CHAPTER 3

MOTION IN A STRAIGHT LINE

3.1 MECHANICS

1. *What is mechanics ? What are its various sub-branches ?*

Mechanics. Mechanics is the branch of physics that *deals with the conditions of rest or motion of the material objects* around us. It is one of the oldest branches of physics.

Sub-branches of mechanics :

(*i*) ***Statics.*** It is the branch of mechanics that *deals with the study of objects at rest or in equilibrium,* even when they are under the action of several forces. The measurement of time is not essential in statics.

(*ii*) ***Kinematics.*** It is the branch of mechanics that *deals with the study of motion of objects without considering the cause of motion.* Here measurement of time is essential. The word *kinematics* comes from Greek word *kinema* which means motion.

(*iii*) ***Dynamics.*** It is the branch of mechanics that *deals with the study of motion of objects taking into consideration the cause of their motion*. The word *dynamics* comes from a Greek word *dynamics* which means power. Dynamics is concerned with the forces which cause motion. Newton contributed a lot to dynamics by postulating his famous laws of motion.

3.2 REST AND MOTION

2. *Define the terms rest and motion. Show that rest and motion are relative terms.*

Rest. *An object is said to be at rest if it does not change its position w.r.t. its surroundings with the passage of time e.g.,* a book lying on a table.

Motion. *An object is said to be in motion if it changes its position w.r.t. its surroundings with the passage of time e.g.,* a train moving on rails.

Rest and motion are relative terms. A passenger sitting in a moving train is at rest with respect to his fellow passengers but he is in motion with respect to the objects outside the train. Thus an object may be at rest w.r.t. one object and at the same time it may be in motion relative to another object. Hence rest and motion are relative terms.

3. *Can a body exist in a state of absolute rest or of absolute motion ? Explain.*

Absolute rest and motion are unknown. In order to know whether the position of an object changes with time or not, a point *absolutely fixed in space has to be chosen as reference point.* But no such point is known in the universe. The earth revolves around the sun, the entire solar system travels through our own galaxy, the milkyway and clusters of galaxies move with respect to other clusters. So *no object in the universe is in a state of absolute rest.*

As no object in the universe is at absolute rest, so the absolute motion cannot be realised. Only relative rest and relative motion can be realised.

3.3 CONCEPT OF A POINT OBJECT

4. *What is meant by a point object ? Give suitable examples.*

Point object. *If the position of an object changes by distances much greater than its own size in a resonable duration of time, then the object may be regarded as a point object.* When a point object moves, its rotational and vibrational motions may be ignored.

Examples :

(*i*) Earth can be regarded as a point object for studying its motion around the sun.

(*ii*) A train under a journey of several hundred kilometres can be regarded as a point object.

3.4 MOTION IN ONE, TWO AND THREE DIMENSIONS

5. *What do you mean by motion in one, two and three dimensions ? Give examples of each type.*

One dimensional motion. *The motion of an object is said to be one dimensional if only one of the three coordinates specifying the position of the object changes with time.* Here the object moves along a straight line. This motion is also called **rectilinear** or **linear motion.** As shown in Fig. 3.1, only the x-coordinate changes from x_1 to x_2 when the particle moves from P_1 to P_2 along a *straight line* path.

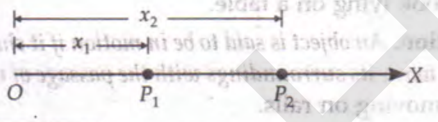


Fig. 3.1 One dimensional motion.

Examples of one dimensional motion :

(*i*) Motion of a train along a straight track.

(*ii*) Motion of a freely falling body.

Two dimensional motion. *The motion of an object is said to be two dimensional if only two of the three coordinates specifying its position change with time.* Here an object moves along a plane. As shown in Fig. 3.2, the coordinates (x_1, y_1) change to (x_2, y_2) as the particle moves from P_1 to P_2 in a *plane.*

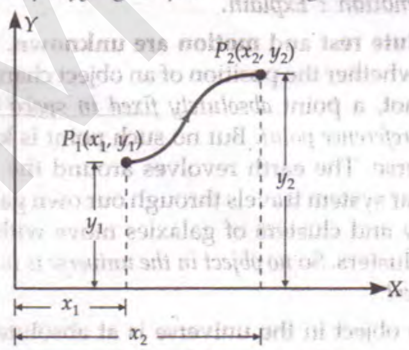


Fig. 3.2 Two dimensional motion.

Examples of two dimensional motion :

(*i*) Motion of planets around the sun.

(*ii*) A car moving along a zig-zag path on a level road.

Three dimensional motion. *The motion of an object is said to be three dimensional if all the three coordinates specifying its position change with time.* Here an object moves in space. As shown in Fig. 3.3, the coordinates (x_1, y_1, z_1) change to (x_2, y_2, z_2) as the particle moves from P_1 to P_2 in *space.*

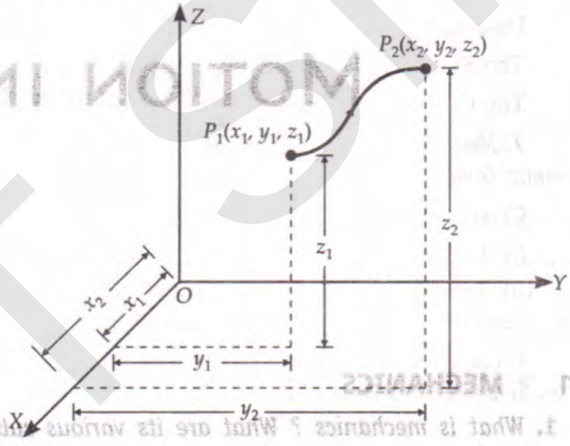


Fig. 3.3 Three dimensional motion.

Examples of three dimensional motion :

(*i*) A kite flying on a windy day.

(*ii*) Motion of an aeroplane in space.

3.5 DISTANCE AND DISPLACEMENT

6. *Distinguish between the terms distance and displacement.*

Distance or path length. *It is the length of the actual path traversed by a body between its initial and final positions.* As shown in Fig. 3.4, suppose a body moves from position A to B through C. Then

Distance covered = $AC + CB$

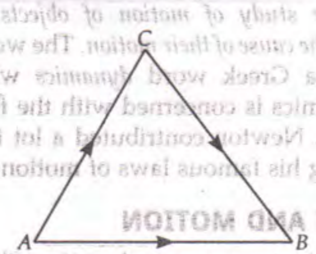


Fig. 3.4

Distance is a *scalar quantity* because it has only magnitude and no direction. Distance covered is always positive or zero.

The SI unit of distance is metre (m).

The CGS unit of distance is centimetre (cm).

Displacement. *The displacement of an object is the change in the position of an object in a fixed direction.* It is the shortest (or the straight line) path measured in the direction from initial point to the final point. As displacement has both magnitude and direction, so it is a vector quantity.

In Fig. 3.4, displacement = $\vec{AB}$.

Displacement may be positive, negative or zero.

The SI unit of displacement is metre (m).

The CGS unit of displacement is centimetre (cm).

7. *Mention some important characteristics of displacement. Give illustrations to support these characteristics.*

Characteristics of Displacement :

(*i*) *Displacement has the units of length.*

(*ii*) *The displacement of an object can be positive, negative or zero.*

When an object moves towards right in time t_1 to t_2, its displacement is positive as shown in Fig. 3.5.

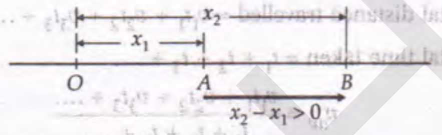


Fig. 3.5 Positive displacement.

When an object moves to the left in time t_1 to t_2, its displacement is negative as shown in Fig. 3.6.

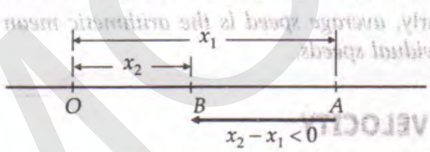


Fig. 3.6 Negative displacement.

When an object remains stationary, or it moves first towards right and then an equal distance towards left, its displacement is zero as shown in Fig. 3.7.

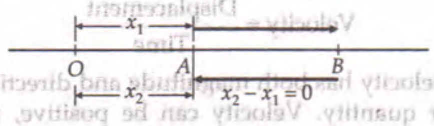


Fig. 3.7 Zero displacement.

(*iii*) *Displacement is not dependent on the choice of the origin O of the position coordinates.* As shown in Fig. 3.8, the shift of the origin from O to O' on the line L produces equal changes in the position coordinates at times t_1 and t_2, so that the difference in the new position coordinates still remains the same. Hence the displacement AB remains unaffected.

Clearly,

$$x_2 - x_1 = AB = x_2' - x_1'$$

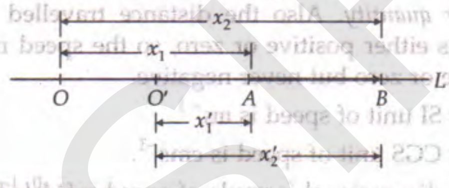


Fig. 3.8 Shifting origin causes no change in displacement.

(*iv*) *The actual distance travelled by an object in a given time interval is greater than or equal to the magnitude of the displacement.* The first situation is shown in Fig. 3.9 and second one in Fig. 3.10.

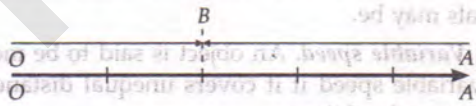


Fig. 3.9 Magnitude of displacement $\vec{OB}$ (2 units) < Actual distance ($OA + AB$) travelled (8 units).

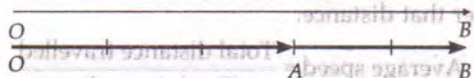


Fig. 3.10. Magnitude of displacement $\vec{OB}$ = Actual distance travelled ($OA + AB$) = 5 units each.

(*v*) *The displacement of an object between two points is the unique path that takes the body from its initial to final position.*

(*vi*) *The displacement of an object between two positions does not give any information regarding the shape of the actual path followed by the object between these two positions.*

(*vii*) *The magnitude of the displacement of an object between two positions gives the shortest distance between these positions.*

(*viii*) *Displacement is a vector quantity. Displacement of an object between two given positions is independent of the actual path followed by the object in moving from one position to another.*

3.6 SPEED

8. *Define the term speed. Is it a scalar or vector quantity ? Give its units and dimensions.*

Speed. *The rate of change of position of an object with time in any direction is called its speed.* It is equal to the distance travelled by the object per unit time.

$$\text{Speed} = \frac{\text{Distance travelled}}{\text{Time taken}}$$

Speed has only magnitude and no direction, so it is a *scalar quantity*. Also the distance travelled by an object is either positive or zero, so the speed may be positive or zero but never negative.

The SI unit of speed is ms^{-1}.

The CGS unit of speed is cms^{-1}.

The dimensional formula of speed is $[M^0L^1T^{-1}]$.

9. *Define : (i) uniform speed (ii) variable speed (iii) average speed and (iv) instantaneous speed.*

Different Types of Speed :

(i) ***Uniform speed.*** An object is said to be moving with uniform speed, if it covers equal distances in equal intervals of time, however small these time intervals may be.

(ii) ***Variable speed.*** An object is said to be moving with variable speed if it covers unequal distances in equal intervals of time.

(iii) ***Average speed.*** For an object moving with variable speed, the average speed is the total distance travelled by the object divided by the total time taken to cover that distance.

$$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

(iv) ***Instantaneous speed.*** *The speed of an object at any particular instant of time or at a particular point of its path is called the instantaneous speed of the object.* Suppose an object covers distance Δx in a small time interval Δt around the instant t, then its average speed is $\Delta x / \Delta t$. The limiting value of this average speed when the time interval Δt approaches zero, gives the instantaneous speed at the instant t. Thus

Instantaneous speed,

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Here $\frac{dx}{dt}$ is the first order derivative of distance x with respect to time t.

The **speedometer** of an automobile indicates its instantaneous speed at any instant.

For Your Knowledge

Average Speed in Different Situations

▲ **A body covering different distances with different speeds.** Suppose a body covers distances $s_1, s_2, s_3, \ldots$ with speeds $v_1, v_2, v_3, \ldots$ respectively, then its average speed will be

$$v_{av} = \frac{\text{Total distance travelled}}{\text{Total time taken}} = \frac{s}{t}$$

$$= \frac{s_1 + s_2 + s_3 + \ldots}{t_1 + t_2 + t_3 + \ldots}$$

or $$v_{av} = \frac{s_1 + s_2 + s_3 + \ldots}{\left(\frac{s_1}{v_1} + \frac{s_2}{v_2} + \frac{s_3}{v_3} + \ldots\right)}$$

Special case If $s_1 = s_2 = s$ *i.e.*, the body covers equal distances with different speeds, then

$$v_{av} = \frac{2s}{s\left(\frac{1}{v_1} + \frac{1}{v_2}\right)} = \frac{2\, v_1 v_2}{v_1 + v_2}$$

Clearly, *the average speed is the harmonic mean of the individual speeds.*

▲ **A body moving with different speeds in different time intervals.** Suppose a body travels with speeds $v_1, v_2, v_3, \ldots$ in time intervals $t_1, t_2, t_3, \ldots$ respectively, then

Total distance travelled $= v_1t_1 + v_2t_2 + v_3t_3 + \ldots$

Total time taken $= t_1 + t_2 + t_3 + \ldots$

$$\therefore \quad v_{av} = \frac{v_1t_1 + v_2t_2 + v_3t_3 + \ldots}{t_1 + t_2 + t_3 + \ldots}$$

Special case It $t_1 = t_2 = t_3 = \ldots = t_n = t$ (say), then

$$v_{av} = \frac{(v_1 + v_2 + v_3 + \ldots + v_n)\, t}{nt} = \frac{v_1 + v_2 + v_3 + \ldots + v_n}{n}$$

Clearly, *average speed is the arithmetic mean of the individual speeds.*

3.7 VELOCITY

10. *Define the term velocity. Is it a scalar or vector quantity ? Give its units and dimensions.*

Velocity. *The rate of change of position of an object with time in a given direction is called its velocity.* It can also be defined as the speed of an object in a given direction. It is equal to the displacement covered per unit time.

$$\text{Velocity} = \frac{\text{Displacement}}{\text{Time}}$$

As velocity has both magnitude and direction, it is a vector quantity. Velocity can be positive, zero or negative depending on the displacement is positive, zero or negative.

The SI unit of velocity is ms^{-1}.

The CGS unit of velocity is cms^{-1}.

The dimensional formula for the velocity is $[M^0L^1T^{-1}]$.

11. *Define (i) uniform velocity (ii) variable velocity (iii) average velocity and (iv) instantaneous velocity.*

Different Types of Velocities :

(*i*) ***Uniform velocity.*** *A body is said to be moving with uniform velocity if it covers equal displacements in equal intervals of time, however small these time intervals may be.*

(*ii*) ***Variable velocity.*** A *body is said to be moving with variable velocity if either its speed changes or direction of motion changes or both change with time.*

(*iii*) ***Average velocity.*** *For an object moving with variable velocity, average velocity is defined as the ratio of its total displacement to the total time interval in which that displacement occurs.*

$$\text{Average velocity} = \frac{\text{Total displacement}}{\text{Total time}}$$

If x_1 and x_2 are the positions of an object at times t_1 and t_2, then the average velocity from time t_1 to t_2 is given by

$$v_{av} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

(*iv*) ***Instantaneous velocity.*** *The velocity of an object at a particular instant of time or at a particular point of its path is called its instantaneous velocity.* It is equal to the limiting value of the average velocity of the object in a small time interval taken around that instant, when the time interval approaches zero. Thus

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{x}}{\Delta t} = \frac{d\vec{x}}{dt}$$

Thus instantaneous velocity of an object is equal to the first order derivative of its displacement with respect to time.

12. *When is the average speed of an object equal to the magnitude of its average velocity ? Give reason also.*

We know that

$$\text{Average speed} = \frac{\text{Total path length}}{\text{Time interval}}$$

$$\text{Average velocity} = \frac{\text{Displacement}}{\text{Time interval}}$$

When an object moves along a straight line and in the same direction, its total path length is equal to the magnitude of its displacement. Consequently, its average speed is equal to the magnitude of its average velocity.

3.8 UNIFORM AND NON-UNIFORM MOTIONS

13. *What is meant by uniform motion ?*

Uniform motion. *An object is said to be in uniform motion if it covers equal distances in equal intervals of time, however small these time intervals may be, in the same fixed direction.* So if an object in uniform motion covers 1200 m in 1 minute in a given direction, it indicates that it covers 600 m in every 30 s, 300 m in every 15 s, 100 m in every 5 s, and so on.

14. *Obtain the formulae for the position of an object moving with uniform velocity v (i) at any time t in terms of its position at t = 0 and (ii) at any time t' in terms of its position at another time t.*

Kinematic formulae for uniform motion. Consider an object moving with uniform velocity v along the positive direction of x-axis. Suppose it occupies positions x_0, x and x' at times $t = 0$, t and t' respectively, as shown in Fig. 3.11.

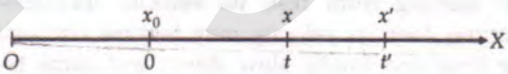


Fig. 3.11 Uniform motion.

Velocity of the object in the time interval 0 to t is

$$v = \frac{\text{Displacement}}{\text{Time}} = \frac{x - x_0}{t - 0}$$

or $$x = x_0 + vt \quad ...(1)$$

This equation gives position of the particle at instant t.

Similarly, velocity of the object in the time interval t to t' is

$$v = \frac{x' - x}{t' - t} \quad \text{or} \quad x' = x + v(t' - t)$$

This equation gives position of the object at instant t' in terms of position x at time t.

From equation (1), we have : $x - x_0 = vt$

But $x - x_0$ = displacement in time interval t

$= s$ (say)

$\therefore \quad s = vt$

This equation can be used to determine the distance travelled by an object in uniform motion.

15. *Give some important features of uniform motion.*

Some Important Features of Uniform Motion :

(*i*) The velocity in uniform motion does not depend on the choice of origin.

(*ii*) The velocity in uniform motion does not depend on the choice of the time interval $(t_2 - t_1)$.

(*iii*) For uniform motion along a straight line in the same direction, the magnitude of the displacement is equal to the actual distance covered by the object.

(*iv*) The velocity is positive if the object is moving towards the right of the origin and negative if the object is moving towards the left of the origin.

(*v*) For an object in uniform motion, no force is required to maintain its motion.

(*vi*) In uniform motion, the instantaneous velocity is equal to the average velocity at all times because velocity remains constant at each instant or at each point of the path.

16. *What is non-uniform motion ?*

Non-uniform motion. *A body is said to be in non-uniform motion if its velocity changes with time.* Here either the speed of the body or its direction of motion or both change with time. For example, when a vehicle starts moving from rest, its velocity increases for sometime, then its velocity may become constant for some time and finally slow down and come to rest again. The velocity of the vehicle is different at different instants and so it has non-uniform motion.

Examples based on Calculation of Distance Covered, Displacement, Average Speed and Average Velocity

FORMULAE USED

1. Distance covered = Length of actual path traversed by the body
2. Displacement = Vector drawn from initial to final position of the body
3. Average speed = $\dfrac{\text{Total distance travelled}}{\text{Total time taken}}$

UNITS USED

Distance and displacement are in metre (or km), average speed and average velocity in ms^{-1} (or $km\ h^{-1}$) and time in second (or hour).

EXAMPLE 1. *In Fig. 3.12, a particle moves along a circular path of radius r. It starts from point A and moves anticlockwise. Find the distance travelled by the particle as it (i) moves from A to B (ii) moves from A to C (iii) moves A to D (iv) completes one revolution. Also find the magnitude of displacement in each case.*

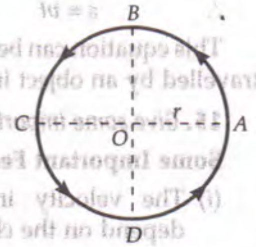


Fig. 3.12

Solution. (*i*) ***From A to B.***

Distance covered $= \frac{1}{4} \times 2\pi r = \frac{1}{2}\pi r.$

Displacement $= |\vec{AB}| = \sqrt{OA^2 + OB^2}$

$= \sqrt{r^2 + r^2} = \sqrt{2}\, r.$

(*ii*) ***From A to C.***

Distance covered $= \frac{1}{2} \times 2\pi r = \pi r.$

Displacement $= |\vec{AC}| = 2r.$

(*iii*) ***From A to D.***

Distance covered $= \frac{3}{4} \times 2\pi r = \frac{3}{2}\pi r.$

Displacement $= |\vec{AD}| = \sqrt{r^2 + r^2} = \sqrt{2}\, r$

(*iv*) ***From A to A.*** Distance covered $= 2\pi r$

As the final position coincides with the initial position, displacement covered = zero.

