nu^{-3} \,.\, v\nu^{-1} \,.\, \nu^2 = \rho v^4 \nu^{-2}$$

$$[\text{Impulse}] = \text{Force} \times \text{time} = \rho v^4 \nu^{-2} . \nu^{-1} = \rho v^4 \nu^{-3}$$

7. Using the dimensions of M, L and T as obtained in Problem 6, we get

$$[p] = [mv] = \text{MLT}^{-1}$$

$$= \rho v^3 \nu^{-3} \,.\, v\nu^{-1} \,.\, \nu = \rho v^4 \nu^{-3}$$

$$\text{Surface tension} = \frac{\text{Force}}{\text{Length}}$$

$$= \frac{\text{MLT}^{-2}}{\text{L}} = \text{MT}^{-2} = \rho v^3 \nu^{-3} \nu^2 = \rho v^3 \nu^{-1}.$$

8. $[E] = \text{ML}^2\ \text{T}^{-2}$, $[\text{M}] = \text{M}$, $[l] = \text{ML}^2\ \text{T}^{-1}$,

$[G] = \text{M}^{-1}\text{L}^3\text{T}^{-2}$

$$\therefore \quad \left[\frac{El^2}{M^5\ G^2}\right] = \frac{\text{ML}^2\text{T}^{-2}[\text{ML}^2\text{T}^{-1}]^2}{\text{M}^5[\text{M}^{-1}\text{L}^3\text{T}^{-2}]^2}$$

$$= \frac{\text{M}^3\text{L}^6\text{T}^{-4}}{\text{M}^3\text{L}^6\text{T}^{-4}} = [\text{M}^0\text{L}^0\text{T}^0]$$

2.22 DIFFERENT TYPES OF VARIABLES AND CONSTANTS

43. *How can we classify variables and constants on the basis of dimensions ? Give examples of each type.*

Different types of variables and constants. On the basis of dimensions, we can classify quantities into *four* categories :

1. *Dimensional variables.* *The physical quantities which possess dimensions and have variable values are called dimensional variables.*

Examples. Area, volume, velocity, force, etc.

2. *Dimensionless variables.* *The physical quantities which have no dimensions but have variable values are called dimensionless variables.*

Examples. Angle, specific gravity, strain, etc.

3. *Dimensional constants.* *The physical quantities which possess dimensions and have constant values are called dimensional constants.*

Examples. Gravitational constant, Planck's constant, etc.

4. *Dimensionless constants.* *The constant quantities having no dimensions are called dimensionless constants.*

Examples. π, e, etc.

2.23 APPLICATIONS OF DIMENSIONAL ANALYSIS

44. *Mention some applications of dimensional analysis.*

Applications of dimensional analysis. *The method of studying a physical phenomenon on the basis of dimensions is called dimensional analysis.* Following are the *three* main uses of dimensional analysis :

1. To convert a physical quantity from one system of units to another.
2. To check the correctness of a given physical relation.
3. To derive a relationship between different physical quantities.

2.24 CONVERSION OF ONE SYSTEM OF UNITS TO ANOTHER

45. *How can a physical quantity be converted from one system of units to another ? Explain it with the help of a suitable example.*

To convert a physical quantity from one system of units to another. It is based on the fact that *the magnitude of a physical quantity remains the same, whatever may be the system of units.* If u_1 and u_2 are the units of measurement of a physical quantity Q and n_1 and n_2 are the corresponding numerical values, then

$$Q = n_1 u_1 = n_2 u_2$$

Let M_1, L_1 and T_1 be the sizes of fundamental units of mass, length and time in one system ; and M_2, L_2, T_2 be corresponding units in another system. If the dimensional formula of quantity Q be $M^a L^b T^c$, then

$$u_1 = M_1^a L_1^b T_1^c$$

and $$u_2 = M_2^a L_2^b T_2^c$$

$\therefore$ $$n_1 [M_1^a L_1^b T_1^c] = n_2 [M_2^a L_2^b T_2^c]$$

or $$n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c.$$

This equation can be used to find the numerical value in the second or new system of units.

Example. Let us convert one joule into erg.

Joule is SI unit of energy and erg is the CGS unit of energy. Dimensional formula of energy is ML^2T^{-2}.

$\therefore \quad a = 1,\ b = 2,\ c = -2.$

SI	CGS
$M_1 = 1\text{ kg} = 1000\text{ g}$	$M_2 = 1\text{ g}$
$L_1 = 1\text{ m} = 100\text{ cm}$	$L_2 = 1\text{ cm}$
$T_1 = 1\text{ s}$	$T_2 = 1\text{ s}$
$n_1 = 1\text{ (joule)}$	$n_2 = ?\text{ (erg)}$

$$n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$$

$$= 1\left[\frac{1000}{1}\right]^1 \left[\frac{100}{1}\right]^2 \left[\frac{1}{1}\right]^{-2}$$

$$= 1 \times 10^3 \times 10^4 = 10^7$$

$\therefore$ **1 joule = 10^7 erg.**

For Your Knowledge

- The above conversion technique is applicable to only absolute systems of units. The gravitational or other practical units must be first converted into absolute units before using the above technique.

Examples based on Conversion of Units from one System to another

FORMULAE USED

1. $n_1 u_1 = n_2 u_2$
2. $n_1 [M_1^a L_1^b T_1^c] = n_2 [M_2^a L_2^b T_2^c]$
3. $n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$

UNITS USED

The SI units of mass, length and time are kg, m and s and the corresponding CGS units are g, cm and s.

EXAMPLE 35. The value G in CGS system is 6.67×10^{-8} dyne $cm^2\ g^{-2}$. Calculate the value in SI units.

Solution. As $F = G\dfrac{m_1 m_2}{r^2}$

$$\therefore \quad G = \frac{Fr^2}{m_1 m_2}$$

$$[G] = \frac{MLT^{-2} . L^2}{MM} = M^{-1}L^3T^{-2}$$

$\therefore \quad a = -1,\ b = 3,\ c = -2$

CGS units	SI units
$n_1 = 6.67 \times 10^{-8}$	$n_2 = ?$
$M_1 = 1\,g$	$M_2 = 1\text{ kg} = 1000\text{ g}$
$L_1 = 1\text{ cm}$	$L_2 = 1\text{ m} = 100\text{ cm}$
$T_1 = 1\text{ s}$	$T_2 = 1\text{ s}$

$$\therefore \quad n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$$

$$= 6.67 \times 10^{-8} \left[\frac{1}{1000}\right]^{-1} \left[\frac{1}{100}\right]^3 \left[\frac{1}{1}\right]^{-2}$$

$$= 6.67 \times 10^{-11}$$

Hence in SI units,

$$\mathbf{G = 6.67 \times 10^{-11} Nm^2 kg^{-2}}.$$

EXAMPLE 36. Find the value of 60 J per min on a system that has 100 g, 100 cm and 1 min as the base units.

Solution. $P = 60\ \dfrac{\text{joule}}{\text{min}} = \dfrac{60\text{ joule}}{60\text{ s}} = 1\text{ watt}$

which is the SI unit of power.

Now $\quad [\text{Power}] = ML^2T^{-3}$

$\therefore \quad a = 1, \quad b = 2, \quad c = -3$

SI	New System
$n_1 = 1$	$n_2 = ?$
$M_1 = 1\text{ kg} = 1000\text{ g}$	$M_2 = 100\text{ g}$
$L_1 = 1\text{ m} = 100\text{ cm}$	$L_2 = 100\text{ cm}$
$T_1 = 1\text{s}$	$T_2 = 1\text{ min} = 60\text{ s}$

$$n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$$

$$= 1\left[\frac{1000}{100}\right]^1 \left[\frac{100}{100}\right]^2 \left[\frac{1}{60}\right]^{-3}$$

$$= 2.16 \times 10^6$$

$\therefore$ 60 J min^{-1} = **2.16×10^6 new units of power.**

EXAMPLE 37. In CGS system, the value of Stefan's constant is $\sigma = 5.67 \times 10^{-5}$ erg s^{-1} cm^{-2} K^{-4}. Find its value in SI units. Given 1 J $= 10^7$ erg.

Solution. In CGS system, $\sigma = 5.67 \times 10^{-5}$ erg $s^{-1}cm^{-2}K^{-4}$. The SI unit of work is joule. We have, 1 erg $= 10^{-7}$ J and 1 cm $= 10^{-2}$ m

$\therefore$ The value of Stefan's constant in SI units is

$$\sigma = 5.67 \times 10^{-5}[10^{-7}\,\text{J}]\,\text{s}^{-1}\,[10^{-2}\,\text{m}]^{-2}\,\text{K}^{-4}$$
$$= 5.67 \times 10^{-5} \times 10^{-7} \times 10^{4}\ \text{Js}^{-1}\text{m}^{-2}\,\text{K}^{-4}$$
$$= \mathbf{5.67 \times 10^{-8}\ Js^{-1}m^{-2}K^{-4}}.$$

EXAMPLE 38. If the unit of force is 1 kN, unit of length 1 km and the unit of time is 100 s, what will be the unit of mass ?

Solution. $$M = \frac{MLT^{-2}.\,T^2}{L} = \frac{FT^2}{L}$$
$$= \frac{1000\ \text{N} \times 10^4\ \text{s}^2}{1000\ \text{m}} = 10^4\,\frac{\text{Ns}^2}{\text{m}}$$
$$= 10^4\,\frac{\text{kg m s}^{-2}.\,\text{s}^2}{\text{m}} = \mathbf{10^4\ kg}.$$

PROBLEMS FOR PRACTICE

1. Convert one dyne into newton. **[Himachal 09]**
(**Ans.** 10^{-5} newton)
2. If the value of universal gravitational constant in SI is 6.6×10^{-11} $Nm^2\,kg^{-2}$, then find its value in CGS system. **[Himachal 09]**
(**Ans.** 6.6×10^{-8} dyne cm^2g^{-2})
3. The density of mercury is 13.6 g cm^{-3} in CGS system. Find its value in SI units.
(**Ans.** 13.6×10^3 kg m^{-3})
4. The surface tension of water is 72 dyne cm^{-1}. Express it in SI units. (**Ans.** 0.072 Nm^{-1})
5. An electric bulb has a power of 500 W. Express it in CGS units. (**Ans.** 5×10^9 erg s^{-1})
6. If the value of atmospheric pressure is 10^6 dyne cm^{-2}, find its value in SI units. (**Ans.** 10^5 Nm^{-2})
7. In SI units, the value of Stefan's constant is $\sigma = 5.67 \times 10^{-8}$ Js^{-1} m^{-2} K^{-4}. Find its value in CGS system. (**Ans.** 5.67×10^{-5} erg s^{-1} cm^{-2} K^{-4})
8. Find the value of 100 J on a system which has 20 cm, 250 g and half minute as fundamental units of length, mass and time. (**Ans.** 9×10^6 new units)
9. If the units of force, energy and velocity are 20 N, 200 J and 5 ms^{-1}, find the units of length, mass and time. (**Ans.** 10 m, 8 kg, 2 s)
10. When 1 m, 1 kg and 1 min are taken as the fundamental units, the magnitude of the force is 36 units. What will be the value of this force in CGS system ?
(**Ans.** 10^3 dyne)

HINTS

3. [Density] $= M^1L^{-3}$

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^1\left[\frac{L_1}{L_2}\right]^{-3}$$
$$= 13.6\left[\frac{1\,\text{g}}{1\,\text{kg}}\right]^1\left[\frac{1\,\text{cm}}{1\,\text{m}}\right]^{-3}$$
$$= 13.6\left[\frac{1\,\text{g}}{1000\,\text{g}}\right]\left[\frac{1\,\text{cm}}{100\,\text{cm}}\right]^{-3} = 13.6 \times 10^3$$
$$\therefore \quad 13.6\ \text{g cm}^{-3} = \mathbf{13.6 \times 10^3\ kgm^{-3}}.$$

4. [Surface Tension] $= M^1T^{-2}$

$$n_2 = 72\left[\frac{1\,\text{g}}{1\,\text{kg}}\right]^1\left[\frac{1\,\text{s}}{1\,\text{s}}\right]^{-2}$$
$$= 72\left[\frac{1\,\text{g}}{1000\,\text{g}}\right]^1\left[\frac{1\,\text{s}}{1\,\text{s}}\right]^{-2} = 0.072$$
$$\therefore \quad 72\ \text{dyne cm}^{-1} = \mathbf{0.072\ Nm^{-1}}.$$

6. [Pressure] $= M^1L^{-1}T^{-2}$

$$n_2 = 10^6\left[\frac{1\,\text{g}}{1\,\text{kg}}\right]^1\left[\frac{1\,\text{cm}}{1\,\text{m}}\right]^{-1}\left[\frac{1\,\text{s}}{1\,\text{s}}\right]^{-2} = 10^5$$
$$\therefore \quad 10^6\ \text{dyne cm}^{-2} = \mathbf{10^5\ Nm^{-2}}.$$

7. $$\sigma = 5.67 \times 10^{-8}\,\text{Js}^{-1}\text{m}^{-2}\text{K}^{-4}$$
$$= 5.67 \times 10^{-8} \times 10^7\ \text{erg s}^{-1}\,(100\,\text{cm})^{-2}\,\text{K}^{-4}$$
$$= \mathbf{5.67 \times 10^{-5}\ erg\ s^{-1}\ cm^{-2}\ K^{-4}}.$$

8. [Work] $= ML^2\,T^{-2}$

$$n_2 = 100\left[\frac{1\,\text{kg}}{250\,\text{g}}\right]^1\left[\frac{1\,\text{m}}{20\,\text{cm}}\right]^2\left[\frac{1\,\text{s}}{0.5\,\text{min}}\right]^{-2}$$
$$= 100\left[\frac{1000\,\text{g}}{250\,\text{g}}\right]^1\left[\frac{100\,\text{cm}}{20\,\text{cm}}\right]^2\left[\frac{1\,\text{s}}{30\,s}\right]^{-2}$$
$$= 100 \times 4 \times 25 \times 30 \times 30 = \mathbf{9 \times 10^6\ new\ units}.$$

9. (*i*) $MLT^{-2} = 20$ N (*ii*) $ML^2\,T^{-2} = 200$ J

(*iii*) $LT^{-1} = 5\ ms^{-1}$

Dividing (*ii*) by (*i*), $L = \frac{200}{20} = \mathbf{10\ m}$

Putting the value of L in (*iii*),
$$10\,T^{-1} = 5 \text{ or } T = \mathbf{2\ s}$$

From (*i*),
$$M \times 10 \times (2)^{-2} = 20 \quad \text{or} \quad M = \mathbf{8\ kg}.$$

10. $$n_2 = 36\left[\frac{1\,\text{kg}}{1\,\text{g}}\right]^1\left[\frac{1\,\text{m}}{1\,\text{cm}}\right]^1\left[\frac{1\,\text{min}}{1\,\text{s}}\right]^{-2}$$
$$= 36\left[\frac{1000\,\text{g}}{1\,\text{g}}\right]\left[\frac{100\,\text{cm}}{1\,\text{cm}}\right]^1\left[\frac{60\,\text{s}}{1\,s}\right]^{-2} = \mathbf{10^3\ dyne}.$$

2.25 CHECKING THE DIMENSIONAL CONSISTENCY OF EQUATIONS

46. *State the principle of homogeneity of dimensions. What is its basis ?*

Principle of homogeneity of dimensions. According to this principle, *a physical equation will be dimensionally correct if the dimensions of all the terms occurring on both sides of the equation are the same.* This principle is based on the fact that only the physical quantities of the same kind can be added, subtracted or compared. Thus, velocity can be added to velocity but not to force.

47. *How can we check the dimensional correctness of a physical equation ? Explain it with a suitable example.*

To check the dimensional correctness of a physical equation. For this purpose we make use of the principle of homogeneity of dimensions. If the dimensions of all the terms on the two sides of the equation are same, then the equation is dimensionally correct.

Example. Let us check the dimensional accuracy of the equation of motion,

$$s = ut + \frac{1}{2}at^2$$

Dimensions of different terms are

$$[s] = [\mathrm{L}]$$
$$[ut] = [\mathrm{LT^{-1}}][\mathrm{T}] = [\mathrm{L}]$$
$$\left[\frac{1}{2}at^2\right] = [\mathrm{LT^{-2}}][\mathrm{T^2}] = [\mathrm{L}]$$

As all the terms on both sides of the equations have the same dimensions, so the given equation is dimensionally correct.

For Your Knowledge

- A dimensionally correct equation need not be actually a correct equation, but a dimensionally inconsistent equation must be wrong. The equation of motion : $s = ut + at^2$ is dimensionally correct but numerically it is wrong.

Examples based on Dimensional Correctness of Physical Relations

CONCEPT USED

By the principle of homogeneity of dimensions, a physical relation will be dimensionally correct if the dimensions of all the terms in the equation are the same.

EXAMPLE 39. *The distance x travelled by a body in time t which starts from the position x_0 with initial velocity v_0 and has uniform acceleration a, is given by $x = x_0 + v_0t + \frac{1}{2}at^2$.*

Check the dimensionally consistency of this equation.

Solution. The dimensions of the various terms are

$$[x] = [\mathrm{L}]$$
$$[x_0] = [\mathrm{L}]$$
$$[v_0t] = [\mathrm{LT^{-1}}][\mathrm{T}] = [\mathrm{L}]$$
$$[\tfrac{1}{2}at^2] = [\mathrm{LT^{-2}}][\mathrm{T^2}] = [\mathrm{L}]$$

Since the dimensions of all the terms are same, hence the given equation is dimensionally correct.

EXAMPLE 40. *Check whether the following equation is dimensionally correct.*

$$\tfrac{1}{2}mv^2 = mgh. \qquad \textbf{[NCERT, Himachal 06C]}$$

Solution. $[\frac{1}{2}mv^2] = [\mathrm{M}][\mathrm{LT^{-1}}]^2 = [\mathrm{ML^2T^{-2}}]$

$$[mgh] = [\mathrm{M}][\mathrm{LT^{-2}}][\mathrm{L}] = [\mathrm{ML^2T^{-2}}]$$

∴ Dimensions of LHS = Dimensions of RHS

Hence the given equation is dimensionally correct.

EXAMPLE 41. *Check the correctness of the equation,*

$$FS = \tfrac{1}{2}mv^2 - \tfrac{1}{2}mu^2$$

where F is the force acting on a body of mass m and S is the distance moved by the body when its velocity changes from u to v. **[Delhi 08]**

Solution.

$$[FS] = \mathrm{MLT^{-2}.L = ML^2T^{-2}}$$
$$[\tfrac{1}{2}mv^2] = \mathrm{M[LT^{-1}]^2 = ML^2T^{-2}}$$
$$[\tfrac{1}{2}mu^2] = \mathrm{M[LT^{-1}]^2 = ML^2T^{-2}}$$

Since dimensions of all the terms in the given equation are same, hence the given equation is dimensionally correct.

EXAMPLE 42. *Check the correctness of the relation $\tau = I\alpha$, where τ is the torque acting on a body, I is the moment of inertia and α is angular acceleration.* **[Punjab 1990]**

Solution. Given $\tau = I\alpha$

As torque, τ = Force × distance

∴ $[\tau] = \mathrm{MLT^{-2}.L = ML^2T^{-2}}$

Moment of inertia,

$$I = \text{Mass} \times \text{distance}^2$$

∴ $[I] = \mathrm{ML^2}$

Angular acceleration,

$$\alpha = \frac{\text{Angle}}{(\text{Time})^2}$$

∴ $$[\alpha] = \frac{1}{\mathrm{T^2}} = \mathrm{T^{-2}}$$

$$[I\alpha] = \mathrm{ML^2T^{-2}}$$

∴ Dimensions of LHS = Dimensions of RHS

Hence the given equation is dimensionally correct.

EXAMPLE 43. *Check the dimensional consistency of the following equations :*

(*i*) *de-Broglie wavelength,* $\lambda = \frac{h}{mv}$

(*ii*) *Escape velocity,* $v = \sqrt{\frac{2GM}{R}}$. **[Himachal 05]**

Solution. (*i*) Given $\lambda = \frac{h}{mv}$

As wavelength is a distance,

$\therefore \quad [\lambda] = L$

Also $\left[\frac{h}{mv}\right] = \frac{\text{Planck's constant}}{\text{Mass} \times \text{velocity}} = \frac{ML^2 T^{-1}}{M \times LT^{-1}} = L$

$\because$ Dimensions of LHS = Dimensions of RHS

Hence the given equation is dimensionally consistent.

(*ii*) Given $v = \sqrt{\frac{2\,GM}{R}}$

$[v] = LT^{-1}$

$$\left[\frac{2GM}{R}\right]^{\frac{1}{2}} = \left[\frac{M^{-1}L^3T^{-2}.\,M}{L}\right]^{\frac{1}{2}} = [L^2\,T^{-2}]^{\frac{1}{2}} = LT^{-1}$$

$\because$ Dimensions of LHS = Dimensions of RHS

Hence the equation is dimensionally correct.

EXAMPLE 44. *Check by the method of dimensions whether the following equations are correct :*

(*i*) $E = mc^2$ (*ii*) $T = 2\pi\sqrt{\frac{l}{g}}$ **[Delhi 98]**

(*iii*) $v = \sqrt{\frac{P}{\rho}}$, *where* v = *velocity of sound,*

P = *pressure and* ρ = *density of medium.*

(*iv*) $\nu = \frac{1}{2l}\sqrt{\frac{T}{m}}$, *where* ν = *frequency of vibration,*

l = *length of the string,* T = *tension in the string and* m = *mass per unit length.*

Solution.

(*i*) $[L.H.S] = [E] = \text{Energy} = ML^2T^{-2}$

$[R.H.S] = [mc^2] = M[LT^{-1}]^2 = ML^2T^{-2}$

Hence the relation is correct.

(*ii*) $[RHS] = \sqrt{\frac{[l]}{[g]}} = \left[\frac{L}{LT^{-2}}\right]^{1/2} = T$

$[LHS] = [T] = T.$

Hence the relation is correct.

(*iii*) $[RHS] = \sqrt{\frac{[P]}{[\rho]}} = \left[\frac{ML^{-1}T^{-2}}{ML^{-3}}\right]^{1/2} = LT^{-1}$

$[LHS] = [v] = LT^{-1}.$

Hence the relation is correct.

(*iv*) $\text{RHS} = \frac{1}{[l]}\sqrt{\frac{[T]}{[m]}} = \frac{1}{L}\left[\frac{MLT^{-2}}{ML^{-1}}\right]^{1/2}$

$= \frac{1}{L}.\,LT^{-1} = T^{-1}$

$[LHS] = [\nu] = T^{-1}.$

Hence the relation is correct.

EXAMPLE 45. *By the method of dimensions, test the accuracy of the equation :* $\delta = \frac{mg\,l^3}{4b\,d^3\,Y}$

where δ *is the depression produced in the middle of a bar of length l, breadth b and depth d, when it is loaded in the middle with mass m. Y is the Young's modulus of the material of the bar.*

Solution. $[LHS] = [\delta] = \text{depression} = L$

$[RHS] = \left[\frac{mg\,l^3}{4bd^3\,Y}\right]$

$$= \frac{\text{Mass} \times \text{acceleration} \times \text{length}^3}{4 \times \text{breadth} \times \text{depth}^3 \times \text{Young's modulus}}$$

$$= \frac{M.\,LT^{-2}.\,L^3}{L.\,L^3.\,ML^{-1}\,T^{-2}} = L$$

$\therefore$ [LHS] = [RHS]. Hence the relation is correct.

EXAMPLE 46. *Find the dimensions of a/b in the equation :* $F = a\sqrt{x} + bt^2$, *where F is force, x is distance and t is time.*

Solution. $[a\sqrt{x}] = [F]$

$\therefore \quad [a] = \frac{[F]}{[\sqrt{x}]} = \frac{MLT^{-2}}{L^{1/2}} = ML^{1/2}\,T^{-2}$

$[bt^2] = [F]$

$\therefore \quad [b] = \frac{[F]}{[t^2]} = \frac{MLT^{-2}}{T^2} = MLT^{-4}$

$[a/b] = \frac{ML^{1/2}\,T^{-2}}{MLT^{-4}} = \mathbf{L^{-1/2}T^{2}}.$

EXAMPLE 47. *Find the dimensions of a × b in the relation :* $P = \frac{b - x^2}{at}$; *where P is power, x is distance and t is time.*

Solution. $[b] = [x^2] = L^2 \quad \therefore \quad [P] = \frac{L^2}{[at]}$

or $\quad [a] = \frac{L^2}{[P][t]} = \frac{L^2}{ML^2T^{-3}.\,T} = M^{-1}\,T^2$

Hence $[a \times b] = \mathbf{M^{-1}\,L^2\,T^2}.$

EXAMPLE 48. *The Vander Wall's equation for a gas is*

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT$$

Determine the dimensions of a and b. Hence write the SI units of a and b. **[Himachal 06C ; Delhi 95]**

Solution. Since dimensionally similar quantities can be added or subtracted, therefore,

$$[P] = \left[\frac{a}{V^2}\right]$$

or $$[a] = [PV^2] = [ML^{-1}T^{-2}][L^3]^2 = ML^5T^{-2}$$

Also $[b] = [V] = L^3$.

The SI unit of a is kg m^5s^{-2} and that of b is m^3.

EXAMPLE 49. When white light travels through glass, the refractive index of glass (μ = velocity of light in air/velocity of light in glass) is found to vary with wavelength as $\mu = A + \frac{B}{\lambda^2}$. Using the principle of homogeneity of dimensions, find the SI units in which the constants A and B must be expressed.

Solution. Here $\mu = \frac{\text{Velocity of light in air}}{\text{Velocity of light in glass}}$

= a dimensionless number

∴ $[A] = [\mu]$ = a dimensionless number

As $\left[\frac{B}{\lambda^2}\right] = [\mu]$

∴ $[B] = [\mu][\lambda^2] = 1 . L^2 = L^2$

Hence A, being dimensionless, has no units and SI unit of B is m^2.

EXAMPLE 50. In the equation : $y = a \sin(\omega t - kx)$, t and x stand for time and distance respectively. Obtain the dimensional formula for ω and k.

Solution. An angle is a dimensionless quantity,

∴ $[\omega t] = 1$ or $[\omega] = \frac{1}{[t]} = \frac{1}{T} = T^{-1}$.

$[kx] = 1$ or $[k] = \frac{1}{[x]} = \frac{1}{L} = L^{-1}$.

EXAMPLE 51. Rule out or accept the following formulae for kinetic energy on the basis of dimensional arguments :

(i) $\frac{3}{16}mv^2$ (ii) $\frac{1}{2}mv^2 + ma$ **[Central Schools 08]**

Solution. K.E. $= \frac{1}{2}mv^2$

Dimensions of K.E. $= [ML^2T^{-2}]$

(i) Dimensions of $\frac{3}{16}mv^2 = [ML^2T^{-2}]$

This formula is dimensionally acceptable for K.E.

(ii) $\left[\frac{1}{2}mv^2\right] + [ma] = [ML^2T^{-2}] + [MLT^{-2}]$

This formula cannot be accepted for K.E. as it is dimensionally inhomogeneous.

PROBLEMS FOR PRACTICE

1. Test the dimensional consistency of the following equations : (i) $v = u + at$ (ii) $s = ut + \frac{1}{2}at^2$ (iii) $v^2 - u^2 = 2as$ **[Himachal 07C]** (**Ans.** All relations are dimensionally correct)
2. The viscous force 'F' acting on a small sphere of radius 'r' moving with velocity v through a liquid is given by $F = 6\pi\eta rv$. Calculate the dimensions of η, the coefficient of viscosity. (**Ans.** $ML^{-1}T^{-1}$)
3. The distance covered by a particle in time t is given by $x = a + bt + ct^2 + dt^3$; find the dimensions of a, b, c and d. (**Ans.** L, LT^{-1}, LT^{-2}, LT^{-3})
4. The critical velocity of the flow of a liquid through a pipe of radius r is given by $v_c = \frac{K\eta}{r\rho}$ where ρ is the density and η is the coefficient of viscosity of the liquid. Check if this relation is dimensionally correct. (**Ans.** Correct)
5. The rate of flow (V) of a liquid flowing through a pipe of radius r and a pressure gradient (P/l) is given by Poiseuille's equation : $V = \frac{\pi}{8}\frac{Pr^4}{\eta l}$ Check the dimensional consistency of this equation. (**Ans.** Correct)
6. Test if the following equation is dimensionally correct : $$h = \frac{2S\cos\theta}{r\rho g}$$ where h = height, S = surface tension, ρ = density, r = radius, and g = acceleration due to gravity. (**Ans.** Correct)
7. Find the dimensions of the quantity v in the equation, $v = \frac{\pi p(a^2 - x^2)}{2\eta l}$, where a is the radius and l is the length of the tube in which the fluid of coefficient of viscosity η is flowing, x is the distance from the axis of the tube and p is the pressure difference. (**Ans.** LT^{-1})
8. Find the dimensions of the quantity q from the expression. $T = 2\pi\sqrt{\frac{ml^3}{3Yq}}$, where T is the time period of a bar of length l, mass m and Young's modulus Y. (**Ans.** L^4)
9. An artificial satellite of mass m is revolving in a circular orbit around a planet of M and radius R. If the radius of the orbit of the satellite be r. Justify by the method of dimensions that the time period of the satellite is given by : $T = \frac{2\pi}{R}\sqrt{\frac{r^3}{g}}$.

10. Find the dimensions of $(a \times b)$ in the equation : $E = \dfrac{b - x^2}{at}$; where E is energy, x is distance and t is time. (**Ans.** $M^{-1} L^2 T$)

11. Find the dimensions of (a/b) in the equation :

$$P = \frac{a - t^2}{bx}$$

where P is pressure, x is distance and t is time. (**Ans.** MT^{-2})

12. Time period of an oscillating drop of radius r, density ρ and surface tension S is : $t = K\sqrt{\dfrac{\rho r^3}{S}}$

Check the correctness of the relation. **[Himachal 04]** (**Ans.** correct)

13. Out of formulae (*i*) $y = a \sin 2\pi t / T$ and (*ii*) $y = a \sin vt$ for the displacement y of particle undergoing a certain periodic motion, rule out the wrong formula on dimensional grounds. (where a = maximum displacement of the particle, v = speed of the particle, T = time period of motion). **[Delhi 09]**

HINTS

1. (*i*) $[v] = LT^{-1}$, $[u] = LT^{-1}$,

$[at] = LT^{-2}.T = LT^{-1}$

(*ii*) $[s] = L$, $[ut] = LT^{-1}.T = L$,

$$\left[\frac{1}{2}at^2\right] = LT^{-2}.T^2 = L$$

(*iii*) $[v^2] = [LT^{-1}]^2 = L^2 T^{-2}$,

$[u^2] = [LT^{-1}]^2 = L^2 T^{-2}$

$[2\,aS] = LT^{-2}.L = L^2 T^{-2}$

4. $[v_c]$ = critical velocity $= LT^{-1}$

$$\left[\frac{K\eta}{r\rho}\right] = \frac{1.\,ML^{-1}\,T^{-1}}{L\,.\,ML^{-3}} = LT^{-1}$$

Hence the relation is correct.

5. $[V]$ = Rate of flow of liquid

$= \text{Volume / Time} = L^3T^{-1}$

$$\left[\frac{\pi}{8}\,\frac{Pr^4}{\eta\, l}\right] = \frac{ML^{-1}\,T^{-2}\,.\,L^4}{ML^{-1}\,T^{-1}\,.\,L} = L^3T^{-1}$$

Hence the relation is correct.

6. $[h] = L$

$$\left[\frac{2\,S\cos\theta}{r\rho g}\right] = \frac{MT^{-2}\,.\,1}{L\,.\,ML^{-3}\,.\,LT^{-2}} = L$$

Hence the relation is correct.

7. $$[v] = \frac{[ML^{-1}T^{-2}]\,.\,[L^2 - L^2]}{[ML^{-1}\,T^{-1}]\,.\,[L]} = \frac{ML^{-1}T^{-2}\,.\,L^2}{MT^{-1}} = \mathbf{LT^{-1}}.$$

8. $$[q] = \left[\frac{4\pi^2\, ml^3}{3YT^2}\right] = \frac{ML^3}{ML^{-1}T^{-2}\,.\,T^2} = \mathbf{L^4}.$$

9. $[T] = T$

$$\left[\frac{2\pi}{R}\sqrt{\frac{r^3}{g}}\right] = \frac{1}{L}\left[\frac{L^3}{LT^{-2}}\right]^{1/2} = T.$$

Hence the relation is correct.

11. $[a] = [t^2] = T^2 \quad \therefore\ [P] = \dfrac{T^2}{[bx]}$

or $\quad [b] = \dfrac{T^2}{[P]\,[x]} = \dfrac{T^2}{ML^{-1}T^{-2}\,.\,L} = M^{-1}T^4$

$$[a/b] = \frac{T^2}{M^{-1}T^4} = \mathbf{MT^{-2}}.$$

12. $$[\text{RHS}] = \left[K\sqrt{\frac{\rho r^3}{S}}\right] = \left[\frac{[ML^{-3}]\,[L^3]}{[MT^{-2}]}\right]^{1/2} = [T] = [\text{LHS}]$$

13. Refer to the answer of NCERT Exercise 2.14.

2.26 DEDUCING RELATION AMONG THE PHYSICAL QUANTITIES

48. *How can the method of dimensions be used to deduce a relation among the physical quantities ? Explain it with the help of a suitable example.*

To derive the relationship among physical quantities. By making use of the homogeneity of dimensions, we can derive an expression for a physical quantity if we know the various factors on which it depends.

Example. Let us derive an expression for the centripetal force F acting on a particle of mass m moving with velocity v in a circle of radius r.

Let $\quad F \propto m^a\, v^b\, r^c \quad$ or $\quad F = K\, m^a\, v^b\, r^c \quad$ ...(1)

where K is a dimensionless constant. Writing the dimensions of various quantities in equation (1), we get

$$[MLT^{-2}] = 1\,[M]^a\,[LT^{-1}]^b\,[L]^c$$

or $\quad M^1L^1T^{-2} = M^a\,L^{b+c}\,T^{-b}$.

Comparing the dimensions of similar quantities on both sides, we get

$$a = 1$$

$$b + c = 1 \quad \text{or} \quad c = 1 - b = 1 - 2 = -1$$

$$-2 = -b \quad \text{or} \quad b = 2$$

From equation (1), we get

$$F = K\, mv^2\, r^{-1} = K\,\frac{mv^2}{r}.$$

This is the required expression for the centripetal force.

Examples based on
Deriving Relationship between Physical Quantities

CONCEPT USED

By using the principle of homogeneity of dimensions, the form of expression for a given physical quantity can be obtained if we know the factors upon which that physical quantity depends.

EXAMPLE 52. *Consider a simple pendulum, having a bob attached to a string, that oscillates under the action of the force of gravity. Suppose that the period of oscillation of the simple pendulum depends on (i) mass m of the bob (ii) length l of the pendulum and (iii) acceleration due to gravity g at the place. Derive the expression for its time period using method of dimensions.* **[NCERT]**

Solution. Let us assume that $T \propto m^a\, l^b\, g^c$

or $\quad T = Km^a\, l^b\, g^c \qquad ...(i)$

where K is a dimensionless constant.

The dimensions of various quantities are

$$[T] = \mathrm{T},\ [m] = \mathrm{M},\ [l] = \mathrm{L},\ [g] = \mathrm{LT^{-2}}$$

Substituting these dimensions in eqn. (*i*), we get

$$\mathrm{T} = [\mathrm{M}]^a\, [\mathrm{L}]^b\, [\mathrm{LT^{-2}}]^c$$

or $\quad \mathrm{M^0L^0T^1} = \mathrm{M^a L^{b+c} T^{-2c}}$

Equating the exponents of M, L and T on both sides, we get

$$a = 0, \quad b + c = 0, \quad -2c = 1$$

On solving, $\quad a = 0, \quad b = \dfrac{1}{2}, \quad c = -\dfrac{1}{2}$

$\therefore \quad T = Km^0 l^{1/2} g^{-1/2} = K\sqrt{\dfrac{l}{g}}$

From experiments, $K = 2\pi$.

Therefore $\quad T = 2\pi\sqrt{\dfrac{l}{g}}$.

EXAMPLE 53. *The velocity 'v' of water waves depends on the wavelength 'λ', density of water 'ρ' and the acceleration due to gravity 'g'. Deduce by the method of dimensions the relationship between these quantities.* **[Central Schools 08]**

Solution. Let $\quad v = K\lambda^a \rho^b g^c \qquad ...(i)$

where $K = a$ dimensionless constant.

Dimensions of the various quantities are

$$[v] = \mathrm{LT^{-1}},\ [\lambda] = \mathrm{L},\ [\rho] = \mathrm{ML^{-3}},\ [g] = \mathrm{LT^{-2}}$$

Substituting these dimensions in equation (*i*), we get

$$[\mathrm{LT^{-1}}] = [\mathrm{L}]^a [\mathrm{ML^{-3}}]^b [\mathrm{LT^{-2}}]^c$$

$$\mathrm{M^0L^1T^{-1}} = \mathrm{M^b L^{a-3b+c} T^{-2c}}$$

Equating the powers of M, L and T on both sides,

$$b = 0, \quad a - 3b + c = 1, \quad -2c = -1$$

On solving, $\quad a = \dfrac{1}{2}, \quad b = 0, \quad c = \dfrac{1}{2}$

$\therefore \quad v = K\lambda^{\frac{1}{2}} \rho^0 g^{\frac{1}{2}} = K\sqrt{\lambda g}$.

EXAMPLE 54. *Assuming that the mass M of the largest stone that can be moved by a flowing river depends upon 'v' the velocity, 'ρ' the density of water and on 'g', the acceleration due to gravity. Show that M varies with the sixth power of the velocity of flow.*

Solution. Let $\quad M = Kv^a \rho^b g^c \qquad ...(i)$

where $K =$ a dimensionless constant.

Dimensions of the various quantities are :

$$[M] = \mathrm{M},\ [v] = \mathrm{LT^{-1}},\ [\rho] = \mathrm{ML^{-3}},\ [g] = \mathrm{LT^{-2}}$$

Substituting these dimensions in equation (*i*), we get

$$[\mathrm{M}] = [\mathrm{LT^{-1}}]^a [\mathrm{ML^{-3}}]^b [\mathrm{LT^{-2}}]^c$$

$$\mathrm{M^1L^0T^0} = \mathrm{M^b L^{a-3b+c} T^{-a-2c}}$$

Equating the powers of M, L and T, we get

$$b = 1, \quad a - 3b + c = 0, \quad -a - 2c = 0$$

On solving, $a = 6, \quad b = 1, \quad c = -3$

$\therefore \quad M = Kv^6 \rho^1 g^{-3}$

Hence $\quad M \propto v^6$.

EXAMPLE 55. *The velocity of sound waves 'v' through a medium may be assumed to depend on :*

(i) the density of the medium 'd' and

(ii) the modulus of elasticity 'E'.

Deduce by the method of dimensions the formula for the velocity of sound. Take dimensional constant K = 1.

Solution. Let the velocity of sound waves be given by

$$v = K\, d^a\, E^b \qquad ...(i)$$

where $K =$ a dimensionless constant.

Dimensions of the various quantities are

$$[v] = \mathrm{LT^{-1}},\ [d] = \mathrm{ML^{-3}}$$

$$[E] = \frac{\text{stress}}{\text{strain}} = \frac{\text{force}}{\text{area} \times \text{strain}} = \frac{\mathrm{MLT^{-2}}}{\mathrm{L^2}.1} = \mathrm{ML^{-1}T^{-2}}$$

Substituting these dimensions in equation (*i*), we get

$$[\mathrm{LT^{-1}}] = [\mathrm{ML^{-3}}]^a [\mathrm{ML^{-1}T^{-2}}]^b$$

or $\quad \mathrm{M^0L^1T^{-1}} = \mathrm{M^{a+b} L^{-3a-b} T^{-2b}}$

Equating the dimensions of M, L and T, we get

$$a + b = 0, \ -3a - b = 1, \ -2b = -1$$

On solving, $\quad a = -\dfrac{1}{2}, \ b = \dfrac{1}{2}$

$\therefore \quad v = Kd^{-1/2}E^{1/2} \quad$ or $\quad v = \sqrt{\dfrac{E}{d}}. \qquad [\because \ K = 1]$

Example 56. *The frequency 'ν' of vibration of a stretched string depends upon :*

(i) its length l,

(ii) its mass per unit length 'm' and

(iii) the tension T in the string.

Obtain dimensionally an expression for frequency ν. **[Delhi 2002]**

Solution. Let the frequency of vibration of the string be given by

$$\nu = K\, l^a\, m^b\, T^c \qquad ...(i)$$

where K = a dimensionless constant.

Dimensions of the various quantities are :

$$[\nu] = \mathrm{T}^{-1}, \quad [l] = \mathrm{L}, \qquad [T] = \text{Force} = \mathrm{MLT}^{-2}$$

$$[m] = \frac{\text{mass}}{\text{length}} = \frac{\mathrm{M}}{\mathrm{L}} = \mathrm{ML}^{-1},$$

Substituting these dimensions in equation (*i*), we get

$$[\mathrm{T}^{-1}] = [\mathrm{L}]^a\, [\mathrm{ML}^{-1}]^b\, [\mathrm{MLT}^{-2}]^c$$

or $$\mathrm{M}^0\mathrm{L}^0\mathrm{T}^{-1} = \mathrm{M}^{b+c}\,\mathrm{L}^{a-b+c}\,\mathrm{T}^{-2c}$$

Equating the dimensions of M, L and T, we get

$$b + c = 0, \quad a - b + c = 0, \quad -2c = -1$$

On solving, $a = -1, \quad b = -\dfrac{1}{2}, \quad c = \dfrac{1}{2}.$

$$\therefore \qquad \nu = Kl^{-1}m^{-1/2}T^{1/2}$$

or $$\nu = \frac{K}{l}\sqrt{\frac{T}{m}}.$$

Example 57. *A planet moves around the sun in nearly circular orbit. Its period of revolution 'T' depends upon :*

(i) radius 'r' of orbit *(ii) mass 'M' of the sun and*

(iii) the gravitational constant G.

Show dimensionally that $T^2 \propto r^3$.

Solution. Let $\qquad T = Kr^a\, M^b\, G^c \qquad ...(i)$

where K = a dimensionless constant.

Dimensions of the various quantities are :

$$[T] = \mathrm{T}, \ [r] = \mathrm{L}, \ [M] = \mathrm{M}$$

$$[G] = \frac{Fr^2}{m_1 m_2} = \frac{\mathrm{MLT}^{-2}.\mathrm{L}^2}{\mathrm{MM}} = \mathrm{M}^{-1}\mathrm{L}^3\mathrm{T}^{-2}.$$

Substituting these dimensions in equation (*i*), we get

$$[\mathrm{T}] = [\mathrm{L}]^a[\mathrm{M}]^b[\mathrm{M}^{-1}\mathrm{L}^3\mathrm{T}^{-2}]^c$$

$$\mathrm{M}^0\mathrm{L}^0\mathrm{T}^1 = \mathrm{M}^{b-c}\,\mathrm{L}^{a+3c}\,\mathrm{T}^{-2c}$$

Equating the dimensions of M, L and T, we get

$$b - c = 0, \quad a + 3c = 0, \quad -2c = 1$$

On solving,

$$a = \frac{3}{2}, \quad b = -\frac{1}{2}, \quad c = -\frac{1}{2}.$$

$$\therefore \qquad T = Kr^{3/2}M^{-1/2}G^{-1/2}$$

or $$T^2 = \frac{K^2 r^3}{MG}$$

$$\therefore \qquad T^2 \propto r^3.$$

Example 58. *Reynold number N_R (a dimensionless quantity) determines the condition of laminar flow of a viscous liquid through a pipe. N_R is a function of the density of the liquid 'ρ', its average speed 'v' and coefficient of viscosity 'η'. Given that N_R is also directly proportional to 'D' (the diameter of the pipe), show from dimensional considerations that $N_R \propto \dfrac{\rho v D}{\eta}$.*

The unit of η in SI system is kg m^{-1} s^{-1}.

Solution. Let $\quad N_R = K\rho^a\, v^b\, \eta^c\, D \qquad ...(i)$

where K = a dimensionless constant.

Dimensions of various quantities are

$$[N_R] = 1 = \mathrm{M}^0\,\mathrm{L}^0\,\mathrm{T}^0$$

$$[\rho] = \mathrm{ML}^{-3}, \quad [v] = \mathrm{LT}^{-1},$$

$$[\eta] = \mathrm{ML}^{-1}\,\mathrm{T}^{-1}, \quad [D] = \mathrm{L}$$

Substituting these dimensions in equation (*i*), we get

$$[\mathrm{M}^0\,\mathrm{L}^0\,\mathrm{T}^0] = [\mathrm{ML}^{-3}]^a\,[\mathrm{LT}^{-1}]^b\,[\mathrm{ML}^{-1}\,\mathrm{T}^{-1}]^c[\mathrm{L}]$$

$$\mathrm{M}^0\mathrm{L}^0\mathrm{T}^0 = \mathrm{M}^{a+c}\,\mathrm{L}^{-3a+b-c+1}\,\mathrm{T}^{-b-c}$$

Equating the powers of M, L and T, we get

$$a + c = 0, \quad -3a + b - c + 1 = 0, \quad -b - c = 0$$

On solving, $a = 1, \quad b = 1, \quad c = -1.$

$$\therefore \qquad N_R = K\rho^1\, v^1\, \eta^{-1}\, D = K\frac{\rho v D}{\eta}$$

or $$N_R \propto \frac{\rho v D}{\eta}.$$

Example 59. *Derive by the method of dimensions, an expression for the volume of a liquid flowing out per second through a narrow pipe. Assume that the rate of flow of liquid depends on*

(i) the coefficient of viscosity 'η' of the liquid

(ii) the radius 'r' of the pipe and

(iii) the pressure gradient $\left(\dfrac{p}{l}\right)$ along the pipe.

Take $K = \dfrac{\pi}{8}$.

Solution. Let volume flowing out per second through the pipe be given by

$$V = K\eta^a\, r^b \left(\frac{p}{l}\right)^c \qquad ...(i)$$

where K = *a* dimensionless constant.

Dimensions of the various quantities are

$$[V]=\frac{\text{volume}}{\text{time}}=\frac{\mathrm{L}^3}{\mathrm{T}}=\mathrm{L^3T^{-1}}$$

$$[\eta]=\mathrm{ML^{-1}\,T^{-1}},\ [r]=\mathrm{L}$$

$$\left[\frac{p}{l}\right]=\frac{\text{pressure}}{\text{length}}=\frac{\text{force}}{\text{area}\times\text{length}}$$

$$=\frac{\mathrm{MLT^{-2}}}{\mathrm{L^2\,.\,L}}=\mathrm{ML^{-2}T^{-2}}$$

Substituting these dimensions in equation (*i*), we get

$$[\mathrm{L^3T^{-1}}]=[\mathrm{ML^{-1}T^{-1}}]^a\,[\mathrm{L}]^b\,[\mathrm{ML^{-2}T^{-2}}]^c$$

or $$\mathrm{M^0L^3T^{-1}}=\mathrm{M}^{a+c}\,\mathrm{L}^{-a+b-2c}\,\mathrm{T}^{-a-2c}$$

Equating the powers of M, L and T, we get

$$a+c=0,\ -a+b-2c=3,\ -a-2c=-1$$

On solving,

$$a=-1,\quad b=4,\quad c=1$$

$$\therefore\qquad V=K\eta^{-1}\,r^4\left(\frac{p}{l}\right)^1$$

or $$V=\frac{\pi\,r^4\,p}{8\eta\,l}\qquad\text{[Poiseuille's equation]}$$

Example 60. *The period of vibration of a tuning fork depends on the length l of its prong, density d and Young's modulus Y of its material. Deduce an expression for the period of vibration on the basis of dimensions.*

Solution. Let $$T=K\,l^a\,d^b\,Y^c\qquad\text{...}(i)$$

where K = a dimensionless constant.

Dimensions of various quantities are

$$[T]=\mathrm{T},\ [l]=\mathrm{L},\ [d]=\mathrm{ML^{-3}},\ [Y]=\mathrm{ML^{-1}T^{-2}}$$

Substituting these dimensions in equation (*i*), we get

$$\mathrm{T}=[\mathrm{L}]^a\,[\mathrm{ML^{-3}}]^b\,[\mathrm{ML^{-1}\,T^{-2}}]^c$$

or $$\mathrm{M^0L^0T^1}=\mathrm{M}^{b+c}\,\mathrm{L}^{a-3b-c}\,\mathrm{T}^{-2c}$$

Equating the powers of M, L and T, we get

$$b+c=0,\ a-3b-c=0,\ -2c=1$$

On solving,

$$a=1,\quad b=\frac{1}{2},\quad c=-\frac{1}{2}$$

$$\therefore\qquad T=K\,l^1d^{1/2}Y^{-1/2}=Kl\sqrt{\frac{d}{Y}}.$$

Example 61. *The frequency ν of an oscillating drop may depend upon radius r of the drop, density ρ of the liquid and surface tension S of the liquid. Establish an expression for ν dimensionally.*

Solution. Let $$\nu=K\,r^a\,\rho^b\,S^c\qquad\text{...}(i)$$

where K = a dimensionless constant.

Dimensions of various quantities are

$$[\nu]=\mathrm{T^{-1}},\ [r]=\mathrm{L},\ [\rho]=\mathrm{ML^{-3}},\ [S]=\mathrm{MT^{-2}}$$

Substituting these dimensions in equation (*i*), we get

$$\mathrm{T^{-1}}=[\mathrm{L}]^a\,[\mathrm{ML^{-3}}]^b\,[\mathrm{MT^{-2}}]^c$$

or $$\mathrm{M^0L^0T^{-1}}=\mathrm{M}^{b+c}\,\mathrm{L}^{a-3b}\,\mathrm{T}^{-2c}$$

Equating the powers of M, L and T on both sides,

$$b+c=0,\ a-3b=0,\ -2c=-1$$

On solving,

$$a=-\frac{3}{2},\quad b=-\frac{1}{2},\quad c=\frac{1}{2}$$

$$\therefore\qquad \nu=Kr^{-3/2}\,\rho^{-1/2}\,S^{1/2}=K\sqrt{\frac{S}{\rho r^3}}.$$

Example 62. *The escape velocity v of a body depends upon (i) the acceleration due to gravity of the planet and (ii) the radius of the planet R. Establish dimensionally the relationship between v, g and R.* **[Himachal 06 ; Delhi 05]**

Solution. Let $v=Kg^a\,R^b$

where K = *a* dimensionless constant.

Putting the dimensions,

$$\mathrm{LT^{-1}}=[\mathrm{LT^{-2}}]^a[\mathrm{L}]^b=\mathrm{L}^{a+b}\mathrm{T}^{-2a}$$

Equating the powers of L and T,

$$a+b=1,\ -2a=-1$$

$$\therefore\qquad a=\frac{1}{2},\ b=\frac{1}{2}$$

Hence $$v=K\,g^{1/2}\,R^{1/2}=K\sqrt{gR}.$$

Example 63. *A large fluid star oscillates in shape under the influence of its own gravitational field. Using dimensional analysis, find the expression for period of oscillation (T) in terms of radius of star (R), mean density of fluid (ρ) and universal gravitational constant G.* **[Chandigarh 04]**

Solution. Let $T=KR^a\rho^bG^c$

where K is a dimensionless constant.

Putting the dimensions,

$$\mathrm{T^1}=[\mathrm{L}]^a[\mathrm{ML^{-3}}]^b[\mathrm{M^{-1}L^3T^{-2}}]^c$$

or $$\mathrm{M^0L^0T^1}=\mathrm{M}^{b-c}\mathrm{L}^{-3b+3c+a}\mathrm{T}^{-2c}$$

Equating the powers of M, L and T, we get

$$b-c=0,\ -3b+3c+a=0,\ -2c=1$$

On solving,

$$a=0,\quad b=-\frac{1}{2},\quad c=-\frac{1}{2}$$

$$\therefore\qquad T=KR^0\rho^{-1/2}G^{-1/2}=K\frac{1}{\sqrt{\rho G}}.$$

PROBLEMS FOR PRACTICE

1. The wavelength λ associated with a moving electron depends on its mass m, its velocity v and Planck's constant h. Prove dimensionally that $\lambda \propto \dfrac{h}{mv}$.
[Himachal 04 ; Chandigarh 08]

2. Obtain an expression for the centripetal force F acting on a particle of mass m moving with velocity v in a circle of radius r. Take dimensionless constant $K = 1$. **[Himachal 2000]**
$\left(\textbf{Ans. } F = \dfrac{mv^2}{r}\right)$

3. The orbital velocity v of a satellite may depend on its mass m, the distance r from the centre of the earth and acceleration due to gravity g. Obtain an expression for its orbital velocity. (**Ans.** $v = K\sqrt{rg}$)

4. A small spherical ball of radius r falls with velocity v through a liquid having coefficient of viscosity η. Find the viscous drag F on the ball assuming it depends on η, r and v. Take $K = 6\pi$.
(**Ans.** $F = 6\pi\eta rv$)

5. The velocity of a freely falling body is a function of the distance fallen through (h) and acceleration due to gravity g. Show by the method of dimensions that $v = K\sqrt{gh}$.

6. Using the method of dimensions, derive an expression for the energy of a body executing SHM ; assuming this energy depends upon its mass m, frequency ν and amplitude of vibration r. **[Himachal 06]**
(**Ans.** $E = Km\nu^2 r^2$)

7. A body of mass m hung at one end of the spring executes SHM. Prove that the relation $T = 2\pi m / k$ is incorrect, where k is the force constant of the spring. Also derive the correct relation. (**Ans.** $T = K\sqrt{m/k}$)

8. Assuming that the critical velocity v_c of a viscous liquid flowing through a capillary tube depends only upon the radius r of the tube, density ρ and the coefficient of viscosity η of the liquid, find the expression for critical velocity. $\left(\textbf{Ans. } v_c = \dfrac{K\eta}{r\rho}\right)$

9. By the method of dimensions, obtain an expression for the surface tension S of a liquid rising in a capillary tube. Assume that the surface tension depends upon (*i*) mass m of the liquid (*ii*) pressure p of the liquid and (*iii*) radius r of the capillary tube. Take $K = 1/2$. (**Ans.** $S = pr/2$)

10. The depth x to which a bullet penetrates a human body depends upon (*i*) coefficient of elasticity η and (*ii*) kinetic energy E_k. By the method of dimensions, show that : $x \propto \left[\dfrac{E_k}{\eta}\right]^{1/3}$.

11. A U-tube of uniform cross-section contains mercury upto a height h in either limb. The mercury in one limb is depressed a little and then released. Obtain an expression for the time period of oscillation assuming that T depends on h, ρ and g.
$\left(\textbf{Ans. } T = K\sqrt{\dfrac{h}{g}}\right)$

12. The critical angular velocity ω_c of a cylinder inside another cylinder containing a liquid at which its turbulence occurs depends on viscosity η, density ρ and the distance d between the walls of the cylinder. Find an expression for ω_c.
$\left(\textbf{Ans. } \omega_c = \dfrac{K\eta}{\rho d^2}\right)$

13. A body of mass m is moving in a circle of radius r with angular velocity ω. Find expression for centripetal force acting on it by the method of dimensions. **[Himachal 03, 09C]** (**Ans.** $F = Kmr\omega^2$)

14. Consider a simple pendulum. The period of oscillation of the simple pendulum depends on its length 'l' and acceleration due to gravity 'g'. Derive the expression for its period of oscillation by the method of dimensions. **[Himachal 06, 06C, 07]**

HINTS

1. Let $\lambda = Km^a v^b h^c$, then

$$M^0L^1T^0 = [M]^a[LT^{-1}]^b[ML^2T^{-1}]^c = M^{a+c}L^{b+2c}T^{-b-c}$$

$\therefore \quad a + c = 0, \quad b + 2c = 1, \quad -b - c = 0$

On solving, $a = -1, \quad b = -1, \quad c = 1$

$\therefore \quad \lambda = k\dfrac{h}{mv}$ or $\lambda \propto \dfrac{h}{mv}$.

4. Let $F = K\eta^a r^b v^c$, then

$$M^1L^1T^{-2} = [ML^{-1}T^{-1}]^a[L]^b[LT^{-1}]^c = M^a L^{-a+b+c}T^{-a-c}$$

$\therefore \quad a = 1, \quad -a + b + c = 1, \quad -a - c = -2$

On solving, $a = b = c = 1$

Hence $F = K\eta rv = 6\pi\eta rv$. (Stoke's law)

6. Let $E = Km^a \nu^b r^c$, then

$$M^1L^2T^{-2} = [M]^a[T^{-1}]^b[L]^c = M^a L^c T^{-b}$$

$\therefore \quad a = 1, \; b = 2, \; c = 2$ Hence $E = Km\nu^2 r^2$.

7. $[\text{LHS}] = [T] = T$

$$[\text{RHS}] = \left[\frac{2\pi m}{k}\right] = \frac{M}{MT^{-2}} = T^2$$

$$\left[\because [k] = \frac{\text{Force}}{\text{Length}} = \frac{MLT^{-2}}{L} = MT^{-2}\right]$$

$[\text{LHS}] \neq [\text{RHS}]$. Hence the relation is incorrect.

To find the correct relation, let $T = K m^a k^b$, then

$$T^1 = [M]^a [MT^{-2}]^b = M^{a+b} T^{-2b}$$

$\therefore \quad a + b = 0, \; -2b = 1$

On solving, $a = \frac{1}{2}, \; b = -\frac{1}{2}$

Hence $T = Km^{1/2} k^{-1/2} = K\sqrt{\frac{m}{k}}$.

8. Let $v_c = Kr^a \rho^b \eta^c$, then

$$LT^{-1} = [L]^a [ML^{-3}]^b [ML^{-1}T^{-1}]^c = M^{b+c} L^{a-3b-c} T^{-c}$$

$\therefore \quad b + c = 0, \; a - 3b - c = 1, \; -c = -1$

On solving,

$$a = -1, \; b = -1, \; c = 1$$

$\therefore \quad v_c = Kr^{-1} \rho^{-1} \eta^1 = \frac{K\eta}{r\rho}$.

10. Let $x = K\eta^a [E_k]^b$, then

$$L^1 = [ML^{-1}T^{-2}]^a [ML^2T^{-2}]^b = M^{a+b} L^{-a+2b} T^{-2a-2b}$$

$\therefore \quad a + b = 0, \; -a + 2b = 1, \; -2a - 2b = 0$

On solving, $a = -\frac{1}{3}, \; b = \frac{1}{3}$

Hence $x = K\eta^{-1/3} E_k^{1/3} = K\left[\frac{E_k}{\eta}\right]^{1/3}$.

12. Let $\omega_c = K\eta^a \rho^b d^c$, then

$$T^{-1} = [ML^{-1}T^{-1}]^a [ML^{-3}]^b [L]^c = M^{a+b} L^{-a-3b+c} T^{-a}$$

$\therefore \quad a + b = 0, \; -a - 3b + c = 0, \; -a = -1$

On solving,

$$a = 1, \; b = -1, \; c = -2$$

Hence $\omega_c = K\eta^1 \rho^{-1} d^{-2} = \frac{K\eta}{\rho d^2}$.

2.27 LIMITATIONS OF DIMENSIONAL ANALYSIS

49. *Mention the limitations of the method of dimensional analysis.*

Limitations of the method of dimensions :

1. The method does not give any information about the dimensionless constant K.
2. It fails when a physical quantity depends on more than three physical quantities.
3. It fails when a physical quantity (*e.g.*, $s = ut + \frac{1}{2}at^2$) is the sum or difference of two or more quantities.
4. It fails to derive relationships which involve trigonometric, logarithmic or exponential functions.
5. Sometimes, it is difficult to identify the factors on which the physical quantity depends. The method becomes more complicated when dimensional constants like G, h, etc. are involved.

2.28 SIGNIFICANT FIGURES

50. *What is meant by significant figures in a measured quantity ?*

Significant figures. *The significant figures are normally those digits in a measured quantity which are known reliably or about which we have confidence in our measurement plus one additional digit that is uncertain.* The larger the number of significant figures in a measurement, the higher is the accuracy of the measurement. Suppose the time period of a simple pendulum is 1.62s. This digits 1 and 6 are reliable and certain, while the digit 2 is uncertain. So the time period has three significant figures. Again, suppose the length of an object is measured as 273. 6 cm. It has four significant figures. The digits 2, 7 and 3 are reliable while the digit 6 is uncertain.

51. *State the rules for counting the number of significant figures in a measured quantity.*

Rules for determining the number of significant figures :

(*i*) *All non-zero digits are significant.* So 13.75 has four significant figures.

(*ii*) *All zeros between two non-zero digits are significant.* Thus 100.05 km has five significant figures.

(*iii*) *All zeros to the right of a non-zero digit but to the left of an understood decimal point are not significant.* For example, 86400 has three significant figures. But *such zeros are significant if they come from a measurement.* For example, 86400 s has five significant figures.

(*iv*) *All zeros to the right of a non-zero digit but to the left of a decimal point are significant.* For example, 648700. has six significant figure.

(*v*) *All zeros to the right of a decimal point are significant.* So 161 cm, 161.0 cm and 161.00 cm have three, four and five significant figures respectively.

(*vi*) *All zeros to the right of a decimal point but to the left of a non-zero digit are not significant.* So 0.161 cm and 0.0161 cm, both have three significant figures. Moreover, zero conventionally placed to the left of the decimal point is not significant.

(*vii*) *The number of significant figures does not depend on the system of units.* So 16.4 cm, 0.164 m and 0.000164 km, all have three significant figures.

For Your Knowledge

- In **scientific notation**, a number is expressed in the power of 10 as $a \times 10^b$, where a is the number between 1 and 10, and b is any positive or negative exponent of 10. The decimal point is written after the first digit. Suppose the length of a rod is reported as 3.500 m. In scientific notation, it can be expressed in different units as

$$3.500 \text{ m} = 3.500 \times 10^2 \text{ cm} = 3.500 \times 10^3 \text{ mm} = 3.500 \times 10^{-3} \text{ km}.$$

- We retain only those zeros in the base number which are the result of a measurement. Now *the power of 10 is not relevant to the determination of significant figures.* Each of the above numbers has four significant figures.

52. *State the rules for rounding off a measurement.*

Rules for rounding off a measurement :

(*i*) If the digit to be dropped is smaller than 5, then the preceding digit is left unchanged.

(*ii*) If the digit to be dropped is greater than 5, then the preceding digit is increased by 1.

(*iii*) If the digit to be dropped is 5 followed by non-zero digits, then the preceding digit is increased by 1.

(*iv*) If the digit to be dropped is 5, then the preceding digit is left unchanged if it is even.

(*v*) If the digit to be dropped is 5, then the preceding digit is increased by 1 if it is odd.

53. *State the rules for determining the significant figures in the sum, difference, product and quotient of two numbers.*

Arithmetic operations with significant figures. The result of an arithmetic operation involving measured values of quantities cannot be more accurate than the measured values themselves. So certain rules have to be followed while doing arithmetic operations with significant figures so as to ensure that the precision of the final result is consistent with the precision of the original measured values.

1. ***Significant figures in the sum or difference of two numbers.*** *In addition or subtraction, the final result should be reported to the same number of decimal places as that of the original number with minimum number of decimal places.*

2. ***Significant figures in the product or quotient of two numbers.*** *In multiplication or division, the final result should be reported to the same number of significant figures as that of the original number with minimum number of significant figures.*

Examples based on Significant Figures

FORMULAE USED

1. **Rules for rounding off a measurement.** Refer to Q. 52.
2. **Rules for counting significant figures.** Refer to Q. 51.
3. **Significant figures in the sum or difference of two numbers.** The final result should be reported to the same number of decimal places as that of the number with minimum number of decimal places.
4. **Significant figures in the product or quotient of two numbers.** The final result should be reported to the same number of significant figures as that of the number with minimum number of significant figures.

EXAMPLE 64. *State the number of significant figures in the following : (i) 2.000 m (ii) 5100 kg (iii) 0.050 cm.*

Solution. (*i*) Four : 2, 0, 0, 0 (*ii*) Four : 5, 1, 0, 0 (*iii*) Two : 5, 0.

EXAMPLE 65. *Round off the following numbers as indicated :*

(*i*) 18.35 *upto 3 digits* (*ii*) 143.45 *upto 4 digits*
(*iii*) 18967 *upto 3 digits* (*iv*) 12.653 *upto 3 digits*
(*v*) 248337 *upto 3 digits* (*vi*) 321.135 *upto 5 digits*
(*vii*) 101.55×10^6 *upto 4 digits*
(*viii*) 31.325×10^{-5} *upto 4 digits.*

Solution. (*i*) 18.4 (*ii*) 143.4 (*iii*) 19000 (*iv*) 12.7 (*v*) 248000 (*vi*) 321.14 (*vii*) 101.6×10^6 (*viii*) 31.32×10^{-5}.

EXAMPLE 66. *Add 7.21, 12.141 and 0.0028, and express the result to an appropriate number of significant figures.*

Solution.
$$\begin{array}{r} 7.21 \\ +\,12.141 \\ +\,0.0028 \\ \hline \end{array}$$

Sum $= 19.3538$

Corrected sum $= 19.35$ [Rounded off upto 2nd decimal place]

Here 7.21 has minimum number of decimal places (two), so result is rounded off upto second place of decimal point.

EXAMPLE 67. *Subtract 4.27153 from 6.807 and express the result to an appropriate number of significant figures.*

Solution.
$$\begin{array}{r} 6.807 \\ -\,4.27153 \\ \hline \end{array}$$

Difference $= 2.53547$

Corrected difference $= 2.535$ [Rounded off upto 3rd decimal place]

Here 6.807 has the lesser number of decimal places (three), so difference is rounded off to 3rd place of decimal point.

EXAMPLE 68. *Subtract 2.5×10^{-6} from 4.0×10^{-4} with due regard to significant figures.*

Solution. Le $x = 2.5 \times 10^{-6} = 0.0000025$

(2 significant figures)

$y = 4.0 \times 10^{-4} = 0.00040$

(2 significant figures)

$\therefore \quad y - x = 0.00040 - 0.0000025 = 0.0003975$

$= 3.975 \times 10^{-4} = \mathbf{4.0 \times 10^{-4}}$

[Rounded off upto 2 significant figures]

EXAMPLE 69. *Solve the following and express the result to an appropriate number of significant figures :*

(*i*) *Add 6.2 g, 4.33 g and 17.456 g.*

(*ii*) *Subtract 63.54 kg from 187.2 kg*

(*iii*) $75.5 \times 125.2 \times 0.51$.

(*iv*) $\dfrac{2.13 \times 24.78}{458.2}$ (*v*) $\dfrac{2.51 \times 10^{-4} \times 1.81 \times 10^{7}}{0.4463}$

Solution.

(*i*) 6.2 g + 4.33 g + 17.456 g = 27.986 = **28.0 g.**

[Rounded off to first decimal place]

(*ii*) 187.2 kg − 63.54 kg = 123.66 kg = **123.7 kg.**

[Rounded off to first decimal place]

(*iii*) $75.5 \times 125.2 \times 0.51 = 4820.826 = \mathbf{4800}$.

[Rounded off upto two significant figures]

(*iv*) $\dfrac{2.13 \times 24.78}{458.2} = 0.115193 = \mathbf{0.115}$.

[Rounded off to 3 significant figures]

(*v*) $\dfrac{2.51 \times 10^{-4} \times 1.81 \times 10^{7}}{0.4463} = 10.1795 \times 10^{3}$

$= \mathbf{10.2 \times 10^{3}}$.

[Rounded off to 3 significant figures]

EXAMPLE 70. *Each side of a cube is measured to be 7.203 m. What are the total surface area and the volume of the cube to appropriate significant figures ?* **[NCERT]**

Solution. Side of the cube = 7.203 m

Total surface area

$= 6 \times \text{side}^2 = 6 \times (7.203)^2$ m

$= 311.299254 \text{ m}^2 = 311.3 \text{ m}^2$

[Rounded off to 4 significant figures]

Volume = $\text{side}^3 = (7.203)^3 = 373.714754 \text{ m}^3$

$= \mathbf{373.7 \text{ m}^3}$.

[Rounded off to 4 significant figures]

EXAMPLE 71. *The radius of a sphere is 1.41 cm. Express its volume to an appropriate number of significant figures.*

Solution. Radius of the sphere, $r = 1.41$ cm

(3 significant figures)

Volume of the sphere $= \dfrac{4}{3}\pi r^3$

$= \dfrac{4}{3} \times 3.14 \times (1.41)^3 \text{ cm}^3 = 11.736 \text{ cm}^3$

$= \mathbf{11.7 \text{ cm}^3}$.

[Rounded off upto 3 significant figures]

EXAMPLE 72. *The length and the radius of a cylinder measured with slide callipers are found to be 4.54 cm and 1.75 cm respectively. Calculate the volume of the cylinder.*

Solution. Length of cylinder, $h = 4.54$ cm

(3 significant figures)

Radius of cylinder, $r = 1.75$ cm

(3 significant figures)

Volume of cylinder

$= \pi r^2 h = 3.14 \times (1.75)^2 \times 4.54 \text{ cm}^3$

$= 43.657775 \text{ cm}^3 = \mathbf{43.7 \text{ cm}^3}$.

[Rounded off upto 3 significant figures]

EXAMPLE 73. *The mass and radius of the earth are 5.975×10^{24} kg and 6.37×10^{6} m respectively. Calculate the average earth's density to correct significant figures. Take $\pi = 3.142$.*

Solution. Radius of earth, $R = 6.37 \times 10^{6}$ m

(3 significant figures)

Mass of earth, $M = 5.975 \times 10^{24}$ kg

(4 significant figures)

Average density

$$= \frac{M}{V} = \frac{M}{\frac{4}{3}\pi R^3} = \frac{5.975 \times 10^{24}}{\frac{4}{3} \times 3.142 \times (6.37 \times 10^{6})^3}$$

$= 0.005517 \times 10^{6} \text{ kgm}^{-3}$

$= \mathbf{5.52 \times 10^{3} \text{ kg m}^{-3}}$.

[Rounded off upto 3 significant figures]

EXAMPLE 74. *5.74 g of a substance occupies 1.2 cm³. Express its density keeping significant figures in view.* **[NCERT]**

Solution. Density

$$= \frac{\text{Mass}}{\text{Volume}} = \frac{5.74 \text{ g}}{1.2 \text{ cm}^3} = 4.783 \text{ gcm}^{-3}$$

$= \mathbf{4.8 \text{ g cm}^{-3}}$.

[Rounded of upto 2 significant figures]

PROBLEMS FOR PRACTICE

1. State the number of significant figures in the following measurements :
(i) 0.009 m^2. (ii) 5.049 Nm^{-2} (iii) 0.1890 g cm^{-3}
(iv) 1.90×10^{11} kg (v) 0.020800 m (vi) 5.308 J.
[Ans. (i) 1 (ii) 4 (iii) 4 (iv) 3 (v) 5 (vi) 4]
2. Subtract 2.5×10^4 from 3.9×10^5 with due regard to significant figures. **(Ans.** 3.7×10^5)
3. Round off the following numbers as indicated :
(i) 15.654 upto 3 digits (ii) 15.75 upto 3 digits
(iii) 15.654 upto 4 digits (iv) 15.65 upto 3 digits
(v) 142667 upto 5 digits (vi) 5.996×10^5 upto 3 digits.
(vii) 0.7995 upto 1 digit
(viii) 2.5946×10^{-4} upto 2 digits.
[Ans. (i) 15.7 (ii) 15.8 (iii) 15.65 (iv) 15.6 (v) 142670 (vi) 6.00×10^5 (vii) 0.8 (viii) 2.6×10^{-4}]
4. A jeweller puts a diamond in a box weighing 1.2 kg. Find the total weight of the box and diamond with due regard to significant figures, if the weight of diamond is 5.42 g. **(Ans.** 1.2 kg)
5. The diameter of a circle is 1.06 m. Calculate the area to an appropriate number of significant figures. Take $\pi = 3.14$. **(Ans.** 0.882 m^2)
6. The radius of a solid sphere is measured as 11.24 cm. What is the surface area of the sphere to appropriate significant figures ? **[Delhi 02]**
(Ans. 1588 cm^2)
7. The mass of a body is 275.32 g and its volume is 36.41 cm^3. Express its density upto appropriate significant figures. **(Ans.** 7.562 gcm^{-3})
8. 9.74 g of a substance occupies 1.2 cm^3. Express its density by keeping the significant figures in view. **[Central Schools 09]**
(Ans. 8.1 gcm^{-3})

2.29 ACCURACY AND PRECISION

54. *Distinguish between the terms precision and accuracy of a measurement.*

Every measurement is limited by the *reliability of the measuring instrument and skill of the person making the measurement.* If we repeat a particular measurement, we usually do not get precisely the same result as each result is subject to some experimental error. This imperfection in measurement can be described in two ways :

1. *Accuracy.* *It refers to the closeness of a measurement to the true value of the physical quantity.* It indicates the relative freedom from errors. As we reduce the errors, the measurement becomes more accurate.

2. *Precision.* *It refers to the resolution or the limit to which the quantity is measured.* Precision is determined by the least count of the measuring instrument. *The smaller the least count, greater is the precision.* If we repeat a particular measurement of a quantity a number of times, then the precision refers to the closeness of the set of values so obtained.