EXAMPLE 2. *A car is moving along X-axis. As shown in Fig. 3.13, it moves from 0 to P in 18 s and returns from P to Q in 6 s. What are the average velocity and average speed of the car in going from (i) from O to P and (ii) from O to P and back to Q ?* **[NCERT]**

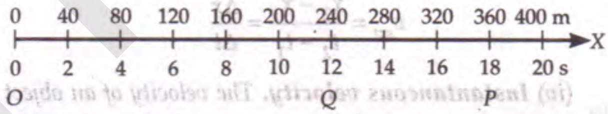


Fig. 3.13

Solution. (*i*) ***From O to P.***

Average velocity $= \dfrac{\text{Displacement}}{\text{Time interval}}$

$= \dfrac{+360\text{ m}}{18\text{ s}} = +\mathbf{20\ ms^{-1}}.$

Average speed $= \dfrac{\text{Path length}}{\text{Time interval}} = \dfrac{360\text{ m}}{18\text{ s}} = \mathbf{20\ ms^{-1}}.$

(*ii*) ***From O to P and back to Q.***

Average velocity $= \dfrac{\text{Displacement}}{\text{Time interval}} = \dfrac{OQ}{18+6}$

$= \dfrac{+240\text{ m}}{24\text{ s}} = \mathbf{10\ ms^{-1}}.$

Average speed $= \dfrac{\text{Path length}}{\text{Time interval}} = \dfrac{OP + PQ}{18+6}$

$= \dfrac{(360+120)}{24\text{ s}}\text{ m} = \mathbf{20\ ms^{-1}}.$

EXAMPLE 3. *A body travels from A to B at 40 ms^{-1} and from B to A at 60 ms^{-1}. Calculate the average speed and average velocity.* **[Himachal 07]**

Solution. Total time taken by the body to travel from A to B and then from B to A,

$$t_1 + t_2 = \frac{AB}{40} + \frac{BA}{60} = AB\left(\frac{1}{40} + \frac{1}{60}\right) = \frac{AB}{24} \text{ s.}$$

Total distance covered

$$= AB + BA = 2\,AB$$

$$\therefore \text{ Average speed} = \frac{2\,AB}{t_1 + t_2} = \frac{2\,AB}{AB/24} = 48 \text{ ms}^{-1}.$$

As the body comes back to its initial position A, its net displacement is zero.

$$\therefore \text{ Average velocity} = \frac{\text{Displacement}}{\text{Time}} = \frac{0}{AB/24} = 0.$$

EXAMPLE 4. On a 60 km track, a train travels the first 30 km with a uniform speed of 30 km h^{-1}. How fast must the train travel the next 30 km so as to average 40 km h^{-1} for the entire trip ?

Solution. Here $s_1 = s_2 = s = 30$ km, $v_1 = 30$ km h^{-1}, $v_{av} = 40$ km h^{-1}, $v_2 = ?$

$$\text{But } v_{av} = \frac{s_1 + s_2}{t_1 + t_2} = \frac{s + s}{\frac{s}{v_1} + \frac{s}{v_2}} = \frac{2s}{s\left(\frac{1}{v_1} + \frac{1}{v_2}\right)} = \frac{2v_1 v_2}{v_1 + v_2}$$

$$\therefore \quad 40 = \frac{2 \times 30 \times v_2}{30 + v_2} \quad \text{or} \quad 30 + v_2 = \frac{3}{2}\, v_2$$

or $v_2 = 60$ km h^{-1}.

EXAMPLE 5. A body covers one-third of its journey with speed 'u', next one-third with speed 'v' and the last one-third with speed 'w'. Calculate the average speed of the body during the entire journey.

Solution. Let total distance $= 3x$. Then,

$$\text{Total time taken} = \frac{x}{u} + \frac{x}{v} + \frac{x}{w} = x\left(\frac{vw + uw + uv}{uvw}\right)$$

$$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

$$= \frac{3x}{x\left(\frac{vw + uw + uv}{uvw}\right)} = \frac{3uvw}{uv + vw + uw}$$

EXAMPLE 6. A body travelling along a straight line traversed one-half of the total distance with a velocity v_0. The remaining part of the distance was covered with a velocity v_1, for half the time and with velocity v_2 for the other half of time. Find the mean velocity averaged over the whole time of motion.

Solution. Let total distance $= x$

Let the time taken to cover first one-half distance $= t_1$.

$$\text{Then} \quad t_1 = \frac{x/2}{v_0} = \frac{x}{2v_0}$$

Let the time taken for next $x/2$ distance $= t_2$.

$$\text{Then} \quad \frac{x}{2} = v_1 \cdot \frac{t_2}{2} + v_2 \cdot \frac{t_2}{2} = \frac{(v_1 + v_2)\, t_2}{2}$$

$$\text{or} \quad t_2 = \frac{x}{v_1 + v_2}$$

Total time taken

$$= t_1 + t_2 = \frac{x}{2v_0} + \frac{x}{v_1 + v_2}$$

$$= \frac{x\,(v_1 + v_2 + 2v_0)}{2v_0\,(v_1 + v_2)}$$

Average velocity

$$= \frac{\text{Total distance}}{\text{Total time}} = \frac{x}{\frac{x\,(v_1 + v_2 + 2v_0)}{2v_0\,(v_1 + v_2)}}$$

$$= \frac{2v_0\,(v_1 + v_2)}{v_1 + v_2 + 2v_0}.$$

PROBLEMS FOR PRACTICE

1. A cyclist moving on a circular track of radius 100 m completes one revolution in 4 minutes. What is his (*i*) average speed (*ii*) average velocity in one full revolution ? [**Ans.** (*i*) 50 π metre/minute (*ii*) 0]

2. A body travels the first half of the total distance with velocity v_1 and the second half with velocity v_2. Calculate the average velocity. $\left(\textbf{Ans. } \frac{2v_1 v_2}{v_1 + v_2}\right)$

3. A car covers the first half of the distance between two places at a speed of 40 kmh^{-1} and the second half at 60 kmh^{-1}. What is the average speed of the car ? **[Central Schools 07]** (**Ans.** 48 kmh^{-1})

4. A train moves with a speed of 30 kmh^{-1} in the first 15 minutes, with another speed of 40 kmh^{-1} the next 15 minutes, and then with a speed of 60 kmh^{-1} in the last 30 minutes. Calculate the average speed of the train for this journey. (**Ans.** 47.5 kmh^{-1})

5. A body travels a distance s_1 with velocity v_1 and distance s_2 with velocity v_2 in the same direction. Calculate the average velocity of the body. $\left[\textbf{Ans. } \frac{(s_1 + s_2)\, v_1 v_2}{s_1 v_2 + s_2 v_1}\right]$

6. A car travels along a straight line for the first half time with speed 50 kmh^{-1} and the second half time with speed 60 kmh^{-1}. Find the average speed of the car. (**Ans.** 55 kmh^{-1})

HINTS

1. Distance travelled in one revolution

$$= 2\pi r = 2\pi \times 100 \text{ m}$$

Time taken = 4 minute

Displacement in 4 minute = 0

(*i*) Average speed $= \dfrac{\text{Distance}}{\text{Time}} = \dfrac{2\pi \times 100 \text{ m}}{4 \text{ min}}$

$$= \mathbf{50\pi \text{ metre / minute.}}$$

(*ii*) Average velocity $= \dfrac{\text{Displacement}}{\text{Time}} = 0.$

2. Let total distance $= 2x$. Then,

Total time taken $= \dfrac{x}{v_1} + \dfrac{x}{v_2} = x\left(\dfrac{v_1 + v_2}{v_1 v_2}\right)$

$\therefore$ Average speed $= \dfrac{2x}{x\left(\dfrac{v_1 + v_2}{v_1 v_2}\right)} = \dfrac{2v_1 v_2}{v_1 + v_2}.$

3. $v_{av} = \dfrac{2v_1 v_2}{v_1 + v_2} = \dfrac{2 \times 40 \times 60}{40 + 60} = \mathbf{48 \text{ km h}^{-1}}.$

4. Total distance covered

$$= 30 \times \frac{15}{60} + 40 \times \frac{15}{60} + 60 \times \frac{30}{60} = 47.5 \text{ km}$$

Total time taken = 15 min + 15 min + 30 min

= 60 min = 1 h

Average speed $= \dfrac{47.5}{1} = \mathbf{47.5 \text{ km h}^{-1}}.$

5. Total distance covered $= s_1 + s_2$

Total time taken

$$= t_1 + t_2 = \frac{s_1}{v_1} + \frac{s_2}{v_2} = \frac{s_1 v_2 + s_2 v_1}{v_1 v_2}$$

Average velocity

$$= \frac{s_1 + s_2}{\dfrac{s_1 v_2 + s_2 v_1}{v_1 v_2}} = \frac{(s_1 + s_2) v_1 v_2}{s_1 v_2 + s_2 v_1}.$$

6. Average speed $= \dfrac{50 \times \dfrac{t}{2} + 60 \times \dfrac{t}{2}}{t} = \mathbf{55 \text{ kmh}^{-1}}.$

3.9 ACCELERATION

17. *Define the term acceleration. Is it a scalar or a vector quantity ? Give its units and dimensions.*

Acceleration. *The rate of change of velocity of an object with time is called its acceleration.* It tells how fast the velocity of an object changes with time.

$$\text{Acceleration} = \frac{\text{Change in velocity}}{\text{Time taken}}$$

Acceleration is a vector quantity. It has the same direction as that of the change in velocity.

The SI unit of acceleration is ms^{-2}.

The CGS unit of acceleration is cm s^{-2}.

The dimensional formula of acceleration is $[\text{M}^0\text{L}^1\text{T}^{-2}]$.

18. *Define (i) uniform acceleration (ii) variable acceleration (iii) average acceleration and (iv) instantaneous acceleration.*

Different Types of Acceleration :

(*i*) ***Uniform acceleration.*** *The acceleration of an object is said to be uniform acceleration if its velocity changes by equal amounts in equal intervals of time, however small these time intervals may be.*

(*ii*) ***Variable acceleration.*** *The acceleration of an object is said to be variable acceleration if its velocity changes by unequal amounts in equal intervals of time.*

(*iii*) ***Average acceleration.*** For an object moving with variable velocity, the *average acceleration is defined as the ratio of the total change in velocity of the object to the total time interval taken.* Suppose v_1 and v_2 are the velocities of an object at times t_1 and t_2 respectively, then its average acceleration is

$$a_{av} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

(*iv*) ***Instantaneous acceleration.*** *The acceleration of an object at a given instant of time or at a given point of its motion, is called its instantaneous acceleration.* It is equal to the limiting value of the average acceleration of the object in small time interval around that instant, when the time interval approaches zero. Thus

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

As $\qquad v = \dfrac{dx}{dt}$

$\therefore \qquad a = \dfrac{d}{dt}\left(\dfrac{dx}{dt}\right) = \dfrac{d^2x}{dt^2}$

Thus, acceleration is the first order derivative of velocity and second order derivative of position with respect to time.

19. *What are positive and negative accelerations ? Give examples.*

Positive acceleration. *If the velocity of an object increases with time, its acceleration is positive.* When a bus leaves a bus-stop, its acceleration is positive.

Negative acceleration. *If the velocity of an object decreases with time, its acceleration is negative.* Negative acceleration is also called **retardation** or **deceleration**. When a bus slows down on approaching a bus-stop, its acceleration is negative.

Examples based on Instantaneous Velocity and Instantaneous Acceleration

FORMULAE USED

1. $v_{av} = \frac{s_2 - s_1}{t_2 - t_1} = \frac{\Delta s}{\Delta t}$
2. $v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$
3. $a_{av} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$
4. $a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d^2 s}{dt^2}$

UNITS USED

All velocities are in ms^{-1} and accelerations in ms^{-2}

EXAMPLE 7. *The position of an object moving along x-axis is given by $x = a + bt^2$, where $a = 8.5$ m, $b = 2.5\ ms^{-2}$ and t is measured in seconds. What is its velocity at $t = 0$ s and $t = 2$ s? What is the average velocity between $t = 2$ s and $t = 4$ s?* **[NCERT ; Himachal 08]**

Solution. Given $x = a + bt^2$

Instantaneous velocity,

$$v = \frac{dx}{dt} = \frac{d}{dt}(a + bt^2) = 0 + b \times 2t = 2bt$$

At $t = 0$, $v = \mathbf{0}$.

At $t = 2$s, $v = 2 \times 2.5 \times 2 = \mathbf{10\ ms^{-1}}$.

At $t = 2$s, $x = a + 4b$

At $t = 4$s, $x = a + 16b$

Average velocity

$$= \frac{x_2 - x_1}{t_2 - t_1} = \frac{(a + 16b) - (a + 4b)}{4 - 2} = 6b$$

$$= 6 \times 2.5 = \mathbf{15.0\ ms^{-1}}.$$

Example 8. *The displacement (in metre) of a particle moving along x-axis is given by $x = 18t + 5t^2$. Calculate :*

(i) *the instantaneous velocity at $t = 2$ s,*
(ii) *average velocity between $t = 2$ s and $t = 3$ s,*
(iii) *instantaneous acceleration.* **[Central Schools 05]**

Solution. Given : $x = 18t + 5t^2$

(i) Velocity, $v = \frac{dx}{dt} = \frac{d}{dt}(18t + 5t^2) = 18 + 10t$

At $t = 2$ s, the instantaneous velocity

$$= 18 + 10 \times 2 = \mathbf{38\ ms^{-1}}.$$

(ii) Displacement at $t = 2$ s is

$$x_1 = 18 \times 2 + 5 \times 2^2 = 56 \text{ m}$$

Displacement at $t = 3$ s is

$$x_2 = 18 \times 3 + 5 \times 3^2 = 54 + 45 = 99 \text{ m}$$

Average velocity,

$$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{99 - 56}{3 - 2} = \mathbf{43\ ms^{-1}}.$$

(iii) Acceleration,

$$a = \frac{dv}{dt} = \frac{d}{dt}(18 + 10t) = \mathbf{10\ ms^{-2}}.$$

EXAMPLE 9. *The displacement x of a particle varies with time t as $x = 4t^2 - 15t + 25$.*

Find the position, velocity and acceleration of the particle at $t = 0$. When will the velocity of the particle become zero? Can we call the motion of the particle as one with uniform acceleration?

Solution. Position, $x = 4t^2 - 15t + 25$

Velocity, $v = \frac{dx}{dt} = 8t - 15$

Acceleration, $a = \frac{dv}{dt} = 8$

At time $t = 0$, we have

$$x = \mathbf{25\ m}, \quad v = \mathbf{-15\ ms^{-1}}, \quad a = \mathbf{8\ ms^{-2}}.$$

Velocity will become zero, when $8t - 15 = 0$

or $t = \frac{15}{8} = \mathbf{1.875\ s}.$

Yes, the particle has a uniform acceleration because a does not depend on time t.

EXAMPLE 10. *The velocity of a particle is given by the equation, $v = 2t^2 + 5\ cms^{-1}$. Find (i) the change in velocity of the particle during the time interval between $t_1 = 2$ s and $t_2 = 4$ s (ii) the average acceleration during the same interval and (iii) the instantaneous acceleration at $t_2 = 4$ s.*

Solution. Given $v = 2t^2 + 5\ cms^{-1}$

(i) When $t_1 = 2$s, $v_1 = 2(2)^2 + 5 = 13\ cms^{-1}$

When $t_2 = 4$ s, $v_2 = 2(4)^2 + 5 = 37\ cms^{-1}$

Change in velocity

$$= v_2 - v_1 = 37 - 13 = \mathbf{24\ cms^{-1}}.$$

(ii) Average acceleration,

$$a_{av} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{37 - 13}{4 - 2} = \mathbf{12\ cms^{-2}}.$$

(iii) Instantaneous acceleration,

$$a = \frac{dv}{dt} = \frac{d}{dt}(2t^2 + 5) = 4t$$

At $t = 4$ s, $a = 4 \times 4 = \mathbf{16\ cms^{-2}}.$

EXAMPLE 11. *The distance x of a particle moving in one dimension, under the action of a constant force is related to time t by the equation, $t = \sqrt{x} + 3$, where x is in metres and t in seconds. Find the displacement of the particle when its velocity is zero.* **[IIT]**

Solution. Given : $t = \sqrt{x} + 3$ or $\sqrt{x} = t - 3$

or $x = (t - 3)^2 = t^2 - 6t + 9$

Velocity, $v = \frac{dx}{dt} = 2t - 6$

Velocity becomes zero when $2t-6=0$ or $t=3$ s

$\therefore$ At $t=3$ s,

$$x=(t-3)^2=(3-3)^2=0$$

Hence displacement of the particle is zero when its velocity is zero.

Example 12. *The acceleration of a particle in ms^{-2} is given by $a=3t^2+2t+2$, where time t is in second. If the particle starts with a velocity $v=2\ ms^{-1}$ at $t=0$, then find the velocity at the end of 2 s.*

Solution. As $a=\dfrac{dv}{dt}=3t^2+2t+2$

or $$dv=(3t^2+2t+2)\,dt$$

Integrating both sides, we get

$$\int dv=\int(3t^2+2t+2)\,dt$$

or $$v=3\frac{t^3}{3}+2\frac{t^2}{2}+2t+C=t^3+t^2+2t+C$$

At $t=0$, $v=2\ ms^{-1}$, therefore

$$2=0+C \quad\text{or}\quad C=2\ ms^{-1}$$

$$v=t^3+t^2+2t+2$$

At $t=2$ s, $v=8+4+4+2=$ **18 ms^{-1}**.

✗ Problems For Practice

1. The displacement x of a particle at time t along a straight line is given by $x=\alpha-\beta t+\gamma t^2$. Find the acceleration of the particle. **(Ans.** 2γ**)**
2. A particle moves along X-axis in such a way that its x-coordinate varies with time t as $x=2-5t+6t^2$. Find the initial velocity of the particle. [MNREC 87] **(Ans.** – 5 units**)**
3. The displacement x of a particle along X-axis is given by $x=3+8t+7t^2$. Obtain its velocity and acceleration at $t=2$s. **(Ans.** 36 ms^{-1}, 14 ms^{-2}**)**
4. The distance traversed by a particle moving along a straight line is given by $x=180\,t+50\,t^2$ metre. Find :
   (*i*) the initial velocity of the particle
   (*ii*) the velocity at the end of 4s and
   (*iii*) the acceleration of the particle.
   [Ans. (*i*) 180 ms^{-1} (*ii*) 580 ms^{-1} (*iii*) 100 ms^{-2}**]**

3.10 ▼ KINEMATIC EQUATIONS FOR UNIFORMLY ACCELERATED MOTION

20. *Derive the following equations of motion for an object moving with constant acceleration along a straight line :*

(*i*) $v=v_0+at$

(*ii*) $x=x_0+v_0t+\frac{1}{2}at^2$ or $s=v_0t+\frac{1}{2}at^2$

(*iii*) $v^2=v_0^2+2a(x-x_0)=v_0^2+2as$

(*iv*) $s_{nth}=v_0+\dfrac{a}{2}(2n-1)$.

Equations for uniformly accelerated motion along a straight line. Consider an object moving with constant acceleration along positive direction of X-axis.

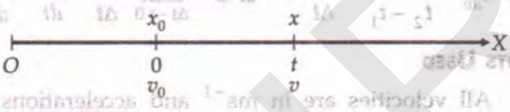


Fig. 3.14 Uniformly accelerated motion.

As shown in Fig. 3.14, let

x_0 = position of the object at instant $t=0$

x = position of the object at instant t

v_0 = velocity of the object at instant $t=0$

v = velocity of the object at instant t

(*i*) *Velocity after a certain time.* By definition,

$$\text{Acceleration}=\frac{\text{Change in velocity}}{\text{Time taken}}$$

$$a=\frac{v-v_0}{t-0}$$

or $$at=v-v_0$$

or $$v=v_0+at \qquad ...(1)$$

This is the required velocity-time relation.

(*ii*) *Distance covered in a certain time.* Average velocity of the object in time interval 0 to t is given by

$$v_{av}=\frac{\text{Displacement}}{\text{Time}}=\frac{x-x_0}{t-0}$$

or $$x-x_0=v_{av}\times t$$

But $$v_{av}=\frac{\text{Initial velocity}+\text{Final velocity}}{2}$$

$$=\frac{v_0+v}{2}$$

$$\therefore\quad x-x_0=\frac{v_0+v}{2}\times t=\frac{2v_0+at}{2}\times t \qquad [\because v=v_0+at]$$

$$=v_0t+\tfrac{1}{2}at^2$$

or $$x=x_0+v_0t+\tfrac{1}{2}at^2 \qquad ...(2)$$

Also, $x-x_0$ = distance travelled in time $t=s$, say

$$\therefore\quad s=v_0t+\tfrac{1}{2}at^2 \qquad ...(3)$$

Equation (2) is the required *position-time relation* and equation (3) is the required *distance-time relation*.

(iii) *Velocity at a certain position.* We know that

$$v = v_0 + at$$

or $$v - v_0 = at \quad ...(4)$$

Also, Average velocity × time = Displacement

or $$\frac{v + v_0}{2} \times t = x - x_0$$

or $$v + v_0 = \frac{2}{t}(x - x_0) \quad ...(5)$$

Multiplying equations (4) and (5), we get

$$(v + v_0)(v - v_0) = \frac{2}{t}(x - x_0)\, at$$

or $$v^2 - v_0^2 = 2a(x - x_0)$$

or $$v^2 = v_0^2 + 2a(x - x_0) \quad ...(6)$$

But $x - x_0 = s$, the distance travelled in time t, therefore

$$v^2 = v_0^2 + 2as \quad ...(7)$$

Equation (6) is the required *velocity-position relation* and equation (7) is the required *velocity-distance relation.*

(iv) *Distance covered in nth second.* As shown in Fig. 3.15, the distance travelled in nth second can be obtained by subtracting the distance travelled in first $(n-1)$ seconds from the distance travelled in first n seconds.

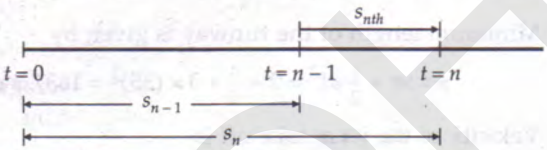


Fig. 3.15 Distance travelled in nth second.