We can illustrate the difference between accuracy and precision with the help of an example. Suppose three students are asked to find the mass of a piece of metal whose mass is known to be 0.520 g. They obtain the data given in Table 2.9.

Table 2.9 *Data to illustrate accuracy and precision*

Student	Measurement 1	Measurement 2	Measurement 3	Average mass
A	0.52 g	0.51 g	0.50 g	0.51 g
B	0.516 g	0.515 g	0.514 g	0.515 g
C	0.521 g	0.520 g	0.520 g	0.520 g

The data obtained by the student *A* are neither very precise nor accurate, the individual values differ widely and also the average value is not accurate. The data for student *B* are more precise, as they vary slightly from one another but the average mass is not accurate. The data for student *C* are both precise and accurate. The resolution for *A* is 0.01 g and that for *B* or *C* is 0.001 g.

2.30 ERRORS IN A MEASUREMENT

55. *What do you mean by error in a measurement ? Briefly explain the different types of errors and their causes. How can these errors be minimised ?*

Error in a measurement. Every measurement is done with the help of some instrument. While making the measurement, some uncertainty gets introduced in the measurement. As a result, the measured value is always somewhat different from the actual or true value. *The error in a measurement is equal to the difference between the true value and the measured value of the quantity.*

Error = True value − Measured value

An error gives an indication of the limits within which the true value may lie. Every measurement has an error. Every calculated value which is based on measured values has an error.

Different types of errors :

1. Constant errors. *The errors which affect each observation by the same amount are called constant errors.* Such errors are due to the faulty calibration of the scale of the measuring instrument. Such errors can be eliminated by measuring the same physical quantity by a number of different methods, apparatus or technique.

2. Systematic errors. *The errors which tend to occur in one direction, either positive or negative, are called systematic errors.* We can eliminate such errors once we know the rule which governs them. These errors may be of the following types :

(*i*) ***Instrumental errors.*** *These errors occur due to the inbuilt defect of the measuring instrument.* For example, wearing off the metre scale at one end, zero error in a vernier callipers (zero of the vernier scale may not coincide with the zero of main scale), etc. This error can be detected by measuring a physical quantity with two different instruments of the same type or by measuring the same physical quantity by two different methods.

(*ii*) ***Imperfections in experimental technique.*** *These errors are due to the limitations of the experimental arrangement.* For example, error due to radiation loss in calorimetric experiments, error due to buoyancy of air when we weigh a body in air. Such errors cannot be eliminated altogether but necessary corrections can be applied for them.

(*iii*) ***Personal errors.*** *These errors arise due to individual's bias, lack of proper setting of apparatus or individual's carelessness in taking observations without observing proper precautions,* etc. For example, when an observer (by habit) holds his head towards right, while reading a scale, he introduces some error due to *parallax*. Such errors can be minimised if measurements are repeated by different persons or removing the personal bias as far as possible.

(*iv*) ***Errors due to external causes.*** *These errors arise due to the change in external conditions* like pressure, temperature, wind, etc. For example, the expansion of a scale due to the increase in temperature. Such errors can be easily detected and necessary corrections may be made accordingly. These errors can also be minimised by controlling the external conditions during the experimentation.

3. Random errors. *The errors which occur irregularly and at random, in magnitude and direction, are called random errors.* Such errors occur by chance and arise due to slight variation in the attentiveness of the observer while taking the readings or because of slight variations in the experimental conditions. For example, if a person repeats the observation a number a times, he may get different readings everytime. Random errors have almost equal chances for both positive and negative errors. Hence the *arithmetic mean* of a large number of observations can be taken as the true value of the measured quantity.

4. Least count error. *This error is due to the limitation imposed by the least count of the measuring instrument.* It is an uncertainty associated with the resolution of the measuring instrument. *The smallest division on the scale of the measuring instrument is called its least count.* For example, a metre scale has a least count of 1 mm, its readings are good only upto this value. The error in its reading will be half of this value *i.e.*, ± 0.5 mm or ± 0.05 cm.

5. Gross errors or mistakes. These errors are due to either carelessness of the person or due to improper adjustment of the apparatus. No corrections can be applied for gross errors.

For Your Knowledge

- Least count errors are random errors but within a limited size ; they occur with both random and systematic errors.
- The accuracy of measurement is related to the systematic errors but its precision is related to the random errors, which include least count error also.

2.31 ABSOLUTE ERROR, RELATIVE ERROR AND PERCENTAGE ERROR

56. ***How is random error eliminated ? What do you mean by (i) absolute error (ii) mean absolute error (iii) relative error and (iv) percentage error ?***

Elimination of error. *Normal* or *Gaussian law* of random errors shows that the probability of occurrence of positive and negative errors is same, so random error can be minimised by repeating measurements a large number of times. Then the *arithmetic mean of all measurements can be taken as the true value of the measured quantity.*

If $a_1, a_2, a_3, ... a_n$ be the n measured values of a physical quantity, then its true value, $\bar{a}$ is given by the arithmetic mean,

$$\bar{a} \text{ or } a_{mean} = \frac{a_1 + a_2 + a_3 + ... + a_n}{n} = \frac{1}{n} \sum_{i=1}^{n} a_i .$$

(*i*) ***Absolute error.*** *The magnitude of the difference between the true value of the quantity measured and the individual measured value is called absolute error.*

If we take arithmetic mean $\bar{a}$ as the true value, then the absolute errors in the individual measured values will be

$$\Delta a_1 = \bar{a} - a_1 \qquad \Delta a_2 = \bar{a} - a_2$$
$$\Delta a_3 = \bar{a} - a_3 \qquad \Delta a_n = \bar{a} - a_n$$

(*ii*) ***Mean or final absolute error.*** *The arithmetic mean of the positive magnitudes of all the absolute errors is called mean absolute error.* It is given by

$$\Delta \bar{a} = \frac{|\Delta a_1| + |\Delta a_2| + ... + |\Delta a_n|}{n} = \frac{1}{n} \sum_{i=1}^{n} |\Delta a_i|$$

Thus the final result of the measure of a physical quantity can be expressed as $a = \bar{a} \pm \Delta\bar{a}$.

Clearly, any measured value of a will be such that

$$\bar{a} - \Delta\bar{a} \le a \le \bar{a} + \Delta\bar{a}$$

(*iii*) ***Relative error.*** *The ratio of the mean absolute error to the true value of the measured quantity is called relative error.*

$$\text{Relative error, } \delta a = \frac{\Delta\bar{a}}{\bar{a}}$$

(*iv*) ***Percentage error.*** *The relative error expressed in percent is called percentage error.*

$$\text{Percentage error} = \frac{\Delta\bar{a}}{\bar{a}} \times 100\%$$

For Your Knowledge

- The unit of absolute error is same as that of the quantity being measured.
- It is the relative error or the percentage error and not the absolute error which truly indicates the accuracy of a measurement.

Examples based on Errors in Measurements

Formulae Used

1. *True value.* If $a_1, a_2, a_3, \ldots\ldots a_n$ are the readings of an experiment, then true value of the quantity is given by the arithmetic mean,
$$\bar{a} = \frac{a_1 + a_2 + a_3 + \ldots\ldots + a_n}{n} = \frac{1}{n}\sum_{i=1}^{n} a_i.$$
2. Absolute error = True value – Measured value
or $\Delta a_i = \bar{a} - a_i$
3. Final absolute error
= Arithmetic mean of absolute errors
$$\Delta\bar{a} = \frac{|\Delta a_1| + |\Delta a_2| + |\Delta a_3| + \ldots + |\Delta a_n|}{n}$$
$$= \frac{1}{n}\sum_{i=1}^{n} |\Delta a_i|$$
4. Relative error or fractional error
$$= \frac{\text{Final absolute error}}{\text{True value}} \quad \text{or} \quad \delta a = \frac{\Delta\bar{a}}{\bar{a}}$$
5. Percentage error $= \frac{\Delta\bar{a}}{\bar{a}} \times 100\%$

Example 75. *The length of a rod as measured in an experiment was found to be 2.48 m, 2.46 m, 2.49 m, 2.50 m, and 2.48 m. Find the average length, the absolute error in each observation and the percentage error.*

Solution. Average length

$$= \frac{2.48 + 2.46 + 2.49 + 2.50 + 2.48}{5} = \frac{12.41}{5}\text{ m}$$

$$= 2.482\text{ m} = 2.48\text{ m}$$

[Rounded off to 2 places of decimal point]

The absolute errors in the different measurements are

$$\Delta L_1 = 2.48 - 2.48 = 0.00\text{ m}$$
$$\Delta L_2 = 2.48 - 2.46 = 0.02\text{ m}$$
$$\Delta L_3 = 2.48 - 2.49 = -0.01\text{ m}$$
$$\Delta L_4 = 2.48 - 2.50 = -0.02\text{ m}$$
$$\Delta L_5 = 2.48 - 2.48 = 0.00\text{ m}$$

$$\text{The absolute error} = \frac{\Sigma|\Delta L|}{5}$$

$$= \frac{0.00 + 0.02 + 0.01 + 0.02 + 0.00}{5} = \frac{0.05}{5} = 0.01\text{ m}$$

$\therefore$ Correct length = **2.48 ± 0.01 m**

$$\text{Percentage error} = \frac{0.01}{2.48} \times 100 = \mathbf{0.40\%}.$$

Example 76. *In successive measurements, the readings of the period of oscillation of a simple pendulum were found to be 2.63 s, 2.56 s, 2.42 s, 2.71 s and 2.80 s in an experiment. Calculate (i) mean value of the period of oscillation (ii) absolute error in each measurement (iii) mean absolute error (iv) relative error (v) percentage error and (vi) express the result in proper form.* **[NCERT]**

Solution. (*i*) Mean period of oscillation,

$$\bar{T} = \frac{2.63 + 2.56 + 2.42 + 2.71 + 2.80}{5}$$

$$= \frac{13.12}{5} = 2.624\text{ s}$$

$= 2.62$ s [Rounded off to 2nd decimal place]

(*ii*) Absolute errors in different measurements are

$$\Delta T_1 = 2.62 - 2.63 = -0.01\text{ s}$$
$$\Delta T_2 = 2.62 - 2.56 = 0.06\text{ s}$$
$$\Delta T_3 = 2.62 - 2.42 = 0.20\text{ s}$$
$$\Delta T_4 = 2.62 - 2.71 = -0.09\text{ s}$$
$$\Delta T_5 = 2.62 - 2.80 = -0.18\text{ s}$$

(*iii*) Mean absolute error $= \frac{\Sigma|\Delta T_i|}{n}$

$$\Delta\bar{T} = \frac{0.01 + 0.06 + 0.20 + 0.09 + 0.18}{5}$$

$$= \frac{0.54}{5} = 0.108\text{ s} = \mathbf{0.11\text{ s}}.$$

[Rounded off to 2nd decimal place]

(*iv*) Relative error,

$$\delta T = \frac{\Delta\bar{T}}{\bar{T}} = \frac{0.11}{2.62} = 0.04198 = \mathbf{0.04}.$$

(*v*) Percentage error in $T = 0.04 \times 100 = 4\%$.

(*vi*) In terms of absolute error, $T = \mathbf{(2.62 \pm 0.11)\text{ s}}$.

In terms of percentage error, $T = \mathbf{(2.62 \pm 4\%)\text{ s}}$.

EXAMPLE 77. *In an experiment, refractive index of glass was observed to be 1.45, 1.56, 1.54, 1.44, 1.54 and 1.53. Calculate (i) Mean value of refractive index ; (ii) Mean absolute error ; (iii) Fractional error ; (iv) Percentage error. Express the result in terms of absolute error and percentage error.* **[Chandigarh 03]**

Solution. (*i*) Mean value of refractive index,

$$\bar{\mu} = \frac{1.45 + 1.56 + 1.54 + 1.44 + 1.54 + 1.53}{6} = \mathbf{1.51.}$$

(*ii*) Absolute errors in different measurements are

$$\Delta\mu_1 = 1.51 - 1.45 = 0.06$$
$$\Delta\mu_2 = 1.51 - 1.56 = -0.05$$
$$\Delta\mu_3 = 1.51 - 1.54 = -0.03$$
$$\Delta\mu_4 = 1.51 - 1.44 = 0.07$$
$$\Delta\mu_5 = 1.51 - 1.54 = -0.03$$
$$\Delta\mu_6 = 1.51 - 1.53 = -0.02$$

Mean absolute error,

$$\Delta\bar{\mu} = \frac{\Sigma|\Delta\mu_i|}{n}$$
$$= \frac{0.06 + 0.05 + 0.03 + 0.07 + 0.03 + 0.02}{6}$$
$$= 0.26/6 = 0.0433 \approx \mathbf{0.04.}$$

(*iii*) Fractional error,

$$\delta\mu = \frac{\Delta\bar{\mu}}{\bar{\mu}} = \frac{0.04}{1.51} = 0.02649 = \mathbf{0.03.}$$

(*iv*) Percentage error $= \frac{\Delta\bar{\mu}}{\bar{\mu}} \times 100 = \mathbf{3\%}$

In terms of absolute error, $\mu = \mathbf{1.51 \pm 0.04}$

In terms of percentage error, $\mu = \mathbf{1.51 \pm 3\%}$.

PROBLEMS FOR PRACTICE

1. The diameter of a wire as measured by a screw gauge was found to be 0.026 cm, 0.028 cm, 0.029 cm, 0.027 cm, 0.024 cm and 0.027 cm. Calculate (*i*) mean value of the diameter (*ii*) mean absolute error (*iii*) relative error (*iv*) percentage error. Also express the result in terms of absolute error and percentage error. [**Ans.** (*i*) 0.027 cm (*ii*) 0.001 cm (*iii*) 0.037 (*iv*) 3.7%; (0.027 ± 0.001) cm, (0.027 ± 3.7%) cm]

2. The refractive index of water as measured by the relation $\mu = \frac{\text{Real depth}}{\text{Apparent depth}}$ was found to have the values 1.29, 1.33, 1.34, 1.35, 1.32, 1.36, 1.30, 1.33. Calculate (*i*) mean value of μ (*ii*) mean value of absolute error (*iii*) relative error (*iv*) percentage error. [**Ans.** (*i*) 1.33 (*ii*) 0.02 (*iii*) 0.015 (*iv*) 1.5%]

3. In an experiment to measure focal length of a concave mirror, the value of focal length in successive observations turns out to be 17.3 cm, 17.8 cm, 18.3 cm, 18.2 cm, 17.9 cm and 18.0 cm. Calculate the mean absolute error and percentage error. Express the result in a proper way. [**Ans.** 0.25 cm, 1.4%, (17.9 ± 0.25) cm]

2.32 COMBINATION OF ERRORS

57. *How can we estimate the error in the (i) sum (ii) difference (iii) product (iv) quotient (v) power of different measured quantities. Deduce the general rule for evaluating the error in a combined calculation.*

Propagation or combination of errors. An experiment involves several measurements and the final result is arrived at using different physical relations. The error in the final result depends on the individual measurements as well as the mathematical operations done to get the final result. Following rules are used to evaluate *maximum permissible* error in a measurement.

(*i*) *Error in the sum of two quantities.* Let ΔA and ΔB be the absolute errors in the two quantities A and B respectively. Then

Measured value of $A = A \pm \Delta A$

Measured value of $B = B \pm \Delta B$

Consider the sum, $Z = A + B$

The error ΔZ in Z is then given by

$$Z \pm \Delta Z = (A \pm \Delta A) + (B \pm \Delta B)$$
$$= (A + B) \pm (\Delta A + \Delta B)$$
$$= Z \pm (\Delta A + \Delta B)$$

or $$\Delta Z = \Delta A + \Delta B$$

Hence the rule. *The maximum possible error in the sum of two quantities is equal to the sum of the absolute errors in the individual quantities.*

(*ii*) *Error in the difference of two quantities.* Consider the difference,

$$Z = A - B$$

The error ΔZ in Z is given by

$$Z \pm \Delta Z = (A \pm \Delta A) - (B \pm \Delta B)$$
$$= (A - B) \pm \Delta A \mp \Delta B$$
$$= Z \pm \Delta A \mp \Delta B$$

For error ΔZ to be maximum, ΔA and ΔB must have the same sign, therefore

$$\Delta Z = \Delta A + \Delta B$$

Hence the rule. *The maximum error in the difference of two quantities is equal to the sum of the absolute errors in the individual quantities.*

(iii) Error in the product of two quantities. Consider the product,

$$Z = AB$$

The error ΔZ in Z is given by

$$Z \pm \Delta Z = (A \pm \Delta A)(B \pm \Delta B)$$
$$= AB \pm A\Delta B \pm B\Delta A \pm \Delta A . \Delta B$$

Dividing L.H.S. by Z and R.H.S. by AB, we get

$$1 \pm \frac{\Delta Z}{Z} = 1 \pm \frac{\Delta B}{B} \pm \frac{\Delta A}{A} \pm \frac{\Delta A}{A} . \frac{\Delta B}{B}$$

As $\frac{\Delta A}{A}$ and $\frac{\Delta B}{B}$ are small quantities, their product term can be neglected. The maximum fractional error in Z is

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

Hence the rule. *The maximum fractional error in the product of two quantities is equal to the sum of the fractional errors in the individual quantities.*

(*iv*) Error in the division or quotient. Consider the quotient,

$$Z = \frac{A}{B}$$

The error ΔZ in Z is given by

$$Z \pm \Delta Z = \frac{A \pm \Delta A}{B \pm \Delta B} = \frac{A\left(1 \pm \frac{\Delta A}{A}\right)}{B\left(1 \pm \frac{\Delta B}{B}\right)}$$

$$= \frac{A}{B}\left(1 \pm \frac{\Delta A}{A}\right)\left(1 \pm \frac{\Delta B}{B}\right)^{-1}$$

or $$Z \pm \Delta Z = Z\left(1 \pm \frac{\Delta A}{A}\right)\left(1 \mp \frac{\Delta B}{B}\right)$$

$[\because (1 + x)^n \simeq 1 + nx$, when $x << 1]$

Dividing both sides by Z, we get

$$1 \pm \frac{\Delta Z}{Z} = \left(1 \pm \frac{\Delta A}{A}\right)\left(1 \mp \frac{\Delta B}{B}\right)$$

$$= 1 \pm \frac{\Delta A}{A} \mp \frac{\Delta B}{B} \pm \frac{\Delta A}{A} . \frac{\Delta B}{B}$$

As the terms $\frac{\Delta A}{A}$ and $\frac{\Delta B}{B}$ are small, their product term can be neglected. The maximum fractional error in Z is given by

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

Hence the rule. *The maximum fractional error in the quotient of two quantities is equal to the sum of their individual fractional errors.*

(v) Error in the power of a quantity. Consider the n^{th} power of A,

$$Z = A^n$$

The error ΔZ in Z is given by

$$Z \pm \Delta Z = (A \pm \Delta A)^n = A^n\left(1 \pm \frac{\Delta A}{A}\right)^n$$

$$= Z\left(1 \pm n\frac{\Delta A}{A}\right)$$

$[\because (1 + x)^n \simeq 1 + nx$, when $x << 1]$

Dividing both sides by Z, we get

$$1 \pm \frac{\Delta Z}{Z} = 1 \pm n\frac{\Delta A}{A}$$

or $$\frac{\Delta Z}{Z} = n . \frac{\Delta A}{A}$$

Hence the rule. *The fractional error in the n^{th} power of a quantity is n times the fractional error in that quantity.*

General rule. If $Z = \frac{A^p B^q}{C^r}$, then maximum fractional error in Z is given by

$$\frac{\Delta Z}{Z} = p\frac{\Delta A}{A} + q\frac{\Delta B}{B} + r\frac{\Delta C}{C}$$

The percentage error in Z is given by

$$\frac{\Delta Z}{Z} \times 100 = p\frac{\Delta A}{A} \times 100 + q\frac{\Delta B}{B} \times 100 + r\frac{\Delta C}{C} \times 100$$

Derivation of general rule using differentiation. We have

$$Z = \frac{A^p B^q}{C^r}$$

Taking logarithms, we get

$$\log Z = p \log A + q \log B - r \log C$$

On differentiating both sides, we get

$$\frac{dZ}{Z} = p\frac{dA}{A} + q\frac{dB}{B} - r\frac{dC}{C}$$

Writing the above equation in terms of fractional errors,

$$\pm\frac{\Delta Z}{Z} = \pm p\frac{\Delta A}{A} \pm q\frac{\Delta B}{B} \pm \frac{\Delta C}{C}$$

The maximum permissible error in Z is given by

$$\frac{\Delta Z}{Z} = p\frac{\Delta A}{A} + q\frac{\Delta B}{B} + r\frac{\Delta C}{C}.$$

For Your Knowledge

- In all mathematical operations, the errors are of additive nature.
- When a quantity appears with a power n greater than one in an expression, its error contribution to the final result increases n times. So quantities with higher power in the expression should be measured with maximum accuracy.
- If a quantity appears with a power less than one in an expression, then its error contribution in the final result is reduced.

Examples based on Combination of Errors

Formulae Used

1. If $Z = A + B$, then the maximum possible error in Z,
$$\Delta Z = \Delta A + \Delta B$$
2. If $Z = A - B$, then the maximum possible error in Z,
$$\Delta Z = \Delta A + \Delta B$$
3. If $Z = AB$, then the maximum fractional error in Z
$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$
4. If $Z = A / B$, then the maximum fractional error in Z
$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$
5. If $Z = A^n$, then the maximum fractional error in Z,
$$\frac{\Delta Z}{Z} = n \cdot \frac{\Delta A}{A}$$
6. If $Z = \frac{A^p B^q}{C^r}$, then the maximum fractional error in Z,
$$\frac{\Delta Z}{Z} = p\frac{\Delta A}{A} + q\frac{\Delta B}{B} + r\frac{\Delta C}{C}$$
The percentage error in Z
$$\frac{\Delta Z}{Z} \times 100 = p\frac{\Delta A}{A} \times 100 + q\frac{\Delta B}{B} \times 100 + r\frac{\Delta C}{C} \times 100$$

Units Used

The maximum possible error has the same units as the quantity itself but fractional error has no units.

Example 78. *Two resistances $R_1 = 100 \pm 3\,\Omega$ and $R_2 = 200 \pm 4\,\Omega$ are connected in series. What is their equivalent resistance ?* **[NCERT]**

Solution. Equivalent resistance,
$$R = R_1 + R_2 = (100 \pm 3) + (200 \pm 4)$$
$$= (100 + 200) \pm (3 + 4) = \mathbf{(300 \pm 7)\,\Omega}.$$

Example 79. *Two different masses are determined as (23.7 ± 0.5) g and (17.6 ± 0.3) g. What is the sum of their masses?*

Solution. Sum of the masses
$$= (23.7 \pm 0.5) + (17.6 \pm 0.3)$$
$$= (23.7 + 17.6) \pm (0.5 + 0.3) = \mathbf{(41.3 \pm 0.8)\ g}.$$

Example 80. *The initial and final temperatures of a water bath are $(18 \pm 0.5)°C$ and $(40 \pm 0.3)°C$. What is the rise in temperature of the bath ?*

Solution. Rise in temperature
$$= \text{Final temperature} - \text{Initial temperature}$$
$$= (40 \pm 0.3) - (18 \pm 0.5) = (40 - 18) \pm (0.3 + 0.5)$$
$$= \mathbf{(22 \pm 0.8)°\ C}.$$

Example 81. *The resistance $R = \frac{V}{I}$, where $V = 100 \pm 5\ V$ and $I = 10 \pm 0.2\ A$. Find the percentage error in R.* **[NCERT]**

Solution. The percentage error in V is 5% and in I it is 2%.

The total percentage error in R is given by
$$\frac{\Delta R}{R} \times 100 = \frac{\Delta V}{V} \times 100 + \frac{\Delta I}{I} \times 100$$
$$= 5\% + 2\% = \mathbf{7\%}.$$

Example 82. *If the errors involved in the measurements of a side and mass of a cube are 3% and 4% respectively, what is the maximum permissible error in the density of the material ?*

Solution. Density, $\rho = \frac{M}{V} = \frac{M}{L^3}$

The percentage error in density is given by
$$\frac{\Delta \rho}{\rho} \times 100 = \frac{\Delta M}{M} \times 100 + 3 \times \frac{\Delta L}{L} \times 100$$
$$= 4\% + 3 \times 3\% = \mathbf{13\%}.$$

Example 83. *The error in the measurement of radius of a sphere is 2%. What would be the error in the volume of the sphere ?*

Solution. Given : $\frac{\Delta r}{r} \times 100 = 2\%$

Volume of sphere, $V = \frac{4}{3}\pi r^3$

Percent error in volume,
$$\frac{\Delta V}{V} \times 100 = 3\frac{\Delta r}{r} \times 100 = 3 \times 2 = \mathbf{6\%}.$$

Example 84. *The percentage errors in the measurement of mass and speed are 2% and 3% respectively. How much will be the maximum error in the estimate of kinetic energy obtained by measuring mass and speed ?* **[NCERT]**

Solution. Given :
$$\frac{\Delta m}{m} \times 100 = 2\% \quad \text{and} \quad \frac{\Delta v}{v} \times 100 = 3\%$$

Kinetic energy, $K = \frac{1}{2}mv^2$

Percent error in K.E.,
$$\frac{\Delta K}{K} \times 100 = \frac{\Delta m}{m} \times 100 + 2\frac{\Delta v}{v} \times 100$$
$$= 2\% + 2 \times 3\% = \mathbf{8\%}.$$

EXAMPLE 85. *The length, breadth and height of a rectangular block of wood were measured to be :*

$l = 12.13 \pm 0.02$ *cm* ; $b = 8.16 \pm 0.01$ *cm*;

$h = 3.46 \pm 0.01$ *cm*

Determine the percentage error in the volume of the block.

Solution. Volume of block, $V = lbh$

The percentage error in the volume is given by

$$\frac{\Delta V}{V} \times 100 = \left(\frac{\Delta l}{l} + \frac{\Delta b}{b} + \frac{\Delta h}{h}\right) \times 100$$

$$= \left(\frac{0.02}{12.13} + \frac{0.01}{8.16} + \frac{0.01}{3.46}\right) \times 100$$

$$= \frac{200}{1213} + \frac{100}{816} + \frac{100}{346}$$

$$= 0.1649 + 0.1225 + 0.2890 = \mathbf{0.58\%}.$$

[Rounded off to 2 significant figures]

EXAMPLE 86. *The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{L/g}$. Measured value of L is 20.0 cm known to 1 mm accuracy and time for 100 oscillations of the pendulum is found to be 90 s using a wrist watch of 1 s resolution. What is the accuracy in the determination of g ?* **[NCERT]**

Solution. As $T = 2\pi\sqrt{\dfrac{L}{g}}$

or $\quad T^2 = 4\pi^2 \dfrac{L}{g} \quad$ or $\quad g = \dfrac{4\pi^2 L}{T^2}$

$$\therefore \quad \frac{\Delta g}{g} \times 100 = \frac{\Delta L}{L} \times 100 + 2 \times \frac{\Delta T}{T} \times 100$$

Now $L = 20.0$ cm, $\Delta L = 1$ mm $= 0.1$ cm, T for 100 oscillations $= 90$ s, $\Delta T = 1$ s

$$\therefore \quad \frac{\Delta g}{g} \times 100 = \frac{0.1}{20.0} \times 100 + 2 \times \frac{1}{90} \times 100$$

$$= 0.5 + 2.22 = 2.72\% = \mathbf{3\%}.$$

EXAMPLE 87. *Find the relative error in Z, if*

$$Z = A^4 B^{1/3} / CD^{3/2}.$$ **[NCERT]**

Solution. The relative error in Z is

$$\frac{\Delta Z}{Z} = 4 \times \frac{\Delta A}{A} + \frac{1}{3} \times \frac{\Delta B}{B} + \frac{\Delta C}{C} + \frac{3}{2}\frac{\Delta D}{D}.$$

EXAMPLE 88. *A physical quantity X is given by $X = \dfrac{a^2 b^3}{c\sqrt{d}}$. If the percentage errors of measurement in a, b, c and d are 4%, 2%, 3% and 1% respectively, then calculate the percentage error in X.*

Solution. Given : $X = \dfrac{a^2 b^3}{c\sqrt{d}}$

The percentage error in X is given by

$$100 \times \frac{\Delta X}{X} = 2\frac{\Delta a}{a} \times 100 + 3\frac{\Delta b}{b} \times 100 + \frac{\Delta c}{c} \times 100 + \frac{1}{2}\frac{\Delta d}{d} \times 100$$

$$= 2 \times 4\% + 3 \times 2\% + 3\% + \tfrac{1}{2} \times 1\% = \mathbf{17.5\%}.$$

EXAMPLE 89. *For the estimation of Young's modulus :*

$$Y = \frac{4\,Mg}{\pi d^2} \cdot \frac{L}{l}$$

for the specimen of a wire, following observations were recorded : $L = 2.890$, $M = 3.00$, $d = 0.082$, $g = 9.81$, $l = 0.087$. Calculate the maximum percentage error in the value of Y and mention which physical quantity causes maximum error.

Solution. Given $Y = \dfrac{4\,Mg}{\pi d^2} \cdot \dfrac{L}{l}$

As 4 and π are constants and a standard value of 'g' is taken, so the percentage error in Y will be

$$\frac{\Delta Y}{Y} \times 100 = \left(\frac{\Delta M}{M} + \frac{\Delta L}{L} + 2\frac{\Delta d}{d} + \frac{\Delta l}{l}\right) \times 100$$

It is obvious from the given data that

$$\Delta M = 0.01, \Delta L = 0.001, \Delta d = 0.001, \Delta l = 0.001$$

$$\therefore \quad \frac{\Delta Y}{Y} \times 100 = \left(\frac{0.01}{3.00} + \frac{0.001}{2.890} + \frac{2 \times 0.001}{0.082} + \frac{0.001}{0.087}\right) \times 100$$

$$= (0.0033 + 0.0003 + 0.0244 + 0.0115) \times 100$$

$$= 0.0395 \times 100 = \mathbf{3.95\%}.$$

Diameter of the wire causes maximum error in the value of Y.

EXAMPLE 90. *The specific resistance σ of a thin wire of radius r cm, resistance R Ω and length L cm is given by*

$$\sigma = \frac{\pi r^2 R}{L}$$

If $r = 0.26 \pm 0.02$ cm, $R = 32 \pm 1\,\Omega$

and $L = 78 \pm 0.01$ cm, find the percentage error in σ.

Solution. The percentage error in specific resistance σ is given by

$$\frac{\Delta\sigma}{\sigma} \times 100 = \left(2\frac{\Delta r}{r} + \frac{\Delta R}{R} + \frac{\Delta L}{L}\right) \times 100$$

$$= \left[\frac{2 \times 0.02}{0.26} + \frac{1}{32} + \frac{0.01}{78}\right] \times 100$$

$$= [0.15 + 0.03 + 0.0001] \times 100$$

$$= 0.1801 \times 100 = 0.18 \times 100 = \mathbf{18\%}.$$

EXAMPLE 91. *If two resistors of resistances $R_1 = (4 \pm 0.5)\,\Omega$ and $R_2 = (16 \pm 0.5)\,\Omega$ are connected (i) in series and (ii) in parallel; find the equivalent resistance in each case with limits of percentage error.*

Solution. (*i*) The equivalent resistance of the series combination,

$$R = R_1 + R_2 = (4 \pm 0.5)\Omega + (16 \pm 0.5)\Omega$$

$$= \mathbf{(20 \pm 1)\Omega} \text{ or } \mathbf{20 \pm 5\%}$$

(*ii*) The equivalent resistance of parallel combination,

$$R' = \frac{R_1 R_2}{R_1 + R_2} = \frac{4 \times 16}{4 + 16} = 3.2\ \Omega$$

From $= \dfrac{1}{R'} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$, we get

$$\frac{\Delta R'}{R'^2} = \frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}$$

$$\therefore \quad \Delta R' = (R'^2)\frac{\Delta R_1}{R_1^2} + (R'^2)\frac{\Delta R_2}{R_2^2}$$

$$= \left(\frac{3.2}{4}\right)^2 \times 0.5 + \left(\frac{3.2}{16}\right)^2 \times 0.5$$

$$= 0.34\ \Omega$$

Here $R' = \mathbf{(3.2 \pm 0.34)\ \Omega = 3.2\ \Omega \pm 10.625\%}$

Problems For Practice

1. If $A = (12.0 \pm 0.1)$ cm and $B = (8.5 \pm 0.5)$ cm, find (*i*) $A + B$ and (*ii*) $A - B$. **[Delhi 04]**
 [Ans. (*i*) 20.5 ± 0.6 cm (*ii*) 3.5 ± 0.6 cm**]**

2. The temperatures of two bodies measured by a thermometer are $t_1 = 20°C \pm 0.5°C$ and $t_2 = 50°C \pm 0.5°C$. Calculate the temperature difference and the error therein. **[NCERT ; Central Schools 12]**
 (Ans. $30°C \pm 1°C$**)**

3. The lengths of two rods are 15.2 ± 0.2 cm and 10.7 ± 0.2 cm. Find difference in lengths of the two rods with the limits of error. **[Himachal 07]**
 (Ans. 4.5 ± 0.4 cm**)**

4. The length and breadth of a rectangle are (5.7 ± 0.1) cm and (3.4 ± 0.2) cm. Calculate area of the rectangle with error limits. **[Chandigarh 04]**
 (Ans. $19.4 \pm 1.5\ \text{cm}^2$**)**

5. If displacement of a body, $s = (200 \pm 5)$ meters and time taken by it $t = (20 \pm 0.2)$ seconds, then find the percentage error in the calculation of velocity.
 [Delhi 05] **(Ans.** 3.5%**)**

6. If the length and time period of an oscillating pendulum have errors of 1% and 2% respectively, what is the error in the estimate of g ?
 [Central Schools 04] **(Ans. 5%)**

7. If $l_1 = (10.0 \pm 0.1)$ cm and $l_2 = (9.0 \pm 0.1)$ cm, find their sum, difference and error in each.
 [Ans. (19.0 ± 0.2) cm, 1% ; (1.0 ± 0.2) cm, 20%**]**

8. The length and breadth of a field are 22.4 cm and 15.8 cm respectively and have been measured to an accuracy of 0.2 cm. Find the percentage error in the area of the field. **[Himachal 07]** **(Ans.** 2.16%**)**

9. The relative density of a material is found by weighing the body first in air and then in water. If the weight in air is (10.0 ± 0.1) N and weight in water is (5.0 ± 0.1) N, what would be the maximum percentage error in relative density ? **(Ans.** 5%**)**

10. The voltage across a lamp is (6.0 ± 0.1) volt and the current passing through it is (4.0 ± 0.2) ampere. Find the power consumed. **[Ans.** (24.0 ± 1.6) watt**]**

11. The radius of a sphere is measured as (2.1 ± 0.5) cm. Calculate its surface area with error limits.
 [Ans. $(55.4 \pm 26.4)\ \text{cm}^2$**]**

12. The radius of a sphere is 5.3 ± 0.1 cm. Calculate the percentage error in its volume. **[Himachal 06]**
 (Ans. 5.7 %**)**

13. The measure of the diameter of a cylinder is (1.60 ± 0.01) cm and its length is (5.0 ± 0.1) cm. Calculate the percentage error in its volume.
 (Ans. 3.25%**)**

14. The measured mass and volume of a body are 2.00 g and 5.0 cm^3 respectively. With possible errors of 0.01 g and 0.1 cm^3, what would be the percent error in density ? **(Ans.** 2.5%**)**

15. A body travels uniformly a distance of (13.8 ± 0.2) m in a time (4.0 ± 0.3) s. Calculate its velocity with error limits. What is percentage error in velocity ?
 [Ans. $(3.5 \pm 0.3)\ \text{ms}^{-1}$, 9%**]**

16. Find the percentage error in Z, if $Z = \dfrac{A^{1/3} B^4}{CD^{2/3}}$.
 [Delhi 08]
 [Ans. % error in Z $= \left(\frac{1}{3}\frac{\Delta A}{A} + 4\frac{\Delta B}{B} + \frac{\Delta C}{C} + \frac{2}{3}\frac{\Delta D}{D}\right)100$**]**

17. A physical quantity X is related to three observables a, b, c as $X = \sqrt{a}\, b^2 / c^2$. The errors of measurement in a, b and c are 2%, 1% and 3% respectively. What is the percentage error in the quantity X ? **(Ans.** 9%**)**

18. A physical quantity Q is given by $Q = \dfrac{A^2\ B^{3/2}}{C^{+4}\ D^{1/2}}$

 The percentage errors in A, B, C and D are 1%, 2%, 4% and 2% respectively. Find the percentage error in Q. **[Central Schools 05]** **(Ans.** 22%**)**

19. Two resistors of resistances $R_1 = 100 \pm 3$ ohm and $R_2 = 200 \pm 4$ ohm are connected (*a*) in series, (*b*) in parallel. Find the equivalent resistance of the (*a*) series combination. (*b*) parallel combination. Use for (*a*) the relation $R = R_1 + R_2$ and for (*b*) $\dfrac{1}{R'} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$ and $\dfrac{\Delta R'}{R'^2} = \dfrac{\Delta R_1}{R_1^{\,2}} + \dfrac{\Delta R_2}{R_2^{\,2}}$. **[NCERT]**
 (Ans. $66.7 \pm 1.8\ \Omega$**)**

HINTS

2. $t' = t_2 - t_1 = (50°\text{C} \pm 0.5°\text{C}) - (20°\text{C} \pm 0.5°\text{C})$
$t' = 30°\text{C} \pm 1°\text{C}$.