Distance travelled in first t seconds is given by

$$s_t = v_0 t + \frac{1}{2} at^2$$

∴ Distance travelled in first $(n-1)$ seconds is

$$s_{n-1} = v_0(n-1) + \frac{1}{2} a(n-1)^2$$

Distance travelled in first n seconds is

$$s_n = v_0 n + \frac{1}{2} an^2$$

Hence the distance travelled in nth second is

$$\begin{aligned} s_{nth} &= s_n - s_{n-1} \\ &= (v_0 n + \tfrac{1}{2} an^2) - [v_0(n-1) + \tfrac{1}{2} a(n-1)^2] \\ &= (v_0 n + \tfrac{1}{2} an^2) - [v_0 n - v_0 + \tfrac{1}{2} a(n^2 - 2n + 1)] \\ &= v_0 n + \tfrac{1}{2} an^2 - v_0 n + v_0 - \tfrac{1}{2} an^2 + an - \frac{a}{2} \end{aligned}$$

or $$s_{nth} = v_0 + \frac{a}{2}(2n - 1).$$

3.11 EQUATIONS OF MOTION BY CALCULUS METHOD

21. *Deduce the following equations for uniformly accelerated motion by using integration technique :*

(i) $v = u + at$ (ii) $s = ut + \frac{1}{2}at^2$

(iii) $v^2 - u^2 = 2as$ (iv) $s_{nth} = u + \frac{a}{2}(2n - 1)$.

First equation of motion. Acceleration is defined as

$$a = \frac{dv}{dt}$$

or $$dv = a\,dt \quad ...(1)$$

When time = 0, velocity = u (say)

When time = t, velocity = v (say)

Integrating equation (1) within the above limits of time and velocity, we get

$$\int_u^v dv = \int_0^t a\,dt$$

or $$[v]_u^v = a\int_0^t dt = a[t]_0^t$$

or $$v - u = a(t - 0)$$

or $$v = u + at \quad ...(2)$$

Second equation of motion. Velocity is defined as

$$v = \frac{ds}{dt}$$

or $$ds = v\,dt = (u + at)\,dt \quad ...(3)$$

When time = 0, distance travelled = 0

When time = t, distance travelled = s (say)

Integrating equation (3) within the above limits of time and distance, we get

$$\int_0^s ds = \int_0^t (u + at)\,dt = u\int_0^t dt + a\int_0^t t\,dt$$

or $$[s]_0^s = u[t]_0^t + a\left[\frac{t^2}{2}\right]_0^t$$

or $$s - 0 = u(t - 0) + a\left[\frac{t^2}{2} - 0\right]$$

or $$s = ut + \frac{1}{2}at^2 \quad ...(4)$$

Third equation of motion. By the definitions of acceleration and velocity,

$$a = \frac{dv}{dt} = \frac{dv}{ds} \times \frac{ds}{dt} = \frac{dv}{ds} \times v$$

or $$a\,ds = v\,dv \quad ...(5)$$

When time = 0, velocity = u, distance travelled = 0

When time = t, velocity = v, distance travelled = s (say)

Integrating equation (5) within the above limits of velocity and distance, we get

$$\int_0^s a\,ds = \int_u^v v\,dv$$

or $$a\int_0^s ds = \int_u^v v\,dv$$

or $$a\,[s]_0^s = \left[\frac{v^2}{2}\right]_u^v$$

or $$a\,[s-0] = \frac{v^2}{2} - \frac{u^2}{2}$$

or $$2as = v^2 - u^2$$

or $$\boxed{v^2 - u^2 = 2as} \quad ...(6)$$

Fourth equation of motion. By definition of velocity,

$$v = \frac{ds}{dt}$$

or $$ds = v dt = (u + at)\,dt \quad ...(7)$$

When time = $(n-1)$ second, distance travelled = s_{n-1} (say)

When time = n second, distance travelled = s_n (say)

Integrating equation (7) within the above limits of time and distance, we get

$$\int_{s_{n-1}}^{s_n} ds = \int_{n-1}^{n} (u + at)\,dt$$

or $$[s]_{s_{n-1}}^{s_n} = u\int_{n-1}^{n} dt + a\int_{n-1}^{n} t\,dt$$

or $$s_n - s_{n-1} = u\,[t]_{n-1}^{n} + a\left[\frac{t^2}{2}\right]_{n-1}^{n}$$

$$= u\,[n-(n-1)] + \frac{a}{2}[n^2 - (n-1)^2]$$

$$= u + \frac{a}{2}[n^2 - (n^2 - 2n + 1)]$$

or $$\boxed{s_{nth} = u + \frac{a}{2}(2n-1)} \quad ...(8)$$

where $s_{nth} = s_n - s_{n-1}$ = distance travelled in n^{th} second.

Examples based on Motion with Uniform Acceleration

Formulae Used

1. Equations of motion in conventional form,
   (i) $v = u + at$
   (ii) $s = ut + \frac{1}{2}at^2$
   (iii) $v^2 - u^2 = 2as$ or $v^2 = u^2 + 2as$
   (iv) $s_{nth} = u + \frac{a}{2}(2n-1)$
2. Equations of motion in Cartesian form,
   (i) $v(t) = v(0) + at$
   (ii) $v(t') = v(t) + a(t'-t)$
   (iii) $x(t) = x(0) + v(0)t + \frac{1}{2}at^2$
   (iv) $x(t') = x(t) + v(t)(t'-t) + \frac{1}{2}a(t'-t)^2$
   (v) $[v(t')]^2 - [v(t)^2] = 2a\,[x(t') - x(t)]$.

Units Used

Displacement is in metre, velocity in ms^{-1}, acceleration in ms^{-2} and time in second.

<u>EXAMPLE 13.</u> *A jet plane starts from rest with an acceleration of 3 ms^{-2} and makes a run for 35 s before taking off. What is the minimum length of the runway and what is the velocity of the jet at take off ?*

Solution. Here $u = 0$, $a = 3\ ms^{-2}$, $t = 35\,s$

Minimum length of the runway is given by

$$s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2}\times 3\times(35)^2 = \mathbf{1837.5\ m.}$$

Velocity of the jet at take off is

$$v = u + at = 0 + 3\times 35 = \mathbf{105\ ms^{-1}}.$$

<u>EXAMPLE 14.</u> *An electron travelling with a speed of 5×10^3 ms^{-1} passes through an electric field with an acceleration of 10^{12} ms^{-2}. (i) How long will it take for the electron to double its speed ? (ii) What will be the distance covered by the electron in this time ?*

Solution. Here $u = 5\times 10^3\ ms^{-1}$,

$v = 2\times 5\times 10^3\ ms^{-1}$, $a = 10^{12}\ ms^{-2}$

(i) Time, $$t = \frac{v-u}{a} = \frac{2\times 5\times 10^3 - 5\times 10^3}{10^{12}}$$

$$= \frac{5\times 10^3}{10^{12}} = \mathbf{5\times 10^{-9}\ s.}$$

(ii) Distance,

$$s = ut + \frac{1}{2}at^2$$

$$= 5\times 10^3\times 5\times 10^{-9} + \frac{1}{2}\times 10^{12}\times(5\times 10^{-9})^2$$

$$= (25 + 12.5)\times 10^{-6} = \mathbf{3.75\times 10^{-5}\ m.}$$

EXAMPLE 15. A driver takes 0.20 s to apply the brakes after he sees a need for it. This is called the reaction time of the driver. If he is driving car at a speed of 54 kmh^{-1} and the brakes cause a deceleration of 6.0 ms^{-2}, find the distance travelled by the car after he sees the need to put the brakes.

Solution. During the time of reaction, the car continues to move with uniform speed of 54 kmh^{-1} or 15 ms^{-1}.

$\therefore$ Distance covered during 0.20 s $= 15 \times 0.20 = 3.0$ m

For motion with deceleration :

$$u = 15 \text{ ms}^{-1}, \quad v = 0, a = -6.0 \text{ ms}^{-2}$$

As $v^2 - u^2 = 2as$

$\therefore \quad 0^2 - 15^2 = 2 \times (-6.0)\, s$

or $$s = \frac{225}{12} = 18.75 \text{ m}$$

Total distance travelled $= 3.0 + 18.75 =$ **21.75 m.**

EXAMPLE 16. On a foggy day two drivers spot each other when they are just 80 metres apart. They are travelling at 72 km h^{-1} and 60 km h^{-1}, respectively. Both of them applied brakes retarding their cars at the rate of 5 ms^{-2}. Determine whether they avert collision or not.

Solution. *For the first car :*

$$u = 72 \text{ kmh}^{-1} = 20 \text{ ms}^{-1}, \; v = 0, a = -5 \text{ ms}^{-2}$$

As $$v^2 - u^2 = 2\,as$$

$\therefore$ $$0^2 - 20^2 = 2\,(-5)\, s_1$$

Distance covered by first car, $s_1 = 40$ m

For the second car :

$$u = 60 \text{ kmh}^{-1} = \frac{60 \times 5}{18}$$
$$= \frac{50}{3} \text{ ms}^{-1}, \; v = 0, a = -5 \text{ ms}^{-2}$$

As $v^2 - u^2 = 2as$

$\therefore$ $$0^2 - \left(\frac{50}{3}\right)^2 = 2\,(-5)\, s_2$$

Distance covered by second car,

$$s_2 = \frac{2500}{9 \times 10} = 27.78 \text{ m}$$

Total distance covered by the two cars

$$= s_1 + s_2 = 40 + 27.78 = 67.78 \text{ m}$$

As this distance is less than the initial distance (= 80 m) between the two cars, so *the collision will be averted.*

EXAMPLE 17. A hundred metre sprinter increases her speed from rest uniformly at the rate of 1 ms^{-2} upto three quaters of the total run and covers the last quater with uniform speed. How much time does she take to cover the first half and the second half of the run ?

Solution. The situation is shown in Fig. 3.16.

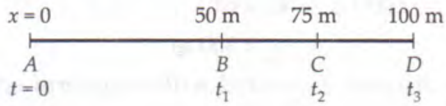


Fig. 3.16

For the motion from A to B :

$$u = 0, \quad a = 1 \text{ ms}^{-2}, \quad s = 50 \text{ m}$$

As $$s = ut + \frac{1}{2}at^2$$

$\therefore$ $$50 = 0 + \frac{1}{2} \times 1 \times t_1^2$$

or $$t_1^2 = 100$$

$\therefore$ Time taken for first half run $AB = t_1 =$ **10 s.**

For the motion from A to C :

$$u = 0, \quad a = 1 \text{ ms}^{-2}, \quad s = 75 \text{ m}$$

As $$s = ut + \frac{1}{2}at^2$$

$\therefore$ $$75 = 0 + \frac{1}{2} \times 1 \times t_2^2$$

or $$t_2 = \sqrt{150} = 12.2 \text{ s}$$

Time taken to run $BC = 12.2 - 10 = 2.2$ s

Again for part AC, $u = 0, a = 1 \text{ ms}^{-2}, \; t = 12.2$ s

$\therefore$ $$v = u + at = 0 + 1 \times 12.2 = \mathbf{12.2 \text{ ms}^{-1}}$$

For motion from C to D : This motion is uniform with speed equal to 12.2 ms^{-1}.

$\therefore$ Time taken $= \dfrac{25 \text{ m}}{12.2 \text{ s}} = 2.04$ s

Total time taken for 2nd half run BD

$$= 2.2 + 2.04 = \mathbf{4.24 \text{ s.}}$$

EXAMPLE 18. A motor car starts from rest and accelerates uniformly for 10 s to a velocity of 20 ms^{-1}. It then runs at a constant speed and is finally brought to rest in 40 m with a constant acceleration. Total distance covered is 640 m. Find the value of acceleration, retardation and total time taken.

Solution. Let s_1, s_2 and s_3 be distances covered in the three parts of the motion.

For first part of the motion :

$$u = 0, \quad t = 10 \text{ s}, \quad v = 20 \text{ ms}^{-1}$$

As $v = u + at \quad \therefore \; 20 = 0 + a \times 10$

Acceleration, $a = 2 \text{ ms}^{-2}$

Distance, $$s_1 = ut + \frac{1}{2}at^2$$
$$= 0 \times 10 + \frac{1}{2} \times 2 \times (10)^2 = 100 \text{ m}$$

For second part of the motion : We have

$$s_1 = 100 \text{ m}, \quad s_3 = 40 \text{ m}$$

As $\quad s_1 + s_2 + s_3 = 640$

$\therefore \quad 100 + s_2 + 40 = 640$

or $\quad s_2 = 500$ m

This distance is covered with a uniform speed of 20 ms^{-1}.

$\therefore$ Time taken $= \dfrac{500}{20} = 25$ s

For third part of the motion :

$$u = 20 \text{ ms}^{-1}, \; v = 0, \; s = s_3 = 40 \text{ m}$$

As $\quad v^2 - u^2 = 2as$

$\therefore \quad 0^2 - 20^2 = 2 \times a \times 40$

or $\quad a = -\dfrac{400}{80} = -5 \text{ ms}^{-2}$

$\therefore$ Retardation = **5 ms^{-2}**

Time taken, $t = \dfrac{v - u}{a} = \dfrac{0 - 20}{-5} = 4$ s

Total time taken = 10 + 25 + 4 = **39 s.**

Example 19. *An athlete runs a distance of 1500 m in the following manner. (i) Starting from rest, he accelerates himself uniformly at 2 ms^{-2} till he covers a distance of 900 m. (ii) He, then runs the remaining distance of 600 m at the uniform speed developed. Calculate the time taken by the athlete to cover the two parts of the distance covered. Also find the time, when he is at the centre of the track.*

Solution. The situation is shown in Fig. 3.17.

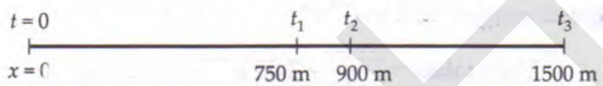


Fig. 3.17

For motion between times t = 0 and t_2 :

$$u = 0, \; a = 2 \text{ ms}^{-2}, \; s = 900 \text{ m}, \; v = v_2 \text{ (say)}$$

As $\quad v^2 - u^2 = 2as$

$\therefore \quad v_2^2 - 0^2 = 2 \times 2 \times 900$

or $\quad v_2 = 60 \text{ ms}^{-1}$

Time, $\quad t_2 = \dfrac{v - u}{a} = \dfrac{60 - 0}{2} = $ **30 s.**

For motion between times t_2 and t_3 : This motion occurs at a constant speed of 60 ms^{-1}.

$\therefore$ Time taken, $t_3 - t_2 = \dfrac{1500 - 900}{60} = $ **10 s.**

For motion between time t = 0 and t_1 :

$$u = 0, \; s = 750 \text{ m}, \; a = 2 \text{ ms}^{-2}, \; t = t_1 \text{ (say)}$$

As $\quad s = ut + \frac{1}{2}at^2 \qquad \therefore \; 750 = 0 \times t_1 + \frac{1}{2} \times 2 \times t_1^2$

or $\quad t_1 = \sqrt{750} = $ **27.4 s.**

Example 20. *A man is s = 9 m behind the door of a train when it starts moving with acceleration a = 2 ms^{-2}. The man runs at full speed. How far does he have to run and after what time does he get into the train ? What is his full speed ?*

Solution. Let v be the full speed of the man. He gets into the train when his velocity becomes equal to the velocity of the train.

$\therefore \quad v = 0 + at$

Also, the man's acceleration is $-a$ relative to train and his initial velocity is v relative to train, so

$$s = vt - \tfrac{1}{2}at^2 = at \, . \, t - \tfrac{1}{2}at^2 = \tfrac{1}{2}at^2$$

or $\quad t = \sqrt{\dfrac{2s}{a}} = \sqrt{\dfrac{2 \times 9}{2}} = $ **3 s**

$$s = vt = at \, . \, t = at^2 = a \, . \, \frac{2s}{a} = 2s = 2 \times 9 = \textbf{18 m}$$

and $\quad v = at = a\sqrt{\dfrac{2s}{a}} = \sqrt{2\,as} = \sqrt{2 \times 2 \times 9} = $ **6 ms^{-1}.**

Example 21. *A car accelerates from rest at a constant rate α for some time, after which it decelerates at a constant rate β to come to rest. If the total time elapsed is t second, then calculate :*

(i) the maximum velocity attained by the car, and

(ii) the total distance travelled by the car in terms of α, β and t.

Solution. (*i*) Let the car accelerate for time t_1 and v be the maximum velocity so attained in time t_1.

As $\quad v = u + at$

$\therefore \quad v = 0 + \alpha t_1$

or $\quad t_1 = \dfrac{v}{\alpha} \qquad$ ...(1)

Now, starting with the maximum velocity v, the car decelerates at constant rate β and comes to rest in time $(t - t_1)$. Therefore,

$$0 = v - \beta(t - t_1)$$

or $\quad t - t_1 = \dfrac{v}{\beta} \qquad$ ...(2)

Adding equations (1) and (2), we get

$$t = \frac{v}{\alpha} + \frac{v}{\beta} = v\left(\frac{\alpha + \beta}{\alpha\beta}\right)$$

or $\quad v = \dfrac{\alpha\beta\, t}{\alpha + \beta} \qquad$ ...(3)

This gives the maximum velocity attained by the car.

(*ii*) Distance covered by the car in time t_1 is

$$x_1 = 0 + \frac{1}{2}\alpha\, t_1^2 = \frac{1}{2}\alpha \, . \, \frac{v^2}{\alpha^2} = \frac{v^2}{2\alpha} \quad \text{[using (1)]}$$

Distance covered by the car in time $(t - t_1)$ is

$$x_2 = v(t - t_1) - \frac{1}{2}\beta(t - t_1)^2$$

$$= v \cdot \frac{v}{\beta} - \frac{1}{2}\beta \cdot \frac{v^2}{\beta^2} = \frac{v^2}{2\beta} \qquad \text{[using (2)]}$$

$\therefore$ Total distance travelled by the car is

$$x = x_1 + x_2 = \frac{v^2}{2\alpha} + \frac{v^2}{2\beta} = \frac{v^2}{2}\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$$

$$= \frac{\alpha^2\beta^2 t^2}{2(\alpha+\beta)^2} \cdot \frac{(\alpha+\beta)}{\alpha\beta} \qquad \text{[using (3)]}$$

or $$x = \frac{\alpha\beta t^2}{2(\alpha+\beta)}.$$

Example 22. *A body covers 12 m in 2nd second and 20 m in 4th second. How much distance will it cover in 4 seconds after the 5th second?* **[Chandigarh 03]**

Solution. Here $s_{2nd} = 12$ m, $s_{4th} = 20$ m

As $$s_{n\,th} = u + \frac{a}{2}(2n - 1)$$

$\therefore$ $$s_{2nd} = u + \frac{a}{2}(2 \times 2 - 1) = 12$$

or $$u + \frac{3}{2}a = 12 \qquad ...(i)$$

Also $$s_{4th} = u + \frac{a}{2}(2 \times 4 - 1) = 20$$

or $$u + \frac{7a}{2} = 20 \qquad ...(ii)$$

Subtracting (*i*) from (*ii*),

$$2a = 8 \quad \text{or} \quad a = 4 \text{ ms}^{-2}$$

From (*i*),

$$u + \frac{3}{2} \times 4 = 12 \quad \text{or} \quad u = 12 - 6 = 6 \text{ ms}^{-1}$$

Distance covered in 4 seconds after 5th second

$$= s_9 - s_5 \qquad \left[\text{Here } s_t = ut + \tfrac{1}{2}at^2\right]$$

$$= (6 \times 9 + \tfrac{1}{2} \times 4 \times 9^2) - (6 \times 5 + \tfrac{1}{2} \times 4 \times 5^2)$$

$$= 216 - 80 = \mathbf{136\ m}.$$

Example 23. *If x, y, z be the distances described by a particle during the pth, qth and rth second respectively, prove that : $(q - r)x + (r - p)y + (p - q)z = 0$*

Solution. As $$s_{nth} = u + \frac{a}{2}(2n - 1)$$

$\therefore$ $$x = u + \frac{a}{2}(2p - 1) \qquad ...(1)$$

$$y = u + \frac{a}{2}(2q - 1) \qquad ...(2)$$

$$z = u + \frac{a}{2}(2r - 1) \qquad ...(3)$$

Subtracting (3) from (2),

$$y - z = \frac{a}{2}(2q - 2r) \quad \text{or} \quad q - r = \frac{y - z}{a}$$

Subtracting (1) from (3),

$$z - x = \frac{a}{2}(2r - 2p) \quad \text{or} \quad r - p = \frac{z - x}{a}$$

Subtracting (2) from (1),

$$x - y = \frac{a}{2}(2p - 2q) \quad \text{or} \quad p - q = \frac{x - y}{a}$$

Hence $(q - r)x + (r - p)y + (p - q)z$

$$= \frac{(y - z)x}{a} + \frac{(z - x)y}{a} + \frac{(x - y)z}{a}$$

$$= \frac{(xy - xz) + (yz - xy) + (xz - yz)}{a} = \mathbf{0}.$$

Example 24. *Two buses A and B are at positions 50 m and 100 m from the origin at time t = 0. They start moving in the same direction simultaneously with uniform velocity of 10 ms^{-1} and 5 ms^{-1}. Determine the time and position at which A overtakes B.*

Solution. Here we use equation of motion for constant velocity in Cartesian form.

Given $x_1(0) = 50$ m, $x_2(0) = 100$ m,

$$v_1 = 10 \text{ ms}^{-1}, \quad v_2 = 5 \text{ ms}^{-1}$$

The positions of the two buses at any instant t are

$$x_1(t) = x_1(0) + v_1 t = 50 + 10t$$

$$x_2(t) = x_2(0) + v_2 t = 100 + 5t$$

When A overtakes B,

$$x_1(t) = x_2(t)$$

$$50 + 10t = 100 + 5t$$

or $$5t = 50$$

$$t = \mathbf{10\ s}$$

$$x_1(10) = x_2(10) = \mathbf{150\ m}.$$

Thus A overtakes B at a position of 150 m from the origin at time $t = 10$ s.