4. Here $l = 5.7 \pm 0.1\,\text{cm}$ and $b = 3.4 \pm 0.2\,\text{cm}$

Area $= lb = 5.7 \times 3.4 = 19.38\,\text{cm}^2 = 19.4\,\text{cm}^2$

$$\Delta A = \left(\frac{\Delta l}{l} + \frac{\Delta b}{b}\right) lb = \left(\frac{0.1}{5.7} + \frac{0.2}{3.4}\right) \times 19.4$$

$$= (0.1017 + 0.059) \times 19.4 = 1.47 = 1.5$$

$\therefore$ $A = \mathbf{(19.4 \pm 1.5)\,cm^2}$.

5. As $v = s/t$

$$\therefore \frac{\Delta v}{v} \times 100 = \frac{\Delta s}{s} \times 100 + \frac{\Delta t}{t} \times 100$$

$$= 2.5\% + 1\% = \mathbf{3.5\%}.$$

6. As $g = \dfrac{4\pi^2 l}{T^2}$

$$\therefore \frac{\Delta g}{g} \times 100 = \frac{\Delta l}{l} \times 100 + 2\frac{\Delta T}{T} = 1 + 2 \times 2 = \mathbf{5\%}.$$

7. Sum $= (10.0 + 9.0) \pm (0.1 + 0.1) = \mathbf{(19.0 \pm 0.2)}$.

Percentage error $= \dfrac{0.2}{19.0} \times 100 = \mathbf{1\%}$

Difference $= (10.0 - 9.0) \pm (0.1 + 0.1) = \mathbf{(1.0 \pm 0.2)\ cm.}$

Percentage error $= \dfrac{0.2}{1.0} \times 100 = \mathbf{20\%}$.

8. Area, $A = l \times b$

$$\%\ \text{Error} = \frac{\Delta A}{A} \times 100 = \frac{\Delta l}{l} \times 100 + \frac{\Delta b}{b}$$

$$= \frac{0.2}{22.4} \times 100 + \frac{0.2}{15.8} \times 100$$

$$= 0.89 + 1.27 = \mathbf{2.16\%}$$

9. Relative density $= \dfrac{\text{Weight in air}}{\text{Loss of weight in water}}$

Loss of weight in water

$= (10.0 - 5.0) \pm (0.1 + 0.1) = 5.0 \pm 0.2\,\text{N}$

Percentage error in relative density

= Percentage error in weight in air

+ Percentage error in loss of weight in water

$$= \left(\frac{0.1}{10.0} + \frac{0.2}{5.0}\right) \times 100 = \mathbf{5\%}.$$

10. Power consumed, $P = VI = 6.0 \times 4.0 = 24.0\,\text{W}$

$$\therefore \frac{\Delta P}{P} = \frac{\Delta V}{V} + \frac{\Delta I}{I} = \frac{0.1}{6.0} + \frac{0.2}{4.0} = \frac{1}{15}$$

$$\therefore \Delta P = \frac{1}{15} \times P = \frac{1}{15} \times 24.0 = 1.6\,\text{W}$$

Power consumed $= \mathbf{(24.0 \pm 1.6)\ W.}$

11. Surface area of a sphere, $A = 4\pi r^2$

$$\therefore \frac{\Delta A}{A} = 2\frac{\Delta r}{r}.$$

13. Volume of a cylinder, $V = \pi r^2 h = \dfrac{\pi d^2 h}{4}$

$$\%\ \text{Error},\ \frac{\Delta V}{V} \times 100 = \left[2\frac{\Delta d}{d} + \frac{\Delta h}{h}\right] \times 100$$

$$= \left[2 \times \frac{0.01}{1.60} + \frac{0.1}{5.0}\right] \times 100 = [0.0125 + 0.02] \times 100$$

$$= 0.0325 \times 100 = \mathbf{3.25\%}.$$

14. $$\frac{\Delta\rho}{\rho} = \left[\frac{\Delta m}{m} + \frac{\Delta V}{V}\right] \times 100$$

$$= \left[\frac{0.01}{2.00} + \frac{0.1}{5.0}\right] \times 100 = \mathbf{2.5\%}.$$

15. Given : $s = (13.8 \pm 0.2)\,\text{m}$, $t = (4.0 \pm 0.3)\,\text{s}$

Velocity, $v = \dfrac{s}{t} = \dfrac{13.8}{4.0} = 3.45\,\text{ms}^{-1} = 3.4\,\text{ms}^{-1}$

[Rounded off to first place of decimal]

$$\frac{\Delta v}{v} = \frac{\Delta s}{s} + \frac{\Delta t}{t} = \frac{0.2}{13.8} + \frac{0.3}{4.0} = \frac{0.8 + 4.14}{13.8 \times 4.0}$$

$$= \frac{4.94}{13.8 \times 4.0} = 0.0895$$

$$\Delta v = 0.0895 \times v = 0.0895 \times 3.45 = 0.3087 = \pm\,0.3$$

$\therefore$ Velocity $= \mathbf{(3.4 \pm 0.3)\ ms^{-1}}$.

$\therefore$ Error, $\dfrac{\Delta v}{v} \times 100 = 0.0895 \times 100 = 8.95\% = \mathbf{9\%}$.

18. $$\frac{\Delta Q}{Q} \times 100 = 2\frac{\Delta A}{A} \times 100 + \frac{3}{2}\frac{\Delta B}{B} \times 100 + 4\frac{\Delta C}{C} \times 100 + \frac{1}{2}\frac{\Delta D}{D} \times 100$$

$$= 2 \times 1 + \frac{3}{2} \times 2 + 4 \times 4 + \frac{1}{2} \times 2$$

$$= 2 + 3 + 16 + 1 = \mathbf{22\%}.$$

19. (*a*) The equivalent resistance of series combination,

$$R = R_1 + R_2 = (100 \pm 3)\,\Omega + (200 \pm 4)\,\Omega$$
$$= \mathbf{300 \pm 7\,\Omega}.$$

(*b*) The equivalent resistance of parallel combination,

$$R' = \frac{R_1 R_2}{R_1 + R_2} = \frac{200}{3} = 66.7\,\Omega$$

From $\dfrac{1}{R'} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$, we get

$$\frac{\Delta R'}{R'^2} = \frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}$$

$$\therefore \Delta R' = (R'^2)\frac{\Delta R_1}{R_1^2} + (R'^2)\frac{\Delta R_2}{R_2^2}$$

$$= \left(\frac{66.7}{100}\right)^2 3 + \left(\frac{66.7}{200}\right)^2 4 = 1.8$$

Hence $R' = \mathbf{66.7 \pm 1.8\,\Omega}$.

2.33 ▼ RULES FOR DETERMINING THE UNCERTAINTY OF NUMBERS IN ARITHMATIC OPERATIONS

58. *State and illustrate the rules for determining the uncertainty of a number in arithmatic operations.*

Rules for determining the uncertainty of a number in arithmatic operations :

1. *The uncertainty in a measured value is equal to half the least count of the measuring instrument.*

Illustration. Suppose the measured values of the length and breadth of a rectangular sheet are 16.2 cm and 10.1 cm respectively. Then its true length and breadth may be expressed as

$$l = 16.2 \pm \frac{1}{2} \times 0.1 = 16.2 \pm 0.05 \text{ cm}$$

or $$l = 16.2 \text{ cm} \pm \frac{0.05}{16.2} \times 100 = 16.2 \text{ cm} \pm 0.3\%.$$

and $$b = 10.1 \pm \frac{1}{2} \times 0.1 = 10.1 \pm 0.05 \text{ cm}.$$

or $$b = 10.1 \text{ cm} \pm \frac{0.05}{10.1} \times 100 = 10.1 \text{ cm} \pm 0.5\%.$$

2. *When two or more measured values are multiplied, the percentage uncertainty of the final result is equal to the square root of the sum of the squares of the percentage uncertainties in the original values.*

Illustration. The true area of the above rectangular sheet can be expressed as

$$lb = 16.2 \times 10.1 \text{ cm}^2 \pm \sqrt{(0.3\%)^2 + (0.5\%)^2}$$

$$= 163.62 \text{ cm}^2 \pm 0.6\%$$

or $$lb = 163.62 \pm \frac{163.62 \times 0.6}{100} = 163.62 \pm 1.0 \text{ cm}^2.$$

3. *If a set of experimental data is specified to n significant figures, then the result obtained by combining this data will also be valid upto n significant figures. But sometimes the number of significant figures get reduced in subtraction.*

Illustration. 12.9 g – 7.06 g = 5.84 g ≃ 5.8 g, the corrected difference on being rounded off to first place of decimal. Both 2.9 g and 7.06 g have three significant figures each but their difference contains two significant figures.

4. *The fractional errors of the value of a number specified to n significant figures depends not only on n but also on the original number.*

Illustration. Accuracy in the measurement of mass 1.02 g = ± 0.005 g

Fractional error in 1.02 g

$$= \pm \frac{0.005}{1.02} \times 100 = \pm 0.5\%.$$

Accuracy in the measurement of mass 9.89 g

$$= \pm 0.005 \text{ g}$$

Fractional error in 9.89 g

$$= \pm \frac{0.005}{9.89} \times 100 = \pm 0.05\%.$$

Both 1.02 g and 9.89 g contain three significant figures each. Both the numbers have same accuracy but different fractional errors.

5. *The results in the intermediate steps of a multi-step calculation should be calculated to one extra significant figure in each measurement than the number of digits in the least precise measurement. This avoids building up of errors in the process of rounding off the numbers.*

Illustration. Reciprocal of 9.58 after being rounded off to 3 significant figures = 0.104.

But the reciprocal of 0.104 calculated to 3 significant figures = 9.62.

However, if we take 1/9.58 = 0.1044, then find the reciprocal to three significant figures, we get back the original number 9.58.

Very Short Answer Conceptual Problems

Problem 1. What is the necessity of selecting some units as fundamental units ?

Solution. The number of physical units required to be measured is very large. If a separate unit is defined for each of them, then it will become very difficult to remember all of them as they will be quite unrelated to each other.

Problem 2. How is SI a coherent system of units ?

Solution.In SI, all derived units can be obtained by multiplying and dividing the basic and supplementary units and no numerical factors are required to be introduced. So SI is a coherent system of units.

Problem 3. In defining the standard of length, (the prototype metre), we have to specify the temperature at which the measurement should be made. Are we justified in calling length a fundamental quantity, if another physical quantity (temperature) has to be specified in choosing a standard ?

Solution. Yes, the choice of length as a fundamental quantity is justified. The modern definition of metre in terms of wavelength of light radiation is not affected by temperature.

Problem 4. Do Å and AU stand for the same unit of length ? *[Himachal 03, 05C, 08]*

Solution. No. 1 Å(angstrom) = 10^{-10} m

1 AU (astronomical unit) = 1.496×10^{11} m.

Problem 5. Why is it convenient to express the distances of stars in terms of light year rather than in metre or kilometre ?

Solution. One light year

$= 9.46 \times 10^{15}$ m $= 9.46 \times 10^{12}$ km.

As the distances of stars are extra-ordinarily large, so it is convenient to express them in light year rather than in metre or kilometre.

Problem 6. Comment on the statement : "To define a physical quantity for which no method of measurement is given or known has no meaning."

Solution. The given statement is not correct. A physical quantity, if it is called so, must have a physical meaning. If it cannot be measured by any direct method, these must be some indirect method for its measurement. Entropy is one such physical quantity.

Problem 7. Is the measure of an angle dependent upon the unit of length ?

Solution. $\theta\,(\text{radian}) = \dfrac{\text{Arc}}{\text{Radius}}$

As an angle is the ratio of the length of an arc and the radius *i.e.*, it is the ratio of two lengths, so the measure of an angle doe not depend upon the unit of length.

Problem 8. What is meant by angular diameter of the moon ? What is its value ?

Solution. The angle subtended by the two diametrically opposite ends of the moon at a point on the earth is called angular diameter of the moon. Its value is about 0.5°.

Problem 9. For a given base line, which will show a greater parallax—a distant star or a nearby star ?

Solution. Parallactic angle, $\theta = \dfrac{\text{base line}}{\text{distance of star}} = \dfrac{b}{S}$

Thus for a given base line *b*, parallax of a star is inversely proportional to its distance *S*. Hence the nearby star will show a greater parallax.

Problem 10. Why is parallax method not useful for measuring the distances of stars more than 100 light years away ?

Solution. For a star more than 100 light years away, the parallax angle is so small that it cannot be measured accurately.

Problem 11. What is the difference between mN, Nm and nm ? *[Himachal 03]*

Solution. 1 mN = 1 millinewton $= 10^{-3}$ newton (unit of force).

1 Nm = 1 Newton metre (unit of work)

1 nm = 1 nanometre $= 10^{-9}$ metre (unit of distance).

Problem 12. Do all physical quantities have dimensions ? If no, name three physical quantities which are dimensionless.

Solution. No, all physical quantities do not have dimensions. The physical quantities like angle, strain and relative density are dimensionless.

Problem 13. If 'slap' times speed equals power, what will be the dimensional equation for 'slap' ?

Solution. As slap × speed = power

$$\therefore \quad \text{Slap} = \frac{\text{Power}}{\text{Speed}} = \frac{\text{ML}^2\text{T}^{-3}}{\text{LT}^{-1}} = \textbf{MLT}^{-2}.$$

Problem 14. Write the dimensions and SI unit of linear momentum.

Solution. Linear momentum = Mass × velocity

$\therefore$ [Linear momentum] $= [\text{M}]\,[\text{LT}^{-1}] = [\text{MLT}^{-1}]$

SI unit of linear momentum = $\textbf{kgms}^{-1}$.

Problem 15. What is the basis of the principle of homogeneity of dimensions ?

Solution. The principle of homogeneity of dimensions is based on the fact that only the physical quantities of the same kind can be added, subtracted or compared.

Problem 16. If $x = a + bt + ct^2$, where x is in metre and t in second; then what is the unit of c ? *[Himachal 05C]*

Solution. $[x] = [ct^2]$ or $[c] = \left[\dfrac{x}{t^2}\right]$

$\therefore$ Unit of $c = \text{ms}^{-2}$.

Problem 17. What are the dimensions of a and b in the relation : $F = a + bx$, where F is force and x is distance ?

Solution. $[a] = [F] = \text{MLT}^{-2}$

$$[b] = \left[\frac{F}{x}\right] = \frac{\text{MLT}^{-2}}{\text{L}} = \text{MT}^{-2}.$$

Problem 18. Name two physical quantities having the dimensions $[\text{ML}^2\text{T}^{-2}]$.

Solution. Work and torque.

Problem 19. Write three physical quantities having dimensions $[\text{ML}^{-1}\text{T}^{-2}]$.

Solution. Pressure, stress and Young's modulus of elasticity.

Problem 20. If the units of force and length each are doubled, then how many times would the unit of energy be affected ?

Solution. Energy = Work done = Force × length

So when the units of force and length each are doubled, the unit of energy will increase four times.

Problem 21. The velocity v of a particle depends on time t as : $v = At^2 + Bt + C$ where v is in m/s and t in second. What are the units of A, B and C ?

Solution. Unit of $A = \dfrac{\text{Unit of } v}{\text{Unit of } t^2} = \dfrac{\text{ms}^{-1}}{\text{s}^2} = \textbf{ms}^{-3}$

Unit of $B = \dfrac{\text{Unit of } v}{\text{Unit of } t} = \dfrac{\text{ms}^{-1}}{\text{s}} = \textbf{ms}^{-2}$.

Unit of C = Unit of v = $\textbf{ms}^{-1}$.

Problem 22. Can a quantity have dimensions but still has no units ?

Solution. No, a quantity having dimensions must have some units of its measurement.

Problem 23. Can a quantity have different dimensions in different systems of units ?

Solution. No, a quantity has same dimensions in all system of units.

Problem 24. Can a quantity have units but still be dimensionless ?

Solution. Yes. For example, a plane angle has no dimensions but has unit like radian for its measurement.

Problem 25. Does the magnitude of a physical quantity depend on the system of units chosen ?

Solution. No. The magnitude of physical quantity remains same in all systems of units.

Problem 26. Justify L + L = L and L – L = L.

Solution. When we add a length to another length, we get length only so L + L = L is justified.

When we subtract a length from another length, again we get length. This justifies L – L = L.

Problem 27. Can there be a physical quantity that has no units and no dimensions ?

Solution. Yes, strain is a physical quantity that has no units and no dimensions.

Problem 28. Can an instrument be called precise without being accurate ? Can it be accurate without being precise ?

Solution. Yes, an instrument can be precise without being accurate but the measurements cannot be accurate without being precise.

Problem 29. Which of the following length measurements is (*i*) most precise and (*ii*) least precise ? Give reason (*i*) l = 5 cm (*ii*) l = 5.00 cm (*iii*) 5.000 cm (*iv*) 5.000 cm (*v*) 5.00000 cm.

Solution. (*i*) The last measurement is most precise, because it has been taken with an instrument whose least count is equal to 0.00001 cm.

(*ii*) The first measurement is least precise because it has been taken with a device having least count equal to 1 cm.

Problem 30. Which of the following readings is the most accurate :

(*i*) 5000 m (*ii*) 5×10^2 m (*iii*) 5×10^3 m ?

Solution. (*i*) 5000 m is most accurate.

Problem 31. Which quantity in a given formula should be measured most accurately ?

Solution. The quantity which has maximum power (say, n) in the formula should be measured more accurately because an error in measurement is multiplied n times in the final result.

Problem 32. Which of the following measurements is more accurate and why ?

(*a*) 0.0002 g (*b*) 20.0 g

Solution. The measurement

(*a*) 0.0002 g is more accurate because it is correct upto fourth decimal place while measurement

(*b*) 20.0 g is correct upto first decimal place.

Short Answer Conceptual Problems

Problem 1. Why do we treat length, mass and time as basic or fundamental quantities in mechanics ?

Solution. In mechanics, the quantities like length, mass and time are taken as fundamental quantities because :

(*i*) these quantities represent basic scientific notations,

(*ii*) there is no other quantity simpler to them,

(*iii*) length, mass and time cannot be expressed in terms of one another, and

(*iv*) all other physical quantities in mechanics can be expressed in terms of these quantities.

Problem 2. SI is a rational system of units while mks system is not so. Justify.

Solution. SI assigns only one unit to a particular physical quantity so it is a rational system. For example, all types of energies are measured in joule in SI. But in mks system, mechanical energy is measured in joule, heat energy in calorie and electrical energy in watt-hour. So mks system is not a rational system of units.

Problem 3. Why it became necessary to redefine metre on atomic standard ?

Solution. It became necessary to redefine metre on atomic standard because the prototype metre offered the following difficulties :

(*i*) It is difficult to preserve a metre bar.

(*ii*) It is difficult to produce replicas of metre bar for their use in different countries.

(*iii*) The techniques used for producing replicas are not of very high accuracy.

Problem 4. What are the advantages of defining metre in terms of the wavelength of light radiation ?

Solution. The advantages of defining metre in terms of wavelength of light radiation are as follows :

(*i*) It can be easily reproduced anywhere and at any time.

(*ii*) It is invariant in time and space.

(*iii*) It is unaffected by environmental conditions like temperature and pressure.

(*iv*) It has an high accuracy of 1 part in 10^9.

Problem 5. Give reasons why is platinum iridium alloy used in making prototype metre and kilogram.

Solution. The reasons for making standard kilogram and metre from platinum-iridium alloy are as follows :

(*i*) The alloy is least affected by temperature variations.

(*ii*) It is non-corrosive and so does not wear out easily.

(*iii*) It is quite hard.

(*iv*) It does not change with time.

Problem 6. Suggest a distance corresponding to each of the following order of length :

(*i*) 10^7 m (*ii*) 10^4 m (*iii*) 10^2 m (*iv*) 10^{-3} m (*v*) 10^{-6} m (*vi*) 10^{-14} m.

Solution. (*i*) 10^7 m = Radius of the earth

(*ii*) 10^4 m = Height of Mount Everest

(*iii*) 10^2 m = Length of Hockey Field

(*iv*) 10^{-3} m = Thickness of a card-board

(*v*) 10^{-6} m = Mean free path of air molecule

(*vi*) 10^{-14} m = Size of atomic nucleus.

Problem 7. Suggest an indirect method for measuring the height of a tree on a sunny day.

Solution. Let AB be the height of a tree, as shown in Fig. 2.15 and BC be its shadow cast by the sun.

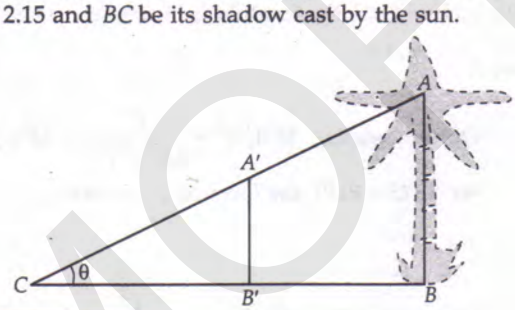


Fig. 2.15

Let $\angle ACB = \theta$.

Clearly, $\tan\theta = \dfrac{AB}{BC}$

Take a rod $A'\,B'$ and fix it at such a point that the tip of its shadow coincides with the point C. Then

$$\tan\theta = \frac{A'B'}{B'C}.$$

Hence $\dfrac{AB}{BC} = \dfrac{A'B'}{B'C}$.

$\therefore$ Height of tree, $AB = \dfrac{A'B'}{B'C} \times BC$.

Problem 8. What is the basic principle of alpha particle scattering method for estimating the size of the nucleus ? *[Pubjab 90]*

Solution. Both the α-particle and nucleus are positively charged. When an α-particle approaches a nucleus, its kinetic energy gradually changes into potential energy due to repulsive forces. At the distance of closest approach r_0, the entire kinetic energy changes into potential energy. This concept can be used to calculate r_0, which gives the order of the size of the nucleus.

Problem 9. If the velocity of light is taken as the unit of velocity and one year as the unit of time, what must be the unit of length ? What is it called ?

Solution. Unit of length = unit of velocity × unit of time

$$= 3 \times 10^8 \text{ ms}^{-1} \times 1 \text{ year}$$
$$= 3 \times 10^8 \text{ ms}^{-1} \times 365 \times 24 \times 60 \times 60 \text{ s}$$
$$= 9.45 \times 10^{15} \text{ ms}^{-1}$$
$$= \mathbf{1 \text{ light year.}}$$

Problem 10. What is common between bar and torr ?

Solution. Both bar and torr are the units of pressure.

1 bar = 1 atmospheric pressure
= 760 mm of Hg column

1 torr = 1 mm of Hg column

$\therefore$ 1 bar = 760 torr.

Problem 11. Distinguish between accuracy and precision.

Solution. By accuracy of a measurement we mean that the measured value of a physical quantity is as close to the true value as possible. On the other hand, a measurement is said to be precise, if same value of the quantity is obtained in each of the various measurements carried out with the given apparatus.

Problem 12. Which of the following measurements is most accurate and which is most precise :

(*i*) 4.00 mm (*ii*) 4.00 cm

(*iii*) 4.00 m (*iv*) 40.00 m ?

Solution.

Length **(*l*)**	***Least Count*** **(Δl)**	***Relative Error*** **($\Delta l / l$)**
(*i*) 4.00 mm	0.01 mm	2.5×10^{-3}
(*ii*) 4.00 cm	0.01 cm	2.5×10^{-3}
(*iii*) 4.00 m	0.01 m	2.5×10^{-3}
(*iv*) 40.00 m	0.01 m	2.5×10^{-4}

Since the relative error in the measurement of 40.00 m is minimum, so this measurement is most accurate. The measurement 4.00 mm is most precise because it is measured with an instrument of minimum least count.

Problem 13. Two clocks are being tested against a standard clock located in a national laboratory. At 12:00:00 noon by the standard clock, the readings of the two clocks are :

Day	*Clock 1*	*Clock 2*
Monday	12:00:05	10:15:06
Tuesday	12:01:15	10:14:59
Wednesday	11:59:08	10:15:18
Thursday	12:01:50	10:15:07
Friday	11:59:15	10:14:53
Saturday	12:01:30	10:15:24
Sunday	12:01:19	10:15:11

If you are doing an experiment that requires precision time interval measurements, which of the two clocks will you prefer ? *[NCERT]*

Solution. The range of variation over the seven days of observations is 162 s for clock 1, and 31 s for clock 2. The average reading of clock 1 is much closer to the standard time than the average reading of clock 2. The important point is that a clock's zero error is not as significant for precision work as its variation, because a 'zero-error' can always be easily corrected. Hence clock 2 is to be preferred to clock 1.

Problem 14. For the determination of 'g' using a simple pendulum, measurements of l and T are required. Error in the measurement of which of these will have larger effect on the value of 'g' thus obtained and why ? What is done to minimize this error ? *[Delhi 04]*

Solution. Time period of a simple pendulum,

$$T = 2\pi\sqrt{\frac{l}{g}} \quad \therefore \quad g = \frac{4\pi^2 l}{T^2}$$

Clearly, the error in the measurement of time period T has larger effect on the value of g than the error in the measurement of length l.

Reasons (*i*) T is very small.

(*ii*) In contrast to l, T^2 appears in the formula for g.

To minimise the error, time period for a large number of oscillations is measured.

Problem. 15. Magnitude of force F experienced by a certain object moving with speed v is given by $F = Kv^2$, where K is a constant. Find the dimensions of K. *[Delhi 04]*

Solution. $[K] = \dfrac{[F]}{[v^2]} = \dfrac{[\text{MLT}^{-2}]}{[\text{LT}^{-1}]^2} = [\text{ML}^{-1}]$.

Problem. 16. The SI unit of energy is $J = \text{kgm}^2\text{s}^{-2}$, that of speed v is ms^{-1} and acceleration a is ms^{-2}. Which of the formulae for kinetic energy (K) given below can you rule out on the basis of dimensional arguments (m stands for the mass of the body) :

(*a*) $K = m^2v^3$ (*b*) $K = (1/2)mv^2$

(*c*) $K = ma$ (*d*) $K = (3/16)mv^2$

(*e*) $K = (1/2)\,mv^2 + ma$ *[NCERT]*

Solution. As SI unit of energy, $J = \text{kgm}^2\text{s}^{-2}$, so

$$[\text{Energy}] = [\text{ML}^2\text{T}^{-2}]$$

(*a*) $[m^2v^2] = [\text{M}^2][\text{LT}^{-1}]^2 = [\text{M}^2\text{L}^2\text{T}^{-2}]$

(*b*) $[1/2\,mv^2] = [\text{M}][\text{LT}^{-1}]^2 = [\text{ML}^2\text{T}^{-2}]$

(*c*) $[ma] = [\text{M}][\text{LT}^{-2}] = [\text{MLT}^{-2}]$

(*d*) $[3/16\,mv^2] = [\text{M}][\text{LT}^{-1}]^2 = [\text{ML}^2\text{T}^{-2}]$

(*e*) The quantities $(1/2)\,mv^2$ and ma have different dimensions and hence cannot be added.

Since the kinetic energy K has the dimensions of $[\text{ML}^2\text{T}^{-2}]$, formulas (*a*), (*c*) and (*e*) are clearly ruled out.

Dimensional analysis cannot tell which of the two, (*b*) or (*d*), is the correct formula. From the actual definition of kinetic energy, only (*b*) is the correct formula for kinetic energy.

Problem. 17. Using the principle of homogeneity of dimensions, find which of the following is correct :

(*i*) $T^2 = 4\pi^2 r^2$ (*ii*) $T^2 = \dfrac{4\pi^2 r^3}{G}$ (*iii*) $T^2 = \dfrac{4\pi^2 r^3}{GM}$

where T is time period, G is gravitational constant, M is mass and r is radius of orbit. *[Central Schools 05]*

Solution. (*i*) $T^2 = 4\pi^2 r^2$

Dimensionally, $\text{M}^0\text{L}^0\text{T}^2 = \text{M}^0\text{L}^2\text{T}^0$

As LHS ≠ RHS, the formula is *incorrect*.

(*ii*) $T^2 = \dfrac{4\pi^2 r^3}{G}$

Dimensionally, $\text{M}^0\text{L}^0\text{T}^2 = \dfrac{\text{L}^3}{\text{M}^{-1}\text{L}^3\text{T}^{-2}} = \text{M}^1\text{L}^0\text{T}^2$

As LHS ≠ RHS, the formula is *incorrect*.

(*iii*) $T^2 = \dfrac{4\pi^2 r^3}{GM}$

Dimensionally, $\text{M}^0\text{L}^0\text{T}^2 = \dfrac{\text{L}^3}{\text{M}^{-1}\text{L}^3\text{T}^{-2}\text{M}} = \text{M}^0\text{L}^0\text{T}^2$

As LHS = RHS, the formula is *correct*.

Problem 18. The mean value of period of oscillation of a simple pendulum in an experiment is 2.825 s. The arithmatic mean of all the absolute errors is 0.11 s. Round off the period of simple pendulum to appropriate number of significant figures. Give reasons. *[Central School 08]*

Solution. The absolute error 0.11 s has only two significant figures.

∴ Period of simple pendulum = 2.9 s

[Rounded off upto 2 significant figures]

HOTS

Problems on Higher Order Thinking Skills

Problem 1. *If nth division of main scale coincides with (n + 1)th division of vernier scale, find the least count of the vernier. Given one main scale division is equal to 'a' units.* **[IIT Mains 03]**

Solution. $(n+1)$ divisions of vernier scale

$$= n \text{ divisions of main scale}$$

$$\therefore \quad 1 \text{ V.S.D.} = \frac{n}{n+1} \text{ M.S.D.}$$

$$\text{Least count} = 1 \text{ M.S.D.} - 1 \text{ V.S.D.}$$

$$= 1 \text{ M.S.D.} - \frac{n}{n+1} \text{ M.S.D.}$$

$$= \frac{1}{n+1} \text{ M.S.D.}$$

$$= \frac{1}{n+1} \times a \text{ units} = \frac{a}{n+1} \text{ units.}$$

Problem 2. *If the velocity of light c, the constant of gravitation G and Planck's constant h be chosen as fundamental units, find the dimensions of mass, length and time in terms of c, G and h.* **[IIT 92]**

Solution. We have,

$$[c] = \text{LT}^{-1}, \quad [G] = \text{M}^{-1}\text{L}^3\text{T}^{-2}, \quad [h] = \text{ML}^2\text{T}^{-1}$$

$$\therefore \quad \frac{[h][c]}{[G]} = \frac{\text{ML}^2\text{T}^{-1}.\,\text{LT}^{-1}}{\text{M}^{-1}\text{L}^3\text{T}^{-2}} = \text{M}^2$$

Hence $[\text{M}] = h^{1/2}c^{1/2}G^{-1/2}$.

$$\text{Again,} \quad \frac{[h]}{[c]} = \frac{\text{ML}^2\text{T}^{-1}}{\text{LT}^{-1}} = \text{ML}$$

$$\therefore \quad [\text{L}] = \frac{h}{c[\text{M}]} = \frac{h}{ch^{1/2}c^{1/2}G^{-1/2}}$$

$$= h^{1/2}c^{-3/2}G^{1/2}$$

$$\text{As} \quad [c] = \text{LT}^{-1}$$

$$\therefore \quad [\text{T}] = \frac{[\text{L}]}{c} = \frac{h^{1/2}c^{-3/2}G^{1/2}}{c} = h^{1/2}c^{-5/2}G^{1/2}.$$

Problem 3. *The velocity of a body which has fallen freely under gravity varies as $g^p h^q$, where g is the acceleration due to gravity at the place and h is the height through which the body has fallen. Determine the values of p and q.* **[NCERT 83]**

Solution. Let $v = \text{K}\, g^p\, h^q$,

where K = a dimensionless constant.

Putting the dimensions of various quantities, we get

$$\text{LT}^{-1} = [\text{LT}^{-2}]^p\, [\text{L}]^q$$

$$\text{or} \quad \text{L}^1\text{T}^{-1} = \text{L}^{p+q}\,\text{T}^{-2p}$$

Equating the powers of L and T on both sides, we get :

$$p + q = 1, \quad -2p = -1$$

$$\text{On solving,} \quad p = \frac{1}{2}, \; q = \frac{1}{2}.$$

Problem 4. *A gas bubble, from an explosion under water, oscillates with a period T proportional to $p^a\, d^b\, E^c$, where p is the static pressure, d is the density of water and E is the total energy of the explosion. Find the values of a, b and c.* **[IIT 81 ; MNREC 90]**

Solution. Let $T = K\,p^a\,d^b\,E^c$,

where K = a dimensionless constant.

Putting the dimensions of various quantities,

$$\text{T} = [\text{ML}^{-1}\text{T}^{-2}]^a\,[\text{ML}^{-3}]^b\,[\text{ML}^2\text{T}^{-2}]^c$$

$$\text{or} \quad \text{M}^0\text{L}^0\text{T} = \text{M}^{a+b+c}\,\text{L}^{-a-3b+2c}\,\text{T}^{-2a-2c}$$

Equating the powers of M, L and T on both sides, we get

$$a + b + c = 0, \quad -a - 3b + 2c = 0, \quad -2a - 2c = 1$$

$$\text{On solving,} \quad a = -\frac{5}{6}, \; b = \frac{1}{2}, \; c = \frac{1}{3}.$$

Problem 5. *A small steel ball of radius r is allowed to fall under gravity through a column of a viscous liquid of coefficient of viscosity η. After some time the velocity of the body attains a constant value v_T. The terminal velocity depends upon (i) the weight of the ball mg (ii) the coefficient of viscosity η and (iii) the radius of the ball r. By the method of dimensions, determine the relation expressing terminal velocity.* **[Chandigarh 07]**

Solution. Let $v_T = K\,(mg)^a\,\eta^b\,r^c$,

where K = a dimensionless constant.

Putting the dimensions of various quantities,

$$\text{LT}^{-1} = [\text{MLT}^{-2}]^a\,[\text{ML}^{-1}\text{T}^{-1}]^b\,[\text{L}]^c$$

$$\text{or} \quad \text{M}^0\text{L}^1\text{T}^{-1} = \text{M}^{a+b}\,\text{L}^{a-b+c}\,\text{T}^{-2a-b}$$

Equating the powers of M, L and T on both sides, we get

$$a + b = 0, \quad a - b + c = 1, \quad -2a - b = -1$$

$$\text{On solving,} \quad a = 1, \quad b = -1, \quad c = -1$$

$$\therefore \quad v_T = K\,(mg)^1\,\eta^{-1}r^{-1} \quad \text{or} \quad v_T \propto \frac{mg}{\eta\, r}.$$

Problem 6. *Derive dimensionally the relation :*

$$S = ut + \tfrac{1}{2}\,at^2.$$

Solution. Let $S = Ku^x\,a^y\,t^z$,

where K = a dimensionless constant.

Putting dimensions of various quantities,

$$L = [LT^{-1}]^x \cdot [LT^{-2}]^y \, [T]^z$$

or $$L^1T^0 = L^{x+y}T^{-x-2y+z}$$

Equating the powers of M, L and T, we get

$$x + y = 1$$

$$-x - 2y + z = 0$$

These two equations cannot be solved for three unknowns x, y and z. The problem is split into two parts.

(*i*) Suppose the body has no acceleration. Then

$$S = K_1 \, u^x \, t^z$$

$$\therefore \quad L = [LT^{-1}]^x \, [T]^z = L^x T^{-x+z}$$

Equating the powers of L and T, $x = 1$, $-x + z = 0$

On solving, $x = 1$, $z = 1$

Hence $S = K_1 \, ut$

(*ii*) Suppose the body has no initial velocity. Then

$$S = K_2 a^y t^z$$

$$\therefore \quad L = [LT^{-2}]^y \, [T]^z = L^y T^{-2y+z}$$

Equating the powers of L and T, $y = 1$, $-2y + z = 0$

On solving, $y = 1$, $z = 2$

Hence $S = K_2 \; at^2$

(*iii*) Suppose the body has both acceleration and initial velocity. Then

$$S = K_1 \, ut + K_2 \, at^2$$

It is found that $K_1 = 1$ and $K_2 = 1/2$. Therefore,

$$S = ut + \tfrac{1}{2} at^2.$$

Problem 7. *The specific heats of a gas are measured as $C_p = (12.28 \pm 0.2)$ units and $C_v = (3.97 \pm 0.3)$ units. Find the value of real gas constant R and percentage error in R.*

Solution. Gas constant,

$$R = C_p - C_v$$

$$= (12.28 \pm 0.2) - (3.97 \pm 0.3)$$

$$= \mathbf{(8.31 \pm 0.5)\ units}$$

% Error in R,

$$\frac{\Delta R}{R} \times 100 = \left[\frac{\Delta C_p + \Delta C_v}{C_p - C_v}\right] \times 100$$

$$= \frac{0.5}{8.31} \times 100 = \mathbf{6.016\%.}$$

Problem 8. *The heat dissipated in a resistance can be determined from the relation :*

$$H = \frac{I^2 Rt}{42} \text{ cal}$$

If the maximum errors in the measurement of current, resistance and time are 2%, 1% and 1% respectively, what would be the maximum error in the dissipated heat ?

Solution. Given : $H = \dfrac{I^2 Rt}{42}$

% Error, $$\frac{\Delta H}{H} \times 100 = \left(2\,\frac{\Delta I}{I} + \frac{\Delta R}{R} + \frac{\Delta t}{t}\right) \times 100$$

$$= 2 \times 2 + 1 + 1 = \mathbf{6\%.}$$

Guidelines to NCERT Exercises

2.1. *Fill in the blanks :*

(*i*) *The volume of a cube of side 1 cm is equal to ...... m^3.*

(*ii*) *The surface area of a solid cylinder of radius 2 cm and height 10 cm is equal to ...... $(mm)^2$.*

(*iii*) *A vehicle moving with a speed of 18 km/h covers ......m in 1 s.*

(*iv*) *The relative density of lead is 11.3. Its density is ...... gcm^{-3} or ...... kgm^{-3}.*

Ans.

(*i*) $V = l^3 = (1\,\text{cm})^3 = (10^{-2}\,\text{m})^3 = \mathbf{10^{-6}\,m^{-3}}$.

(*ii*) $r = 2\,\text{cm} = 20\,\text{mm}$, $h = 10\,\text{cm} = 100\,\text{mm}$

$$S = 2\pi r(r + h) = 2 \times 3.14 \times 20\,(20 + 100)$$

$$= \mathbf{15072\ mm^2}.$$

(*iii*) $v = 18\,\dfrac{\text{km}}{\text{h}} = \dfrac{18 \times 1000\,\text{m}}{60 \times 60\,\text{s}} = \mathbf{5\ ms^{-1}}$.

(*iv*) Density = Relative density × density of water at 4°C

$$= 11.3 \times 1\,\text{gcm}^{-3} = \mathbf{11.3\ gcm^{-3}}$$

$$= 11.3 \times 10^3\ \text{kg m}^{-3} = \mathbf{11300\ kgm^{-3}}.$$

2.2. *Fill in the blanks by suitable conversion of units :*

(*i*) $1\,kg\,m^2s^{-2} = ...\,g\,cm^2s^{-2}$

(*ii*) $1\,m =$ ...... *light year* (*iii*) $3\,ms^{-2} = ...kmh^{-2}$

(*iv*) $G = 6.67 \times 10^{-11}\ Nm^2kg^{-2} = ...\,cm^3s^{-2}g^{-1}$

Ans. (*i*) $1\,\text{kg m}^2\text{s}^{-2} = 1\,(10^3\,\text{g})\,(10^2\,\text{cm})^2\,\text{s}^{-2}$

$$= \mathbf{10^7\,g\,cm^2s^{-2}}.$$

(*ii*) As 1 light year $= 9.46 \times 10^{15}$ m

$$\therefore \quad 1\,\text{m} = \frac{1}{9.46 \times 10^{15}} \text{ light year}$$

$$= 1.053 \times 10^{-16} \text{ light year} \simeq \mathbf{10^{-16}\ light\ year.}$$

(*iii*) $3\ \text{ms}^{-2} = 3(10^{-3}\ \text{km})\left(\dfrac{1}{60\times60}\text{h}\right)^{-2}$

$= 3\times10^{-3}\times3600\times3600\ \text{kmh}^{-2}$

$= 3.888\times10^{4}\ \text{kmh}^{-2} \simeq \mathbf{3.9\times10^{4}\ kmh^{-2}}$.

(*iv*) $G = 6.67\times10^{-11}\ \text{Nm}^2\text{kg}^{-2}$

$= 6.67\times10^{-11}\ \text{kg m s}^{-2}.\ \text{m}^2\ \text{kg}^{-2}$

$= 6.67\times10^{-11}\text{m}^3\text{s}^{-2}\text{kg}^{-1}$

$= 6.67\times10^{-11}\ (10^2\text{cm})^3\text{s}^{-2}(1000\ \text{g})^{-1}$

$= \mathbf{6.67\times10^{-8}\,cm^3s^{-2}g^{-1}}$.

2.3. *A calorie is a unit of heat energy and it equals about 4.2 J, where 1 J = 1 kg m² s⁻². Suppose we employ a system of units in which the unit of mass equals α kg, the unit of length equals β m, the unit of time is γ s. Show that a calorie has a magnitude $4.2\ \alpha^{-1}\beta^{-2}\gamma^{2}$ in terms of the new units.*

Ans. As 1calorie = 4.2 J, where $1\ \text{J} = 1\ \text{kg m}^2\text{s}^{-2}$

Clearly, $[\text{Energy}] = \text{ML}^2\text{T}^{-2}$

$\therefore\ a = 1,\ b = 2,\ c = -2$

SI	New System
$n_1 = 4.2$	$n_2 = ?$
$M_1 = 1\ \text{kg}$	$M_2 = \alpha\ \text{kg}$
$L_1 = 1\ \text{m}$	$L_2 = \beta\ \text{m}$
$T_1 = 1\text{s}$	$T_2 = \gamma\ \text{s}$

$\therefore\ n_2 = n_1\left[\dfrac{M_1}{M_2}\right]^a\left[\dfrac{L_1}{L_2}\right]^b\left[\dfrac{T_1}{T_2}\right]^c$

$= 4.2\left[\dfrac{1\ \text{kg}}{\alpha\ \text{kg}}\right]^1\left[\dfrac{1\ \text{m}}{\beta\ \text{m}}\right]^2\left[\dfrac{1\ \text{s}}{\gamma\ \text{s}}\right]^{-2}$

$= 4.2\ \alpha^{-1}\beta^{-2}\gamma^{2}$

Hence 1 calorie = 4.2 J = $\mathbf{4.2\alpha^{-1}\beta^{-2}\gamma^2}$ **new units of energy.**

2.4. (*i*) *Explain the statement clearly : To call a dimensional quantity 'large' or 'small' is meaningless without specifying a standard for comparison.*

(*ii*) *In view of this, reframe the following statements, wherever necessary :*

(*a*) *Atoms are very small objects*

(*b*) *A jet plane moves with great speed*

(*c*) *The mass of jupiter is very large*

(*d*) *The air inside this room contains a large number of molecules*

(*e*) *A proton is much more massive than an electron*

(*f*) *The speed of sound is much smaller than the speed of light.*

Ans. (*i*) The given statement is correct. Measurement is basically a comparison process. Without specifying a standard of comparison, it is not possible to get an exact idea about the magnitude of a dimensional quantity. For example, the statement that the mass of the earth is very large, is meaningless. To correct it, we can say that the mass of the earth is large in comparison to any object lying on its surface.

(*ii*) (*a*) The size of an atom is much smaller than even the sharp tip of a pin.

(*b*) A jet planet moves with a speed greater than that of a superfast train.

(*c*) The mass of jupiter is very large compared to that of the earth.

(*d*) The air inside this room contains more number of molecules than in one mole of air.

(*e*) This is a correct statement.

(*f*) This is a correct statement.

2.5. *A new unit of length is chosen such that the speed of light in vacuum is unity. What is the distance between the sun and the earth in terms of the new unit if light takes 8 min and 20 s to cover this distance ?*

Ans. Speed of light

= 1 new unit of length / s

Time = 8 min 20 s = $8\times60 + 20 = 500$ s

Distance between the earth and the sun

= Speed of light × time = 1×500

= **500 new units of length.**

2.6. *Which of the following is the most precise device for measuring length :*

(*a*) *a vernier calliper with 20 divisions on the sliding scale,*

(*b*) *a screw gauge of pitch 1 mm and 100 divisions on the circular scale*

(*c*) *an optical instrument that can measure length to within a wavelength of visible light ?*

Ans. The device that has minimum least count will be more precise for measuring length.

(*a*) Least count of vernier callipers

$= 1\ \text{MSD} - 1\ \text{VSD} = 1\ \text{MSD} - \dfrac{19}{20}\ \text{MSD} = \dfrac{1}{20}\ \text{MSD}$

$= \dfrac{1}{20}\times1\ \text{mm} = \dfrac{1}{200}\ \text{cm} = 0.005\ \text{cm}.$

(*b*) Least count of screw gauge

$= \dfrac{\text{Pitch}}{\text{No. of divisions on circular scale}}$

$= \dfrac{1.0\ \text{mm}}{100} = \dfrac{1}{1000}\ \text{cm} = 0.001\ \text{cm}$

(*c*) Least count of optical instrument

= Wavelength of visible (red) light

$= 6000\ \text{Å} = 6000\times10^{-8}\ \text{cm}$

$= 0.00006\ \text{cm}.$

Hence the most precise device for measuring length is the given optical instrument.

2.7. *A student measures the thickness of a human hair by looking at it through a microscope of magnification 100. He makes 20 observations and finds that the average width of the hair in the field of view of the microscope is 3.5 mm. What is the estimate on the thickness of hair ?*

Ans. Average thickness of hair as observed through microscope = 3.5 mm

Magnification produced by the microscope = 100

Actual thickness of hair

$$= \frac{\text{Observed thickness}}{\text{Magnification}} = \frac{3.5}{100} = \mathbf{0.035\ mm.}$$

2.8. *Answer the following :*

(*a*) *You are given a thread and a metre scale. How will you estimate the diameter of the thread ?*

(*b*) *A screw gauge has a pitch of 1.0 mm and 200 divisions on the circular scale. Do you think it is possible to increase the accuracy of the screw gauge arbitrarily by increasing the number of divisions on the circular scale ?*

(*c*) *The mean diameter of a thin brass rod is to be measured by vernier calipers. Why is a set of 100 measurements of the diameter expected to yield a more reliable estimate than a set of 5 measurements only ?*

Ans. (*a*) The thread is wound on the metre scale so that its turns are as close as possible. Thickness '*l*' of the thread coil is measured and the number of turns '*n*' of the thread coil is counted.

$$\therefore \quad \text{Thickness of thread} = \frac{l}{n}\ \text{cm.}$$

(*b*) Least count of a screw gauge

$$= \frac{\text{Pitch}}{\text{Number of divisions on circular scale}}$$

Theoretically, it appears that the least count can be decreased by increasing the number of divisions on the circular scale. Practically, it may not be possible to take the reading precisely due to low resolution of human eye.

(*c*) Larger the number of readings, closer is the arithmetic mean to the true value and hence smaller the random error. Hence result with a set of 100 measurements is more reliable than that with a set of 5 measurements.

2.9. *The photograph of a house occupies an area of 1.75 cm² on a 35 mm slide. The slide is projected on to a screen and the area of the house on the screen is 1.55 m². What is the linear magnification of the projector-screen arrangement ?*

Ans. Size of object $= 1.75\ \text{cm}^2 = 1.75 \times 10^{-4}\ \text{m}^2$

Size of image $= 1.55\ \text{m}^2$

Areal magnification

$$= \frac{\text{Size of image}}{\text{Size of object}} = \frac{1.55}{1.75 \times 10^{-4}} = 8857$$

Linear magnification

$$= \sqrt{\text{Areal magnification}} = \sqrt{8857} = \mathbf{94.1.}$$

2.10. *State the number of significant figures in the following :*

(*i*) $0.007\ m^2$ (*ii*) $2.64 \times 10^{24}\ kg$

(*iii*) $0.2370\ g\ cm^{-3}$ (*iv*) $6.320\ J$

(*v*) $6.032\ Nm^{-2}$ (*vi*) $0.0006032\ m^2$

Ans. (*i*) One : 7 (*ii*) Three : 2, 6, 4

(*iii*) Four : 2, 3, 7, 0 (*iv*) Four : 6, 3, 2, 0

(*v*) Four : 6, 0, 3, 2 (*vi*) Four 6, 0, 3, 2

2.11. *The length, breadth and thickness of a rectangular sheet of metal are 4.234 m, 1.005 m and 2.01 cm, respectively. Give the area and volume of the sheet to correct significant figures.*

Ans. Here $l = 4.234$ m, $b = 1.005$ m,

$$h = 2.01\ \text{cm} = 0.0201\ \text{m}$$

Area of sheet $= 2\,(lb + bh + hl)$

$$= 2\,(4.234 \times 1.005 + 1.005 \times 0.0201 + 0.0201 \times 4.234)\ \text{m}^2$$
$$= 2\,(4.25517 + 0.0202005 + 0.0851034)\ \text{m}^2$$
$$= 2 \times 4.3604739\ \text{m}^2 = 8.7209478\ \text{m}^2$$
$$= 8.72\ \text{m}^2 \quad \text{[Rounded off upto 3 significant figures]}$$

Volume of the sheet

$$= lbh = 4.234 \times 1.005 \times 0.0201\ \text{m}^3$$
$$= 0.0855289\ \text{m}^3 = \mathbf{0.0855\ m^3.}$$

[Rounded off upto three significant figures]

2.12. *The mass of a box measured by a grocer's balance is 2.3 kg. Two gold pieces of masses 20.15 g and 20.17 g are added to the box. What is (a) the total mass of the box and (b) the difference in the masses of the pieces to correct significant figures ?*

Ans. (*a*) Total mass of the box

$$= 2.3\ \text{kg} + 0.02015\ \text{kg} + 0.02017\ \text{kg}$$
$$= 2.34032\ \text{kg} = \mathbf{2.3\ kg.}$$

The result has been rounded off to first place of decimal because mass (2.3 kg) of box has digits upto this place of decimal.

(*b*) Difference in masses of 2 gold pieces

$$= 20.17\ \text{g} - 20.15\ \text{g} = \mathbf{0.02\ g.}$$

2.13. *A physical quantity P is related to four observations : a, b, c and d as follows :* $P = a^3 b^2 / \sqrt{c}\,d$

The percentage errors of measurement in a, b, c and d are 1%, 3%, 4% and 2% respectively. What is the percentage error in the quantity P ? If the value of P calculated using the above relation turns out to be 3.763 , to what value should you round off the result ? **[Central Schools 12 ; Delhi 09]**

Ans. Given : $P = \dfrac{a^3\, b^2}{\sqrt{c}\; d}$

The percentage error in the quantity P is given by

$$100 \times \frac{\Delta P}{P} = 3 \times 100 \cdot \frac{\Delta a}{a} + 2 \times 100 \cdot \frac{\Delta b}{b} + \frac{1}{2} \times 100 \cdot \frac{\Delta c}{c} + 100 \times \frac{\Delta d}{d}$$

$$= 3 \times 1\% + 2 \times 3\% + \frac{1}{2} \times 4\% + 2\% = \mathbf{13\%.}$$

Since 13% = 0.13, so there are two significant figures in the percentage error. Hence P should also be rounded off to 2 significant figures.

$\therefore \quad P = 3.763 = \mathbf{3.8}.$

2.14. *A book with many printing errors contains four different formulae for the displacement y of a particle undergoing a certain periodic motion :*

(*i*) $y = a \sin \dfrac{2\pi t}{T}$, $\qquad$ (*ii*) $y = a \sin vt$,

(*iii*) $y = \left(\dfrac{a}{T}\right) \sin \dfrac{t}{a}$

(*iv*) $y = \left(\dfrac{a}{\sqrt{2}}\right)\left(\sin \dfrac{2\pi t}{T} + \cos \dfrac{2\pi t}{T}\right)$

(a = maximum displacement of the particle, v = speed of the particle, T = time-period of motion). Rule out the wrong formula on dimensional grounds.

Ans. Dimensions of LHS in all the cases (*i*) to (*iv*) = L

Dimensions of RHS in different cases are as follows :

(*i*) $\left[a \sin \dfrac{2\pi t}{\mathrm{T}}\right] = \mathrm{L} \,.\, \sin \dfrac{\mathrm{T}}{\mathrm{T}} = \mathrm{L}$

(Angle $\dfrac{2\pi t}{\mathrm{T}}$ is dimensionless)

This relation is dimensionally correct.

(*ii*) $[a \sin vt] = \mathrm{L} \sin (\mathrm{LT}^{-1}\ \mathrm{T}) = \mathrm{L} \sin (\mathrm{L})$

(Angle is not dimensionless)

This relation is dimensionally wrong.

(*iii*) $\left[\left(\dfrac{a}{\mathrm{T}}\right) \sin \dfrac{t}{a}\right] = \dfrac{\mathrm{L}}{\mathrm{T}} \sin \dfrac{\mathrm{T}}{\mathrm{L}}$

(Angle is not dimensionless)

This relation is dimensionally wrong.

(*iv*) $\left[\dfrac{a}{\sqrt{2}}\left(\sin \dfrac{2\pi t}{\mathrm{T}} + \cos \dfrac{2\pi t}{\mathrm{T}}\right)\right] = \mathrm{L}\left[\sin \dfrac{\mathrm{T}}{\mathrm{T}} + \cos \dfrac{\mathrm{T}}{\mathrm{T}}\right]$

(Angle is dimensionless)

This relation is dimensionally correct.

Hence formulae (*ii*) and (*iii*) are dimensionally wrong.

2.15. *A famous relation in physics relates 'moving mass' m to the 'rest mass' m_0 of a particle in terms of its speed v and the speed of light c. (This relation first arose as a consequence of special relatively due to Albert Einstein). A boy recalls the relation almost correctly but forgets where to put the constant c. He writes :* $\quad m = \dfrac{m_0}{(1 - v^2)^{1/2}}$

Guess where to put the missing c.

Ans. Since quantities of similar nature can only be added or subtracted, v^2 cannot be subtracted from dimensionless constant 1. It should be divided by c^2 so as to make it dimensionless. Hence the corrected relation is

$$m = \frac{m_0}{\left(1 - \dfrac{v^2}{c^2}\right)^{1/2}}.$$

2.16. *The radius of a hydrogen atom is about 0.5 Å. What is the total atomic volume in m^3 of a mole of hydrogen atoms ?*

Ans. Radius of a hydrogen atom,

$$r = 0.5\,\text{Å} = 0.5 \times 10^{-10}\ \mathrm{m}$$

Volume of one atom $= \dfrac{4}{3}\pi r^3$

No. of atoms in 1 mole $= 6.023 \times 10^{23}$

Volume of 1 mole of H-atoms $= N \times \dfrac{4}{3}\pi r^3$

$= 6.023 \times 10^{23} \times \dfrac{4}{3} \times 3.14 \times (0.5 \times 10^{-10})^3$

$= 3.154 \times 10^{-7}\ \mathrm{m}^3 \simeq \mathbf{3 \times 10^{-7}\ m^3}.$

2.17. *One mole of an ideal gas at S.T.P. occupies 22.4 L. What is the ratio of molar volume to the atomic volume of a mole of hydrogen ? Why is this ratio so large ? Take the radius of hydrogen molecule to be 1 Å.*

Ans. Radius of a hydrogen molecule

$= 1\,\text{Å} = 10^{-10}\ \mathrm{m}$

Atomic volume of a mole of hydrogen

= Avogadro's no.

× Volume of a hydrogen molecule

$= 6.023 \times 10^{23} \times \dfrac{4}{3} \times 3.14 \times (10^{-10})^3$

$= 25.2 \times 10^{-7}\ \mathrm{m}^3$

Molar volume $= 22.4\ \mathrm{L} = 22.4 \times 10^{-3}\ \mathrm{m}^3$

$$\therefore \quad \frac{\text{Molar volume}}{\text{Atomic volume}} = \frac{22.4 \times 10^{-3}}{25.2 \times 10^{-7}} \simeq 0.89 \times 10^4 \simeq \mathbf{10^4}.$$

This ratio is large because the actual size of the gas molecules is negligibly small in comparison with the intermolecular separation.

2.18. *Explain why on looking through the window of a fast moving train, the nearby trees and electric poles etc. appear to run in direction opposite to that of motion of the train, while far off houses, hilltops, Moon, stars etc. appear stationary.*

Ans. The line joining the object and the eye is called the line of slight. The direction of the line of sight of the nearby objects like trees, poles etc. ; changes very rapidly due to fast motion of the train and accordingly they appear to be moving opposite to the direction of motion of the train. But the line of sight of a distant object almost does not change its direction due to its extremely large distance from the eye. Hence the distant objects like hilltops, moon, stars etc. appear stationary.

2.19. *A parsec is a convenient unit of length on the astronomical scale. It is the distance of an object that will show a parallax of 1″ (second) of arc from opposite ends of a baseline equal to the distance from the earth to the sun. How much is parsec in terms of metres ?*

Ans. One parsec is the distance at which an arc of length 1 AU makes an angle of 1 second of an arc.

As $\theta(\text{rad}) = \dfrac{\text{Arc}}{\text{Radius}} = \dfrac{l}{r} \quad \therefore \quad r = \dfrac{l}{\theta}$.

Here $l = 1\text{ AU} = 1.496 \times 10^{11}$ m

$$\theta = 1\text{ s of arc} = \frac{\pi}{60 \times 60 \times 180}\text{ rad}$$

$$= 4.85 \times 10^{-6}\text{ rad}$$

$$\therefore \quad 1\text{ parsec} = r = \frac{1.496 \times 10^{11}}{4.85 \times 10^{-6}} = \mathbf{3.08 \times 10^{16}\ m.}$$

Order of magnitude of parsec = **16**.

2.20. *The nearest star (Alpha Centauri) to our solar system is 4.29 light years away. How much is this distance in terms of parsec ? How much parallax would this star show when viewed from two locations of the earth six months apart in its orbit around the sun ?*

Ans. As 1 light year $= 9.46 \times 10^{15}$ m,

$$1\text{ parsec} = 3.08 \times 10^{16}\text{ m}$$

$\therefore$ Distance of Alpha Centauri from the earth,

$$S = 4.29\text{ light years} = 4.29 \times 9.46 \times 10^{15}\text{ m}$$

$$= \frac{4.29 \times 9.46 \times 10^{15}}{3.08 \times 10^{16}}\text{ parsec} = 1.32\text{ parsec}$$

In an orbit around the sun, the distance between the two locations of the earth six months apart,

b = Diameter of the earth's orbit = 2 AU

Parallax of the star,

$$\theta = \frac{\text{Arc}}{\text{Radius}} = \frac{b}{S} = \frac{2\text{ AU}}{1.32\text{ parsec}} = \mathbf{1.515\ s\ of\ arc.}$$

2.21. *Precise measurements of physical quantities are a need of science. For example, to ascertain the speed of an aircraft, one must have an accurate method to find its positions at closely separated instants of time. This was the actual motivation behind the discovery of radar in World War II. Think of different examples in modern science where precise measurements of length, time, mass etc. are needed. Also, wherever you can, give a quantitative idea of the precision needed.*

Ans. Some of the examples of modern science, where precise measurements play an important role, are as follows :

1. Electron microscope uses an electron beam of wavelength 0.2 Å to study very minute objects like viruses, microbes and the crystal structure of solids.
2. The successful launching of artificial satellites has been made possible only due to the precise technique available for accurate measurement of time-intervals.
3. The precision with which the distances are measured in Michelson-Morley Interferometer helped in discarding the idea of hypothetical medium ether and in developing the Theory of Relativity by Einstein.

2.22. *Just as precise measurements are necessary in science, it is equally important to be able to make rough estiamtes of quantities using rudimentary ideas and common observations. Think of ways by which you can estimate the following (where an estimate is difficult to obtain, try to get an upper bound on the quantity) :*

(*a*) *the total mass of rain-bearing clouds over India during the Monsoon*

(*b*) *the mass of an elephant*

(*c*) *the wind speed during a storm*

(*d*) *the number of strands of hair on your head*

(*e*) *the number of air molecules in your classroom.*

Ans. (*a*) The average rainfall during the Monsoon in India is about 100 cm or 1 m.

Total surface area of India

$$= 3.3 \times 10^6\text{ km}^2 = 3.3 \times 10^6 \times (10^3\text{ m})^2$$
$$= 3.3 \times 10^{12}\text{ m}^2.$$

Volume of rain water,

$$V = Ah = 3.3 \times 10^{12}\text{ m}^2 \times 1\text{ m} = 3.3 \times 10^{12}\text{ m}^3$$

Density of water, $\rho = 10^3\text{ kg m}^{-3}$

Hence total mass of rain-bearing clouds over India,

$$m = V\rho = 3.3 \times 10^{12} \times 10^3 = \mathbf{3.3 \times 10^{15}\ kg.}$$

(*b*) To estimate the mass of an elephant, consider a boat of base area A in a river. Let x_1 be the depth of the boat inside water. Now move the elephant into the boat. Again measure the depth x_2 of the boat inside water.

Volume of water displaced by elephant

$$V = A(x_2 - x_1)$$

According to Archimedes' principle, mass of the elephant is

m = Mass of water displaced by the elephant

$$= V\rho = A(x_2 - x_1)\rho$$

Mass of an elephant is about 10^3 kg.

(*c*) The wind speed during a storm can be measured by floating a gas filled balloon in air. When there is no wind storm, suppose the balloon is at vertical height $OA = h$, from the ground. Due to the wind storm, suppose the balloon moves to position B in a small time interval t, as shown in Fig. 2.16.

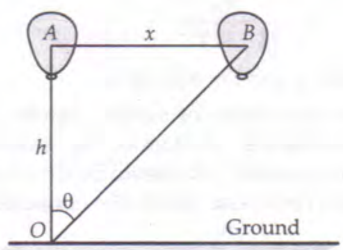


Fig. 2.16

If θ is the angle of drift of the balloon, then from right angled ΔOAB,

$$\tan\theta = \frac{AB}{OA} = \frac{x}{h} \qquad [AB = x, \text{ say}]$$

or $\qquad x = h\tan\theta$

Hence the wind speed during the storm,

$$v = \frac{x}{t} = \frac{h\tan\theta}{t}.$$

(*d*) First we count the strands of hair of 1 cm^2 area of the head. Then by multiplying it by the total area of the head, we can estimate the total number of stands of hair on the head. Its order of magnitude may be as large as 10^8.

(*e*) The dimensions of a typical classroom are

$$8\text{ m} \times 6\text{ m} \times 4\text{ m}$$

Volume of the class room $= 8 \times 6 \times 4 = 192\,\text{m}^3$

Now one mole of air molecules occupy a volume of 22.4 litres or 22.4×10^{-3} m^3.

$\therefore$ Number of molecules in 22.4×10^{-3} m^3

$$= 6.023 \times 10^{23}$$

Number of molecules in the classroom

$$= \frac{6.023 \times 10^{23}}{22.4 \times 10^{-3}} \times 192 = 51.6 \times 10^{26} \text{ molecules}$$

$$\approx \mathbf{10^{28}} \textbf{ molecules.}$$

2.23. *The sun is a hot plasma (ionized matter) with its inner core at a temperature exceeding 10^7 K, and its outer surface at a temperature of about 6000 K. At these high temperatures no substance remains in a solid or liquid phase. In what range do you expect the mass density of the sun to be ? In the range of densities of solids and liquids or gases ? Check if your guess is correct from the following data : mass of the sun = 2.0×10^{30} kg, radius of the sun = 7.0×10^8 m.*

Ans. Mass of the sun, $\quad M = 2.0 \times 10^{30}$ kg

Radius of the sun, $\quad R = 7.0 \times 10^8$ m

Volume of the sun,

$$V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi \times (7.0 \times 10^8)^3$$

$$= 1.437 \times 10^{27} \text{ m}^3$$

Density of the sun,

$$\rho = \frac{M}{V} = \frac{2.0 \times 10^{30}}{1.437 \times 10^{27}}$$

$$= 1391.8 \text{ kg m}^{-3} \approx \mathbf{1.4 \times 10^3 \text{ kg m}^{-3}}.$$

The density of the sun is in the range of the densities of the solids and liquids but not gases. The high density is due to the invard gravitational attraction on the outer layers due to the inward layers of the sun.

2.24. *When the planet Jupiter is at a distance of 824.7 million kilometres from the earth, its angular diameter is measured to be 35.72 s of arc. Calculate the diameter of Jupiter.*

Ans. Distance of Jupiter from the earth,

$$S = 824.7 \times 10^6 \text{ km}$$

Angular diameter of Jupiter,

$$\theta = 35.72'' = \left(\frac{35.72}{60 \times 60}\right)^\circ = \frac{35.72}{3600} \times \frac{\pi}{180} \text{ rad}$$

Diameter of Jupiter,

$$D = S \times \theta = 824.7 \times 10^6 \times \frac{35.72}{3600} \times \frac{\pi}{180}$$

$$= \mathbf{1.428 \times 10^5 \text{ km}}.$$

2.25. *A man walking briskly in rain with speed v must slant his umbrella forward making an angle θ with the vertical. A student derives the following relation between θ and v : $\tan\theta = v$ and checks that the relation has a correct limit : as $v \to 0$, $\theta \to 0$, as expected. (We are assuming there is no strong wind and that the rain falls vertically for a stationary man). Do you think this relation can be correct ? If not, guess at the correct relation.*

Ans. Since trigonometric functions are dimensionless,

$\therefore \qquad [\tan\theta] = 1$

But $\qquad [v] = \text{LT}^{-1}$

$\therefore$ Dimensions of LHS $\neq$ Dimensions of RHS

Hence the given relation is dimensionally wrong.

This relation can be corrected by dividing RHS by the speed 'u' of the rainfall. So the corrected relation is

$$\tan\theta = \frac{v}{u}.$$

2.26. *It is claimed that two cesium clocks, if allowed to run for 100 years, free from any disturbance, may differ by only about 0.02 s. What does this imply for the accuracy of the standard cesium clock in measuring a time-interval of 1 s ?*

Ans. Here $\Delta t = 0.02$ s,

$$t = 100 \text{ years} = 100 \times 365.25 \times 86{,}400 \text{ s}$$

Fractional error

$$= \frac{\Delta t}{t} = \frac{0.02}{100 \times 365.25 \times 86400} = 0.63 \times 10^{-11}$$

So there is an accuracy of 10^{-11} part in 1s or 1s in 10^{11} s.

2.27. *Estimate the average density of a sodium atom assuming its radius to be about 2.5 Å. Compare it with the density of sodium in crystalline phase : 970 kg m^{-3}. Are the two densities of the same order of magnitude ? If so, why ?*

Ans. Radius of a sodium atom,

$$r = 2.5 \text{ Å} = 2.5 \times 10^{-10} \text{ m}$$

Volume of a sodium atom,

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times (2.5 \times 10^{-10})^3$$

$$= 65.42 \times 10^{-30} \text{ kg}$$

Mass of a sodium atom

$$= \frac{\text{Mass number}}{\text{Avogadro's number}} = \frac{23}{6.02 \times 10^{23}} \text{ g}$$

$$= 3.82 \times 10^{-23} \text{ g} = 3.82 \times 10^{-26} \text{ kg}$$

Average density of sodium atom

$$= \frac{\text{Mass}}{\text{Volume}} = \frac{3.82 \times 10^{-26}}{65.42 \times 10^{-30}}$$

$$= 0.58 \times 10^3 \text{ kg m}^{-3}$$

Density of sodium in crystalline phase

$$= 970 \text{ kg m}^{-3} = 0.970 \times 10^3 \text{ kg m}^{-3}$$

Hence the average mass density of sodium atom and the density of crystalline sodium are of the same order of magnitude (10^3). This is because sodium atoms in crystalline phase are closely packed.

2.28. *The unit of length convenient on the nuclear scale is a fermi : 1 $f = 10^{-15}$ m. Nuclear sizes obey roughly the following empirical relation $r = r_0 A^{1/3}$; where r is the radius of the nucleus, A its mass number and r_0 is a constant equal to about 1.2 f. Show that the rule implies that nuclear mass density is nearly constant for different nuclei. Estimate the mass density of sodium nucleus. Compare it with the average mass density of a sodium atom obtained in Exercise 2.27.*

Ans. Radius of a nucleus, $r = r_0 A^{1/3}$

$$\text{Mass of a nucleus} = \frac{\text{Mass number}}{\text{Avogadro's number}} = \frac{A}{N_A}$$

$$\text{Nuclear mass density} = \frac{\text{Mass of a nucleus}}{\text{Volume of a nucleus}}$$

or
$$\rho = \frac{A}{N_A \cdot \frac{4}{3}\pi r^3} = \frac{A}{N_A \cdot \frac{4}{3}\pi (r_0 A^{1/3})^3} = \frac{3}{4\pi N_A r_0^3}$$

As ρ is independent of A, so nuclear mass density is same for different nuclei.

For a kilomole,

$$N_A = 6.02 \times 10^{26}, \; r_0 = 1.2 \; f = 1.2 \times 10^{-15} \text{ m}$$

$$\therefore \quad \rho = \frac{3}{4\pi \times 6.02 \times 10^{26} \times (1.2 \times 10^{-15})^3}$$

$$= \mathbf{2.3 \times 10^{17} \text{ kg m}^{-3}}$$

Density of sodium nucleus should also be

$$= 2.3 \times 10^{17} \text{ kg m}^{-3}$$

From Exercise 2.27, density of sodium atom

$$= 0.58 \times 10^3 \text{ kg m}^{-3}$$

$$\therefore \quad \frac{\text{Nuclear mass density}}{\text{Atomic mass density}} = \frac{2.3 \times 10^{17}}{0.58 \times 10^3} = 3.96 \times 10^{14}.$$

Nuclear density is typically 10^{15} times atomic density of matter.

2.29. *A laser is a source of very intense, monochromatic, and unidirectional beam of light. These properties of a laser light can be exploited to measure long distances. The distance of the Moon from the Earth has been already determined very precisely using a laser as a source of light. A laser light beamed at the moon takes 2.56 s to return after reflection at the moon's surface. How much is the radius of the lunar orbit around the earth ?*

Ans. Here $t = 2.56$ s, $\quad c = 3 \times 10^8 \text{ ms}^{-1}$

Radius of the lunar orbit around the earth

= Distance of the moon from the earth

$$= \frac{c \times t}{2} = \frac{3 \times 10^8 \times 2.56}{2} = \mathbf{3.84 \times 10^8 \text{ m.}}$$

2.30. *A SONAR (Sound Navigation and Ranging) uses ultrasonic waves to detect and locate objects under water. In a submarine equipped with a SONAR, the time delay between generation of a probe wave and the reception of its echo after reflection from an enemy submarine is found to be 77 s. What is the distance of the enemy submarine ?*

(Speed of sound in water = 1450 ms^{-1}).

Ans. Here $t = 77$ s, $\quad c = 1450 \text{ ms}^{-1}$

Distance of enemy submarine

$$= \frac{c \times t}{2} = \frac{1450 \times 77}{2} = \mathbf{55825 \text{ m.}}$$

2.31. *The farthest objects (known as quasers) in our universe are so distant that light emitted by them takes billion of years to reach the earth. What is the distance in km of a quaser from which light takes 3.0 billion years to reach us ?*

Ans. Here t = 3.0 billion years

$$= 3.0 \times 10^9 \times 365.25 \times 24 \times 60 \times 60 \text{ s}$$

Speed of light, $c = 3 \times 10^5 \text{ kms}^{-1}$

Distance of quaser

$$= ct = 3 \times 10^5 \times 3.0 \times 10^9 \times 365.25 \times 24 \times 60 \times 60$$

$$= \mathbf{2.84 \times 10^{22} \text{ km.}}$$

2.32. *It is a well known fact that during a solar eclipse the disc of the moon almost completely covers the disc of the sun. From this fact and from the information that sun's angular distance α is measured to be 1920'', determine the approximate diameter of the moon. Given earth-moon distance = 3.8452×10^8 m.*

Ans. During total solar eclipse, the disc of the moon completely covers the disc of the sun, so the angular diameters of both the sun and the moon must be equal.

∴ Angular diameter of the moon,

θ = Angular diameter of the sun

$$= 1920'' = 1920 \times 4.85 \times 10^{-6} \text{ rad}$$

$$[\because \; 1'' = 4.85 \times 10^{-6} \text{ rad}]$$

Earth-moon distance, $S = 3.8452 \times 10^8$ m

Diameter of the moon,

$$D = \theta \times S = 1920 \times 4.85 \times 10^{-6} \times 3.8452 \times 10^8$$

$$= 3.581 \times 10^6 \text{ m} = \mathbf{3581 \text{ km.}}$$

2.33. *A great physicist of this century (P.A.M. Dirac) loved playing with numerical values of Fundamental constants of nature. This led him to an interesting observation. Dirac found that from the basic constants of atomic physics (c, e, mass of electron, mass of proton) and the gravitational constant G, he could arrive at a number with the dimension of time. Further, it was a very large number, its magnitude being close to the present estimate on the age of the universe (≈ 15 billion years).*

From the table of fundamental constants in this book, try to see if you too can construct this number (or any other interesting number you can think of). If its coincidence with the age of the universe were significant, what would this imply for the constancy of fundamental constant ?