Example 25. *An object is moving along +ve x-axis with a uniform acceleration of 4 ms^{-2}. At time t = 0, x = 5 m and v = 3 ms^{-1}.*

(*a*) *What will be the velocity and position of the object at time t = 2 s?*

(*b*) *What will be the position of the object when it has a velocity of 5 ms^{-1}?*

Solution. Here we use equations of motion for constant acceleration in Cartesian form.

$$x(0) = 5 \text{ m}, \quad v(0) = 3 \text{ ms}^{-1}, \quad a = 4 \text{ ms}^{-2}$$

(*i*) Velocity of the object at time $t = 2$ s is given by

$$v(t) = v(0) + at$$
$$v(2) = 3 + 4 \times 2 = \mathbf{11\ ms^{-1}}$$

Position of the object at time $t = 2$ s is given by

$$x(t) = x(0) + v(0)\,t + \frac{1}{2}at^2$$
$$x(2) = 5 + 3 \times 2 + \frac{1}{2} \times 4 \times (2)^2 = \mathbf{19\ m.}$$

(*ii*) The position $x(t)$ of the body when its velocity is $5\ ms^{-1}$ is given by

$$2a\,[x(t) - x(0)] = v(t)^2 - v(0)^2$$
$$2 \times 4[x(t) - 5] = 5^2 - 3^2$$

or
$$x(t) - 5 = \frac{16}{8}$$

or
$$x(t) = 2 + 5 = \mathbf{7\ m.}$$

Problems For Practice

1. A race car accelerates on a straight road from rest to a speed of $180\ kmh^{-1}$ in 25 s. Assuming uniform acceleration of the car throughout, find the distance covered in this time. **[Delhi 02]** (**Ans.** 625 m)
2. A bullet travelling with a velocity of $16\ ms^{-1}$ penetrates a tree trunk and comes to rest in 0.4 m. Find the time taken during the retardation. (**Ans.** 0.05 s)
3. A car moving along a straight highway with a speed of $72\ kmh^{-1}$ is brought to a stop within a distance of 100 m. What is the retardation of the car and how long does it take for the car to stop ? **[Delhi 95, 05C]** (**Ans.** $2\ ms^{-2}$, 10 s)
4. On turning a corner a car driver driving at $36\ kmh^{-1}$, finds a child on the road 55 m ahead. He immediately applies brakes, so as to stop within 5 m of the child. Calculate the retardation produced and the time taken by the car to stop. (**Ans.** $1\ ms^{-2}$, 10 s)
5. The reaction time for an automobile driver is 0.6 s. If the automobile can be decelerated at $5\ ms^{-2}$, calculate the total distance travelled in coming to stop from an initial velocity of $30\ kmh^{-1}$, after a signal is observed. (**Ans.** 11.94 m)
6. A car starts from rest and accelerates uniformly for 10 s to a velocity of $8\ ms^{-1}$. It then runs at a constant velocity and is finally brought to rest in 64 m with a constant retardation. The total distance covered by the car is 584 m. Find the value of acceleration, retardation and total time taken. (**Ans.** $0.8\ ms^{-2}$, $0.5\ ms^{-2}$, 86 s)
7. Two trains–one travelling at $72\ kmh^{-1}$ and other at $90\ kmh^{-1}$ are heading towards one another along a straight level track. When they are 1.0 km apart, both the drivers simultaneously see the other's train and apply brakes which retard each train at the rate of $1.0\ ms^{-2}$. Determine whether the trains would collide or not. (**Ans.** No)
8. A burglar's car had a start with an acceleration of $2\ ms^{-2}$. A police vigilant party came after 5 seconds and continued to chase the burglar's car with a uniform velocity of $20\ ms^{-1}$. Find the time in which the police van overtakes the burglar's car. (**Ans.** 5 s)
9. A ball rolls down an inclined track 2 m long in 4 s. Find (*i*) acceleration (*ii*) time taken to cover the second metre of the track and (*iii*) speed of the ball at the bottom of the track. [**Ans.** (*i*) $0.25\ ms^{-2}$ (*ii*) 1.17 s (*iii*) $1\ ms^{-2}$]
10. A bus starts from rest with a constant acceleration of $5\ ms^{-2}$. At the same time a car travelling with a constant velocity of $50\ ms^{-1}$ overtakes and passes the bus. (*i*) Find at what distance will the bus overtake the car ? (*ii*) How fast will the bus be travelling then ? [**Ans.** (*i*) 1000 m (*ii*) $100\ ms^{-1}$]
11. A body starting from rest accelerates uniformly at the rate of $10\ cms^{-2}$ and retards uniformly at the rate of $20\ cms^{-2}$. Find the least time in which it can complete the journey of 5 km if the maximum velocity attained by the body is $72\ kmh^{-1}$. (**Ans.** 400 s)
12. A body covers a distance of 20 m in the 7th second and 24 m in the 9th second. How much shall it cover in 15th s ? (**Ans.** 36 m)
13. A body covers a distance of 4 m is 3rd second and 12 m in 5th second. If the motion is uniformly accelerated, how far will it travel in the next 3 seconds ? (**Ans.** 60 m)
14. An object is moving with uniform acceleration. Its velocity after 5 seconds is $25\ ms^{-1}$ and after 8 seconds, it is $34\ ms^{-1}$. Find the distance travelled by the object in 12th second. (**Ans.** 44.5 m)

Hints

1. Here $u = 0$, $t = 25$ s,

$$v = 180\ kmh^{-1} = \frac{180 \times 5}{18} = 50\ ms^{-1}$$

Acceleration,

$$a = \frac{v - u}{t} = \frac{50 - 0}{25} = 2\ ms^{-2}$$

Distance,

$$s = ut + \frac{1}{2}at^2 = 0 \times 25 + \frac{1}{2} \times 2 \times (25)^2$$
$$= \mathbf{625\ m.}$$

2. Here $u = 16\ \text{ms}^{-1}, v = 0, s = 0.4\ \text{m}, t = ?$

As $v^2 - u^2 = 2as$

$\therefore \quad 0^2 - 16^2 = 2a \times 0.4$

or $a = -\dfrac{16 \times 16}{2 \times 0.4} = -320\ \text{ms}^{-2}$

Time, $t = \dfrac{v-u}{a} = \dfrac{0-16}{-320} = \mathbf{0.05\ s.}$

5. Distance covered in 0.6 s $= 30 \times \dfrac{5}{18} \times 0.6 = 5.0$ m

During deceleration,

$$v^2 - u^2 = 2as$$

or $0^2 - \left(\dfrac{25}{3}\right)^2 = 2 \times (-5)\, s$

or $s = 6.94$ m

Total distance travelled $= 5.0 + 6.94 = \mathbf{11.94\ m.}$

8. Suppose the police van overtakes the burglar's car in time t second after it starts chasing.

Distance covered by police van in t seconds $= 20\,t$

Distance covered by burglar's car in $(t+5)$ seconds,

$$s = u(t+5) + \tfrac{1}{2}a(t+5)^2$$

$$= 0\,(t+5) + \tfrac{1}{2} \times 2\,(t+5)^2 = (t+5)^2$$

Now $20\,t = (t+5)^2$

or $t^2 - 10\,t + 25 = 0$

or $(t-5)^2 = 0$

$\therefore \quad t = \mathbf{5\ s.}$

9. (*i*) Here $s = 2$ m, $t = 4$ s, $u = 0$

As $s = ut + \tfrac{1}{2}at^2$

$\therefore \quad 2 = 0 + \tfrac{1}{2} \times a \times 4^2$

or $a = \mathbf{0.25\ ms^{-2}}.$

(*ii*) Velocity v at the end of 1 m is given by

$$v^2 - u^2 = 2as$$

or $v^2 - 0^2 = 2 \times 0.25 \times 1 = 0.5$

$\therefore \quad v = \sqrt{0.5} = 0.707\ \text{ms}^{-1}$

Time taken to cover first 1 m is given by

$$t = \frac{v-u}{a} = \frac{0.707 - 0}{0.25} = 2.83\ \text{s}$$

Hence time taken to cover the second metre

$$= 4 - 2.83 = \mathbf{1.17\ s.}$$

(*iii*) Speed at the bottom,

$$v = u + at = 0 + 0.25 \times 4 = \mathbf{1\ ms^{-1}}.$$

10. (*i*) Suppose the bus overtakes the car after covering distance s.

When the two meet, time taken t is same.

For bus, $s = ut + \tfrac{1}{2}at^2 = 0 + \tfrac{1}{2} \times 5t^2$

For car, $s = 50\,t$

$\therefore \quad \dfrac{5}{2}t^2 = 50\,t$ or $t = 20$ s

Hence $s = 50\,t = 50 \times 20 = \mathbf{1000\ m.}$

(*ii*) $v^2 = u^2 + 2as = 0 + 2 \times 5 \times 1000 = 10{,}000$

or $v = \mathbf{100\ ms^{-1}}.$

11. (*i*) *For motion with uniform acceleration :*

$u = 0,\ v = 72\ \text{kmh}^{-1} = 20\ \text{ms}^{-1},$

$a = 10\ \text{cms}^{-2} = 0.1\ \text{ms}^{-2},\ t = t_1 = ?,\ s = ?$

As $v = u + at$

$\therefore \quad 20 = 0 + 0.1 \times t_1$ or $t_1 = 200$ s

Also, $v^2 = u^2 + 2as$

$\therefore \quad 20^2 = 0^2 + 2 \times 0.1 \times s$ or $s = 2000$ m $= 2$ km

(*ii*) *For motion with uniform retardation :*

$u = 20\ \text{ms}^{-1},\ v = 0,$

$a = -20\ \text{cms}^{-2} = -0.2\ \text{ms}^{-2},\ t = t_2 = ?\quad s = ?$

As $v = u + at$

$\therefore \quad 0 = 20 - 0.2 \times t_2$ or $t_2 = 100$ s

Again, $v^2 = u^2 + 2as$

$\therefore \quad 0 = 20^2 + 2 \times (-0.2)\, s$

or $s = 1000$ m $= 1$ km

Remaining part of journey

$$= 5 - (2 + 1) = 2\ \text{km} = 2000\ \text{m}$$

This journey occurs at the uniform maximum velocity of $20\ \text{ms}^{-1}$.

(*iii*) *For motion with uniform velocity :*

$u = 20\ \text{ms}^{-1},\ s = 2000\ \text{m},\ t = t_3 = ?$

Time, $t_3 = \dfrac{s}{u} = \dfrac{2000}{20} = 100$ s

Total time $= t_1 + t_2 + t_3 = 200 + 100 + 100 = \mathbf{400\ s.}$

12. Here $s_{7th} = u + \dfrac{a}{2}(2 \times 7 - 1) = 20$

or $u + \dfrac{13}{2}a = 20$ m ...(*i*)

$s_{9th} = u + \dfrac{a}{2}(2 \times 9 - 1) = 24$ m

or $u + \dfrac{17}{2}a = 24$ ...(*ii*)

Subtracting (*i*) from (*ii*), we get

$2a = 4$ or $a = 2\ \text{ms}^{-2}$

From (*i*),

$u + \dfrac{13}{2} \times 2 = 20$ or $u = 7\ \text{ms}^{-1}.$

Hence,

$$s_{15th} = 7 + \frac{2}{2}(2 \times 15 - 1) = 7 + 29 = \mathbf{36\ m.}$$

13. $$s_{3rd} = u + \frac{a}{2}(2 \times 3 - 1) = 4$$

or $$u + \frac{5}{2}a = 4$$

$$s_{5th} = u + \frac{a}{2}(2 \times 5 - 1) = 12$$

or $$u + \frac{9}{2}a = 12$$

On solving,

$$u = -6 \text{ ms}^{-1}, \quad a = 4 \text{ ms}^{-2}$$

Distance travelled in next 3 seconds

$$= s_8 - s_5 = \left[-6 \times 8 + \frac{1}{2} \times 4 \times (8)^2\right] - \left[-6 \times 5 + \frac{1}{2} \times 4 \times (5)^2\right]$$

$$= 80 - 20 = \mathbf{60 \text{ m.}}$$

14. As $v = u + at$

$\therefore$ In first case, $25 = u + 5a$

In second case, $34 = u + 8a$

On solving,

$$u = 10 \text{ ms}^{-1}, \quad a = 3 \text{ ms}^{-2}$$

$$s_{12th} = 10 + \frac{3}{2}(2 \times 12 - 1)$$

$$= 10 + 34.5 = \mathbf{44.5 \text{ m.}}$$

3.12 MOTION UNDER GRAVITY

22. *Discuss the motion under free fall. Write the equations of motion for a freely falling body.*

Free fall. A body released near the surface of the earth is accelerated downward under the influence of force of gravity. In the absence of air resistance, all bodies fall with the same acceleration near the surface of the earth. This motion of body falling towards the earth from a small height is called *free fall*. The acceleration with which a body falls is called acceleration due to gravity and is denoted by g.

Near the surface of the earth, $g = 9.8 \text{ ms}^{-2}$

Equations of motion for a freely falling body. For a freely falling body, the following equations of motion hold good :

(i) $v = u + gt$ (ii) $s = ut + \frac{1}{2}gt^2$ (iii) $v^2 - u^2 = 2gs$

When a body falls freely under the action of gravity, it velocity increases and the value of g is taken positive.

When a body is thrown vertically upward, its velocity decreases and the value of g is taken negative.

Examples based on Motion under Gravity

FORMULAE USED

1. For a freely falling body, the equations of motion are
(i) $v = u + gt$ (ii) $s = ut + \frac{1}{2}gt^2$ (iii) $v^2 - u^2 = 2gs$
2. For a body falling freely under the action of gravity, g is taken *positive*.
3. For a body thrown vertically upward, g is taken *negative*.
4. When a body is just dropped, $u = 0$
5. For a body thrown vertically up with initial velocity u,
(i) Maximum height reached, $h = \frac{u^2}{2g}$
(ii) Time of ascent = Time of descent = $\frac{u}{g}$
(iii) Total time of flight = $\frac{2u}{g}$
(iv) Velocity of fall at the point of projection = u
(v) Velocity attained by a body dropped from height h, $v = \sqrt{2gh}$.

UNITS USED

Velocities u, v are in ms^{-1}, acceleration due to gravity g is in ms^{-2}, distances s and h are in metres.

EXAMPLE 26. *A ball thrown vertically upwards with a speed of 19.6 ms⁻¹ from the top of a tower returns to the earth in 6 s. Find the height of the tower.*

Solution. Here : $u = 19.6 \text{ ms}^{-1}$, $g = -9.8 \text{ ms}^{-2}$, $t = 6 \text{ s}$

Net displacement,

$$s = -h$$

Negative sign is taken because displacement is in the opposite direction of initial velocity.

As $s = ut + \frac{1}{2}gt^2$

$$\therefore -h = 19.6 \times 6 + \frac{1}{2} \times (-9.8) \times 6^2$$

$$= 117.6 - 176.4 = -58.8$$

or $$h = \mathbf{58.8 \text{ m.}}$$

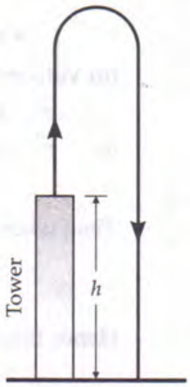


Fig. 3.18

EXAMPLE 27. *A ball is thrown vertically upwards with a velocity of 20 ms⁻¹ from the top of a multistoreyed building. The height of the point from where the ball is thrown is 25 m* *from the ground. (i) How high will the ball rise ? (ii) How long will it be before the ball hits the ground ? (iii) Trace the trajectory of this ball.* **[Central Schools 08, NCERT]**

Solution. (*i*) Here : $u = +20\text{ ms}^{-1}$, $g = -10\text{ ms}^{-2}$

At the highest point, $v = 0$

Suppose the ball rises to the height h from the point of projection.

As $v^2 - u^2 = 2gs$

$\therefore$ $0^2 - 20^2 = 2 \times (-10) \times h$

or $h = \mathbf{+20\text{ m}}$.

(*ii*) Net displacement, $s = -25\text{ m}$

Negative sign is taken because displacement is in the opposite direction of initial velocity.

As $s = ut + \frac{1}{2}gt^2$

$\therefore$ $-25 = 20\,t + \frac{1}{2} \times (-10) \times t^2$

or $5t^2 - 20\,t - 25 = 0$

or $t^2 - 4t - 5 = 0$

or $(t+1)(t-5) = 0$

As $t \neq -1$, so $t = \mathbf{5\text{ s}}$.

(*iii*) The trajectory of the ball will be a vertical line.

$$y = ut + \tfrac{1}{2}gt^2, \quad x = 0,\ y \neq 0.$$

EXAMPLE 28. *A ball thrown up is caught by the thrower after 4 s. How high did it go and with what velocity was it thrown ? How far was it below the highest point 3 s after it was thrown ?*

Solution. As time of ascent = time of descent

$\therefore$ Time taken by the ball to reach the highest point

$= 2\text{ s}$

For upward motion of the ball :

$u = ?,\ v = 0,\ t = 2\text{s},\ g = -9.8\text{ ms}^{-2}$

As $v = u + gt$

$\therefore$ $0 = u - 9.8 \times 2$

or $u = \mathbf{19.6\text{ ms}^{-1}}$.

Maximum height attained by the ball is given by

$$s = ut + \tfrac{1}{2}gt^2$$
$$= 19.6 \times 2 + \tfrac{1}{2} \times (-9.8) \times 2^2 = \mathbf{19.6\text{ m}}.$$

Distance covered by the ball in 3 s,

$$s = 19.6 \times 3 + \tfrac{1}{2} \times (-9.8) \times 3^2$$
$$= 58.8 - 44.1 = 14.7\text{ m}.$$

Distance of the ball from the highest point 3 s after it was thrown

$$= 19.6 - 14.7 = \mathbf{4.9\text{ m}}.$$

EXAMPLE 29. *A balloon is ascending at the rate of 9.8 ms^{-1} at a height of 39.2 m above the ground when a food packet is dropped from the balloon. After how much time and with what velocity does it reach the ground ? Take $g = 9.8\text{ ms}^{-2}$.*

Solution. Initially the food packet shares the upward velocity of the balloon, so

$$u = 9.8\text{ ms}^{-1},\ g = -9.8\text{ ms}^{-2},\ s = -39.2\text{ m}$$

Here s is taken negative because it is in the opposite direction of initial velocity.

Using, $s = ut + \frac{1}{2}gt^2$, we get

$$-39.2 = 9.8\,t - \tfrac{1}{2} \times 9.8\,t^2$$

or $4.9\,t^2 - 9.8\,t - 39.2 = 0$ or $t^2 - 2t - 8 = 0$

or $(t-4)(t+2) = 0$ or $t = 4\text{ s}$ or -2 s

As time is never negative, so $t = \mathbf{4\text{ s}}$.

Velocity with which the food packet reaches the ground is

$$v = u + gt = 9.8 - 9.8 \times 4 = \mathbf{-29.4\text{ ms}^{-1}}.$$

Negative sign shows that the velocity is directed vertically downwards.

EXAMPLE 30. *A food packet is released from a helicopter which is rising steadily at 2 ms^{-1}. After two seconds (i) What is the velocity of the packet ? (ii) How far is it below the helicopter ? Take $g = 9.8\text{ ms}^{-2}$.*

Solution. Here $u = 2\text{ ms}^{-1}$,

$$g = -9.8\text{ ms}^{-2},\quad t = 2\text{ s}$$

(*i*) $v = u + gt = 2 - 9.8 \times 2 = \mathbf{-17.6\text{ ms}^{-1}}$.

Negative sign shows that the velocity is directed vertically downwards.

(*ii*) Distance covered by the food packet in 2 s,

$$s = ut + \tfrac{1}{2}at^2 = 2 \times 2 - \tfrac{1}{2} \times 9.8 \times 2^2$$
$$= 4 - 19.6 = -15.6\text{ m}$$

Thus the food packet falls through a distance of 15.6 m in 2 s but in the mean time the helicopter rises up through a distance

$$= 2\text{ ms}^{-1} \times 2\text{ s} = 4\text{ m}$$

$\therefore$ After 2 s, the distance of the food packet from the helicopter

$$= 15.6 + 4 = \mathbf{19.6\text{ m}}.$$

EXAMPLE 31. *A parachutist bails out from an aeroplane and after dropping through a distance of 40 m, he opens the parachute and decelerates at 2 ms^{-2}. If he reaches the ground with a speed of 2 ms^{-1}, how long is he in the air ? At what height did he bail out from the plane ?* **[AIEEE 05]**

Solution. *When the parachutist falls freely :*

$$u = 0,\ g = 9.8\text{ ms}^{-2},\ s = 40\text{ m},\ t = ?,\ v = ?$$

As $s = ut + \frac{1}{2}gt^2$

$\therefore \quad 40 = 0 + \frac{1}{2} \times 9.8 \times t^2$

or $t = \sqrt{\frac{80}{9.8}} = \frac{20}{7}$ s $=$ **2.86 s**

Also, $v = u + gt = 0 + 9.8 \times \frac{20}{7} = 28 \text{ ms}^{-1}$

When the parachutist decelerates uniformly :

$u = 28 \text{ ms}^{-1}, \; a = -2 \text{ ms}^{-2}, \; v = 2 \text{ ms}^{-1}$

Time taken,

$$t = \frac{v-u}{a} = \frac{2-28}{-2} = 13 \text{ s}$$

Distance, $s = ut + \frac{1}{2}at^2 = 28 \times 13 - \frac{1}{2} \times 2 \times (13)^2$

$= 364 - 169 = 195$ m

Total time of parachutist in air

$= 2.86 + 13 =$ **15.86 s**

Height at which parachutist bails out

$= 40 + 195 =$ **235 m.**

EXAMPLE 32. *Two balls are thrown simultaneously, A vertically upwards with a speed of 20 ms^{-1} from the ground, and B vertically downwards from a height of 40 m with the same speed and along the same line of motion. At what points do the two balls collide ? Take $g = 9.8 \text{ ms}^{-2}$.*

Solution. Suppose the two balls meet at a height of x metre from the ground after time t s from the start.