Ans. Using basic constants such as speed of light (c), charge on electron (e), mass of electron (m_e), mass of proton (m_p) and gravitational constant (G), we can construct the quantity,

$$\tau = \left(\frac{e^2}{4\pi\varepsilon_0}\right)^2 \times \frac{1}{m_p m_e^2 c^3 G}$$

$$\text{Now } \left[\frac{e^2}{4\pi\varepsilon_0}\right] = \left[\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}r^2\right] = [Fr^2]$$

$$= [MLT^{-2}.L^2] = [ML^3T^{-2}]$$

$$\therefore \quad [\tau] = \frac{[ML^3T^{-2}]^2}{[M][M]^2[LT^{-1}]^3[M^{-1}L^3T^{-2}]} = [T]$$

Clearly, the quantity τ has the dimensions of time.

Put $G = 6.67 \times 10^{-11}$ Nm^2 kg^{-2}, $c = 3 \times 10^8$ ms^{-1}, $e = 1.6 \times 10^{-19}$ C, $m_e = 9.1 \times 10^{-31}$ kg, $m_p = 1.67 \times 10^{-27}$ kg and $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ Nm^{2}C^{-2}

$$\therefore \quad \tau = \frac{[9\times10^9 \times (1.6\times10^{-19})^2]^2}{1.67\times10^{-27} \times (9.1\times10^{-31})^2 \times (3\times10^8)^3 \times 6.67\times10^{-11}}$$

$$= 2.13 \times 10^{16} \text{ s}$$

$$= \frac{2.13\times10^{16}}{3.156\times10^7} \text{ years} = 0.667\times10^9 \text{ years.}$$

$$= 0.667 \text{ billion years.}$$

This time is slightly less than the age of the universe ($\approx$ 15 billion years). It implies that the values of the basic constants of physics should change with time because the age of the universe increases with time.

Text Based Exercises

Type A : Very Short Answer Questions

1 Mark Each

1. What is a physical unit ? **[Himachal 01]**
2. Write the relationship between numerical value of a quantity and the size of the unit.
3. Does the magnitude of a physical quantity change with the change in the system of units.
4. Define length.
5. Define a prototype metre.
6. Define standard metre in terms of the wavelength of light.
7. What is the accuracy of the metre defined in terms of wavelength of light radiation ?
8. Define standard metre in terms of velocity of light.
9. Why was metre redefined in terms of velocity of light in 1983 ?
10. Is light year a unit of time ?
11. Define light year and express it in metres. **[Himachal 2K]**
12. Define one astronomical unit. **[Himachal 06C]**
13. Name the unit used to express the distances of stars.
14. Define parsec and express it in metres. **[Himachal 02, 05]**
15. Express parsec in light years.
16. Arrange in ascending order-astronomical unit, parsec and light year.
17. Name two commonly units used to express wavelength of light.
18. Express one micron in metre. **[Delhi 10]**
19. Name the unit used to measure size of a nucleus and express it in metre.
20. Express nanometre in terms of metre and angstrom units.
21. Express the wavelength of yellow light (5893 Å) in terms of nm.
22. How many nanometres are there in one metre ? **[Himachal 07C, 08C]**
23. How many angstrom are there in one metre ? **[Himachal 07C, 08C]**
24. How many fermi are there in one metre ? **[Himachal 07C]**
25. How many light years are there in one metre ? **[Himachal 06C]**
26. Write the full name of the technique used in locating (*a*) an under-water obstacle (*b*) position of an aeroplane in space. **[Chandigarh 07]**
27. How much far away is the nearest star alpha centuri from us ?
28. What is the order of size of our galaxy ?
29. Give the order of mean free path of an air molecule.
30. What is the shortest distance measured indirectly so far ?
31. What is the estimated radius of the universe ?
32. Suggest a distance corresponding to 10^7 m.
33. Name the unit used for measuring nuclear cross-sections.
34. Name the device used for measuring directly the lengths (*i*) from 10^{-3} m to 10^2 m (*ii*) to an accuracy of 10^{-4} m (*iii*) to an accuracy of 10^{-5} m.

35. Name the device that can be used to measure the number of wavelengths of light in a given distance.
36. What does the word RADAR stand for ?
37. What does the word SONAR stand for ?
38. What does the word LASER stand for ?
39. What is a laser ?
40. Express light year in metres. What is its order of magnitude ?
41. What types of waves are used in a SONAR ?
42. Define international standard of mass.
43. Define atomic mass unit. Express it in kg.
44. Are the inertial and gravitational masses of an object different from one another ?
45. It is said that common balance compares masses and spring balance measures weight. If a body is 2 kg in common balance, it is 2 kg in spring balance, does it mean that mass is equal to weight ?
46. How many times larger is kg than a mg ? **[Himachal 03, 06C]**
47. How many metric tons are there in a teragram ?
48. What is the order of mass of an electron ?
49. What is the order of mass of universe ?
50. What is the smallest mass measured indirectly so far ?
51. Define second in terms of casium-133 vibrations.
52. Human heart is an in-built clock. Comment.
53. Which is the most accurate atomic clock ?
54. Name the unit used for measuring very small time intervals. How is it related to second ?
55. Which technique is used for measuring age of fossils, rocks, etc.
56. How many times is millisecond larger than a microsecond ?
57. Are there more microseconds in a second than the number of seconds in a year ?
58. What is the order of age of the earth ?
59. Give the order of average life of a human being.
60. What is the shortest time interval measured indirectly so far ?
61. Name the SI unit of (*i*) temperature and (*ii*) electric current.
62. Name and define the SI unit of luminous intensity.
63. Define one mole.
64. What is leap year ?
65. Name the year in which there is total solar eclipse.
66. The density of wood is $0.5\ \text{cm}^{-3}$. Write this value in SI units.
67. The radius of the sun is 696 000 000 m. Express it in scientific notation (in powers of 10).
68. Express 0.00000538 in powers of 10.
69. Define dimensions of a physical quantity.
70. State the principle of homogeneity of dimensions.
71. Which of the following has the same dimensions as 'Planck's constant' : Torque, gravitational constant and angular momentum ?
72. What are the dimensions of rate of flow ?
73. Give names of a scalar quantity and a vector quantity which have same dimensions.
74. What is the difference between the measurements 4.0 cm and 4.000 cm ?
75. What importance is attached to the final zeros in a number without any decimal point ?
76. If $f = x^3$, then relative error in f would be how many times the relative error in x ?
77. A junior research fellow takes 100 careful readings in an experiment. If he repeats the same experiment by taking 500 readings, then by what factor will the probable error be reduced ?
78. Give two examples of non-dimensional variables.
79. Write the names of three dimensional constants.
80. Write two examples of non-dimensional constants.
81. If g is the acceleration due to gravity and λ is wavelength, then which physical quantity does $\sqrt{\lambda g}$ represent ?
82. Write the dimensional formula of wavelength and frequency of a wave. **[Delhi 09]**
83. If $x = a + bt^2$, where x is in metres and t in seconds, find the units of a and b. **[Himachal 08]**
84. If $x = at + bt^2$, where x is in metres and t in hours (hr), what will be the units of 'a' and 'b' ? **[Central Schools 08 ; Delhi 10]**
85. Give the no. of significant figures in 6.200×10^0 sec. **[Central Schools 07]**
86. How can a systematic error be eliminated ?
87. How can we eliminate constant error ?
88. Name the SI unit used to express the amount of substance. **[Central Schools 04]**
89. Write the number of significant figures in each of the following measurements :
(*a*) 1.67×10^{-27} kg ; (*b*) 0.270 cm **[Delhi 04]**
90. Add 8.2 and 10.163 and round off the sum to two significant figures. **[Delhi 06]**
91. Give approximate ratio of 1 AU and 1 light year. **[Delhi 01]**
92. How many kilograms are there in 1 amu ? **[Himachal 2K]**
93. What do you understand by absolute error ? **[Himachal 04]**

Answers

1. The standard amount of a physical quantity chosen to measure the physical quantity of the same kind is called a physical unit.
2. The numerical value (n) of a physical quantity is inversely proportional to the size of the unit (u) *i.e.*,
$$n \propto \frac{1}{u} \quad \text{or} \quad nu = \text{constant}.$$
3. No, the magnitude of a physical quantity does not change with the change in the system of units.
4. Length may be defined as distance of separation between two points in space.
5. In 1899, the general conference of weights and measures defined metre as the distance between two transverse lines marked on a platinum-iridium rod preserved at a constant temperature of 273.16 K at 1 bar pressure in the International Bureau of Weights and Measures at Sevres, near Paris in France.
6. Standard metre is defined as the distance which contains 1650,763.73 wavelengths of certain orange-red radiation emitted by Kr-86 atom in its transition between $2p_{10}$ and $2d_5$ levels.
7. Standard metre defined in terms of the wavelength of light radiation has an accuracy of 1 part in 10^9.
8. The velocity of light in vacuum, $c = 299, 792, 458\ \text{ms}^{-1}$. So one metre is defined as the length of the path travelled by light in vacuum in 1/ 299, 792, 458 of a second.
9. Because of the tremendous accuracy (1 part in 10^{13}) achieved in the measurement of time, metre has been redefined in terms of velocity of light since 1983.
10. No, light year is a unit of distance.
11. One light year is defined as the distance travelled by light in one year.
$$1 \text{ light year} = 9.46 \times 10^{15} \text{ m}$$
12. One astronomical unit (AU) is the mean distance of the earth from the sun.
$$1 \text{ AU} = 1.496 \times 10^{11} \text{ m}.$$
13. Parallactic second (parsec).
14. One parsec is defined as the distance at which an arc of length one astronomical unit subtends an angle of one second of an arc.
$$1 \text{ parsec} = 3.08 \times 10^{16} \text{ m}.$$
15. 1 parsec = 3.26 light years.
16. 1 AU < 1 light year < 1 parsec.
17. nm and Å.
18. One micron is one millionth part of a metre.
$$1 \text{ micron} = 1\mu = 10^{-6} \text{ m}.$$
19. The unit used to measure nuclear size is fermi.
$$1 \text{ fermi} = 10^{-15} \text{ m}.$$
20. 1 nanometre or 1 nm = 10^{-9} m = 10 Å.
21. As 10 Å = 1 nm
$$\therefore 5893 \text{ Å} = \frac{5893}{10} = 589.3 \text{ nm}.$$
22. 1 nanometre (nm) = 10^{-9} m $\quad \therefore$ 1 m = 10^9 nm.
23. 1 angstrom (Å) = 10^{-10} m $\quad \therefore$ 1 m = 10^{10} Å
24. 1 fermi (f) = 10^{-15} m $\quad \therefore$ 1 m = 10^{15} f
25. 1 light year (ly) = 9.46×10^{15} m
$$\therefore 1 \text{ m} = \frac{1}{9.46 \times 10^{15}} \text{ ly} = \mathbf{1.057 \times 10^{-16}\ ly.}$$
26. (*a*) Sound navigation and ranging (SONAR)
(*b*) Radio navigation and ranging (RADAR)
27. Alpha centauri is 4.26 light years away from us.
28. The order of size of our galaxy is 10^{20} m.
29. The order of mean free path of an air molecule is 10^{-6} m.
30. The shortest distance measured indirectly so far is the radius of a proton ($\approx 10^{-15}$ m).
31. The estimated radius of the universe is 10^{25} m.
32. Radius of the earth is the order of 10^7 m.
33. Barn is used to measure nuclear cross-section.
$$1 \text{ barn} = 10^{-28} \text{ m}^2.$$
34. (*i*) Metre-scale (*ii*) Vernier callipers (*iii*) Screw gange or spherometer.
35. Optical interferometer.
36. The word RADAR stands for *radio detection and ranging.*
37. The word SONAR stands for *sound navigation and ranging.*
38. The word LASER stands for *light amplification by stimulated emission of radiation.*
39. A laser a source of very intense, highly monochromatic and highly directional beam of light.
40. 1 light year = 9.46×10^{15} m = 0.946×10^{16} m.
$\therefore$ Order of magnitude of light year = 10^{16} m.
41. Ultrasonic waves.
42. The international standard of mass is kilogram (kg). One kilogram is the mass of a platinum iridium (90% Pt and 10% Ir) cylinder of diameter equal to its height preserved in the International Bureau of Weight and Measures at Sevres.
43. One atomic mass unit is the mass of $\frac{1}{12}$th of the mass of an atom of carbon-12 isotope.
$$1 \text{ amu} = 1.66 \times 10^{-27} \text{ kg}$$
44. No, the inertial and gravitational masses of an object are equivalent.

45. No, mass and weight are two different physical quantities. If mass is 2 kg in common balance, it is 2 kilogram force (kg f) in spring balance.

46. 1 kg = 10^6 mg.

47. 1 Teragram = 10^{12} g = 10^9 kg = 10^6 metric tons.

48. Mass of an electron $\simeq 10^{-30}$ kg

49. Mass of universe $\simeq 10^{55}$ kg

50. The smallest mass measured so far is the mass of an electron ($\approx 10^{-30}$ kg).

51. One second is defined as the duration of 9192, 631, 770 vibrations corresponding to the transition between two hyperfine levels of cesium-133 atom in the ground state.

52. Yes, because human heart beats at a regular rate.

53. Cesium atomic clock.

54. Shake is the unit used for measuring very small time-intervals.

1 shake = 10^{-8} second

55. Radioactive dating.

56. 1 millisecond = 10^3 microsecond.

57. No. As 1 microsecond = 10^{-6} s, therefore

1 second = 10^6 microseconds

Also, no. of seconds in a year

$= 365 \times 24 \times 60 \times 60 = 31.536 \times 10^6$.

Clearly, the number of microseconds in a second is smaller than the number of seconds in a year.

58. Age of earth = 10^{17} second.

59. Average life of a human being $\approx$ 100 years = 10^9 s.

60. The shortest time interval measured so far is the life-span of most unstable nucleus ($\approx 10^{-24}$ s).

61. (*i*) The SI unit of temperature is kelvin (K).

(*ii*) The SI unit of electric current is ampere (A).

62. The SI unit of luminous intensity is candela (cd). One candela is defined as the luminous intensity, in a given direction, of a source that emits a mochromatic radiation of frequency 540×10^{12} hertz and has a radiant intensity in that direction of 1/683 watt per steradian.

63. One mole is that amount of a substance which contains as many elementary entities as there are atoms in 12 g of carbon-12 isotope. These entities may be atoms, molecules, ions, etc.

64. Leap year is the year which is divisible by four. The February month of a leap year has 29 days.

65. Tropical year.

66. Density $= 0.5 \dfrac{\text{g}}{\text{cm}^3} = \dfrac{0.5 \times 10^{-3}\ \text{kg}}{10^{-6}\ \text{m}^3} = 500\ \text{kg m}^{-3}$.

67. Radius of the sun = 696000 000 m

$= \mathbf{6.96 \times 10^8\ m}$.

68. $0.00000538 = 5.38 \times 10^{-6}$.

69. The dimensions of a physical quantity are the powers to which the fundamental quantities must be raised to represent that quantity completely.

70. The principle of homogeneity of dimensions states that a physical equation will be dimensionally correct if the dimensions of all the terms occurring on both sides of the equation are the same.

71. Angular momentum.

72. $[\text{Rate of flow}] = \dfrac{[\text{Volume}]}{[\text{Time}]} = \dfrac{[L^3]}{[T]} = [L^3T^{-1}]$.

73. Speed is a scalar quantity and velocity is a vector quantity. Both have the same dimensions of $[LT^{-1}]$.

74. (*i*) 4.0 cm has two significant figures while 4.000 cm has four significant figures.

(*ii*) 4.0 cm is correct upto first decimal place while 4.000 cm is correct upto third place of decimal.

75. All such zeros are not significant. For example 86,400 has three significant figures.

76. When $f = x^3$, $\dfrac{\Delta f}{f} = 3 \times \dfrac{\Delta x}{x}$

$\therefore$ Relative error in f

= Three times the relative error in x.

77. The probable error will be reduced by a factor of 1/5.

78. Specific gravity and strain.

79. Gravitational constant, Planck's constant and Boltzmann's constant.

80. π, e, all trigonometric functions.

81. $\sqrt{\lambda g} = \sqrt{L \, . \, LT^{-2}} = LT^{-1}$

$\therefore$ $\sqrt{\lambda g}$ represents velocity.

82. $[\lambda] = [M^0L^1T^0]$ and $[v] = [M^0L^0T^{-1}]$

83. Unit of a = unit of x = metre

Unit of b = unit of $(x/t^2) = \text{m}/\text{s}^2$.

84. Unit of a = unit of $(x/t) = \text{m}/\text{h}$

Unit of b = unit of $(x/t^2) = \text{m}/\text{h}^2$.

85. Four : 6, 2, 0, 0

86. The errors which tend to occur in one direction, positive or negative, are called systematic errors. Such an error can be eliminated by detecting the source of error and the rule governing this error.

87. The errors which affect each observation by the same amount are called constant errors. Such an error can be eliminated measuring the same physical quantity by number of different methods, apparatus or technique.

88. Mole

89. (*a*) Three : 1, 6, 7

(*b*) Three : 2, 7, 0

90.
$$\begin{array}{r} 8.2 \\ +\ 10.163 \\ \hline \end{array}$$
Sum = 18.363

Corrected sum = **18**

[Rounded off upto 2 significant figures]

91. $\dfrac{1\text{ AU}}{1\text{ light year}} = \dfrac{1.496 \times 10^{11}\text{ m}}{9.46 \times 10^{15}\text{ m}} \approx 10^{-5}$.

92. $1\text{ amu} = 1.66 \times 10^{-27}$ kg.

93. Refer answer to Q.56 on page 2.40.

Type B : Short Answer Questions — 2 or 3 Marks Each

1. Define the following : (*i*) Light year (*ii*) Parsecond (*iii*) Astronomical unit. **[Himachal 05]**

2. Explain the triangulation method to measure the height of an inaccessible object. **[Himachal 07]**

3. Distinguish between mass and weight. **[Himachal 01, 04]**

4. Distinguish between inertial mass, gravitational mass and weight of a body.

5. Which technique is used for measuring large time intervals ?

6. Define dimensional formula. Give uses of dimensional analysis. Write down the limitations of dimensional analysis. **[Himachal 05]**

7. In what way is the knowledge of the dimensions of a physical quantity useful ?

8. How is a dimensional formula different from a differential equation ?

9. Distinguish between the dimensions and unit of a physical quantity.

10. Differentiate between dimensional and non-dimensional variables.

11. Distinguish between dimensional and non-dimensional constants.

12. Name any three physical quantities having the same dimensions and also give their dimensions. **[Central Schools 07]**

13. What are the dimensional formulae of the following : (*i*) pressure (*ii*) power (*iii*) density (*iv*) angle ? **[Himachal 01]**

14. Write the dimensional formulae of the following physical quantities : (*i*) work (*ii*) angular velocity (*iii*) pressure (*iv*) Planck's constant. **[Himachal 05]**

15. Mention the various sources of occurrence of errors, while taking measurements.

16. State the errors other than random and instrumental errors.

17. Show that the maximum error in the sum of the two quantities is equal to the sum of the absolute errors in the two individual quantities.

18. Show that the absolute error in the difference of two quantities is equal to the sum of the absolute errors in the individual quantities.

19. Show that the maximum fractional error in the product of two quantities is equal to the sum of the fractional errors in the individual quantities.

20. Show that the maximum error in the quotient of two quantities is equal to the sum of their individual fractional errors.

21. Show that the fractional error in the nth power of a quantity is equal to n times the fractional error in the quantity itself. State the general rule for evaluating the error in a combined calculation.

22. What is meant by the term "measurement of a physical quantity ? How is the result of measurement of a physical quantity expressed ? **[Delhi 03C]**

23. State the principle of homogeneity of dimensions. Test the dimensional homogeneity of the following equation :
$$h = h_0 + v_0 t + \frac{1}{2} g t^2$$ **[Delhi 03]**

24. Define least count error. What is the value of least count error associated with the scale in your geometry box ? **[Central Schools 03]**

25. Describe the parallax method to find the distance of an inferior planet from earth. **[Delhi 06]**

26. What is meant by RADAR and SONAR ? How are long distances measured using these techniques ? **[Central Schools 04]**

Answers

1. Refer answer to Q.13 on page 2.5.

2. Refer answer to Q.17 on page 2.8.

3. Refer answer to Q.32 on page 2.16.

4. Inertial mass of a body is the measure of its inertia in translatory motion. It is equal to the force required to produce unit acceleration in the body.

The mass of a body which determines the gravitational pull acting upon it due to the earth is called its gravitational mass. It is equal to force experienced by the body in a gravitational field of unit intensity.

Weight of a body is the force with which a body is attracted towards the centre of the earth.

5. The technique of radioactive dating is used to measure long time intervals by finding the ratio of the number of radioactive atoms that have undergone decay to the number of atoms left undecayed.

6. Refer answer to Q. 44 on page 2.25 and Q.49 on page 2.36.

7. The dimensions of a physical quantity represent the powers of the fundamental units, on which the given physical quantity depends. So we can express its units in terms of the fundamental units. Suppose the dimensions of a physical quantity are 1 in mass, 1 in length and – 2 in time, then its SI unit is kgms^{-2}.

8. Dimensional formula is an expression which shows how and which of the fundamental units of mass, length and time occur in the derived unit of a physical quantity. The equation obtained by equating a physical quantity to its dimensional formula is called its dimensional equation.

9. Dimensions of a physical quantity are the powers to which the fundamental units must be raised to represent the unit of the given physical quantity. The unit is compact mathematical expression involving fundamental units. The unit may also be given some name. *e.g.*, $[F] = [MLT^{-2}]$ and unit of force = kg ms^{-2} or newton.

10. The quantities which have dimensions but do not possess a constant value are called dimensional variables *e.g.*, velocity, force etc. On the other hand, the quantities which have neither dimensions nor they have a constant value are called non-dimensional variables *e.g.*, relative density, strain, etc.

11. The quantities which have dimensions as well as a constant value are called dimensional constants *e.g.*, Planck's constant, Boltzmann's constant. On the other hand, the quantities which have no dimensions but a constant value are called non-dimensional constants *e.g.*, π, $\sin\theta$, $\cos\theta$ etc.

12. Work, energy and torque. Each quantity has the dimensions $[ML^2T^{-2}]$.

13. (*i*) $[ML^{-1}T^{-2}]$ (*ii*) $[ML^2T^{-3}]$
(*iii*) $[ML^{-3}T^0]$ (*iv*) dimensionless.

14. (*i*) $[ML^2T^{-2}]$ (*ii*) $[M^0L^0T^{-1}]$
(*iii*) $[ML^{-1}T^{-2}]$ (*iv*) $[ML^2T^{-1}]$

15. While taking measurements, the sources of occurrence of errors may be as follows :

(*i*) *Instrumental errors* due to faulty graduations, zero error, etc.

(*ii*) *Personal errors* due to personal peculiarities of the experimenter.

(*iii*) *Error due to imperfection* in experimental arrangement, such as loss of heat due to radiation in calorimetry.

(*iv*) *Errors due to external causes* such as expansion of scale due to rise in temperature.

16. The errors other than random and instrumental errors are (*i*) absolute error (*ii*) relative error (*iii*) percentage error and (*iv*) gross error. The gross error may be due to incorrect reading of the instrument or due to incorrect recording of the reading.

17. Refer answer to Q. 57 on page 2.42.

18. Refer answer to Q. 57 on page 2.42.

19. Refer answer to Q. 57 on page 2.42.

20. Refer answer to Q. 57 on page 2.42.

21. Refer answer to Q. 57 on page 2.42.

22. Refer answer to Q. 3 on page 2.2.

23. For principle of homogeneity of dimensions, refer answer to Very Short Answer Q. 70 on page 2.64.

$$[h] = L, \quad [h_0] = L$$

$$[u_0 t] = LT^{-1} \cdot T = L$$

$$[\frac{1}{2} g t^2] = LT^{-2} \cdot T^2 = L$$

As all the terms have the same dimensions, so the given equation is dimensionally homogeneous.

24. Refer answer to Q. 55 on page 2.39.

25. Refer answer to Q. 22 on page 2.10.

26. Refer answer to Q. 27 and Q. 28 on page 2.11.

Competition Section

Units and Measurements

GLIMPSES

1. **Physical quantities.** All those quantities which can be measured directly or indirectly and in terms of which the laws of physics can be expressed are called *physical quantities.*

2. (*i*) **Physical unit.** The standard amount of a physical quantity chosen to measure the physical quantity of the same kind is called a *physical unit.*

   Measure of a physical quantity = Numerical value of the quantity × size of the unit = nu

   (*ii*) **Relationship between the numerical value and the size of the unit.** The numerical value (n) of a physical quantity is inversely proportional to the size of the unit.

   $$n \propto \frac{1}{u} \quad \text{or} \quad nu = \text{constant} \quad \text{or} \quad n_1 u_1 = n_2 u_2 .$$

3. **Fundamental quantities.** The physical quantities which can be treated as independent of other physical quantities and are not usually defined in terms of other physical quantities are called *fundamental quantities.* The seven fundamental quantities are mass, length, time, electric current, temperature, luminous intensity and amount of substance.

4. **Derived quantities.** The physical quantities whose defining operations are based on other physical quantities are called derived quantities.

5. **Fundamental units.** The physical units which can neither be derived from one another, nor they can be further resolved into more simpler units are called *fundamental units.* The units of length, mass, time, etc. are fundamental units.

6. **Derived units.** All other units which can be expressed in terms of fundamental units are called *derived units.*

7. **System of units.** A complete set of units which is used for measuring all kinds of fundamental and derived quantities is called a *system of units.*

   (*i*) ***The CGS or the metric system.*** In this system the fundamental units of length, mass and time are centimetre, gram and second respectively.

   (*ii*) ***The FPS or the British system.*** In this system the fundamental units of length, mass and time are foot, pound and second respectively.

   (*iii*) ***The MKS system.*** In this system the fundamental units of length, mass and time are metre, kilogram and second respectively.

   (*iv*) ***The SI.*** SI is the abbreviation for *Systeme Internationale d'Unites,* which is the French equivalent for *international system of units.* In this system the fundamental units of length, mass, time, electric current, temperature, luminous intensity and amount of substance are metre, kilogram, second, ampere, kelvin, candela and mole, respectively.

8. **The seven basic SI units.**

   (*i*) ***Metre (m).*** One metre is defined as the length of the path travelled by light in vacuum during a time interval of 1/299,792,458 of a second (1983).

   (*ii*) ***Kilogram (kg).*** It is the mass of a platinum-iridium cylinder (90% Pt and 10% Ir) whose height is equal to its diameter (each = 3.9 cm) preserved at International Bureau of Weights and Measures at Sevres (1889).

   (*iii*) ***Second (s).*** One second is defined as the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom (1967).

   (*iv*) ***Ampere (A).*** One ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 metre apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per metre of length (1948).

(*v*) ***Kelvin (K).*** One kelvin is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water (1967).

(*vi*) ***Candela (cd).*** Candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540×10^{12} hertz and that has a radiant intensity in that direction of 1/683 watt per steradian (1979).

(*vii*) ***Mole (mol).*** One mole is the amount of substance of a system, which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon-12 (1971).

9. **The two supplementary SI units.**

(*i*) ***Radian (rad).*** It is defined as the plane angle subtended at the centre of a circle by an arc equal in length to the radius of the circle.

$$\theta \text{ (in radians)} = \frac{\text{Arc}}{\text{Radius}} = \frac{l}{r}$$

(*ii*) ***Steradian (sr).*** It is the solid angle subtended at the centre of a sphere by a surface of the sphere equal in area to that of a square, having each side equal to the radius of the sphere.

$$\Omega \text{(in steradian)} = \frac{\text{Surface area}}{\text{Radius}^2}$$

10. **Coherent system.** A coherent system of units is a system based on a certain set of fundamental units from which all derived units are obtained by multiplication or division without introducing any numerical factor.

11. **Advantages of SI.** It is a metric, rational, coherent and internationally acceptable system of units which covers all branches of physics.

12. **Some Practical Units.**

(*i*) 1 fermi = 10^{-15} m

(*ii*) 1 angstrom (Å) = 10^{-10} m

(*iii*) 1 nanometre (nm) = 10^{-9} m

(*iv*) 1 micron (μm) = 10^{-6} m

(*v*) 1 light year (ly) = 9.46×10^{15} m

(*vi*) 1 astronomical unit (AU) = 1.496×10^{11} m

(*vii*) 1 parallactic second (parsec) = 3.08×10^{16} m

(*viii*) 1 barn = 10^{-28} m^2

(*ix*) 1 tonne = 1000 kg

(*x*) 1 quintal = 100 kg

(*xi*) 1 slug = 14.57 kg

(*xii*) 1 atomic mass unit (amu) = 1.66×10^{-27} kg

(*xiii*) 1 solar year = 365.25 average solar days
= 366.25 sidereal days

(*xiv*) 1 lunar month = 27.3 days

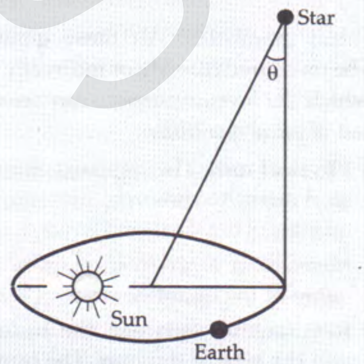
(*xv*) 1 shake = 10^{-8} s

(*xvi*) 1 Torr = 1 mm of Hg

(*xvii*) 1 bar = 1 atmospheric pressure
= 760 mm of Hg = 10^5 Nm^{-2}.

13. **Order of magnitude.** The order of magnitude of a physical quantity is that power of ten which is closest to its magnitude. To determine the order of magnitude of a number N, we express it as $N = n \times 10^x$. If $0.5 < n \leq 5$, then x will be the order of magnitude of N.

14. **Annual parallax.** It is the angle (θ) at which the semi-major axis of the earth's orbit perpendicular to the star's direction is seen from the star.



15. **Indirect methods for measuring long distances.**

(*i*) ***Reflection or echo method.*** The distance (S) of a hill can be determined by noting the time interval t of sending a sound wave towards the hill and receiving back its echo.

$$S = \frac{c \times t}{2},$$

where c = Speed of sound waves

The same principle is used in radars and sonars.

(*ii*) ***Triangulation method.*** The height (h) of an accessible object like a tower or a tree can be found by measuring the angle θ subtended by the object at the observation point. If x is the distance of the observation point from the foot of the object, then

$$h = x \tan \theta$$

To find the height of an inaccessible object like a hill, its angles of elevation θ_1 and θ_2 are measured at two points separated by distance d.

$$h = \frac{d}{\cot \theta_2 - \cot \theta_1}.$$

(*iii*) ***Parallax method.*** By measuring the angle θ (called parallax angle or parallactic angle) subtended by the astronomical object at two locations on the earth separated by a large distance b (called basis), the distance S of the object can be determined.

$$\theta = \frac{\text{Arc}}{\text{Radius}} = \frac{b}{S} \quad \text{or} \quad S = \frac{\text{Basis}}{\text{Parallactic angle}} = \frac{b}{\theta}$$

(*iv*) ***Size of an astronomical object.*** The angle θ (called angular diameter) subtended by the diameter of the planet is measured at a point on the surface of the earth. Knowing the distance S of the planet from the earth, its diameter D can be determined.

$$\theta = \frac{\text{Arc}}{\text{Radius}} = \frac{D}{S} \quad \text{or} \quad D = S \times \theta$$

Linear diameter = Distance × Angular diameter.

(*v*) ***Copernicus method.*** This method is used to determine the distance of an inferior planet from the sun. The angle formed at the earth between the earth-planet direction and the earth-sun direction is called the planet's elongation (ε).

Distance of inferior planet from the sun,

$$r_{ps} = \sin \varepsilon \,.\, r_{es} = \sin \varepsilon \text{ AU}$$

Distance of inferior planet from the earth,

$$r_{pe} = \cos \varepsilon \,.\, r_{es} = \cos \varepsilon \text{ AU}.$$

(*vi*) ***By applying Kepler's third law.*** If T_1 and T_2 be the periods of revolution of two planets and a_1 and a_2 be their semi-major axes, then

$$\frac{a_2^3}{a_1^3} = \frac{T_2^2}{T_1^2}$$

or
$$a_2 = \left[\frac{T_2}{T_1}\right]^{2/3} . \, a_1$$

16. **Molar Volume.** One mole of every gas occupies a volume of 22.4 litres at S.T.P. This is called molar volume of the gas. It contains an Avogadro's number ($N = 6.023 \times 10^{23}$) of molecules.

17. **Indirect methods for measuring small distances.**

(*i*) ***By using Avogadro's hypothesis.*** According to Avogadro's hypothesis, the actual volume occupied by the atoms in one gram of a substance is two-third of the volume of one gram of the substance.

Radius of atom,

$$r = \left[\frac{M}{2\pi N \rho}\right]^{1/3}$$

where N, M and ρ are the Avogadro's number, molecular weight and density of the substance.

(*ii*) ***Molecular size of oleic acid.*** A large and very thin film of oil is formed on water surface and its area is measured.

Thickness of oil film,

$$t = \frac{\text{Volume of the film}}{\text{Area of the film}}$$

Assuming that the film is of one molecular thickness, then t is approximately the size of oil molecule.

19. **Mass.** The mass of a body is the quantity of matter contained in it. Its SI unit is kg.

20. **Weight.** The weight of a body is the force with which a body is attracted towards the centre of the earth. $W = mg$.

21. **Inertial mass.** The mass of a body which determines its inertia in translatory motion is called its inertial mass. It is measured from Newton's second law of motion.

$$m_i = \frac{F}{a}$$

22. **Gravitational mass.** The mass of a body which determines the gravitational pull upon it due to the earth is called its gravitational mass. It is measured by using Newton's law of gravitation.

$$m_g = \frac{FR^2}{GM}$$

23. **Dimensions of the derived quantity.** These are the powers to which the fundamental units of mass, length and time must be raised in order to represent a derived quantity completely.

24. **Dimensional formula.** It is an expression which shows how and which of the fundamental units of mass, length and time occur in the derived unit of a physical quantity.

25. **Dimensional equation.** The equation which expresses a physical quantity in terms of the fundamental units of mass, length and time, is called dimensional equation.

26. **Principle of homogeneity of dimensions.** This principle states that a physical equation will be dimensionally correct if the dimensions of all the terms occurring on both sides of the equation are the same.

27. **Uses of dimensional equations :**

(*i*) To convert a physical quantity from one system of units to another.

Suppose a physical quantity has dimensional formula $M^a L^b T^c$. Let n_1 and n_2 be its numerical values when the units are u_1 and u_2. Then

$$n_1 u_1 = n_2 u_2$$

$$n_1 [M_1^a \, L_1^b \, T_1^c] = n_2 [M_2^a \, L_2^b \, T_2^c]$$

$$\therefore \; n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$$

(*ii*) To check the correctness of a given physical relation.

(*iii*) To derive a relationship between different physical quantities.

28. Significant figures. The significant figures are normally those digits in a measured quantity which are known reliably plus one additional digit that is uncertain.

29. Rules for counting the number of significant figures in a measured quantity :

(*i*) All non-zero digits are significant.

(*ii*) All zeros between two non-zero digits are significant.

(*iii*) All zeros to the right of a non-zero digit but to left of an understood decimal point are not significant. But such zeros are significant if they come from a measurement.

(*iv*) All zeros to the right of a non-zero digit but to the left of a decimal point are significant. For example, 648700. has six significant figures.

(*v*) All zeros to the right of a decimal point are significant.

(*vi*) All zeros to the right of a decimal point but to the left of a non-zero digit are not significant. Single zero conventionally placed to the left of the decimal point is not significant.

(*vii*) The number of significant figures does not depend on the system of units.

30. Significant figures in the sum or difference of two numbers. In addition or subtraction the result should be reported to the same number of decimal places as that of the number with minimum number of decimal places.

31. Significant figures in the product or quotient of two numbers. In multiplication or division, the result should be reported to the same number of significant figures as that of the number with minimum of significant figures.

32. Rules for rounding off a measurement :

(*i*) If the digit to be dropped is less than 5, then the preceding digit is left unchanged.

(*ii*) If the digit to be dropped is greater than 5, then the preceding digit is increased by 1.

(*iii*) If the digit to be dropped is 5 followed by non-zero digits, then the preceding digit is increased by 1.

(*iv*) If the digit to be dropped is 5, then the preceding digit is left unchanged if it is even.

(*v*) If the digit to be dropped is 5, then the preceding digit is increased by 1 if it is odd.