For upward motion of ball A :

$u = 20 \text{ ms}^{-1}, \; g = -9.8 \text{ ms}^{-2}$

$s = ut + \frac{1}{2}gt^2$

$x = 20\,t - \frac{1}{2} \times 9.8t^2$

$= 20\,t - 4.9\,t^2 \quad ...(i)$

For downward motion of ball B :

$40 - x = 20 \times t + \frac{1}{2} \times 9.8\,t^2$

$= 20t + 4.9\,t^2 \quad ...(ii)$

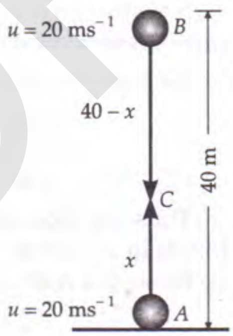


Fig. 3.19

Adding (*i*) and (*ii*),

$40 = 40\,t$ or $t = $ **1 s**

From (*i*), $x = 20 \times 1 - 4.9 \times (1)^2 =$ **15.1 m**

Hence the two balls will collide after 1 s at a height of 15.1 m from the ground.

EXAMPLE 33. *A tennis ball is dropped on to the floor from a height of 4 m. It rebounds to a height of 3 m. If the ball was in contact with the floor for 0.01 s, what was its average acceleration during contact ?*

Solution. *For downward motion of the ball.* Suppose the ball falls from height h_1 and strikes the floor with velocity v_1. Then

$u = 0, \quad v = v_1, \quad a = +g, \quad s = h_1$

As $v^2 - u^2 = 2as$

$\therefore \quad v_1^2 - 0^2 = 2gh_1$ or $v_1 = \sqrt{2gh_1}$

For upward motion of the ball. Suppose the ball gets rebounded from the floor with velocity v_2 and rises to height h_2. Then

$u = v_2, \quad v = 0, \quad a = -g, \quad s = h_2$

As $v^2 - u^2 = 2as$

$\therefore \quad 0^2 - v_2^2 = 2 \times (-g)\,h_2$ or $v_2 = \sqrt{2gh_2}$

Average acceleration,

$$a = \frac{\Delta v}{\Delta t} = \frac{v_2 - (-v_1)}{\Delta t}$$

$$= \frac{v_2 + v_1}{\Delta t} = \frac{\sqrt{2gh_2} + \sqrt{2gh_1}}{\Delta t}$$

But $h_1 = 4m, \; h_2 = 3m, \; \Delta t = 0.01\text{s}, \; g = 9.8 \text{ ms}^{-2}$

$$\therefore \quad a = \frac{\sqrt{2 \times 9.8 \times 3} + \sqrt{2 \times 9.8 \times 4}}{0.01} = \mathbf{1652 \text{ ms}^{-2}}.$$

EXAMPLE 34. *A stone falls from a cliff and travels 24.5 m in the last second before it reaches the ground at the foot of the cliff. Find the height of the cliff.*

Solution. Here $u = 0, \; g = 9.8 \text{ ms}^{-2}$

Let nth second be the last second. Then

$$s_{nth} = u + \frac{g}{2}(2n-1)$$

or $24.5 = 0 + \frac{9.8}{2}(2n-1)$

or $2n - 1 = \frac{24.5}{4.9} = 5$

or $n = 3$ or $t = 3s$

Distance, $s = ut + \frac{1}{2}gt^2 = 0 + \frac{1}{2} \times 9.8 \times 3^2 =$ **44.1 m**

Hence the height of cliff is 44.1 m.

PROBLEMS FOR PRACTICE

1. A stone is thrown vertically upwards with a velocity of 4.9 ms^{-1}. Calculate (*i*) the maximum height reached (*ii*) the time taken to reach the maximum height (*iii*) the velocity with which it returns to the ground and (*iv*) the time taken to reach the ground.

   [**Ans.** (*i*) 1.225 m (*ii*) 0.5 s (*iii*) 4.9 ms^{-1} (*iv*) 1 s]

2. A stone thrown upwards from the top of a tower 85 m high, reaches the ground in 5 s. Find (*i*) the

greatest height above the ground (*ii*) the velocity with which it reaches the ground and (*iii*) the time taken to reach the maximum height. Take $g = 10\ \text{ms}^{-2}$. [**Ans.** (*i*) 88.2 m (*ii*) 42 ms^{-1} (*iii*) 0.8 s]

3. From the top of a multi-storeyed building, 39.2 m tall, a boy projects a stone vertically upwards with an initial velocity of 9.8 ms^{-1} such that it finally drops to the ground. (*i*) When will the stone reach the ground ? (*ii*) When will it pass through the point of projection ? (*iii*) What will be its velocity before striking the ground ? Take $g = 9.8\ \text{ms}^{-2}$.

(**Ans.** 4s, 2s, 29.4 ms^{-1})

4. A rocket is fired vertically from the ground with a resultant vertical acceleration of 10 ms^{-2}. The fuel is finished in 1 minute and it continues to move up. What is the maximum height reached ?

(**Ans.** 36.4 km)

5. A balloon is ascending at the rate of 14 ms^{-1} at a height of 98 m above the ground when the food packet is dropped from the balloon. After how much time and with what velocity does it reach the ground ? Take $g = 9.8\ \text{ms}^{-2}$.

(**Ans.** 6.12 s, 45.98 ms^{-1})

6. A stone is dropped from a balloon rising upwards with a velocity of 16 ms^{-1}. The stone reaches the ground in 4 s. Calculate the height of the balloon when the stone was dropped. (**Ans.** 14.4 m)

7. From the top of a tower 100 m in height a ball is dropped and at the same time another ball is projected vertically upwards from the ground with velocity of 25 ms^{-1}. Find when and where the two balls will meet. Take $g = 9.8\ \text{ms}^{-2}$. **[Delhi 05]**

(**Ans.** 78.4 from top, 4 s)

8. A body is dropped from rest at a height of 150 m, and simultaneously, another body is dropped from rest from a point 100 m above the ground. What is their difference in height after they have fallen (*i*) 2 s (*ii*) 3 s ? How does the difference in height vary with time ? Take $g = 10\ \text{ms}^{-2}$.

(**Ans.** 50 m, difference in height remains constant at 50 m)

9. A body falling freely under gravity passes two points 30 m apart in 1s. Find from what point above the upper point it began to fall ? Take $g = 9.8\ \text{ms}^{-2}$.

(**Ans.** 32.1 m)

10. Four balls are dropped from the top of a tower at intervals of one-one second. The first ball reaches the ground after 4 s of dropping. What are the distances between first and second, second and third, third and fourth balls at this instant ?

(**Ans.** 34.3 m, 24.5 m, 14.7 m)

✖ HINTS

2. Net displacement, $s = -85$ m, $t = 5$ s, $g = -10\ \text{ms}^{-2}$

As $s = ut + \frac{1}{2} g t^2$

$\therefore$ $-85 = u \times 5 - \frac{1}{2} \times 10 \times 25$

or $u = 8\ \text{ms}^{-1}$.

(*i*) Height covered in upward motion is given by

$$v^2 = u^2 + 2gs$$

or $0 = 8^2 + 2 \times (-10)\, s$

or $s = 3.2$ m

$\therefore$ The greatest height above the ground

$= 85 + 3.2 = \mathbf{88.2}$.

(*ii*) *For the downward motion* :

$u = 0$, $g = +10\ \text{ms}^{-2}$, $s = 88.2$ m

$v^2 = u^2 + 2gs = 0 + 2 \times 10 \times 88.2 = 1764$

$\therefore$ Velocity on reaching the ground, $v = \mathbf{42\ ms^{-1}}$.

(*iii*) Let t be the time taken to reach the maximum height.

For upward motion,

$u = 8\ \text{ms}^{-1}$, $v = 0$, $g = -10\ \text{ms}^{-2}$

$$\therefore \quad t = \frac{v-u}{g} = \frac{0-8}{-10} = \mathbf{0.8\ s.}$$

4. Height covered in 1 min,

$$s_1 = ut + \frac{1}{2} g t^2 = 0 + \frac{1}{2} \times 10 \times (60)^2 = 18000 \text{ m}$$

Velocity attained after 1 min,

$$v = u + at = 0 + 10 \times 60 = 600\ \text{ms}^{-1}$$

After the fuel is finished, $u = 600\ \text{ms}^{-1}$, $v = 0$,

$$v^2 - u^2 = 2gs$$

or $0 - (600)^2 = 2 \times (-9.8) \times s_2$

or $s_2 = \dfrac{(600)^2}{2 \times 9.8} = 18367.3$ m

Maximum height reached

$= s_1 + s_2 = 36367.3 \text{ m} \simeq \mathbf{36.4\ km.}$

6. Here $u = 16\ \text{ms}^{-1}$, $g = -9.8\ \text{ms}^{-2}$, $t = 4$ s, $s = -h$

$$s = ut + \frac{1}{2} g t^2$$

$-h = 16 \times 4 - \frac{1}{2} \times 9.8 \times 16 = -14.4$ or $h = \mathbf{14.4\ m}$.

8. *For downward motion of the first body* :

$u = 0$, $t = 2$ s, $g = 10\ \text{ms}^{-2}$

$$s = ut + \frac{1}{2} g t^2 = 0 \times 2 + \frac{1}{2} \times 10 \times 4 = 20 \text{ m}$$

Height from the ground, $h_1 = 150 - 20 = 130$ m

For downward motion of the second body :

$u = 0$, $t = 2$ s, $g = 10\ \text{ms}^{-2}$.

$$s = 0 \times 2 + \frac{1}{2} \times 10 \times 4 = 20 \text{ m}$$

Height from the ground, $h_2 = 100 - 20 = 80$ m
Difference in heights $= h_1 - h_2 = 130 - 80 = 50$ m
Similarly at $t = 3$ s, difference in heights = 50 m

Hence the difference in heights does not depend on time so long as both the bodies are in air.

9. Suppose the body passes the upper point at t second and lower point at $(t + 1)$ s, then

$$s_2 - s_1 = \frac{1}{2} g (t+1)^2 - \frac{1}{2} g t^2 = \frac{1}{2} g (2t + 1)$$

or $\quad 30 \text{ m} = \frac{1}{2} \times 9.8 (2t + 1)$

or $\quad t = 2.56$ s

$\therefore \quad s_1 = \frac{1}{2} g t^2 = \frac{1}{2} \times 9.8 \times (2.56)^2 = \mathbf{32.1 \text{ m}}.$

10. Here we need to find the distances travelled by the first ball in 4 s, by the second ball in 3 s, by the third ball in 2 s and by the fourth ball in 1 s.

$$h_1 = \frac{1}{2} g (4)^2 = 8 \times 9.8 = 78.4 \text{ m};$$
$$h_2 = \frac{1}{2} g (3)^2 = \frac{9}{2} \times 9.8 = 44.1 \text{ m};$$
$$h_3 = \frac{1}{2} g (2)^2 = 2 \times 9.8 = 19.6 \text{ m};$$
$$h_4 = \frac{1}{2} g (1)^2 = \frac{9.8}{2} = 4.9 \text{ m}.$$

$\therefore$ Distance between first and second balls is

$$h_1 - h_2 = 78.4 - 44.1 = \mathbf{34.3 \text{ m}}$$

Distance between second and third balls is

$$h_2 - h_3 = 44.1 - 19.6 = \mathbf{24.5 \text{ m}}$$

Distance between third and fourth balls is

$$h_3 - h_4 = 19.6 - 4.9 = \mathbf{14.7 \text{ m}}.$$

3.13 POSITION-TIME GRAPHS

23. *Draw the position-time graph for a stationary object.*

Position-time graph for a stationary object. The position of a stationary object does not change with time. The object remains at a constant distance $x = x_0$ from the origin at all times. So the position-time $(x - t)$ graph for a stationary object is a straight line parallel to the time-axis, as shown in Fig. 3.20.

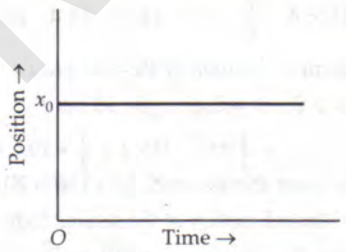


Fig. 3.20 Position-time graph for a stationary object.

24. *Draw the position-time graph for an object in uniform motion. Show that the slope of the position-time graph gives the velocity of the object.*

Position-time graph for uniform motion. An object in uniform motion covers equal distances in equal intervals of time. So the *position-time graph for an object in uniform motion along a straight line path is a straight line inclined to the time-axis,* as shown in Fig. 3.21.

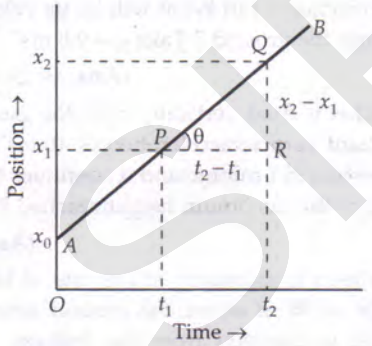


Fig. 3.21. Position-time graph for uniform motion.

Slope of position-time graph AB

$$= \tan\theta = \frac{QR}{PR} = \frac{x_2 - x_1}{t_2 - t_1}$$

$$= \frac{\text{Displacement}}{\text{Time}} = \text{Velocity } (v).$$

Hence the slope of the position-time graph gives velocity of the object.

25. *Draw the position-time graph for uniformly accelerated motion. What does its slope give ?*

Position-time graph for uniformly accelerated motion. The position-time relation for uniformly accelerated motion along a straight line is

$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$

Clearly, $x \propto t^2$ *i.e.*, x is a quadratic function of t. So the *position-time graph for uniformly accelerated motion is a parabola,* as shown in Fig. 3.22.

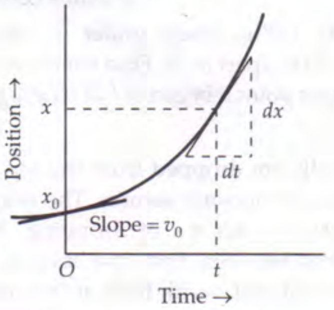


Fig. 3.22 Position-time graph for uniform acceleration.

Slope of position-time graph

$$= \frac{\text{Small change in vertical coordinate}}{\text{Small change in horizontal coordinate}}$$

$$= \frac{dx}{dt} = \text{velocity at instant } t$$

Thus the slope of the position-time graph gives the instantaneous velocity of the object. Moreover, the slope of the x-t graph at time $t=0$ gives the initial velocity v_0 of the object.

3.14 VELOCITY-TIME GRAPHS

26. *Draw the velocity-time graph for an object in uniform motion. Show that the area under the velocity-time graph gives the displacement of the object in the given time interval.*

Velocity-time graph for uniform motion. When an object has uniform motion, it moves with uniform velocity v in the same fixed direction. So *the velocity-time graph for uniform motion is a straight line parallel to the time-axis,* as shown in Fig. 3.23

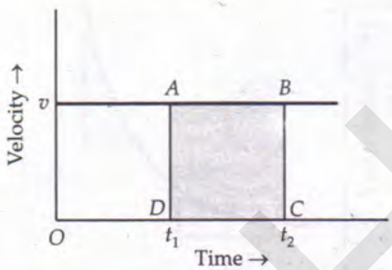


Fig. 3.23 Velocity-time graph for uniform motion.

Area under the velocity-time graph between times t_1 and t_2

$$= \text{Area of rectangle } ABCD$$
$$= AD \times DC = v\,(t_2 - t_1)$$
$$= \text{Velocity} \times \text{time}$$
$$= \text{Displacement}$$

Hence *the area under the velocity-time graph gives the displacement of the object in the given time interval.*

27. *Draw the velocity-time graph for uniformly accelerated motion. Show that slope of the velocity-time graph gives acceleration of the object.*

Velocity-time graph for uniformly accelerated motion. When a body moves with a uniform acceleration, its velocity changes by equal amounts in equal intervals of time. So the *velocity-time graph for a uniformly accelerated motion is a straight line inclined to the time-axis,* as shown in Fig. 3.24.

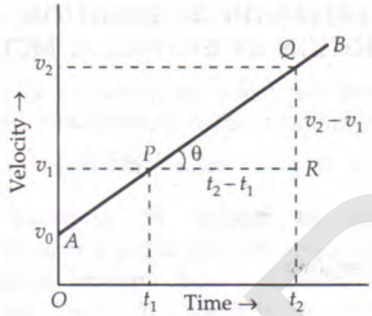


Fig. 3.24 Velocity-time graph for uniform acceleration.

Slope of velocity-time graph AB

$$= \tan\theta = \frac{QR}{PR} = \frac{v_2 - v_1}{t_2 - t_1}$$

$$= \frac{\text{Change in velocity}}{\text{Time interval}}$$

$$= \text{Acceleration } (a)$$

Hence the *slope of the velocity-time graph gives the acceleration of the object.*

28. *Show that the area under the velocity-time graph of an object moving with constant acceleration in a straight line in certain time interval is equal to the distance covered by the object in that interval.*

Distance covered as area under the velocity-time graph. In Fig. 3.25, the straight line AB is the velocity-time graph of an object moving along a straight line path with uniform acceleration a. Let its velocities be v_0 and v at times 0 and t respectively.

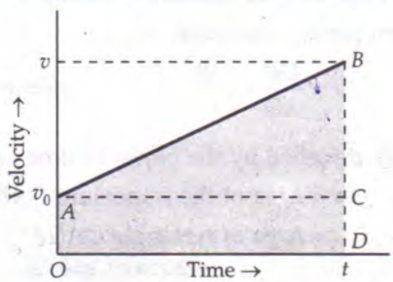


Fig. 3.25. Area under velocity-time graph.

Area under the velocity-time graph AB

$$= \text{Area of trapezium } OABD$$

$$= \frac{1}{2}(OA + BD) \times OD = \frac{1}{2}(v_0 + v) \times (t - 0)$$

$$= \text{Average velocity} \times \text{time interval}$$

$$= \text{Distance travelled in time } t$$

Hence *the area under the velocity-time graph gives the distance travelled by the object in the given time interval.*

3.15 DERIVATION OF EQUATIONS OF MOTION BY GRAPHICAL METHOD

29. *Derive the following equations of motion for uniformly accelerated motion from velocity-time graph :*

(*i*) $v = u + at$ (*ii*) $s = ut + \frac{1}{2}at^2$ (*iii*) $v^2 - u^2 = 2as$.

Equations of motion by graphical method. Consider an object moving along a straight line path with initial velocity u and uniform acceleration a. Suppose it travels distance s in time t. As shown in Fig. 3.26, its velocity-time graph is straight line. Here $OA = ED = u$, $OC = EB = v$ and $OE = t = AD$.

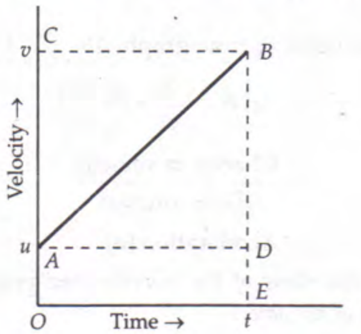


Fig. 3.26 Velocity-time graph for uniform acceleration.

(*i*) We know that,

Acceleration = Slope of velocity-time graph AB

or $$a = \frac{DB}{AD} = \frac{DB}{OE} = \frac{EB - ED}{OE} = \frac{v - u}{t}$$

or $v - u = at$ or $\boxed{v = u + at}$

This proves the *first equation of motion*.

(*ii*) From part (*i*), we have

$$a = \frac{DB}{AD} = \frac{DB}{t} \quad \text{or} \quad DB = at$$

Distance travelled by the object in time t is

s = Area of the trapezium $OABE$

= Area of rectangle $OADE$ + Area of triangle ADB

$= OA \times OE + \frac{1}{2} DB \times AD$

$= ut + \frac{1}{2} at \times t$ or $\boxed{s = ut + \frac{1}{2}at^2}$

This proves the *second equation of motion*.

(*iii*) Distance travelled by object in time t is

s = Area of trapezium $OABE$.

$= \frac{1}{2}(EB + OA) \times OE = \frac{1}{2}(EB + ED) \times OE$

Acceleration,

a = Slope of velocity-time graph AB

or $$a = \frac{DB}{AD} = \frac{EB - ED}{OE} \quad \text{or} \quad OE = \frac{EB - ED}{a}$$

$$\therefore \quad s = \frac{1}{2}(EB + ED) \times \frac{(EB - ED)}{a}$$

$$= \frac{1}{2a}(EB^2 - ED^2) = \frac{1}{2a}(v^2 - u^2)$$

or $\boxed{v^2 - u^2 = 2as}$

This proves the *third equation of motion*.

For Your Knowledge

- A straight line graph has a *single slope*. So if the displacement-time graph is a straight line, it represents a constant velocity. If the velocity-time graph is a straight line, it represents a constant acceleration.
- A curved graph has *multiple slopes*. In Fig. 3.27, as the displacement-time graph bends upwards with the passage of time, the value of θ increases, slope (= tan θ) of the curve increases, consequently the velocity increases and hence the motion is accelerated.

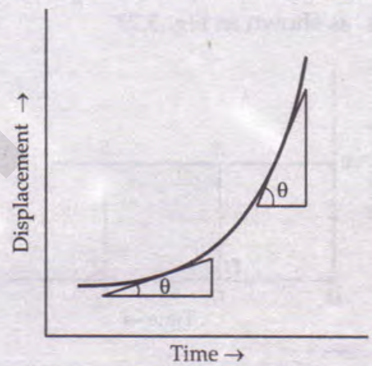


Fig. 3.27 Displacement-time graph for accelerated motion.

- In Fig. 3.28, as the displacement-time graph bends downwards with the passage of time, the value of θ decreases, the slope of the curve decreases, consequently the velocity decreases and hence the motion is decelerated.

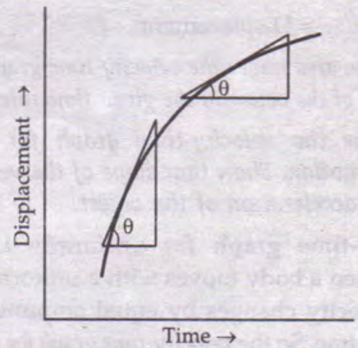


Fig. 3.28 Displacement-time graph for decelerated motion.

3.16 ANALYSING NATURE OF MOTION FROM VARIOUS GRAPHS

30. *Discuss the nature of motion from the given distance-time graphs.*

Different Types of Distance-Time Graphs :

(*i*) For a body at rest, the distance-time graph is a straight line *AB*, as shown in Fig. 3.29. As the slope of *AB* is zero, so speed of the body is zero.

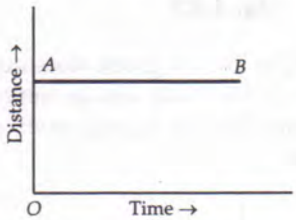


Fig. 3.29

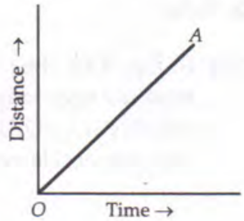


Fig. 3.30

(*ii*) For a body moving with uniform speed, the distance-time graph is a straight line inclined to the time-axis, as shown in Fig. 3.30. As the graph passes through *O*, so distance travelled at $t = 0$ is also zero.

(*iii*) The distance-time graph in Fig. 3.31 represents accelerated (speeding up) motion, because the slope of the graph is increasing with time.

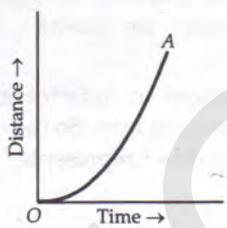


Fig. 3.31

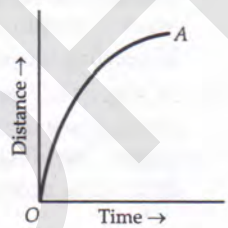


Fig. 3.32

(*iv*) The distance-time graph in Fig. 3.32 represents decelerated (speeding down) motion, because slope of the graph is decreasing with time.

(*v*) In Fig. 3.33, distance-time graph is a straight line parallel to distance-axis. It represents infinite speed which is not possible.

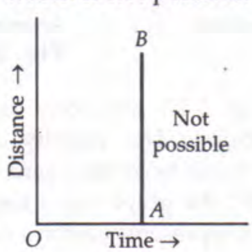


Fig. 3.33

(*vi*) The distance covered by a body cannot decrease with the increase of time. So the distance-time graph of the type shown in Fig. 3.34 is not possible.

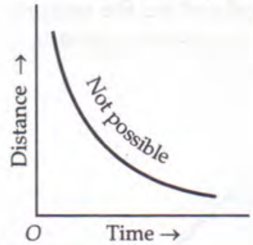


Fig. 3.34

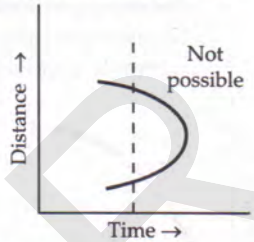


Fig. 3.35

(*vii*) The distance-time graph shown in Fig. 3.35 is not possible because it represents two different positions of the body at the same instant which is not possible.

31. *Discuss the nature of the motion from the given displacement-time graphs.*

Different Types of Displacement-Time Graphs :

(*i*) For a stationary body, the displacement-time graph is a straight line *AB* parallel to time-axis. The zero slope of line *AB* indicates zero velocity.

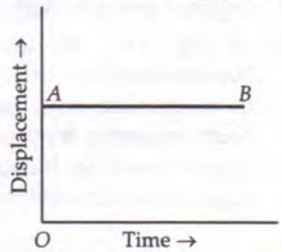


Fig. 3.36

(*ii*) In Fig. 3.37, the displacement-time graph is a straight line *OA* inclined to time-axis. It has a single slope. So it represents a constant velocity and hence zero acceleration.

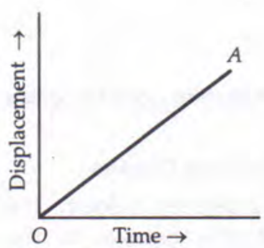


Fig. 3.37

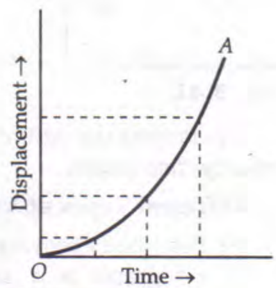


Fig. 3.38

(*iii*) In Fig. 3.38, greater displacements are taking place in equal intervals of time. So the displacement-time curve *OA* represents an increasing velocity or an accelerated motion. For constant acceleration, the displacement-time graph is a parabola bending upwards.

(*iv*) In Fig. 3.39, decreasing displacements are taking place in equal intervals of time. So the displacement-time curve OA represents a decreasing velocity or deceleration. For uniform deceleration, the displacement-time graph is a parabola bending downwards.

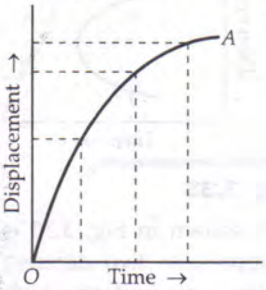


Fig. 3.39

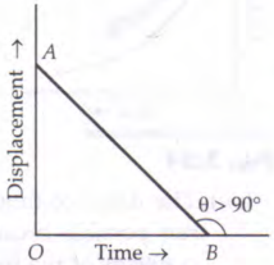


Fig. 3.40

(*v*) In Fig. 3.40, displacement is decreasing uniformly with time and becomes zero after a certain time. Displacement-time graph has a negative slope ($\theta > 90°$ and $\tan\theta < 0$) and represents a uniform negative velocity. It indicates that the body is returning back to its original position with a uniform velocity.

(*vi*) In Fig. 3.41, the displacement of the body becomes negative after time t and then increases in magnitude with time. It indicates that the body returning from position A, moves past the original position B and then moves towards C with a uniform velocity.

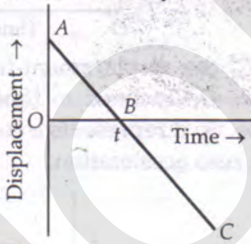


Fig. 3.41

32. *Discuss the nature of the motion from the given velocity-time graphs.*

Different Types of Velocity-Time Graphs :

(*i*) For a body moving with a uniform velocity, the v-t graph is a straight line parallel to the time-axis as shown in Fig. 3.42. The zero slope of line AB indicates zero acceleration.

(*ii*) When a body starts from rest and moves with uniform acceleration, its v-t graph is straight line OA inclined to the time-axis and passing through the origin O, as shown in Fig. 3.43. Greater is the slope of the v-t graph, greater will be the acceleration.

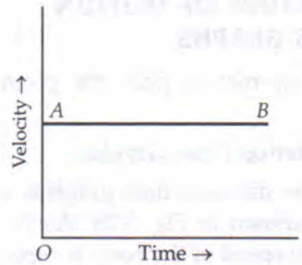


Fig. 3.42

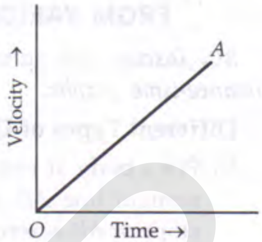


Fig. 3.43

(*iii*) In Fig. 3.44, the straight line v-t graph does not pass through origin O. The body has an initial velocity $u (= OA)$ and then it moves with a uniform acceleration.

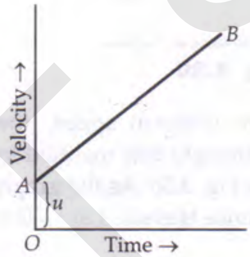


Fig. 3.44

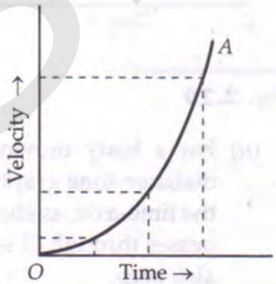


Fig. 3.45

(*iv*) In Fig. 3.45, greater changes in velocity are taking place in equal intervals of time. So the v-t graph bending upwards represents an increasing acceleration.

(*v*) In Fig. 3.46, smaller changes in velocity are taking place in equal intervals of time. So the v-t graph bending downwards represents a decreasing acceleration.

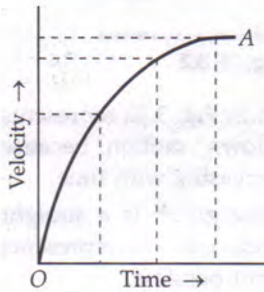


Fig. 3.46

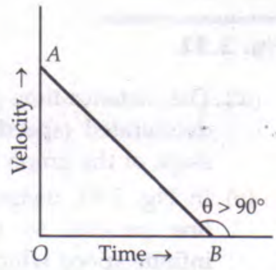


Fig. 3.47

(*vi*) In Fig. 3.47, the body starts with an initial velocity u. The velocity decreases uniformly with time, becoming zero after some time. As $\theta > 90°$, the graph has a negative slope. The v-t graph represents uniform negative acceleration.

(*vii*) In Fig. 3.48, the *v-t* graph represents a body projected upwards with an initial velocity *u*. The velocity decreases with time (negative uniform acceleration), becoming zero after certain time *t*. Then the velocity becomes negative and increases in magnitude, showing body is returning to original position with positive uniform acceleration.

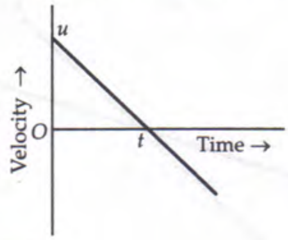


Fig. 3.48

(*viii*) The area between the velocity-time graph and the time-axis gives the displacement. In Fig. 3.49, the *v-t* graph represents variable acceleration.

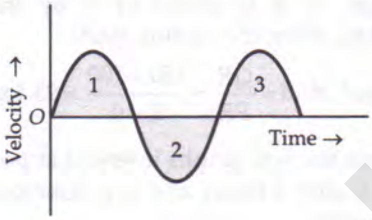


Fig. 3.49

Displacement covered = Area 1 – Area 2 + Area 3

Distance covered = Area 1 + Area 2 + Area 3.

34. *Discuss the nature of motion from the given speed-time graphs.*

Different Types of Speed-Time Graphs :

(*i*) For a body projected upwards, the speed-time graph is of the type shown in Fig. 3.50. When the body moves, its speed decreases uniformly, becoming zero at the highest point. As the body moves down, its speed increases uniformly. It returns with the same speed with which it was thrown up.

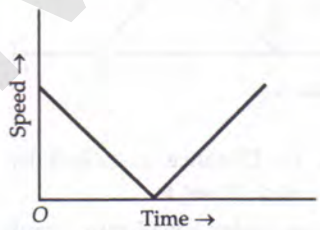


Fig. 3.50

(*ii*) For a ball dropped on the ground from a certain height, the speed-time graph is of the type shown in Fig. 3.51. As the ball falls, its speed increases. As the ball bounces back, its speed decreases uniformly and becomes zero at the highest point.

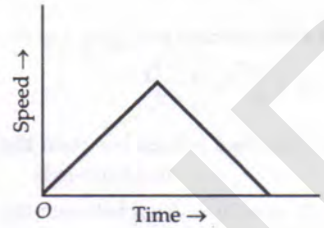


Fig. 3.51

34. *Discuss the nature of motion from the given acceleration-time graphs.*

Different Types of Acceleration-Time Graphs :

(*i*) For a body moving with constant acceleration, the acceleration-time graph is a straight line *AB* parallel to the time-axis, as shown in Fig. 3.52.

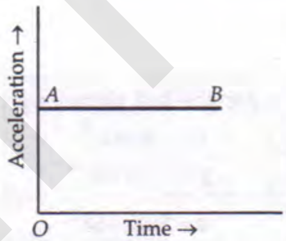


Fig. 3.52

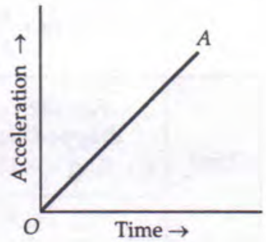


Fig. 3.53

(*ii*) When the acceleration of a body increases uniformly with time, its *a-t* graph is a straight line *OA* inclined to the time-axis as shown in Fig. 3.53.

(*iii*) For a body moving with variable acceleration, the *a-t* graph is a curve. The area between the *a-t* graph and the time-axis gives the change in velocity, as shown in Fig. 3.54.

Change in velocity = Area 1 – Area 2 + Area 3

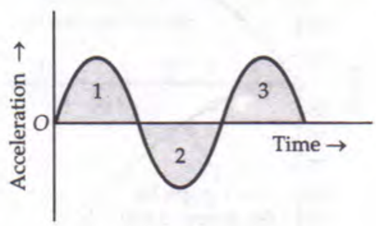


Fig. 3.54

Examples based on Position-Time and Velocity-Time Graphs

FORMULAE USED

1. Slope of position-time (x-t) graph gives velocity.
$$v = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}.$$
2. Slope of velocity-time (v-t) graph gives acceleration.
$$a = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$$
3. Distance travelled = Area between the (v-t) graph and time-axis.
4. Change in velocity = Area between the (a-t) graph and time-axis.

Example 35. *From the top of a tower, a ball is dropped to fall freely under gravity and at the same time, another ball is thrown up with a velocity of 50 ms^{-1}. Plot the position-time graph for the motion of the two balls during the time interval $t = 0$ to $t = 5$ s. Take $g = 10\ ms^{-2}$.*

Solution. We can find the distances covered by the two balls at different instants of time by using the formula,
$$s = ut + \frac{1}{2}gt^2$$

Time	For the ball dropped down $u = 0,\ g = +10\ ms^{-2}$	For the ball thrown up $u = 50\ ms^{-1}$, $g = -10\ ms^{-2}$
	$s = 5t^2$	$s = 50t - 5t^2$
$t = 1$ s	$s = 5$ m	$s = 45$ m
$t = 2$ s	$s = 20$ m	$s =$ [illegible] m
$t = 3$ s	$s = 45$ m	$s = 105$ m
$t = 4$ s	$s = 80$ m	$s = 120$ m
$t = 5$ s	$s = 125$ m	$s = 125$ m

The position-time graphs for the motion of the two balls are as shown in Fig. 3.55.

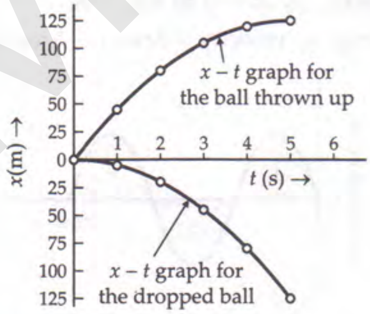


Fig. 3.55

EXAMPLE 36. *Fig. 3.56 shows the distance-time graphs of two trains, which start moving simultaneously in the same direction. From the graphs, find :*

(*i*) *How much ahead of A is B when the motion starts ?*
(*ii*) *What is the speed of B ?*
(*iii*) *When and where will A catch B ?*
(*iv*) *What is the difference between the speeds of A and B ?*

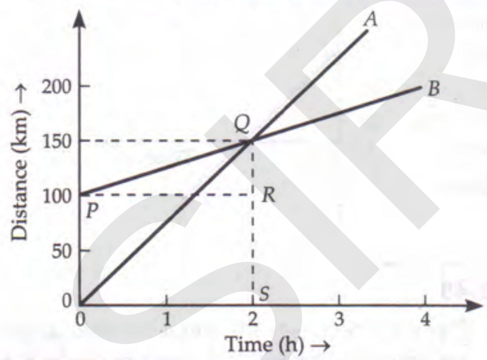


Fig. 3.56

Solution. (*i*) B is ahead of A by the distance OP = **100 km,** when the motion starts.

(*ii*) Speed of $B = \frac{QR}{PR} = \frac{150 - 100}{2 - 0} = \mathbf{25\ kmh^{-1}}$.

(*iii*) Since the two graphs intersect at point Q, so A will catch B after 2 hours and at a distance of 150 km from the origin.

(*iv*) Speed of $A = \frac{QS}{OS} = \frac{150 - 0}{2 - 0} = 75\ kmh^{-1}$

$\therefore$ Difference in speeds $= 75 - 25 = \mathbf{50\ kmh^{-1}}$.

Example 37. *The speed-time graph of a particle moving along a fixed direction is shown in figure. Find :*

(*i*) *distance travelled by the particle between 0 sec to 10 sec*
(*ii*) *average speed between this interval*
(*iii*) *the time when the speed was minimum*
(*iv*) *the time when speed was maximum.* **[Delhi 06]**

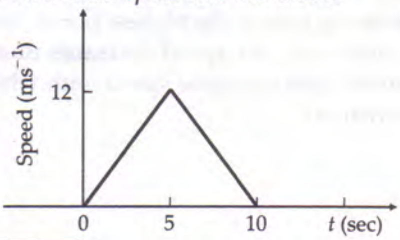


Fig. 3.57

Solution. (*i*) Distance travelled by the particle between 0 sec and 10 sec is

s = Area under speed-time graph
$= \frac{1}{2}(10 - 0)(12 - 0) = \mathbf{60\ m.}$

(*ii*) Average speed $= \dfrac{\text{Total distance travelled}}{\text{Time taken}}$

$$= \frac{60}{10} = \mathbf{6\ ms^{-1}}.$$

(*iii*) Speed is minimum at $t = 0$ **s** and $t = 10$ **s.**

(*iv*) Speed is maximum at $t = 5$ **s.**

Example 38. *A body starting from rest accelerates uniformly along a straight line at the rate of 10 ms^{-2} for 5 seconds. It moves for 2 second with uniform velocity of 50 ms^{-1}. Then it retards uniformly and comes to rest in 3 s. Draw velocity-time graph of the body and find the total distance travelled by the body.* **[Central Schools 04]**

Solution. The velocity-time graph is shown below.

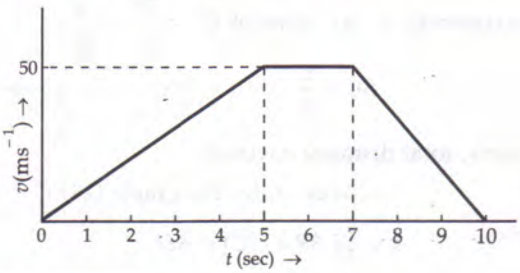


Fig. 3.58

Distance travelled by the body

$= $ Area under v-t graph

$$= \frac{1}{2}(2 + 10) \times 50 = \mathbf{300\ m.}$$

Example 39. *A train moves from one station to another in two hours' time. Its speed-time graph during the motion is shown in Fig. 3.59. (i) Determine the maximum acceleration during the journey (ii) Also calculate the distance covered during the time interval from 0.75 hour to 1 hour.*

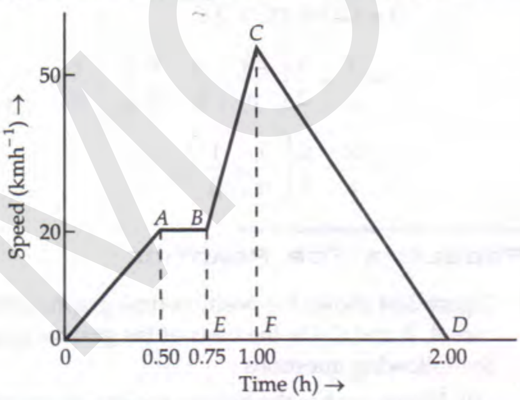


Fig. 3.59

Solution. (*i*) As the part BC of the speed-time graph has maximum slope, so acceleration is maximum during this interval.

Maximum acceleration

$= $ Slope of BC

$$= \frac{(50 - 20)\ \text{kmh}^{-1}}{(1.00 - 0.75)\ \text{h}} = \frac{30}{0.25} = \mathbf{120\ kmh^{-2}}.$$

(*ii*) Distance covered from 0.75 h to 1.00 h

$= $ Area of trapezium $BEFC$

$$= \tfrac{1}{2}[BE + CF] \times EF$$

$$= \tfrac{1}{2}[20 + 50] \times (1.00 - 0.75)\ \text{km} = \mathbf{8.75\ km.}$$

Example 40. *A ball is thrown upward with an initial velocity of 100 ms^{-1}. After how much time will it return? Draw velocity-time graph for the ball and find from the graph (i) the maximum height attained by the ball and (ii) height of the ball after 15 s. Take $g = 10$ ms^{-2}.*

Solution. Here $u = 100\ \text{ms}^{-1}, g = -10\ \text{ms}^{-2}$

At highest point, $v = 0$

As $v = u + gt \quad \therefore \quad 0 = 100 - 10 \times t$

$\therefore$ Time taken to reach highest point,

$$t = \frac{100}{10} = 10\ \text{s}$$

The ball will return to the ground at $t = $ **20 s.**

Velocities of the ball at different instants of time will be as follows :

At $t = 0$, $\quad v = 100 - 10 \times 0 = 100\ \text{ms}^{-1}$

At $t = 5$ s, $\quad v = 100 - 10 \times 5 = 50\ \text{ms}^{-1}$

At $t = 10$ s, $\quad v = 100 - 10 \times 10 = 0$

At $t = 15$ s, $\quad v = 100 - 10 \times 15 = -50\ \text{ms}^{-1}$

At $t = 20$ s, $\quad v = 100 - 10 \times 20 = -100\ \text{ms}^{-1}$

The velocity–time graph will be as shown in Fig. 3.60.