33. Accuracy and precision of a measurement. Accuracy refers to the closeness of a measurement to the true value of the physical quantity. Precision refers to the resolution or the limit to which the quantity is measured.

34. Error in a measurement. It is the difference between the measured value and the true value of a physical quantity. It gives an indication of the limits within which the true value may lie.

35. Elimination of random error. The errors which occur irregularly and at random, in magnitude and direction, are called random errors. To eliminate random error, a large number of readings are taken and their arithmetic mean is taken as the true value.

$$\bar{a} = \frac{a_1 + a_2 + a_3 + \ldots.. + a_n}{n} = \frac{1}{n} \sum_{i=1}^{n} a_i$$

36. Absolute error. The magnitude of the difference between the true value and the measured value is called absolute error.

Such errors are given by

$$\Delta a_1 = \bar{a} - a_1 \qquad \Delta a_2 = \bar{a} - a_2$$

$$\Delta a_3 = \bar{a} - a_3 \qquad \Delta a_n = \bar{a} - a_n$$

37. Mean absolute error. The arithmetic mean of the positive magnitudes of all the absolute errors is called mean absolute error

$$\Delta \bar{a} = \frac{|\Delta a_1| + |\Delta a_2| + \ldots.. + |\Delta a_n|}{n} = \frac{1}{n} \sum_{i=1}^{n} |\Delta a_i|$$

38. Relative error. It is the ratio of the mean absolute error to the true value.

$$\delta a = \frac{\Delta \bar{a}}{\bar{a}}$$

39. Percentage error. The relative error expressed in percent is called percentage error.

$$\text{Percentage error} = \frac{\Delta \bar{a}}{\bar{a}} \times 100\%.$$

40. Error combination in a sum or a difference. When two quantities are added or subtracted, the absolute error in the final result is the sum of the absolute errors associated with the individual quantities.

$$\Delta Z = \Delta A + \Delta B$$

41. Error combination in a product or a quotient. When two quantities are multiplied or divided, the fractional error in the final result is the sum of the fractional errors of the two quantities.

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

42. Error due to the power of a measured quantity. The fractional error in the nth power of a quantity is equal to n times the fractional error in the quantity itself.

If $Z = A^n$, then $\dfrac{\Delta Z}{Z} = n\dfrac{\Delta A}{A}$

43. General rule. If $Z = \dfrac{A^p B^q}{C^r}$, then the maximum fractional or relative error in Z will be

$$\frac{\Delta Z}{Z} = p\frac{\Delta A}{A} + q\frac{\Delta B}{B} + r\frac{\Delta C}{C}$$

$$\% \text{ Error in } Z = \frac{\Delta Z}{Z} \times 100$$

$$= p\frac{\Delta A}{A} \times 100 + q\frac{\Delta B}{B} \times 100 + r\frac{\Delta C}{C} \times 100$$

44. Propagation or combination of errors. It is summarised in the adjacent table.

Operation	Formula Z	Absolute error ΔZ	Relative error $\Delta Z / Z$	Percentage error $100\Delta Z/Z$
Sum	$A + B$	$\Delta A + \Delta B$	$\dfrac{\Delta A + \Delta B}{A + B}$	$\dfrac{\Delta A + \Delta B}{A + B} \times 100$
Difference	$A - B$	$\Delta A + \Delta B$	$\dfrac{\Delta A + \Delta B}{A - B}$	$\dfrac{\Delta A + \Delta B}{A - B} \times 100$
Multiplication	$A \times B$	$A\Delta B + B\Delta A$	$\dfrac{\Delta A}{A} + \dfrac{\Delta B}{B}$	$\left(\dfrac{\Delta A}{A} + \dfrac{\Delta B}{B}\right) \times 100$
Division	$\dfrac{A}{B}$	$\dfrac{B\Delta A + A\Delta B}{B^2}$	$\dfrac{\Delta A}{A} + \dfrac{\Delta B}{B}$	$\left(\dfrac{\Delta A}{A} + \dfrac{\Delta B}{B}\right) \times 100$
Power	A^n	$n A^{n-1}\Delta A$	$n\dfrac{\Delta A}{A}$	$n\dfrac{\Delta A}{A} \times 100$
Root	$A^{1/n}$	$\dfrac{1}{n} A^{1/n - 1}\Delta A$	$\dfrac{1}{n}\dfrac{\Delta A}{A}$	$\dfrac{1}{n}\dfrac{\Delta A}{A} \times 100$

IIT Entrance Exam

MULTIPLE CHOICE QUESTIONS WITH ONE CORRECT ANSWER

1. The equation of state for a real gas is given by

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT$$

The dimensions of the constant a are

(a) $[ML^5T^{-2}]$ (b) $[M^{-1}L^5T^2]$

(c) $[ML^{-5}T^{-1}]$ (d) $[ML^5T^{-1}]$

[IIT 97 ; CBSE PMT 96]

2. Pressure depends on distance as $P = \dfrac{\alpha}{\beta}\exp\left(\dfrac{-\alpha z}{k\theta}\right)$, where α, β are constants, z is distance, k is Boltzmann's constant and θ is temperature. The dimensions of β are

(a) $[M^0L^0T^0]$ (b) $[M^{-1}L^{-1}T^{-1}]$

(c) $[M^0L^2T^0]$ (d) $[M^{-1}L^{-1}T^2]$ [IIT 04]

3. The dimensions of $\left(\dfrac{1}{2}\right)\varepsilon_0 E^2$ are

(ε_0 : permittivity of free space, E : electric field)

(a) $[MLT^{-1}]$ (b) $[ML^2T^{-2}]$

(c) $[ML^{-1}T^{-2}]$ (d) $[ML^2T^{-1}]$ [IIT 2K]

4. In the formula : $X = 3YZ^2$, X and Z have dimensions of capacitance and magnetic induction respectively. What are the dimensions of Y in MKSQ system ?

(a) $[M^{-3}L^{-1}T^3Q^4]$ (b) $[M^{-3}L^{-2}T^4Q^4]$

(c) $[M^{-2}L^{-2}T^4Q^4]$ (d) $[M^{-3}L^{-2}T^4Q^1]$ [IIT 95]

5. A quantity X is given by $\varepsilon_0 L\dfrac{\Delta V}{\Delta t}$ where ε_0 is the permittivity of the free space, L is a length, ΔV is a potential difference and Δt is a time interval. The dimensional formula for X is the same as that of

(a) resistance (b) charge

(c) voltage (d) current [IIT 01]

6. Which of the following sets have different dimensions ?

(a) Pressure, Young's modulus, stress

(b) Emf, potential difference, electric potential

(c) Heat, work done, energy

(d) Dipole moment, electric flux, electric field

[IIT 05]

7. A cube has a side of length 1.2×10^{-2} m. Calculate its volume.

(a) $1.7 \times 10^{-6}\,m^3$ (b) $1.73 \times 10^{-6}\,m^3$

(c) $1.0 \times 10^{-6}\,m^3$ (d) $1.732 \times 10^{-6}\,m^3$ [IIT 03]

8. A student performs an experiment for determination of $g\left(= \dfrac{4\pi^2 l}{T^2}\right)$. The error in length l is Δl and in time T is ΔT and n is number of times the reading is taken. The measurement of g is most accurate for

	Δl	ΔT	n
(a)	5 mm	0.2 sec	10
(b)	5 mm	0.2 sec	20
(c)	5 mm	0.1 sec	10
(d)	1 mm	0.1 sec	50

[IIT 06]

9. A student performs an experiment to determine the Young's modulus of a wire, exactly 2 m long, by Searle's method. In a particular reading, the student measures the extension in the length of the wire to be 0.8 mm with an uncertainty of ± 0.05 mm at a load of exactly 1.0 kg. The student also measures the diameter of the wire to be 0.4 mm with an uncertainty of ± 0.01mm. Take $g = 9.8\ \text{m/s}^2$ (exact). The Young's modulus obtained from the reading is

(*a*) $(2.0 \pm 0.3) \times 10^{11}\ \text{N/m}^2$
(*a*) $(2.0 \pm 0.2) \times 10^{11}\ \text{N/m}^2$
(*c*) $(2.0 \pm 0.1) \times 10^{11}\ \text{N/m}^2$
(*d*) $(2.0 \pm 0.05) \times 10^{11}\ \text{N/m}^2$ [IIT 07]

10. A wire has a mass 0.3 ± 0.003 g, radius 0.5 ± 0.005 mm and length 6 ± 0.06 cm. The maximum percentage error in the measurement of its density is

(*a*) 1 (*b*) 2
(*c*) 3 (*d*) 4 [IIT 04]

11. Students I, II and III perform an experiment for measuring the acceleration due to gravity (*g*) using a simple pendulum. They use different lengths of the pendulum and/or record time for different number of oscillations. The observations are shown in the table.

Least count for length = 0.1 cm

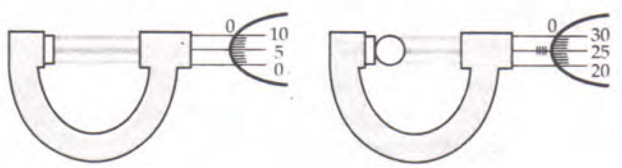
Least count for time = 0.1 s

Student	Length of the pendulum (cm)	No. of oscillations (n)	Total time for (n) oscillation (s)	Time period (s)
I	64.0	8	128.0	16.0
II	64.0	4	64.0	16.0
III	20.0	4	36.0	9.0

If E_I, E_{II} and E_{III} are the percentage errors in g *i.e.*, $\left(\frac{\Delta g}{g} \times 100\right)$ for students I, II and III respectively,

(*a*) $E_I = 0$ (*b*) E_I is maximum
(*c*) $E_I = E_{II}$ (*d*) E_{II} is maximum [IIT 08]

12. In a screw gauge, the zero of main scale coincides with fifth division of circular scale in figure (*i*). The circular divisions of screw gauge are 50.



(*i*) (*ii*)

It moves 0.5 mm on main scale in one rotation. The diameter of the ball in figure (*ii*) is

(*a*) 2.25 mm (*b*) 2.20 mm
(*c*) 1.20 mm (*d*) 1.25 mm [IIT 06]

13. A Vernier calipers has 1 mm marks on the main scale. It has 20 equal divisions on the Vernier scale which match with 16 main scale divisions. For this Vernier calipers, the least count is

(*a*) 0.02 mm (*b*) 0.05 mm
(*c*) 0.1 mm (*d*) 0.2 mm [IIT 2010]

14. The density of a solid ball is to be determined in an experiment. The diameter of the ball is measured with a screw gauge, whose pitch is 0.5 mm and there are 50 divisions on the circular scale. The reading on the main scale is 2.5 mm and that on the circular scale is 20 divisions. If the measured mass of the ball has a relative error of 2%, the relative percentage error in the density is

(*a*) 0.9% (*b*) 2.4%
(*c*) 3.1% (*d*) 4.2% [IIT 2011]

Multiple Choice Questions with One or More than One Correct Answer

15. The dimensions of the quantities in one (or more) of the following pairs are the same. Identify the pair(s)

(*a*) Torque and work
(*b*) Angular momentum and work
(*c*) Energy and Young's modulus
(*d*) Light year and wavelength [IIT 86]

16. The pairs of physical quantities that have the same dimensions are

(*a*) Reynold number and coefficient of friction
(*b*) Curie and frequency of a light wave
(*c*) Latent heat and gravitational potential
(*d*) Planck's constant and torque [IIT 95]

17. The dimensions of length are expressed as $G^x c^y h^z$ where G, c and h are the universal gravitational constant, speed of light and Planck's constant respectively, then

(*a*) $x = (1/2)$, $y = (1/2)$
(*b*) $x = (1/2)$, $z = (1/2)$
(*c*) $y = (-3/2)$, $z = (1/2)$
(*d*) $y = (1/2)$, $z = (3/2)$ [IIT 92]

18. The SI unit of inductance, the henry can be written as

(*a*) weber/ampere (*b*) volt-sec/amp

(*b*) joule/(ampere)^2 (*d*) ohm-second [IIT 98]

19. If L, C, R represent inductance, capacitance and resistance respectively, the combinations having dimensions of frequency are

(*a*) $\frac{1}{\sqrt{CL}}$ (*b*) $\frac{L}{C}$

(*c*) $\frac{R}{L}$ (*d*) $\frac{R}{C}$ [IIT 84]

20. Which of the following combinations have the dimensions of time ? L, C, R represent inductance, capacitance and resistance respectively.

(*a*) RC (*b*) $\sqrt{LC}$

(*c*) R/L (*d*) C/L [IIT 86]

21. Let $[\varepsilon_0]$ denote the permittivity of the vacuum, and $[\mu_0]$ denote the permeability of the vacuum. If M = mass, L = length, T = time and I = electrical current, then the dimensional formulae of ε_0 and μ_0 are

(*a*) $[\varepsilon_0] = M^{-1}L^{-3}T^2I$ (*b*) $[\varepsilon_0] = M^{-1}L^{-3}T^4I^2$

(*c*) $[\mu_0] = MLT^{-2}I^{-2}$ (*d*) $[\mu_0] = ML^2T^{-1}I$ [IIT 98]

22. A student uses a simple pendulum of exactly 1 m length to determine g, the acceleration due to gravity. He uses a stop watch with the least count of 1 second for this and records 40 seconds for 20 oscillations. For this observation, which of the following statement(s) is (are) true ?

(*a*) Error ΔT in measuring T, the time period, is 0.05 second

(*b*) Error ΔT in measuring T, the time period, is 1 second

(*c*) Percentage error in the determination of g is 5%

(*d*) Percentage error in the determination of g is 2.5% [IIT 2010]

MATRIX–MATCH TYPE

23. Match the physical quantities given in column I with dimensions expressed in terms of mass (M), length (L), time (T), and charge (Q) given in column II. [IIT 83]

I	*II*
(*a*) Angular momentum	(*p*) ML^2T^{-2}
(*b*) Torque	(*q*) ML^2T^{-1}
(*c*) Inductance	(*r*) $M^{-1}L^{-2}T^2Q^2$
(*d*) Latent heat	(*s*) ML^2Q^{-2}
(*e*) Capacitance	(*t*) $ML^3T^{-1}Q^{-2}$
(*f*) Resistivity	(*u*) L^2T^{-2}

24. Column I gives three physical quantities. Select the appropriate units for the choices given in column II. Some of the physical quantities may have more than one choice correct. [IIT 90]

I	*II*
(*a*) Capacitance	(*p*) ohm-second
(*b*) Inductance	(*q*) coulomb^2-joule^{-1}
(*c*) Magnetic Induction	(*r*) coulomb(volt)$^{-1}$
	(*s*) newton (ampere m)$^{-1}$
	(*t*) volt-second (ampere)$^{-1}$

25. Some physical quantities are given in column I and some possible SI units in which these quantities may be expressed are given in column II. Match the physical quantities in column I with the units in column II. [IIT 07]

I	*II*
(*a*) GM_eM_s ; G = universalgravitation constant, M_e = mass of earth, M_s = mass of sun	(*p*) (volt) (coulomb) (metre)
(*b*) $\frac{3RT}{M}$; R = universal gas constant, T = absolute temperature, M = molar mass	(*q*) (kilogram) (metre)^3 (second)$^{-2}$
(*c*) $\frac{F^2}{q^2B^2}$; F = force, q = charge, B = magnetic field	(*r*) (metre)^2 (second)$^{-2}$
(*d*) $\frac{GM_e}{R_e}$; G = universal gravitational constant M_e = mass of earth, R_e = radius of earth	(*s*) (farad) (volt)^2 (kg)$^{-1}$

Answers and Explanations

1. (*a*) As quantities with similar dimensions can be added only, so

$$\left[\frac{a}{V^2}\right] = [P]$$

or $$[a] = [P][V^2] = [\text{ML}^{-1}\text{T}^{-2}][\text{L}^3]^2$$

$$= \mathbf{[ML^5T^{-2}]}$$

2. (*c*) Here k = Boltzmann's constant

$$\therefore \quad [k] = \frac{[\text{joule}]}{[\text{kelvin}]} = [\text{ML}^2\text{T}^{-2}\theta^{-1}]$$

As an exponential factor is dimensionless,

$$\therefore \quad \left[\frac{\alpha z}{k\theta}\right] = [\text{M}^0\text{L}^0\text{T}^0]$$

or $$[\alpha] = \left[\frac{k\theta}{z}\right] = \frac{\text{ML}^2\text{T}^{-2}\theta^{-1}\cdot\theta}{\text{L}} = [\text{MLT}^{-2}]$$

$$\beta = \frac{\alpha}{P}\exp\left(\frac{\alpha z}{k\theta}\right)$$

$$\therefore \quad [\beta] = \frac{[\alpha]}{[P]} \times [\text{M}^0\text{L}^0\text{T}^0] = \frac{[\text{MLT}^{-2}]}{[\text{ML}^{-1}\text{T}^{-2}]} = \mathbf{[L^2]}$$

3. (*c*) $\frac{1}{2}\varepsilon_0 E^2 = \frac{\text{Energy}}{\text{Volume}}$

$$\therefore \quad \left[\frac{1}{2}\varepsilon_0 E^2\right] = \frac{[\text{ML}^2\text{T}^{-2}]}{[\text{L}^3]} = \mathbf{[ML^{-1}T^{-2}]}$$

4. (*b*) $[X] = [C] = [\text{M}^{-1}\text{L}^{-2}\text{T}^2Q^2]$

$$[Z] = [B] = [\text{MT}^{-1}Q^{-1}]$$

$$\therefore \quad [Y] = \frac{[X]}{[Z^2]} = \frac{[\text{M}^{-1}\text{L}^{-2}\text{T}^2Q^2]}{[\text{MT}^{-1}Q^{-1}]^2}$$

$$= \mathbf{[M^{-3}L^{-2}T^4Q^4]}$$

5. (*d*) $[\varepsilon_0 L] = [\text{capacitance}]$

$$\therefore \quad \varepsilon_0 L\frac{\Delta V}{\Delta t} = \frac{C\Delta V}{\Delta t} = \frac{\Delta q}{\Delta t} = \frac{\text{charge}}{\text{time}} = \textbf{current}.$$

6. (*d*) Dipole moment = Charge × distance

Electric flux = Electric field × area

∴ Dipole moment, electric flux and electric field have different dimensions.

7. (*a*) Volume = (side)^3 = $(1.2 \times 10^{-2}\text{m})^3$

$$= 1.728 \times 10^{-6}\text{m}^3 = 1.7 \times 10^6 \text{ m}^3$$

As the side has 2 significant figures only, so the expressed volume must have 2 significant figures only.

8. (*d*) Given $g = \frac{4\pi^2 l}{T^2}$

$$\therefore \quad \frac{\Delta g}{g} = \frac{\Delta l}{l} + 2\frac{\Delta T}{T}$$

Δl is minimum in option (*d*),

ΔT is minimum in options (*c*) and (*d*),

n is maximum in option (*d*)

Hence error will be minimum or the measurement of g will be most accurate in option (*d*).

9. (*b*) Young's modulus,

$$Y = \frac{F}{A}\cdot\frac{L}{l} = \frac{4\,Mg}{\pi d^2}\cdot\frac{L}{l}$$

$$\therefore \quad Y = \frac{4 \times 1.0 \times 9.8 \times 2}{\pi(0.4 \times 10^{-3})^2 \times 0.8 \times 10^{-3}} = 2.0 \times 10^{11} \text{ Nm}^{-2}$$

As the values of L, M and g are exactly given, so

$$\frac{\Delta Y}{Y} = 2\frac{\Delta d}{d} + \frac{\Delta l}{l}$$

or $$\Delta Y = \left(\frac{2 \times 0.01}{0.4} + \frac{0.05}{0.8}\right) \times 2 \times 10^{11} \text{ Nm}^{-2}$$

$$= 0.2 \times 10^{11} \text{ Nm}^{-2}$$

∴ Measured value of $Y = \mathbf{(2.0 \pm 0.2) \times 10^{11}\ Nm^{-2}}$.

10. (*d*) Density $= \frac{\text{Mass}}{\text{Volume}}$

$$\rho = \frac{m}{\pi r^2 l}$$

The maximum percentage error in density,

$$\frac{\Delta\rho}{\rho} \times 100 = \left[\frac{\Delta m}{m} + 2\frac{\Delta r}{r} + \frac{\Delta l}{l}\right] \times 100$$

$$= \left[\frac{0.003}{0.3} + \frac{2 \times 0.005}{0.5} + \frac{0.06}{6}\right] \times 100$$

$$= \left[\frac{1}{100} + \frac{2}{100} + \frac{1}{100}\right] \times 100 = \mathbf{4}.$$

11. (*b*) $g = 4\pi^2\left(\frac{l}{T^2}\right)$

$$\frac{\Delta g}{g} = \frac{\Delta l}{l} + 2\frac{\Delta T}{T}$$

$$\Rightarrow \quad E = \frac{\Delta l}{l} + 2\frac{\Delta t}{t},$$

greater the value of t, lesser the error.

Hence, fractional error in the 1st observation is minimum.

12. (c) Least count of screw gauge

$$= \frac{\text{Pitch}}{\text{No. of circular divisions}} = \frac{0.5\,\text{mm}}{50}$$

$$= 0.01 \text{ mm}$$

Initial reading $= 0 + 5 \times 0.01 = 0.05$ mm

Final reading $= 2 \times 0.5 + 25 \times 0.01 = 1.25$ mm

Diameter of ball $= 1.25 - 0.05 =$ **1.20 mm.**

13. (d) $20\,\text{VSD} = 16\,\text{MSD}$

$$1\,\text{VSD} = \frac{16}{20}\text{MSD} = \frac{4}{5}\text{MSD}$$

$$\text{LC} = 1\text{MSD} - 1\text{VSD} = 1\text{MSD} - \frac{4}{5}\text{MSD}$$

$$= \frac{1}{5}\text{MSD} = \frac{1}{5} \times 1\text{mm} = \mathbf{0.2\ mm}$$

14. (c) Least count of screw gauge

$$= \frac{0.5\,\text{mm}}{50} = \frac{1}{100}\text{mm} = 0.01 \text{ mm}$$

$\therefore$ Diameter D

$$= \text{MSR} + \text{CSR} \times \text{LC}$$

$$= 2.5 + 20 \times 0.01 = 2.70 \text{ mm}$$

Now, $\quad$ density $= \dfrac{m}{\frac{4}{3}\pi\left(\frac{D}{2}\right)^3}$

$\therefore$ % error in density

$$= \frac{\Delta m}{m} \times 100 + 3\frac{\Delta D}{D} \times 100$$

$$= 2\% + 3 \times \frac{0.01}{2.70} \times 100 = \mathbf{3.1\%}$$

15. (a), (d)

(a) Torque and work both have the dimensions $[ML^2T^{-2}]$.

(d) Lightyear and wavelength both have the dimensions [L].

16. (a), (b), (c)

(a) Reynold's number and coefficient of friction both are dimensionless quantities.

(b) $\quad$ Curie $= \dfrac{\text{Number of atoms}}{\text{Time}} = [T^{-1}]$

Frequency $= \dfrac{\text{No. of vibrations}}{\text{Time}} = [T^{-1}]$

(c) [Latent heat] = [Gravitational potential]

$$= \frac{\text{Energy}}{\text{Mass}}$$

17. (b) and (c).

Here $L = G^x c^y h^z$.

$$[L] = [M^{-1}L^3T^{-2}]^x [LT^{-1}]^y [ML^2T^{-1}]^z$$

$$M^0L^1T^0 = M^{-x+z} L^{3x+y+2z} T^{-2x-y-z}$$

$\therefore \quad -x + z = 0,\ 3x + y + 2z = 1,\ -2x - y - z = 0$

Hence $x = \dfrac{1}{2}$, $y = -\dfrac{3}{2}$, $z = \dfrac{1}{2}$.

18. (a), (b), (c), (d)

(a) $L = \dfrac{\text{flux}}{\text{current}} \quad \therefore\ 1\,\text{H} = \dfrac{1\,\text{Wb}}{1\,\text{A}}$

(b) $L = \dfrac{\varepsilon}{dI/dT} \quad \therefore\ 1\,\text{H} = \dfrac{1\,\text{V}}{\text{A/s}} = 1\,\text{Vs}\,\text{A}^{-1}$

(c) $U = \dfrac{1}{2}LI^2$ or $L = \dfrac{2U}{I^2} \quad \therefore\ 1\,\text{H} = \dfrac{1\,\text{J}}{1\,\text{A}^2}$

(d) $1\,\text{H} = \dfrac{1\,\text{V.s}}{1\,\text{A}} = 1\,\Omega\,\text{s}.$

19. (a) and (c)

$$[R] = [ML^2T^{-3}A^{-2}]$$

$$[L] = [ML^2T^{-2}A^{-2}]$$

$$[C] = [M^{-1}L^{-2}T^4A^2]$$

$$\therefore \quad \frac{1}{\sqrt{LC}} = \frac{1}{[T]} = [\text{frequency}]$$

and $\quad \left[\dfrac{R}{L}\right] = \dfrac{1}{[T]} = [\text{frequency}]$

20. (a) and (b)

$$[RC] = [ML^2T^{-3}A^{-2}][M^{-1}L^{-2}T^4A^2] = [T]$$

$$[LC] = [ML^2T^{-2}A^{-2}][M^{-1}L^{-2}T^4A^2] = [T^2]$$

$\therefore \quad [\sqrt{LC}] = [T]$

21. (c) $\quad F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1q_2}{r^2}$ or $\varepsilon_0 = \dfrac{1}{4\pi F}\dfrac{q_1q_2}{r^2}$

$$\therefore \quad [\varepsilon_0] = \frac{[IT][IT]}{[MLT^{-2}][L^2]} = [M^{-1}L^{-3}T^4I^2]$$

Also, $\quad \dfrac{1}{\mu_0\varepsilon_0} = c^2$

or $\quad [\mu_0] = \dfrac{1}{[\varepsilon_0][c^2]} = \dfrac{1}{[M^{-1}L^{-3}T^4I^2][LT^{-1}]^2}$

$$= \mathbf{[MLT^{-2}I^{-2}]}.$$

22. (a), (c) Relative error in measurement of time

$$= \frac{1\,\text{second}}{40\,\text{second}} = \frac{1}{40}$$

Time period $= \dfrac{40}{20} = 2$ second

$\therefore$ Error in measurement of time period

$$= 2 \times \frac{1}{40} = \frac{1}{20}\text{ second} = 0.05 \text{ second}$$

Now, $g \propto \frac{1}{T^2}$ [For constant l]

$\Rightarrow \frac{\Delta g}{g} = \frac{2\Delta T}{T} = \frac{2 \times 1}{40} = \frac{1}{20}$

$\Rightarrow \frac{\Delta g}{g} \times 100 = \mathbf{5\%}$

23.

(*a*) [Angular momentum] = $[ML^2T^{-1}]$

(*d*) [Latent heat] = $[L^2T^{-2}]$

(*b*) [Torque] = $[ML^2T^{-2}]$

(*e*) [Capacitance] = $[M^{-1}L^{-2}T^2Q^2]$

(*c*) [Inductance] = $[ML^2Q^{-2}]$

(*f*) [Resistivity] = $[ML^3T^{-1}Q^{-2}]$

$\therefore$ (*a*) $\rightarrow q$; (*b*) $\rightarrow p$; (*c*) $\rightarrow s$;
(*d*) $\rightarrow u$; (*e*) $\rightarrow r$; (*f*) $\rightarrow t$.

24. (*a*) Capacitance, $C = \frac{q}{V}$ = coulomb volt^{-1}

or $C = \frac{q}{W/q} = \frac{q^2}{W}$ = coulomb^{2}joule^{-1}

(*b*) Inductance,

$L = \frac{\varepsilon}{dI/dt} = \frac{\text{volt}}{\text{ampere / sec}}$

= volt sec ampere^{-1} = ohm sec.

(*c*) Magnetic induction,

$B = \frac{F}{Il} = \frac{\text{newton}}{\text{ampere metre}}$

= newton (ampere metre)$^{-1}$

$\therefore$ (*a*) $\rightarrow q, r$; (*b*) $\rightarrow p, t$; (*c*) $\rightarrow s$.

25. (*a*) $\rightarrow p, q$; (*b*) $\rightarrow r, s$;
(*c*) $\rightarrow r, s$; (*d*) $\rightarrow r, s$.

AIEEE

1. Which of the following represents the correct dimensions of the coefficient of viscosity ?

(*a*) $[ML^{-1}T^{-2}]$ (*b*) $[ML^{-2}T^{-2}]$
(*c*) $[ML^{-1}T^{-1}]$ (*d*) $[MLT^{-1}]$ [AIEEE 04]

2. Dimensions of $\frac{1}{\mu_0\varepsilon_0}$, where symbols have their usual meanings, are :

(*a*) $[L^{-1}T]$ (*b*) $[L^2T^2]$
(*c*) $[L^2T^{-2}]$ (*d*) $[LT^{-1}]$ [AIEEE 03]

3. Identify the pair whose dimensions are equal.

(*a*) torque and work (*b*) stress and energy
(*c*) force and stress (*d*) force and work [AIEEE 02]

4. Out of the following pairs, which one does not have identical dimensions

(*a*) Moment of inertia and moment of a force
(*b*) Work and torque
(*c*) Angular momentum and Planck's constant
(*d*) Impulse and momentum [AIEEE 05]

5. The physical quantities not having the same dimensions are

(*a*) torque and work
(*b*) momentum and Planck's constant
(*c*) stress and Young's modulus
(*d*) speed and $1/\sqrt{\mu_0\varepsilon_0}$ [AIEEE 03]

6. Which of the following units denotes the dimensions ML^2Q^{-2}, where Q denotes the electric charge ?

(*a*) henry (H) (*b*) weber (Wb)
(*c*) Wb m^{-2} (*d*) Hm^{-2} [AIEEE 06]

7. The dimensions of magnetic field in M, L, T and C (Coulomb) are given as

(*a*) $[MLT^{-1}C^{-1}]$ (*b*) $[MT^2C^{-2}]$
(*c*) $[MT^{-1}C^{-1}]$ (*d*) $[MT^{-2}C^{-1}]$ [AIEEE 08]

8. Two full turns of the circular scale of a screw gauge cover a distance of 1 mm on its main scale. The total number of divisions on the circular scale is 50. Further, it is found that the screw gauge has a zero error of – 0.03 mm. While measuring the diameter of a thin wire, a student notes the main scale reading of 3 mm and the number of circular scale divisions in line with the main scale is 35. The diameter of the wire is

(*a*) 3.32 mm (*b*) 3.73 mm
(*c*) 3.67 mm (*d*) 3.38 mm [AIEEE 08]

9. In an experiment the angles are required to be measured using an instrument. 29 divisions of the main scale exactly coincide with the 30 divisions of the vernier scale. If the smallest division of the main scale is half-a-degree (=0.5°), then the least count of the instrument is

(*a*) one minute (*b*) half minute
(*c*) one degree (*d*) half degree [AIEEE 09]

10. The respective number of significant figures for the numbers 23.023, 0.0003 and 2.1×10^{-3} are

(a) 4, 4, 2 (b) 5, 1, 2

(c) 5, 1, 5 (d) 5, 5, 2 [AIEEE 2010]

11. A screw gauge gives the following reading when used to measure the diameter of a wire.

Main scale reading : 0 mm

Circular scale reading : 52 divisions

Given that 1 mm on main scale corresponds to 100 divisions of the circular scale.

The diameter of wire from the above data is

(a) 0.52 cm

(b) 0.052 cm

(c) 0.026 cm

(d) 0.005 cm [AIEEE 2011]

Answers and Explanations

1. (c) $\eta = \dfrac{\text{Force}}{\text{Area}} \times \dfrac{\text{Distance}}{\text{Velocity}}$

$$[\eta] = \frac{[MLT^{-2}]}{[L^2]} \cdot \frac{[L]}{[LT^{-1}]} = \mathbf{[ML^{-1}T^{-1}]}.$$

2. (c) $\dfrac{1}{\mu_0 \varepsilon_0} = c^2$, c = speed of light

$$\therefore \quad \left[\frac{1}{\mu_0 \varepsilon_0}\right] = [LT^{-1}]^2 = \mathbf{[L^2T^{-2}]}$$

3. (a) $[\text{Torque}] = [\text{Work}] = [ML^2T^{-2}]$

4. (a) $[\text{Moment of inertia}] = [ML^2T^0]$

$$[\text{Moment of force}] = \text{Force} \times \text{distance} = [ML^2T^{-2}]$$

5. (b) $[\text{Momentum}] = [MLT^{-1}]$

$$[\text{Planck's constant}] = [ML^2T^{-1}].$$

6. (a) $\text{Inductance} = \dfrac{\text{EMF}}{\text{Current / time}}$

$$= \frac{\text{Work}}{\text{Charge}} \cdot \frac{\text{Time}}{\text{Current}}$$

$$[\text{Inductance}] = \frac{[ML^2T^{-2}]}{[Q]} \cdot \frac{[T]}{[Q/T]}$$

$$[\text{Henry}] = [ML^2Q^{-2}]$$

Henry is the SI unit of inductance.

7. (c) $F = IlB$ or $B = \dfrac{F}{Il}$

$$\therefore \quad [B] = \frac{[MLT^{-2}]}{[CT^{-1}][L]} = \mathbf{[MT^{-1}C^{-1}]}.$$

8. (d) Pitch of screw gauge $= \dfrac{1}{2}$ mm $= 0.5$ mm

$$\text{Least count} = \frac{0.5\,\text{mm}}{50} = 0.01 \text{ mm}$$

$$\text{Observed diameter} = 3\,\text{mm} + 35 \times 0.01\,\text{mm} = 3.35 \text{ mm}$$

Corrected diameter

$$= \text{Observed diameter} - \text{Zero error}$$
$$= 3.35 - (-0.03) = \mathbf{3.38\ mm}.$$

9. (a) $29 \text{ MSD} = 30 \text{ VSD}$

$$1 \text{ VSD} = \frac{29}{30} \text{ MSD}$$

$$\text{Least count} = 1 \text{ MSD} - 1 \text{ VSD}$$

$$= \left(1 - \frac{29}{30}\right) \text{MSD} = \frac{1}{30} \text{ MSD}$$

$$= \frac{1}{30}\left(\frac{1}{2}\right)^0 = \left(\frac{1}{60}\right)^0 = \mathbf{1\ minute}.$$

10. (b) $23.023 \rightarrow 5$ (2, 3, 0, 2, 3) ;

$0.0003 \rightarrow 1$ (3) ; $2.1 \times 10^{-3} \rightarrow 2$ (2, 1).

11. (b) $d = \text{MSR} + \text{CSR}$

$$= 0 + 52 \times \frac{1}{100} = 0.52 \text{ mm} = \mathbf{0.052\ cm}$$

DCE and G.G.S. Indraprastha University Engineering Entrance Exam

(Similar Questions)

1. Out of the following four dimensional quantities, which one qualifies to be called a dimensional constant?

(a) Acceleration due to gravity

(b) Surface tension of water

(c) Weight of a standard kilogram mass

(d) The velocity of light in vacuum. [IPUEE 06]

2. The dimensions of torque are

(*a*) $[ML^2T^2]$ (*b*) $[ML^2T^{-2}]$

(*c*) $[M^2L^2T^{-2}]$ (*d*) $[MLT^{-1}]$ [DCE 00]

3. $[ML^2T^{-2}]$ are dimensions of

(*a*) force (*b*) moment of force

(*c*) momentum (*d*) power [DCE 04]

4. The dimensions of Planck's constant are

(*a*) $[M^2L^2T^{-2}]$ (*b*) $[MLT^{-2}]$

(*c*) $[ML^2T^{-2}]$ (*d*) $[ML^2T^{-1}]$

[DPM 01 ; DCE 07]

5. Dimensions of bulk modulus are

(*a*) $[M^{-1}LT^{-2}]$ (*b*) $[ML^{-1}T^{-2}]$

(*c*) $[ML^{-2}T^{-2}]$ (*d*) $[M^2L^2T^{-1}]$ [DCE 07]

6. The unit of a in van der Waal's gas equation is

(*a*) atm $L^{-2}mol^2$ (*b*) atm L^2per mol

(*c*) atm $L^{-1}mol^{-2}$ (*d*) atm L^2mol^{-2} [DCE 97]

7. In the relation, $y = r\sin(\omega t + kx)$, the dimensional formula for kx or ωt is same as

(*a*) r/ω (*b*) r/y

(*c*) $\omega t/r$ (*d*) $yr/\omega t$ [DCE 03]

8. Which of the following quantities can be written in SI units in $kgm^2A^{-2}s^{-3}$?

(*a*) Resistance (*b*) Inductance

(*c*) Capacitance (*d*) Magnetic flux [DCE 07]

9. The time period T of a small drop of liquid (due to surface tension) depends on density ρ, radius r and surface tension S.