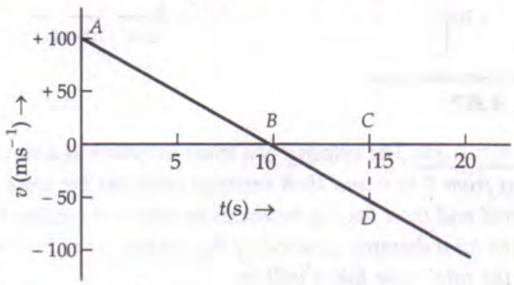


Fig. 3.60

(*i*) Maximum height attained by the ball

$= $ Area of ΔAOB

$$= \tfrac{1}{2} \times 10\ \text{s} \times 100\ \text{ms}^{-1} = \mathbf{500\ m.}$$

(*ii*) Height attained after 15 s

$= $ Area of $\Delta AOB + $ Area of ΔBCD

$$= 500 + \tfrac{1}{2}(15 - 10) \times (-50) = 500 - 125$$

$$= \mathbf{375\ m.}$$

EXAMPLE 41. The velocity-time graph for a vehicle is shown in Fig. 3.61. Draw acceleration-time graph from it.

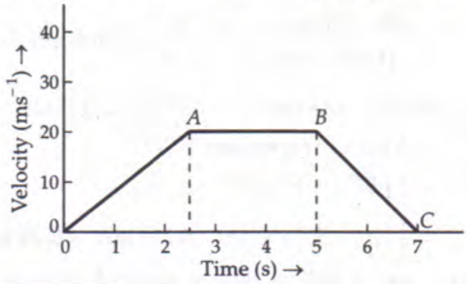


Fig. 3.61

Solution. In time 0 to 2.5 s, acceleration of the vehicle,

$$a = \text{Slope of } OA = \frac{20-0}{2.5-0} = \mathbf{8\ ms^{-2}}$$

In time 2.5 to 5 s, velocity of the vehicle is uniform. So acceleration = 0.

In time 5 to 7 s, acceleration of the vehicle,

$$a = \text{Slope of } BC = \frac{0-20}{7-5} = \mathbf{-10\ ms^{-2}}$$

Hence the acceleration-time graph will be as shown in Fig. 3.62.

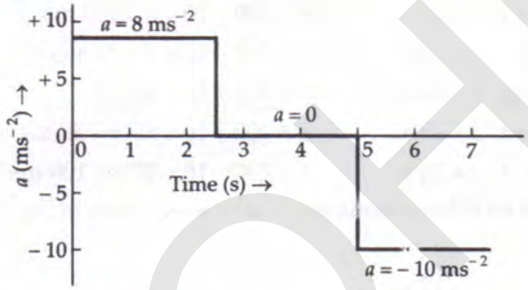


Fig. 3.62

EXAMPLE 42. The velocity of a train increases at a constant rate α from 0 to v and then remains constant for some time interval and then finally decreases to zero at a constant rate β. If the total distance covered by the particle be x, then show that the total time taken will be

$$t = \frac{x}{v} + \frac{v}{2}\left[\frac{1}{\alpha} + \frac{1}{\beta}\right].$$

Solution. The given motion can be represented by the velocity-time graph $OABC$ shown in Fig. 3.63. The portion OA represents the motion with constant acceleration α, the straight line AB parallel to time-axis represents the motion with uniform velocity v and the line BC represents the motion with constant acceleration β.

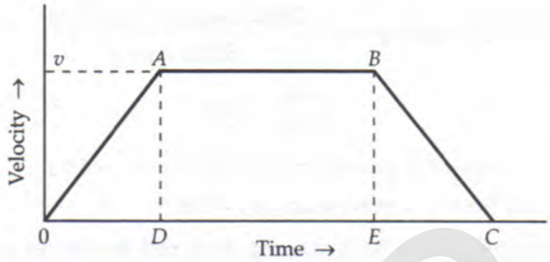


Fig. 3.63

Acceleration, $\alpha = \text{slope of } OA = \frac{AD}{OD} = \frac{v}{OD}$

$\therefore \qquad OD = \frac{v}{\alpha}$

Acceleration, $\beta = \text{slope of } BC = \frac{BE}{EC} = \frac{v}{EC}$

$\therefore \qquad EC = \frac{v}{\beta}$

Now, total distance covered

= Area under the graph $OABC$

or $\qquad x = \frac{1}{2}[AB + OC] \times AD$

$= \frac{1}{2}[DE + OD + DE + EC] \times AD \qquad [\because\ AB = DE]$

$$= \frac{1}{2}\left[2\,DE + \frac{v}{\alpha} + \frac{v}{\beta}\right] \times v$$

or $$\frac{2x}{v} = 2\,DE + \frac{v}{\alpha} + \frac{v}{\beta}$$

or $$DE = \frac{1}{2}\left[\frac{2x}{v} - \frac{v}{\alpha} - \frac{v}{\beta}\right]$$

Now, total time taken is given by

$$t = OD + DE + EC$$

$$= \frac{v}{\alpha} + \frac{1}{2}\left[\frac{2x}{v} - \frac{v}{\alpha} - \frac{v}{\beta}\right] + \frac{v}{\beta}$$

or $$t = \frac{x}{v} + \frac{v}{2}\left[\frac{1}{\alpha} + \frac{1}{\beta}\right].$$

✖ PROBLEMS FOR PRACTICE

1. Figure 3.64 shows the position-time graphs of three cars A, B and C. On the basis of the graphs, answer the following questions :
   (*i*) Which car has the highest speed and which the lowest ?
   (*ii*) Are the three cars ever at the same point on the road ?
   (*iii*) When A passes C, where is B ?

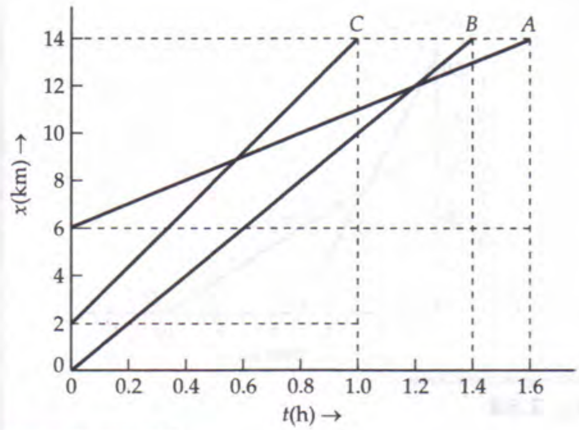


Fig. 3.64

(*iv*) How far did car A travel between the time it passed cars B and C ?

(*v*) What is the relative velocity of car C with respect to car A ?

(*vi*) What is the relative velocity of car B with respect to car C ?

[**Ans.** (*i*) C has the highest speed and A has the lowest speed (*ii*) No (*iii*) 6 km from the origin (*iv*) 3 km (*v*) 7 kmh^{-1} (*vi*) -2 kmh^{-1}]

2. An insect crawling up a wall crawls 5 cm upwards in the first minute but then slides 3 cm downwards in the next minute. It again crawls up 5 cm upwards in the third minute but again slides 3 cm downwards in the fourth minute. How long will the insect take to reach a crevice in the wall at a height of 24 cm from its starting point ? How does the position-time graph of the insect look like ?

(**Ans.** 21 min)

3. A driver of a car travelling at 52 km h^{-1} applies the brakes and decelerates uniformly. The car stops in 5 seconds. Another driver going at 34 km h^{-1} applies his brakes slower and stops after 10 seconds. On the same graph paper, plot the speed versus time graph for two cars. Which of the two cars travelled farther after the brakes were applied ?

(**Ans.** Second car travelled farther)

4. A motor car, starting from rest, moves with uniform acceleration and attains a velocity of 8 ms^{-1} in 8 s. It then moves with uniform velocity and finally brought to rest in 32 m under uniform retardation. The total distance covered by the car is 464 m. Find (*i*) the acceleration (*ii*) the retardation and (*iii*) the total time taken. [**Ans.** (*i*) 1 ms^{-2} (*ii*) 1 ms^{-2} (*iii*) 66 s]

5. Starting from rest a car accelerates uniformly with 3 ms^{-2} for 5 s and then moves with uniform velocity. Draw the distance-time graph of the motion of the car upto $t = 7$s.

6. The velocity-time graph of an object moving along a straight line is as shown in Fig. 3.65. Find the net distance covered by the object in time interval between $t = 0$ to $t = 10$ s. Also find the displacement in time 0 to 10 s. (**Ans.** 100 m, 60 m)

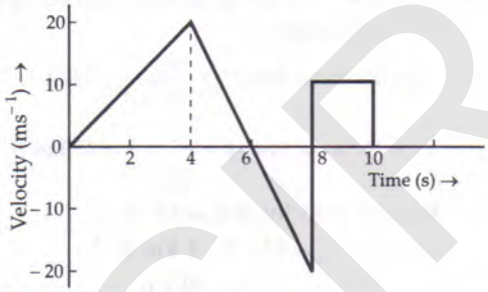


Fig. 3.65

7. As soon as a car just starts from rest in a certain direction, a scooter moving with a uniform speed overtakes the car. Their velocity-time graphs are shown in Fig. 3.66. Calculate (*i*) the difference

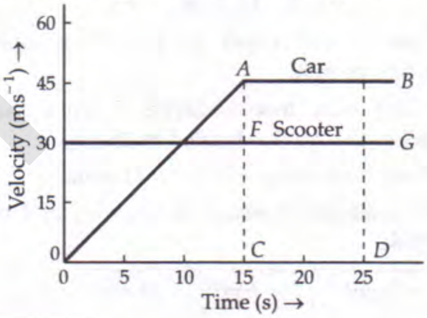


Fig. 3.66

between the distances travelled by the car and the scooter in 15 s (*ii*) the time when the car will catch up the scooter and (*iii*) the distance of car and scooter from the starting point at that instant.

[**Ans.** (*i*) 112.5 m (*ii*) 22.5 s (*iii*) 675 m]

8. The velocity-time graph of an object moving along a straight line is as shown below :

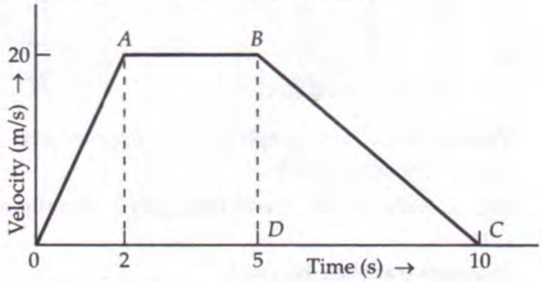


Fig. 3.67

Calculate the distance covered by object between :

(*i*) $t = 0$ to $t = 5$ s (*ii*) $t = 0$ to $t = 10$ s.

[Chandigarh 08] [**Ans.** (*i*) 80 m (*ii*) 130 m]

✖ HINTS

1. (*i*) Slope of line C is highest and that of A is lowest.

   (*ii*) The lines do not meet at one point at any time.

   (*iii*) A passes C at 0.6 h, at this time B is at 6 km from the origin.

   (*v*) v_A = Slope of line $A = \dfrac{14-6}{1.6-0} = 5 \text{ km h}^{-1}$

   v_C = Slope of line $C = \dfrac{14-2}{1.0-0} = 12 \text{ km h}^{-1}$

   Relative velocity of C w.r.t. A

   $= v_C - v_A = 12 - 5 = \mathbf{7 \text{ km h}^{-1}}$.

   (*vi*) v_B = Slope of line $B = \dfrac{14-0}{1.4-0} = 10 \text{ km h}^{-1}$

   Relative velocity of B w.r.t. C

   $= v_B - v_C = 10 - 12 = \mathbf{-2 \text{ km s}^{-1}}$.

2. Net distance crawled upwards in 2 min time

   $= 5 - 3 = 2$ cm

   $\therefore$ Insect will crawl $2 \times 10 = 20$ cm upward in $2 \times 10 = 20$ min.

   In 21st min, insects covers 5 cm upward and reaches the crevice located at 25 cm.

   $\therefore$ Total time taken $= 20 + 1 =$ **21 min.**

   The positions of insect at intervals of 1 min each will be

t (min)	*x (cm)*	*t (min)*	*x (cm)*
0	0	11	15
1	5	12	12
2	2	13	17
3	7	14	14
4	4	15	19
5	9	16	16
6	6	17	21
7	11	18	18
8	8	19	23
9	13	20	20
10	10	21	25

   Position-time (x-t) graph can be drawn with the help of the above table.

3. Fig. 3.68 shows the speed-time graphs for the two cars.

   Distance-travelled by car 1,

   s_1 = Area of ΔAOB

   $= \frac{1}{2} OA \times OB = \frac{1}{2} \times 52 \text{ km h}^{-1} \times 5 \text{ s}$

   $= \frac{1}{2} \times 52 \times \frac{5}{18} \text{ ms}^{-1} \times 5 \text{ s} =$ **36.1 m.**

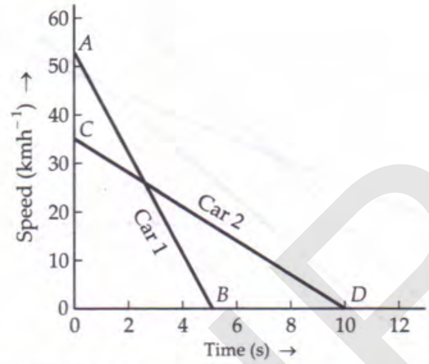


Fig. 3.68

   Distance travelled by car 2,

   s_2 = Area of ΔCOD

   $= \dfrac{1}{2} OC \times OD = \dfrac{1}{2} \times 34 \times \dfrac{5}{18} \text{ ms}^{-1} \times 10 \text{ s}$

   = **47.2 m.**

   Hence second car travelled farther than first after the brakes were applied.

4. Fig. 3.69 shows the velocity-times graph for the motion of the motor car.

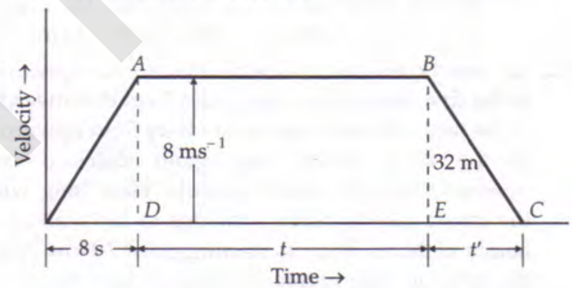


Fig. 3.69

   Distance covered by the motor car,

   Area of ΔAOD + Area of rect. $ADEB$

   + Area of $\Delta BEC = 464$ m

   $\frac{1}{2} \times 8 \times 8 + 8 \times t + 32 = 464$

   or $t = 50$ s.

   Suppose the motor car retards for time t'.

   Area of $\Delta BEC = \frac{1}{2} \times 8 \times t' = 32$

   or $t' = 8$ s

   $\therefore$ Total time taken $= 8 + t + t' = 8 + 50 + 8 =$ **66 s.**

   Acceleration = Slope of $OA = \dfrac{AD}{OD} = \dfrac{8}{8} = \mathbf{1 \text{ ms}^{-2}}$.

   Retardation = Slope of $BC = \dfrac{BE}{EC} = \dfrac{8}{8} = \mathbf{1 \text{ ms}^{-2}}$.

5. Distance covered by car in first 5 s is given by

   $s = ut + \frac{1}{2} at^2 = 0 + \frac{1}{2} \times 3 \times t^2 = \frac{3}{2} t^2$

   At $t = 5$ s, $v = u + at = 0 + 3 \times 5 = 15 \text{ ms}^{-1}$

After $t = 5$ s, car covers a distance 15 m in each second upto $t = 7$ s.

Hence positions of the car at different instants of time will be

t (s)	0	1	2	3	4	5	6	7
x (m)	0	1.5	6.0	13.5	24.0	37.5	52.5	67.5

Fig. 3.70 shows the distance-time (x-t) graph for the car. For accelerated motion (0 to 5 s), the graph is a parabola (OA) and for uniform velocity (5 s to 7 s), the graph is straight line AB.

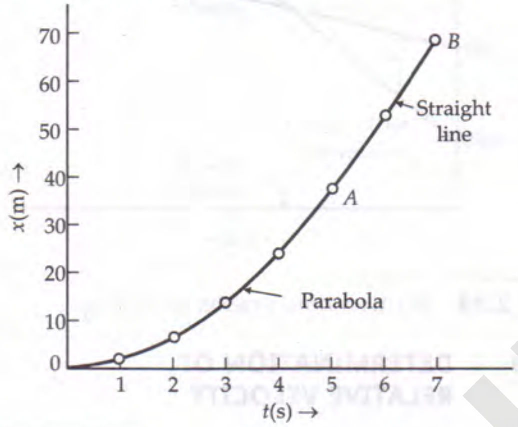


Fig. 3.70

6. Total distance covered in 10 s

$$= \frac{1}{2} \times 6 \times 20 + \frac{1}{2} \times 2 \times 20 + 2 \times 10 = \mathbf{100\ m.}$$

Displacement is positive from 0 to 6 s, negative from 6 to 8 s and positive from 8 to 10 s.

$\therefore$ Total displacement in 10 s

$$= \frac{1}{2} \times 6 \times 20 - \frac{1}{2} \times 2 \times 20 + 2 \times 10 = \mathbf{60\ m.}$$

7. (*i*) Distance travelled by the car in 15 s

$= \text{Area of } \Delta OAC$

$$= \frac{1}{2} \times OC \times AC = \frac{1}{2} \times 15 \times 45 = 337.5 \text{ m}$$

Distance travelled by the scooter in 15 s

$= \text{Area of rect. } OEFC$

$= 15 \times 30 = 450$ m

Difference in the distances travelled

$= 450 - 337.5 = \mathbf{112.5\ m.}$

(*ii*) After $t = 15$ s, relative velocity of the car w.r.t. the scooter

$= 45 - 30 = 15 \text{ ms}^{-1}$

$\therefore$ Time taken in covering a difference of 112.5 m

$$= \frac{112.5 \text{ m}}{15 \text{ ms}^{-1}} = 7.5 \text{ s}$$

$\therefore$ Time after which car will catch up the scooter

$= 15 + 7.5 = \mathbf{22.5\ s.}$

(*iii*) Distance travelled by the scooter in 22.5 s

$= 30 \text{ ms}^{-1} \times 22.5 \text{ s} = \mathbf{675\ m.}$

So the car catches the scooter when both are at 675 m from the starting point.

8. (*i*) Distance covered between $t = 0$ to $t = 5$ s

$$= \text{Area } OABDO = \frac{1}{2}(3+5) \times 20 = \mathbf{80\ m.}$$

(*ii*) Distance covered between $t = 0$ to $t = 10$ s

$$= \text{Area } OABCO = \frac{1}{2}(3+10) \times 20 = \mathbf{130\ m.}$$

3.17 RELATIVE VELOCITY

35. ***Define relative velocity. Deduce an expression for relative velocity of one object with respect to another in terms of their velocities relative to the earth.***

Relative velocity. *The relative velocity of an object 2 with respect to object 1, when both are in motion, is the time rate of change of position of object 2 with respect to that of object 1.*

Expression for relative velocity. As shown in Fig. 3.71, consider the objects 1 and 2 moving along the same direction with constant velocities v_1 and v_2 (relative to the earth) respectively.

Fig. 3.71 Relative velocity.

Suppose the position coordinates of the two objects are $x_1(0)$ and $x_2(0)$ at time $t = 0$. At time $t = t$, their position coordinates will be

$$x_1(t) = x_1(0) + v_1 t \quad ...(1)$$

$$x_2(t) = x_2(0) + v_2 t \quad ...(2)$$

Subtracting (1) from (2), we find that

$$x_2(t) - x_1(t) = x_2(0) - x_1(0) + (v_2 - v_1)\,t$$

or $[x_2(t) - x_2(0)] - [x_1(t) - x_1(0)] = (v_2 - v_1)\,t$

or Displacement of object 2 in time t

– Displacement of object 1 in time $t = (v_2 - v_1)\,t$

or Relative displacement of object 2 w.r.t. object 1 in time $t = (v_2 - v_1)\,t$

or $$\frac{\text{Relative displacement of object 2 w.r.t. object 1}}{\text{Time } t} = v_2 - v_1$$

or Relative velocity of object 2 w.r.t. object 1,

$$v_{21} = v_2 - v_1$$

Similarly, relative velocity of object 1 w.r.t. object 2,

$$v_{12} = v_1 - v_2$$

3.18 RELATIVE VELOCITY IN TERMS OF POSITION-TIME GRAPHS

36. *Draw and discuss the position-time graphs of two objects moving along a straight line, when their relative velocity is (i) zero (ii) positive and (iii) negative.*

Analysis of relative velocity in terms of position-time graphs. The relative velocity, $v_{21} = v_2 - v_1$ may be zero, positive or negative.

Case 1. ***When the two objects move with same velocity in the same direction.*** That is $v_1 = v_2$ and relative velocity, $v_2 - v_1 = 0$

Then, $\quad x_2(t) - x_1(t) = x_2(0) - x_1(0)$

Thus the two objects remain a constant distance apart throughout their motion. Their position-time graphs are parallel straight lines as shown in Fig. 3.72. Here the initial positions of the objects are $x_1(0) = 10$ m and $x_2(0) = 30$ m. Velocities are $v_1 = v_2 = 5\ \text{ms}^{-1}$. The two objects remain 20 m apart at all instants.