The relation is

(*a*) $T \propto \left(\frac{\rho r^3}{S}\right)^{1/2}$ (*b*) $T \propto \rho r S$

(*c*) $T \propto \frac{\rho r}{S}$ (*d*) $T \propto \frac{S}{\rho r}$

10. Which one of the following pairs of quantities has the same dimension ?

(*a*) force and work done

(*b*) momentum and impulse

(*c*) pressure and force

(*d*) surface tension and stress. [DCE 09]

11. If $L = 2.331$ cm, $B = 2.1$ cm, then $L + B = ?$

(*a*) 4.431 cm (*b*) 4.43 cm

(*c*) 4.4 cm (*d*) 4 cm [DCE 03]

12. If error in radius is 3%, what is error in volume of sphere ?

(*a*) 3% (*b*) 27%

(*c*) 9% (*d*) 6% [DCE 06]

13. In an experiment, on the measurement of g, using a simple pendulum, the time period was measured with an accuracy of 0.2% while the length was measured with an accuracy of 0.5%. The percentage accuracy in the value of g thus obtained is

(*a*) 0.7% (*b*) 0.1%

(*c*) 0.25% (*d*) 0.9% [DCE 07]

Answers and Explanations

1. (*d*) The velocity of light in vacuum ($c = 3 \times 10^8$ m/s) is a dimensional constant.

2. (*b*) [Torque] = [Force] [Distance]

$$= [MLT^{-2}][L] = [ML^2T^{-2}]$$

3. (*b*) [Moment of force] = [Torque] = $[ML^2T^{-2}]$

4. (*d*) $[h] = \frac{\text{Energy}}{\text{Frequency}} = \frac{[ML^2T^{-2}]}{[T^{-1}]} = [ML^2T^{-1}]$

5. (*b*) [Bulk modulus] $= \frac{\text{Stress}}{\text{Strain}} = \frac{[ML^{-1}T^{-2}]}{1}$

$$= [ML^{-1}T^{-2}]$$

6. (*b*) Van der Wall's equation is

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT$$

$$\therefore \quad [P] = \left[\frac{a}{V^2}\right] \quad \text{or} \quad [a] = [P][V^2]$$

Unit of a = atm L^2 per mole.

7. (*b*) ωt and kx both are dimensionless. Out of the given options, only r/y is dimensionless.

8. (*a*) [Resistance] = $[ML^2T^{-3}A^{-2}]$

$\therefore$ Unit of resistance = kg $m^2A^{-2}s^{-3}$.

9. (*a*) Let $T = K\rho^a r^b S^c$

$$\therefore \quad [T] = [ML^{-3}]^a [L]^b [MT^{-2}]^c$$

$$\text{or} \quad M^0L^0T^1 = M^{a+c}L^{-3a+b}T^{-2c}$$

$$\therefore \quad a + c = 0, \quad -3a + b = 0, \quad -2c = 1$$

On solving, $a = \frac{1}{2}, \; b = \frac{3}{2}, \; c = -\frac{1}{2}$.

Hence $T \propto \left(\frac{\rho r^3}{S}\right)^{1/2}$.

10. (b) [Change in momentum] = [Impulse]
$= [MLT^{-1}]$.

11. (c) $L + B = 2.331 + 2.1 = 4.431 = 4.4$ cm

$\because$ B has 2 significant figures.

$\therefore$ $L + B$ must have only 2 significant figures.

12. (c) $V = \frac{4}{3}\pi r^3$

$\therefore \frac{\Delta V}{V} \times 100 = 3 \times \frac{\Delta r}{r} \times 100 = 3 \times 3 = 9\%$.

13. (d) As $T = 2\pi\sqrt{\frac{l}{g}}$

$\therefore \quad g = 4\pi^2 \frac{l}{T^2}$

$$\frac{\Delta g}{g} \times 100 = \frac{\Delta l}{l} \times 100 + 2 \times \frac{\Delta T}{T} \times 100$$

$$= 0.5\% + 2 \times 0.2\% = \mathbf{0.9\%}.$$

AIIMS Entrance Exam

1. How many wavelengths of Kr^{86} are there in one metre ?
(a) 15,53,164.13 (b) 16,50,763.73
(c) 23,48,123.73 (d) 6,52,189.63 [AIIMS 94]

2. One nanometre is equal to
(a) 10^9 mm (b) 10^{-6} cm
(c) 10^{-7} cm (d) 10^{-9} m [AIIMS 94]

3. Light year is the unit of
(a) time (b) distance
(c) velocity (d) intensity of light [AIIMS 96]

4. Parsec is the unit of
(a) time (b) distance
(c) frequency (d) angular momentum [AIIMS 96]

5. Length cannot be measured by
(a) fermi (b) debye
(c) micron (d) light year [AIIMS 02]

6. The difference in the length of a mean solar day and a sidereal day is about
(a) 1 minute (b) 4 minutes
(c) 15 minutes (d) 56 minutes [AIIMS 03]

7. Gravitational mass is proportional to gravitational
(a) field (b) force
(c) intensity (d) all of these [AIIMS 08]

8. SONAR emits which of the following waves ?
(a) radio (b) light
(c) ultrasound (d) none of these [AIIMS 99]

9. The dimension of torque is
(a) $[MLT^{-2}]$ (b) $[ML^2T^{-2}]$
(c) $[ML^{-1}T^{-1}]$ (d) $[ML^3T^{-3}]$ [AIIMS 02]

10. What is the dimensional formula of gravitational constant G ?
(a) $[M^{-1}L^3T^{-2}]$ (b) $[M^{-2}L^3T^{-2}]$
(c) $[M^{-1}L^2T^{-2}]$ (d) $[M^{-1}L^3T^{-1}]$ [AIIMS 00]

11. The dimension of angular velocity is
(a) $[MLT^{-2}]$ (b) $[M^2L^0T^{-1}]$
(c) $[M^0L^0T^{-1}]$ (d) $[ML^2T^{-2}]$ [AIIMS 98]

12. Which of the following physical quantity has the dimension of $[ML^2T^{-3}]$?
(a) work (b) power
(c) pressure (d) impulse [AIIMS 94]

13. The dimension of the modulus of rigidity, is
(a) $[MLT^{-2}]$ (b) $[ML^{-1}T^{-2}]$
(c) $[ML^{-2}T^{-2}]$ (d) $[ML^{-1}T^{-1}]$ [AIIMS 94]

14. The dimension of Planck's constant is
(a) $[ML^2T^{-1}]$ (b) $[ML^{-3}T^{-1}]$
(c) $[ML^{-2}T^{-1}]$ (d) $[M^0L^{-1}T^{-3}]$ [AIIMS 97]

15. Dimensions $[ML^{-1}T^{-1}]$ are related to
(a) work (b) torque
(c) energy (d) coefficient of viscosity [AIIMS 99]

16. Which of the following is a dimensionless quantity ?
(a) Strain (b) Stress
(c) Specific heat (d) Quantity of heat [AIIMS 94]

17. Which of the following pair does not have similar dimensions ?
(a) stress and pressure
(b) angle and strin
(c) tension and surface tension
(d) Planck's constant and angular momentum [AIIMS 01, 07]

18. Dimensions of electrical resistance are

(*a*) $[ML^2T^{-3}A^{-1}]$ (*b*) $[ML^2T^{-3}A^{-2}]$

(*c*) $[ML^3T^3A^{-2}]$ (*d*) $[ML^{-1}T^3A^2]$ [AIIMS 94]

19. The magnetic moment has the dimensions of

(*a*) [LA] (*b*) $[L^2A]$

(*c*) $[LT^{-1}A]$ (*d*) $[L^2T^{-1}A]$ [AIIMS 06]

20. Which of the following pairs does not have same dimensions ?

(*a*) impulse and momentum

(*b*) moment of inertia and moment of force

(*c*) angular momentum and Planck's constant

(*d*) work and torque [AIIMS 2010]

21. If the energy, $E = G^p h^q c^r$, where G is the universal gravitational constant, h is the Planck's constant and c is the velocity of light, then the values of p, q and r are, respectively

(*a*) $-1/2, 1/2$ and $5/2$

(*b*) $1/2, -1/2$ and $-5/2$

(*c*) $-1/2, 1/2$ and $3/2$

(*d*) $1/2, -1/2$ and $-3/2$ [AIIMS 2010]

Answers and Explanations

1. (*b*) One metre contains 1650763.73 wavelengths of certain orange radiation of Kr^{86}.

2. (*d*) $1 \text{ nm} = 10^{-9}$ m.

3. (*b*) One light year is the *distance* travelled by light in one year.

4. (*b*) One parse is the *distance* at which an arc of length 1 AU subtends an angle of 1 second of arc.

5. (*b*) Debye is the unit of electric dipole moment not of length or distance.

6. (*b*) The sidereal day is about 4 minutes (more precisely, 3 minutes 56 seconds) shorter than a mean solar day (of 24 hours).

7. (*b*) Gravitational mass is proportional to gravitational force.

8. (*c*) SONAR emtis ultrasound.

9. (*b*) [Torque] = $\mathbf{[ML^2T^{-2}]}$.

10. (*a*) $[G] = \dfrac{Fr^2}{m_1 m_2} = \dfrac{[MLT^{-2}][L^2]}{[M][M]} = \mathbf{[M^{-1}L^3T^{-2}]}$.

11. (*c*) $[\omega] = \dfrac{[\theta]}{[t]} = \dfrac{1}{T} = \mathbf{[M^0L^0T^{-1}]}$

12. (*b*) $[\text{Power}] = \dfrac{\text{work}}{\text{time}} = \dfrac{[ML^2T^{-2}]}{[T]}$

$$= \mathbf{[ML^2T^{-3}]}.$$

13. (*b*) $[\text{Modulus of rigidity}] = \dfrac{\text{Stress}}{\text{Strain}} = \dfrac{[ML^{-1}T^{-2}]}{1}$

$$= \mathbf{[ML^{-1}T^{-2}]}.$$

14. (*a*) $[h] = \dfrac{\text{Energy}}{\text{Frequency}} = \dfrac{[ML^2T^{-2}]}{[T^{-1}]} = \mathbf{[ML^2T^{-1}]}$

15. (*d*) [Coefficient of viscosity] = $\mathbf{[ML^{-1}T^{-1}]}$.

16. (*a*) Strain is a dimensionless quantity.

17. (*a*) [Tension] = [Force] = $[MLT^{-2}]$

$$[\text{Surface tension}] = \frac{[\text{Force}]}{[\text{Length}]} = \frac{[MLT^{-2}]}{[L]} = [MT^{-2}].$$

18. (*b*) $\text{Resistance} = \dfrac{\text{P.D.}}{\text{Current}} = \dfrac{\text{Work}}{\text{Charge} \times \text{Current}}$

$$[\text{Resistance}] = \frac{[ML^2T^{-2}]}{[TA][A]} = \mathbf{[ML^2T^{-3}A^{-2}]}.$$

19. (*b*) [Magnetic moment] = [Current] [Area]

$$= [A][L^2] = \mathbf{[L^2A]}.$$

20. (*b*) Moment of inertia,

$$[I] = \text{Mass} \times \text{distance}^2 = [ML^2]$$

Moment of force,

$$[\tau] = \text{Force} \times \text{distance}$$
$$= ML^{-2}T \cdot L = [ML^2T^{-2}]$$

$$\therefore \quad [I] \neq [\tau].$$

21. (*a*) Given $E = G^p h^q c^r$

$$\therefore \quad [M^1L^2T^{-2}] = [M^{-1}L^3T^{-2}]^p [ML^2T^{-1}]^q [LT^{-1}]^r$$

or $$M^1L^2T^{-2} = M^{-p+q} L^{3p+2q+r} T^{-2p-q-r}$$

Equating the powers of M, L and T, we get

$$-p + q = 1$$
$$3p + 2q + r = 2$$
$$-2p - q - r = -2$$

On solving, we get

$$p = -\frac{1}{2}, \quad q = \frac{1}{2}, \quad r = \frac{5}{2}$$

CBSE PMT Prelims and Final Exams

1. Which of the following has the dimensions of pressure ?

(a) $[MLT^{-2}]$ (b) $[ML^{-1}T^{-2}]$
(c) $[ML^{-2}T^{-2}]$ (d) $[M^{-1}L^{-1}]$ [CBSE PMT 90, 94]

2. The dimensional formula of torque is

(a) $[ML^2T^{-2}]$ (b) $[ML^2T^{-1}]$
(c) $[ML^{-1}T^{-2}]$ (d) $[ML^{-2}T^{-2}]$ [CBSE PMT 89]

3. The dimensions of universal gravitational constant are

(a) $[M^{-1}L^3T^{-2}]$ (b) $[ML^2T^{-1}]$
(c) $[M^{-2}L^3T^{-2}]$ (d) $[M^{-2}L^2T^{-1}]$

[CBSE PMT 92, 04]

4. The dimensions of impulse are equal to that of

(a) pressure (b) linear momentum
(c) force (d) angular momentum [CBSE PMT 89]

5. According to Newton, the viscous force acting between liquid layers of area A and velocity gradient $\Delta v / \Delta x$ is given by

$$F = -\eta A \frac{\Delta v}{\Delta x},$$

where η is constant called coefficient of viscosity. The dimensional formula of η is

(a) $[ML^{-2}T^{-2}]$ (b) $[M^0L^0T^0]$
(c) $[ML^2T^{-2}]$ (d) $[ML^{-1}T^{-1}]$ [CBSE PMT 90]

6. Turpentine oil is flowing through a tube of length l and radius r. The pressure difference between the two ends of the tube is P. The viscosity of oil is given by $\eta = \dfrac{P(r^2 - x^2)}{4vl}$, where v is the velocity of oil at a distance x from the axis of the tube. The dimensions of η are

(a) $[M^0L^0T^0]$ (b) $[MLT^{-1}]$
(c) $[ML^2T^{-2}]$ (d) $[ML^{-1}T^{-1}]$ [CBSE PMT 93]

7. The dimensional formula of angular momentum is

(a) $[ML^2T^{-2}]$ (b) $[ML^{-2}T^{-1}]$
(c) $[MLT^{-1}]$ (d) $[ML^2T^{-1}]$ [CBSE PMT 88]

8. The dimensions of Planck's constant equals to that of

(a) energy (b) momentum
(c) angular momentum (d) power [CBSE PMT 01]

9. The ratio of the dimensions of Planck constant and that of moment of inertia is the dimensions of

(a) time (b) frequency
(c) angular momentum (d) velocity [CBSE PMT 05]

10. Dimensions of resistance in an electrical circuit, in terms of dimension of mass M, of length L, of time T and of current I, would be

(a) $[ML^2T^{-2}]$ (b) $[ML^2T^{-1}I^{-1}]$
(c) $[ML^2T^{-3}I^{-2}]$ (d) $[ML^2T^{-3}I^{-1}]$

[CBSE PMT 07]

11. Dimensional formula of self inductance is

(a) $[MLT^{-2}A^{-2}]$ (b) $[ML^2T^{-1}A^{-2}]$
(c) $[ML^2T^{-2}A^{-2}]$ (d) $[ML^2T^{-2}A^{-1}]$

[CBSE PMT 89]

12. If C and R denote capacitance and resistance, the dimensional formula of CR is

(a) $[M^0L^0T^1]$ (b) $[M^0L^0T^0]$
(c) $[M^0L^0T^{-1}]$
(d) not expressible in terms of MLT [CBSE PMT 89]

13. The dimension of RC is

(a) square of time (b) square of inverse time
(c) time (d) inverse time.

[CBSE PMT 95]

14. Which of the following dimensions will be the same as that of time ?

(a) $\dfrac{L}{R}$ (b) $\dfrac{C}{L}$
(c) LC (d) $\dfrac{R}{L}$ [CBSE PMT 96]

15. The unit of permittivity of free space ε_0 is

(a) coulomb/newton-metre
(b) newton-metre^2/coulomb^2
(c) coulomb^2/newton-metre^2
(d) coulomb^2/(newton-metre)^2 [CBSE PMT 04]

16. The dimensional formula of magnetic flux is

(a) $[M^0L^{-2}T^{-2}A^{-2}]$ (b) $[ML^0T^{-2}A^{-2}]$
(c) $[ML^2T^{-2}A^{-1}]$ (d) $ML^2T^{-1}A^3$

[CBSE PMT 99]

17. The dimensional formula of permeability of free space μ_0 is

(a) $[MLT^{-2}A^{-2}]$ (b) $[M^0L^1T]$
(c) $[M^0L^2T^{-1}A^2]$ (d) none of these.

[CBSE PMT 91]

18. Which pairs do not have equal dimensions ?
(*a*) Energy and torque (*b*) Force and impulse
(*c*) Angular momentum and Planck's constant
(*d*) Elastic modulus and pressure. **[CBSE PMT 2K]**

19. Of the following quantities, which one has dimensions different from the remaining three ?
(*a*) Energy per unit volume
(*b*) Force per unit area
(*c*) Product of voltage and charge per unit volume
(*d*) Angular momentum. **[CBSE PMT 89]**

20. If $x = at + bt^2$, where x is the distance travelled by the body in kilometres while t is the time in seconds, then the unit of b is
(*a*) km/s (*b*) kms
(*c*) km/s^2 (*d*) kms^2 **[CBSE PMT 89]**

21. An equation is given here, $\left(P + \frac{a}{V^2}\right) = b\frac{\theta}{V}$, where P = pressure, V = volume and θ = absolute temperature. If a and b are constants, the dimensions of a will be
(*a*) $[ML^{-5}T^{-1}]$ (*b*) $[ML^5T^1]$
(*c*) $[ML^5T^{-2}]$ (*d*) $[M^{-1}L^5T^2]$
[CBSE PMT 96]

22. The velocity v of a particle at time t is given by $v = at + \frac{b}{t+c}$, where a, b and c are constants. The dimensions of a, b and c are
(*a*) [L], [LT] and $[LT^{-2}]$
(*b*) $[LT^{-2}]$, [L] and [T]
(*c*) $[L^2]$, [T] and $[LT^{-2}]$
(*d*) $[LT^{-2}]$, [LT] and [L]. **[CBSE PMT 06]**

23. The time dependence of a physical quantity p is given by $p = p_0 \exp(-\alpha t^2)$, where α is a constant and t is the time. The constant α
(*a*) is dimensionless (*b*) has dimensions $[T^{-2}]$
(*c*) has dimensions $[T^2]$
(*d*) has dimensions of p. **[CBSE PMT 93]**

24. Which of the following five physical parameters have the same dimensions ?

1. energy density	2. refractive index
3. dielectric constant	4. Young's modulus
5. magnetic field	

(*a*) 2 and 4 (*b*) 3 and 5
(*c*) 1 and 4 (*d*) 1 and 5 **[CBSE PMT 08]**

25. The frequency of vibration f of a mass m suspended from a spring of spring constant k is given by a relation $f = am^xk^y$, where a is a dimensionless constant. The values of x and y are
(*a*) $x = \frac{1}{2}, y = \frac{1}{2}$ (*b*) $x = -\frac{1}{2}, y = -\frac{1}{2}$
(*c*) $x = \frac{1}{2}, y = -\frac{1}{2}$ (*d*) $x = -\frac{1}{2}, y = \frac{1}{2}$
[CBSE PMT 90]

26. P represents radiation pressure, c represents speed of light and S represents radiation energy striking per unit area per sec. The non-zero integers x, y, z such that $P^x S^y c^z$ is dimensionless, are
(*a*) $x = 1, y = 1, z = 1$ (*b*) $x = -1, y = 1, z = 1$
(*c*) $x = 1, y = -1, z = 1$ (*d*) $x = 1, y = 1, z = -1$
[CBSE PMT 92]

27. If the dimensions of a physical quantity are given by $M^aL^bT^c$, then the physical quantity will be
(*a*) Velocity if $a = 1, b = 0, c = -1$
(*b*) Acceleration if $a = 1, b = 1, c = -2$
(*c*) Force if $a = 0, b = -1, c = -2$
(*d*) Pressure if $a = 1, b = -1, c = -2$. **[CBSE PMT 09]**

28. The density of a material in CGS system of units is 4 g / cm^2. In a system of units in which unit of length is 10 cm and unit of mass is 100 g, the value of density of material will be
(*a*) 400 (*b*) 0.04
(*c*) 0.4 (*d*) 40 **[CBSE Final 2011]**

29. Which of the following is a dimensional constant ?
(*a*) relative density (*b*) gravitational constant
(*c*) refractive index (*d*) poisson ratio.
[CBSE PMT 95]

30. Percentage errors in the measurement of mass and speed are 2% and 3% respectively. The error in the estimate of kinetic energy obtained by measuring mass and speed will be
(*a*) 8% (*b*) 2%
(*c*) 12% (*d*) 10% **[CBSE PMT 95]**

31. The density of a cube is measured by measuring its mass and length of its sides. If the maximum errors in the measurement of mass and lengths are 3% and 2% respectively, the maximum error in the measurement of density would be
(*a*) 12% (*b*) 14%
(*c*) 7% (*d*) 9% **[CBSE PMT 89]**

32. A certain body weighs 22.42 g and has a measured volume of 4.7 cc. The possible errors in the measurement of mass and volume are 0.01 g and 0.1 cc. Then maximum error in the density will be

(*a*) 22% (*b*) 2%
(*c*) 0.2% (*d*) 0.02% **[CBSE PMT 91]**

33. If the error in the measurement of radius of a sphere is 2%, then the error in the determination of volume of the sphere will be

(*a*) 4% (*b*) 6%
(*c*) 8% (*d*) 2% **[CBSE PMT 08]**

Answers and Explanations

1. (*b*) [Pressure] = $[\mathbf{ML^{-1}T^{-2}}]$

Refer to point 11 of Table 2.8 on page 2.20.

2. (*a*) [Torque] = $[\mathbf{ML^2T^{-2}}]$

Refer to point 12 of Table 2.8 on page 2.20.

3. (*a*) $[G] = [\mathbf{M^{-1}L^3T^{-2}}]$

Refer to point 13 of Table 2.8 on page 2.20.

4. (*b*) [Impulse] = $[\mathrm{MLT^{-1}}]$

Refer to point 14 of Table 2.8 on page 2.20.

5. (*d*) $[\eta] = \left[\dfrac{F\Delta x}{A\Delta v}\right] = \dfrac{[\mathrm{MLT^{-2}}][\mathrm{L}]}{[\mathrm{L^2}][\mathrm{LT^{-1}}]}$

$= [\mathbf{ML^{-1}T^{-1}}]$

6. (*d*) $[\eta] = \dfrac{P(r^2 - x^2)}{4vl} = \dfrac{[\mathrm{ML^{-1}T^{-2}}][\mathrm{L^2}]}{[\mathrm{LT^{-1}}][\mathrm{L}]}$

$= [\mathbf{ML^{-1}T^{-1}}]$

7. (*d*) $[L] = [mvr] = [\mathrm{M}][\mathrm{LT^{-1}}][\mathrm{L}]$

$= [\mathbf{ML^2T^{-1}}]$.

8. (*c*) $[h] = \dfrac{\text{Energy}}{\text{Frequency}}$

$= \dfrac{[\mathrm{ML^2T^{-2}}]}{[\mathrm{T^{-1}}]} = [\mathrm{ML^2T^{-1}}]$.

9. (*b*) $\left[\dfrac{h}{I}\right] = \dfrac{E}{\nu I} = \dfrac{[\mathrm{ML^2T^{-2}}]}{[\mathrm{T^{-1}}][\mathrm{ML^2}]}$

$= [\mathrm{T^{-1}}] = [\text{Frequency}]$

10. (*c*) $[R] = \dfrac{V}{I} = \dfrac{W}{qI} = \dfrac{[\mathrm{ML^2T^{-2}}]}{[\mathrm{IT}][\mathrm{I}]} = [\mathrm{ML^2T^{-3}I^{-2}}]$

11. (*c*) [Self inductance] = $[\mathrm{ML^2T^{-2}A^{-2}}]$

Refer to point 49 of Table 2.8 on page 2.22.

12. (*a*) $[CR] = \dfrac{q}{V}\cdot\dfrac{V}{I} = \dfrac{q}{I} = \dfrac{[\mathrm{IT}]}{[\mathrm{I}]} = [\mathrm{M^0L^0T^1}]$

13. (*c*) Dimension of RC is same as that of time, as proved in the above problem.

14. (*a*) $\left[\dfrac{L}{R}\right] = \dfrac{[\mathrm{ML^2T^{-2}A^{-2}}]}{[\mathrm{ML^2T^{-3}A^{-2}}]} = [\mathbf{T}]$.

15. (*c*) From Coulomb's law,

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}$$

or $$\varepsilon_0 = \frac{q_1q_2}{4\pi Fr^2}$$

SI unit of $\varepsilon_0 = \dfrac{\mathrm{C\,.\,C}}{\mathrm{Nm^2}} = \mathbf{C^2/Nm^2}$

16. (*c*) $[\phi] = BA = \dfrac{F}{qv\sin\theta}A$

$= \dfrac{[\mathrm{MLT^{-2}}][\mathrm{L^2}]}{[\mathrm{AT}][\mathrm{LT^{-1}}].1}$

$= [\mathbf{ML^2T^{-2}A^{-1}}]$

17. (*a*) $[\mu_0] = \dfrac{4\pi rF}{I_1I_2l} = \dfrac{\mathrm{L.MLT^{-2}}}{\mathrm{A^2.L}}$

$= [\mathbf{MLT^{-2}A^{-2}}]$

18. (*b*) [Force] = $[\mathrm{MLT^{-2}}]$, [Impulse] = $[\mathrm{MLT^{-1}}]$

19. (*d*) $\left[\dfrac{E}{V}\right] = \dfrac{\mathrm{ML^2T^{-2}}}{\mathrm{L^3}} = [\mathrm{ML^{-1}T^{-2}}]$

$\left[\dfrac{F}{A}\right] = \dfrac{\mathrm{MLT^{-2}}}{\mathrm{L^2}} = [\mathrm{ML^{-1}T^{-2}}]$

$\left[\text{voltage}\dfrac{q}{V}\right] = \dfrac{\mathrm{ML^2T^{-3}A^{-1}\cdot AT}}{\mathrm{L^3}} = [\mathrm{ML^{-1}T^{-2}}]$

$[L] = [\mathrm{ML^2T^{-1}}]$

Dimensions of angular momentum L are different from remaining three.

20. (*c*) Unit of b = unit of $\dfrac{x}{t^2}$ = $\mathbf{km/s^2}$

21. (*c*) $\left[\frac{a}{V^2}\right] = [P]$

$\therefore \quad [a] = [P][V^2] = [ML^{-1}T^{-2}][L^3]^2$

$= [ML^5T^{-2}]$

22. (*b*) $V = at + \frac{b}{t+c}$

As *c* is added to *t*, so

$[c] = [T].$

$[at] = [v]$

$\therefore \quad [a] = \frac{[v]}{[t]} = \frac{LT^{-1}}{T} = [LT^{-2}]$

$\frac{[b]}{[T]} = [v]$

or $\quad [b] = LT^{-1}.T = [L]$

23. (*b*) Given $p = p_0 e^{-\alpha t^2}$

αt^2 must be dimensionless

$\therefore \quad [\alpha] = \frac{1}{[t^2]} = \frac{1}{T^2} = [T^{-2}]$

24. (*c*) Energy density

$= \frac{\text{Energy}}{\text{Volume}} = \frac{[ML^2T^{-2}]}{[L^3]}$

$= [ML^{-1}T^{-2}]$

Young's modulus,

$= \frac{F}{A}.\frac{L}{l} = \frac{MLT^{-2}}{L^2}.\frac{L}{L} = [ML^{-1}T^{-2}].$

25. (*d*) Given : $f = am^x k^y$

Putting the dimensions of various quantities,

$T^{-1} = 1.[M]^x[MT^{-2}]^y \quad \left[\because k = \frac{\text{Force}}{\text{Distance}}\right]$

$M^0T^{-1} = M^{x+y}T^{-2y}$

$x + y = 0$ and $-2y = -1$

$y = \frac{1}{2}$ and $x = -\frac{1}{2}$

26. (*c*) $[P] = \frac{\text{Force}}{\text{Area}} = \frac{MLT^{-2}}{L^2} = [ML^{-1}T^{-2}]$

$[S] = \frac{\text{Energy}}{\text{Area} \times \text{Time}}$

$= \frac{ML^2T^{-2}}{L^2.T} = [MT^{-3}]$

$[c] = [LT^{-1}]$

Dimensionless constant,

$k = P^x S^y c^z$

$\therefore \quad M^0L^0T^0 = [ML^{-1}T^{-2}]^x[MT^{-3}]^y[LT^{-1}]^z$

$M^0L^0T^0 = M^{x+y}L^{-x+z}T^{-2x-3y-z}$

$x + y = 0, \; -x + z = 0, \; -2x - 3y - z = 0$

On solving, $x = 1, \; y = -1, \; z = 1.$

27. (*d*) Pressure, $[P] = \frac{[F]}{[A]} = \frac{MLT^{-2}}{L^2}$

$= [M^1L^{-1}T^{-2}]$

$\therefore a = 1, b = -1, c = -2.$

28. (*d*) $n_2 = n_1\left[\frac{M_1}{M_2}\right]\left[\frac{L_1}{L_2}\right]^{-3}$

$= 4\left[\frac{1}{100}\right]\left[\frac{1}{10}\right]^{-3}$

$= 40$

29. (*b*) Only G has dimensions of $[M^{-1}L^3T^{-2}]$. Remaining three quantities are pure ratios.

30. (*a*) $K = \frac{1}{2}mv^2$

$\therefore \quad \frac{\Delta K}{K} \times 100 = \frac{\Delta m}{m} \times 100 + 2\frac{\Delta v}{v} \times 100$

$= 2\% + 2 \times 3\%$

$= 8\%.$

31. (*d*) $\rho = \frac{M}{V} = \frac{M}{L^3}$

$\therefore \quad \frac{\Delta\rho}{\rho} \times 100 = \frac{\Delta M}{M} \times 100 + 3\frac{\Delta L}{L} \times 100$

$= 3\% + 3 \times 2\% = 9\%.$

32. (*b*) $\rho = \frac{M}{V}$

$\therefore \quad \frac{\Delta\rho}{\rho} \times 100 = \frac{\Delta M}{M} \times 100 + \frac{\Delta V}{V} \times 100$

$= \left(\frac{0.01}{22.42} + \frac{0.1}{4.7}\right) \times 100 = 2\%.$

33. (*b*) Here $V = \frac{4}{3}\pi r^3$

$\therefore \quad \frac{\Delta V}{V} \times 100 = 3\frac{\Delta r}{r} \times 100$

$= 3 \times 2\% = 6\%.$

Delhi PMT and VMMC Entrance Exam

(Similar Questions)

1. Light year is used to meaure
(a) distance between stars
(b) distance between atoms
(c) stationary charge
(d) none of these. [DPMT 07]

2. Which of the following is true for the solid angle ?
(a) $\delta\omega = \dfrac{\delta A \cos\theta}{r^2}$ (b) $d\omega = \dfrac{\delta A \cos^2\theta}{r^2}$
(c) $\delta\omega = \dfrac{\delta A \cos\theta}{r^3}$ (d) $\delta\omega = \dfrac{\delta A \cos^2\theta}{r^3}$ [DPMT 99]

3. Dimensions of coeffcient of viscosity are
(a) $[ML^{-1}T^{-1}]$ (b) $[ML^{-3}T^{-4}]$
(c) $[ML^{-1}T^{-2}]$ (d) $[MT^2]$ [DPMT 99, 04]

4. The dimensions of surface tension are
(a) Nm (b) Nm^2
(c) Nm^{-1} (d) N-s. [DPMT 01]

5. Dimensions of Hubble's constant are
(a) $[T^{-1}]$ (b) $[MLT^4]$
(c) $[M^0L^0T^{-2}]$ (d) $[MLT^1]$ [DPMT 97]

6. The units of Planck's constant are
(a) J/s (b) Js^2
(c) Js (d) Js^{-2} [DPMT]

7. Which of the following have same dimensions ?
(a) torque and force
(b) torque and potential energy
(c) potential energy and force
(d) Planck's constant and momentum [DPMT 98]

8. Force F is given by

$$F = at + bt^2,$$

where t is time. What are the dimensions of a and b ?
(a) $[MLT^{-3}]$ and $[MLT^{-4}]$
(b) $[MLT^{-1}]$ and $[MLT^0]$
(c) $[MLT^{-3}]$ and $[MLT^4]$
(d) $[MLT^{-4}]$ and $[MLT^1]$ [DPMT 93]

9. The dimensional formula of the constant a in van der Waal's gas equation

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT \text{ is}$$

(a) $[ML^3T^2]$ (b) $[ML^2T^{-1}]$
(c) $[ML^5T^{-2}]$ (d) $[ML^5T^{-3}]$ [DPMT 92]

10. The equation $\left(P + \dfrac{a}{V^2}\right)(V - b) =$ Constant. The units of a are
(a) dyne × cm^5 (b) dyne × cm^4
(c) dyne cm^{-3} (d) dyne cm^{-2}. [DPMT 06]

11. In a system of units, the units of mass, length and time are 1 quintal, 1 km and 1 h respectively. In this system 1 N force will be equal to
(a) 1 new unit (b) 129.6 new units
(c) 125.7 new units
(d) 10^3 new units.

12. What is the dimension of surface tension ?
(a) $[ML^1T^0]$ (b) $[ML^1T^{-1}]$
(c) $[ML^0T^{-2}]$ (d) $[M^1L^1T^{-2}]$ [DPMT 2011]

13. The significant figures of the number 6.0023 are
(a) 1 (b) 5
(c) 4 (d) 2 [DPMT 95]

14. The angle subtended by a coin of radius 1 cm held at a distance of 80 cm from your eyes is
(a) 1.43° (b) 0.72°
(c) 0.0125° (d) 0.025° [DPMT 09]

Answers and Explanations

1. (a) Light year is the *distance* travelled by light is one year.

2. (a) Solid angle is normal projection of area divided by square of radius of the curved surface

$$\delta\omega = \frac{\delta A.\cos\theta}{r^2}$$

3. (a) [coefficient of viscosity] $= [ML^{-1}T^{-1}]$.

4. (c) Surface tension $= \dfrac{\text{Force}}{\text{Length}}$

∴ SI unit of surface tension $= Nm^{-1}$.

5. (*a*) According to Hubble's law, speed of recession of a galaxy,

$$v = Hr$$

$$\therefore \quad [H] = \frac{[v]}{[r]} = \frac{[LT^{-1}]}{[L]} = [T^{-1}]$$

6. (*c*) $h = \dfrac{\text{Energy}}{\text{Frequency}}$

Unit of $h = \dfrac{J}{s^{-1}} = Js.$

7. (*b*) [Torque] = [Potential energy]

$$= [ML^2T^{-2}]$$

8. (*a*) $[a] = \dfrac{[F]}{[t]} = \dfrac{[MLT^{-2}]}{[T]} = [MLT^{-3}]$

$$[b] = \frac{[F]}{[t]^2} = \frac{[MLT^{-2}]}{[T]^2}$$

$$= [MLT^{-4}]$$

9. (*c*) $[P] = \left[\dfrac{a}{V^2}\right]$

$$\therefore \quad [a] = [P][V^2] = [ML^{-1}T^{-2}][L^3]^2$$

$$= [ML^5T^{-2}]$$

10. (*b*) Unit of a = Unit of $P \times$ Unit of V^2

$$= \text{dyne cm}^{-2} \times (\text{cm}^3)^2$$

$$= \textbf{dyne cm}^4.$$

11. (*b*) [Force] = [MLT^{-2}]

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^1\left[\frac{L_1}{L_2}\right]^1\left[\frac{T_1}{T_2}\right]^{-2}$$

$$= 1\left[\frac{1\text{ kg}}{1\text{ quintal}}\right]^1\left[\frac{1\text{m}}{1\text{km}}\right]^1\left[\frac{1\text{s}}{1\text{h}}\right]^{-2}$$

$$= 1\left[\frac{1}{100}\right]\left[\frac{1}{1000}\right]\left[\frac{1}{3600}\right]^{-2}$$

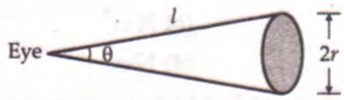
$$= \frac{3600 \times 3600}{100 \times 1000} = \textbf{129.6 new units.}$$

12. (*c*) Surface tension

$$= \frac{\text{Force}}{\text{Length}} = \frac{MLT^{-2}}{L} = [ML^0T^{-2}]$$

13. (*b*) The number of significant figures in 6.0023 is five (6, 0, 0, 2, 3).

14. (*a*) Here $l = 80$ cm, $2r = 2$ cm



$$\theta = \frac{\text{Arc}}{\text{Radius}} = \frac{2r}{l} = \frac{2}{80} = \frac{1}{40} \text{ rad}$$

$$= \left(\frac{180}{\pi} \times \frac{1}{40}\right)^\circ = \mathbf{1.43^\circ}.$$