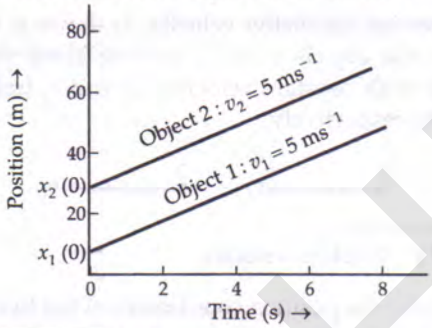


Fig. 3.72 Position-time graphs for $v_1 = v_2$.

Case 2. ***When $v_2 > v_1$ or relative velocity $(v_2 - v_1)$ is positive.*** The relative separation $x_2(t) - x_1(t)$ increases by the amount $(v_2 - v_1)$ after every second. So the position-time graphs gradually move apart, as shown in Fig. 3.73.

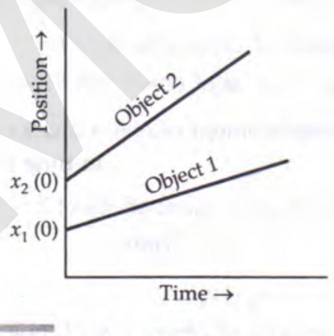


Fig. 3.73 Position-time graphs for $v_2 > v_1$.

Case 3. ***When $v_2 < v_1$ or relative velocity $(v_2 - v_1)$ is negative.*** Initially the object 2 is ahead of object 1 and $x_2(t) - x_1(t)$ is positive. The relative separation $x_2(t) - x_1(t)$ first decreases till the two objects meet at the position $x_1(t) = x_2(t)$. Then the separation $x_2(t) - x_1(t)$ becomes negative. The object 1 overtakes the object 2 and the relative separation between them again begins to increase.

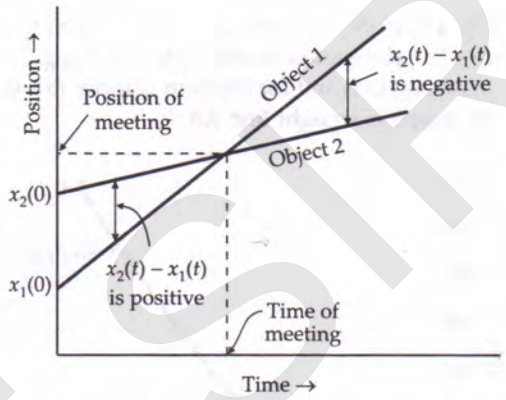


Fig. 3.74 Position-time graphs for $v_2 < v_1$.

3.19 DETERMINATION OF RELATIVE VELOCITY

37. *Two bodies A and B are moving with velocities v_A and v_B, making an angle θ with each other. Determine the relative velocity of A w.r.t. B. What will be the relative velocity (i) when the two bodies move in same direction and (ii) when the two bodies move in opposite directions?*

Rule to determine relative velocity. The relative velocity of a body A with respect to another body B when both are in motion can be obtained by adding to the velocity of A, a velocity equal and opposite to that of B. Thus

$$v_{AB} = v_A + (-v_B)$$

This law can be applied even to the bodies moving in directions inclined to each other.

Consider two bodies A and B moving with velocities v_A and v_B respectively, making an angle θ with each other as shown in Fig. 3.75(*a*).

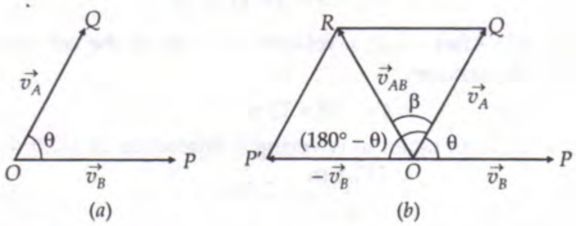


Fig. 3.75 Determination of relative velocity.

To find the relative velocity v_{AB} of the body A with respect to B, draw $OP' = -v_B$ as shown in Fig. 3.75(*b*). Now we add v_A and $(-v_B)$ which make an angle $(180° - \theta)$ with each other. The relative velocity of A with respect to B is given by the diagonal OR of the parallelogram $OQRP'$.

The magnitude of the relative velocity v_{AB} is

$$v_{AB} = \sqrt{v_A^2 + v_B^2 + 2\, v_A\, v_B \cos(180° - \theta)}$$

$$= \sqrt{v_A^2 + v_B^2 - 2\, v_A\, v_B \cos\theta}$$

Suppose the relative velocity v_{AB} makes angle β with v_A. Then

$$\tan\beta = \frac{v_B \sin(180° - \theta)}{v_A + v_B \cos(180° - \theta)}$$

$$= \frac{v_B \sin\theta}{v_A - v_B \cos\theta}$$

or $$\beta = \tan^{-1}\left(\frac{v_B \sin\theta}{v_A - v_B \cos\theta}\right)$$

This gives the direction of the relative velocity v_{AB}.

Special cases :

(*i*) **When both the bodies are moving along parallel straight lines in the same direction.** We have $\theta = 0°$.

$$\therefore \quad v_{AB} = \sqrt{v_A^2 + v_B^2 - 2\, v_A\, v_B \cos 0°}$$

$$= \sqrt{v_A^2 + v_B^2 - 2\, v_A\, v_B}$$

$$= \sqrt{(v_A - v_B)^2} = v_A - v_B.$$

Thus *the relative velocity of A with respect to B is equal to the difference between the magnitudes of their velocities.*

(*ii*) **When the two bodies are moving along parallel straight lines in the opposite directions.** We have $\theta = 180°$.

$$\therefore \quad v_{AB} = \sqrt{v_A^2 + v_B^2 - 2 v_A\, v_B \cos 180°}$$

$$= \sqrt{v_A^2 + v_B^2 + 2 v_A\, v_B}$$

$$= \sqrt{(v_A + v_B)^2} = v_A + v_B$$

Thus *the relative velocity of body A w.r.t. body B is equal to the sum of the magnitudes of their velocities.* That is why when two fast trains cross each other in opposite directions, each appears to go very fast relative to the other.

Examples based on Relative Velocity

FORMULAE USED

1. Relative velocity of object A w.r.t. object B,
$$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$$
2. Relative velocity of object B w.r.t. object A,
$$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$$
where $\vec{v}_A$ and $\vec{v}_B$ are the velocities w.r.t. the ground
3. When the objects A and B move in the same direction,
$$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$$
4. When the object B moves in the opposite direction of A,
$$v_{AB} = v_A + v_B$$

UNITS USED

All velocities are in ms^{-1} or kmh^{-1}.

EXAMPLE 43. *A car A moving at 10 ms^{-1} on a straight road, is ahead of car B moving in the same direction at 6 ms^{-1}. Find the velocity of A relative to B and vice versa.*

Solution. Here $v_A = 10\ ms^{-1}$, $v_B = 6\ ms^{-1}$

Velocity of A relative to B,

$$v_{AB} = v_A - v_B = 10 - 6 = 4\ ms^{-1}$$

Positive velocity indicates that the driver of car B sees car A moving ahead from him at the rate of 4 ms^{-1}.

Velocity of B relative to A

$$v_{BA} = v_B - v_A = 6 - 10 = -4\ ms^{-1}$$

Negative velocity indicates that the driver of car A (when looks back) sees the car B lagging behind at the rate of 4 ms^{-1}.

EXAMPLE 44. *Two parallel rail tracks run north south. Train A moves north with a speed of 54 kmh^{-1} and train B moves south with a speed of 90 kmh^{-1}. What is the*

(*i*) *relative velocity of B with respect to A ?*

(*ii*) *relative velocity of ground with respect to B ?*

(*iii*) *velocity of a monkey running on the roof of the train A against its motion (with a velocity of 18 $km\,h^{-1}$ with respect to the train A) as observed by a man standing on the ground ?* **[NCERT ; Delhi 05C]**

Solution. Taking south to north direction as the positive direction of x-axis, we have

$$v_A = +54\ km/h = \frac{54 \times 1000}{3600}\ ms^{-1} = 15\ ms^{-1}$$

$$v_B = -90\ km/h = \frac{-90 \times 1000}{3600}\ ms^{-1} = -25\ ms^{-1}$$

(*i*) Relative velocity of B with respect to A

$$= v_B - v_A = -25 - 15 = \mathbf{-40\ ms^{-1}}.$$

So to an observer in train A, the train B appears to move with a speed of 40 ms^{-1} from north to south.

(*ii*) Relative velocity of ground with respect to B

$$= 0 - v_B = 0 + 25 = \mathbf{25\ ms^{-1}}.$$

So to an observer in train B, the earth appears to move with a speed of 25 ms^{-1} from south to north.

(*iii*) Let velocity of monkey with respect to ground

$$= v_M$$

$\therefore$ Relative velocity of monkey with respect to train A

$$= v_M - v_A = -18\ \text{km h}^{-1} = -5\ \text{ms}^{-1}$$

or $\quad v_M = v_A - 5 = 15 - 5 = \mathbf{10\ ms^{-1}}.$

EXAMPLE 45. *Two trains 120 m and 80 m in length are running in opposite directions with velocities 42 kmh^{-1} and 30 kmh^{-1}. In what time they will completely cross each other ?*

Solution. Relative velocity of one train w.r.t. the other

$$= 42 - (-30) = 72\ \text{kmh}^{-1} = 20\ \text{ms}^{-1}$$

Total distance to be travelled by each train to cross other train

$$= 120 + 80 = 200\ \text{m}$$

Time taken by each train to cross other train

$$= \frac{200}{20} = \mathbf{10\ s.}$$

EXAMPLE 46. *The speed of a motor launch with respect to still water = 7 ms^{-1} and the speed of stream is u = 3 ms^{-1}. When the launch began travelling upstream, a float was dropped from it. The launch travelled 4.2 km upstream, turned about and caught up with the float. How long is it before the launch reaches the float ?*

Solution. *For upstream motion of launch :*

Relative velocity $= 7 - 3 = 4\ \text{ms}^{-1}$

Distance moved = 4.2 km = 4200 m

Time taken, $\quad t_1 = \frac{4200}{4} = 1050$ s

For downstream motion of launch :

Distance moved downstream by float in 1050 s

$$= 3 \times 1050 = 3150\ \text{m}$$

Distance between float and launch turned about

$$= 4200 + 3150 = 7350\ \text{m}$$

This distance is to be covered by launch with its own velocity (7 ms^{-1}) because stream velocity is being shared by both.

$\therefore$ Time taken, $\quad t_2 = \frac{7350}{7} = 1050$ s

Total time taken, $\quad t = t_1 + t_2 = 1050 + 1050$

$$= 2100\ \text{s} = \mathbf{35\ min.}$$

PROBLEMS FOR PRACTICE

1. A jet airplane travelling at the speed of 450 kmh^{-1} ejects the burnt gases at the speed of 1200 kmh^{-1} relative to the jet airplane. Find the speed of the burnt gases *w.r.t.* a stationary observer on earth. (**Ans.** 750 kmh^{-1})

2. Two cars A and B are moving with velocities of 60 kmh^{-1} and 45 kmh^{-1} respectively. Calculate the relative velocity of A *w.r.t.* B, if (*i*) both cars are travelling eastwards and (*ii*) car A is travelling eastwards and car B is travelling westwards. [**Ans.** (*i*) 15 kmh^{-1} eastwards (*ii*) 105 km h^{-1} eastwards]

3. An open car is moving on a road with a speed of 100 kmh^{-1}. A man sitting in the car fires a bullet from the gun in the opposite direction. If the speed of the bullet is 250 kmh^{-1} relative to the car, then find its (bullet's) speed with respect to an observer on the ground. (**Ans.** 150 kmh^{-1})

4. A car A is moving with a speed of 60 kmh^{-1} and car B is moving with a speed of 75 kmh^{-1}, along parallel straight paths, starting from the same point. What is the position of car A *w.r.t.* B after 20 minutes ? (**Ans.** 5 km behind)

5. Two buses start simultaneously towards each other from towns A and B which are 480 km apart. The first bus takes 8 hours to travel from A to B while the second bus takes 12 hours to travel from B to A. Determine when and where the buses will meet. (**Ans.** 4.8 h, 288 km from A)

6. Two trains A and B, each of length 100 m, are running on parallel tracks. One overtakes the other in 20 s and one crosses the other in 10 s. Calculate the velocities of each train. (**Ans.** 15 ms^{-1}, 5 ms^{-1})

7. A man swims in a river with and against water at the rate of 15 kmh^{-1} and 5 kmh^{-1}. Find the man's speed in still water and the speed of the river. (**Ans.** 10 kmh^{-1}, 5 kmh^{-1})

8. A motorboat covers the distance between the two spots on the river in 8 h and 12 h downstream and upstream respectively. Find the time required by the boat to cover this distance in still water. (**Ans.** 9.6 h)

9. A car A is travelling on a straight level road with a speed of 60 kmh^{-1}. It is followed by another car B which is moving with a speed of 70 kmh^{-1}. When the distance between them is 2.5 km, the car B is given a deceleration of 20 kmh^{-2}. After what distance and time will the car B catch up with car A ? (**Ans.** 32.5 km, 0.5 h)

✖ HINTS

4. Relative speed of A w.r.t. B

$$= 60 - 75 = -15 \text{ kmh}^{-1}$$

Distance of A from B after 20 min

$$= -15 \times \frac{20}{60} = \mathbf{-5 \text{ km.}}$$

5. Speed of first bus $= \frac{480}{8} = 60 \text{ kmh}^{-1}$

Speed of second bus $= \frac{480}{12} = 40 \text{ kmh}^{-1}$

Suppose the two buses meet after time t. Then

$$60t + 40t = 480$$

or $t = \mathbf{4.8 \text{ h}}$

Distance from $A = 60 \times 4.8 = \mathbf{288 \text{ km.}}$

6. Let u and v be the velocities of trains A and B respectively.

During overtaking, relative velocity of A w.r.t. B

$$= u - v$$

During crossing, relative velocity of A w.r.t B

$$= u + v$$

Total distance to be covered by A during overtaking or crossing

$$= 100 + 100 = 200 \text{ m}$$

$\therefore \quad \frac{200}{u-v} = 20$ and $\frac{200}{u+v} = 10$

or $u - v = 10$ and $u + v = 20$

On solving, $u = \mathbf{15 \text{ ms}^{-1}}$, $v = \mathbf{5 \text{ ms}^{-1}}$.

7. Let u be the speed of man in still water and v be the speed of river. Then

$$u + v = 15 \text{ kmh}^{-1} \text{ and } u - v = 5 \text{ kmh}^{-1}$$

On solving,

$$u = \mathbf{10 \text{ kmh}^{-1}},\ v = \mathbf{5 \text{ kmh}^{-1}}.$$

8. Let u and v be the velocity of boat in still water and velocity of river respectively. If x is the distance between the two spots, then

$$u + v = \frac{x}{8} \quad \text{(for upstream)}$$

$$u - v = \frac{x}{12} \quad \text{(for downstream)}$$

On adding,

$$2u = \frac{20}{96}x \quad \text{or} \quad u = \frac{10}{96}x$$

Time required by boat in still water

$$= \frac{x}{u} = \frac{x}{10x/96} = \mathbf{9.6 \text{ h.}}$$

9. Relative velocity of car B w.r.t. A

$$= 70 - 60 = 10 \text{ kmh}^{-1}$$

$\therefore$ For car B,

$$u = 10 \text{ kmh}^{-1},\ s = 2.5 \text{ km},\ a = -20 \text{ kmh}^{-1}$$

As $\quad s = ut + \frac{1}{2}at^2$

$\therefore \quad 2.5 = 10t - \frac{1}{2} \times 20 \times t^2$ or $t = 0.5 \text{ h}$

Actual distance travelled by car B during this time,

$$s = ut + \frac{1}{2}at^2 = 70 \times 0.5 - \frac{1}{2} \times 20 \times (0.5)^2$$

$$= 35 - 2.5 = \mathbf{32.5 \text{ km.}}$$

Very Short Answer Conceptual Problems

Problem 1. When does a cyclist appear to be stationary with respect to another moving cyclist ?

Solution. When both the cyclists are moving in the same direction with the same velocity parallel to each other.

Problem 2. Can the earth be regarded as point object when it is describing its yearly journey around the sun ?

Solution. Yes, because size of the earth is much smaller than the distance from the sun.

Problem 3. Can the displacement be greater than the distance travelled by an object ? Give reason.

Solution. No, the displacement of an object can be either equal to or less than the distance travelled by the object. This is because displacement is shortest distance between the initial and final positions of the object while distance travelled is the length of the actual path traversed by the object.

Problem 4. Can the speed of a body be negative ?

Solution. No, because the speed of an object is the distance travelled per unit time and distance travelled is never negative.

Problem 5. Can a body have a constant speed and still have a varying velocity ? *[Himachal 06C]*

Solution. Yes. A particle in uniform circular motion has a constant speed but varying velocity because of the change in its direction of motion at every point.

Problem 6. Can a body have a constant velocity and still have a varying speed ? *[Himachal 04, 06]*

Solution. No, a body cannot have a constant velocity while having a varying speed. Whenever the speed changes, velocity also changes.

Problem 7. Can a particle in one-dimensional motion have zero speed and a non-zero velocity ?

Solution. No. If the speed is zero, the velocity will be necessarily zero.

Problem 8. Can a body have zero velocity and still be accelerating ? *[Delhi 05C]*

Solution. Yes. A body thrown vertically upwards has zero velocity at its highest point but has acceleration equal to the acceleration due to gravity.

Problem 9. Can an object have an eastward velocity while experiencing a westward acceleration ?

Solution. Yes. A pendulum oscillating in east-west direction will have eastward velocity and westward acceleration in half cycle of its oscillation.

Problem 10. Can the direction of velocity of an object change, when acceleration is constant ?

Solution. Yes. For an object thrown vertically upwards, the direction of velocity changes during its rise and fall. But acceleration acts always downwards and remains constant.

Problem 11. Is it possible for a body to be accelerated without speeding up or slowing down ? If so, give an example. *[Himachal 01, 04]*

Solution. Yes. An object in uniform circular motion is accelerating but its speed neither decreases nor increases.

Problem 12. Under what condition is the average velocity equal to the instantaneous velocity ?

Solution. When a body moves with a constant velocity, its average velocity over any time interval is same as is instantaneous velocity.

Problem 13. Why is the speed, in general, greater than the magnitude of the velocity ?

Solution. Because of the change in the direction of motion, the length of the path traversed by a body is generally greater than the magnitude of its displacement. So speed is generally greater than the magnitude of velocity.

Problem 14. Is the direction of acceleration same as the direction of velocity ?

Solution. Not necessarily. If velocity increases, acceleration acts in the direction of velocity and if velocity decreases, then acceleration acts in the opposite direction of velocity.

Problem 15. Can we use the equations of kinematics to find the height attained by a body projected upwards with any velocity ?

Solution. No. The equations of kinematics are applicable only so long as the acceleration is uniform. The acceleration due to gravity is uniform only near the surface of the earth.

Problem 16. Two balls of different masses (one lighter and other heavier) are thrown vertically upward with same initial speed. Which one will rise to the greater height ? *[I.I.T.]*

Solution. Both the balls will rise to the same heights. It is because, for a body moving with given initial velocity and acceleration, the distance covered by the body does not depend on the mass of the body.

Problem 17. Two balls of different masses (one lighter and other heavier) are thrown vertically upwards with the same speed. Which one will pass through the point of projection in their downward direction with the greater speed ? *[I.I.T.]*

Solution. In case of motion under gravity, the speed with which a body returns back is always equal to the speed with which it is thrown up. Since expression for final speed does not involve mass, both the balls will acquire the same speed.

Problem 18. Can the relative velocity of two bodies be greater than the absolute velocity of either body ?

Solution. Yes. When two bodies move in opposite directions, the relative velocity of each is greater than the individual velocity of either body.

Problem 19. A car travelling with a velocity of 50 km h^{-1} on a straight road is ahead of a motor-cycle travelling with a speed of 75 kmh^{-1}. How would the relative velocity be altered if motor cycle is ahead of car ?

Solution. The relative velocity will remain same as it does not depend on the position of the two bodies.

Problem 20. Even when rain is falling vertically downwards, the front screen of a moving car gets wet while the back screen remains dry. Why ?

Solution. This is because the rain strikes the car in the direction of relative velocity of rain with respect to car.

Problem 21. Is it possible that the brakes of a car are so perfect that the car stops instantaneously. If not, why ?

Solution. No, it is not possible. In order to make velocity zero in an infinitesimally small interval of time, the car needs an infinite declaration which is not possible.

Problem 22. When an observer is standing on earth, the trees and houses appear stationary to him. However, when he is sitting in a moving train, all these objects appear to move in backward direction. Why ?

Solution. For the stationary observer, the relative velocity of trees and houses is zero. For the observer sitting in the moving train, the relative velocity of houses and trees is negative. So these objects appear to move in backward direction.

Problem 23. The displacement of a body is given to be proportional to the cube of time elapsed. What is the nature of the acceleration of the body ?

Solution. Given : $s \propto t^3$ or $s = kt^3$

$\therefore$ Velocity $= \dfrac{ds}{dt} = 3kt^2$

Acceleration $= \dfrac{dv}{dt} = 6\,kt$ *i.e.,* acceleration $\propto t$.

Clearly, the acceleration increases uniformly with time.

Problem 24. The average velocity of a particle is equal to its instantaneous velocity. What is the nature of its displacement-time graph ?

Solution. When the average velocity is equal to the instantaneous velocity, the particle has a uniform motion. So its displacement-time graph is a straight line inclined to the time-axis.

Problem 25. Can the position-time graph have a negative slope ?

Solution. Yes, when the velocity of the object is negative.

Problem 26. What is the nature of the displacement-time curve of a body moving with constant velocity ?

Solution. Parabola.

Problem 27. Can the direction of motion of a body change if its velocity is changing at uniform rate ?

Solution. Yes. This can happen in a retarded motion. A body thrown up moves under constant retardation and from the highest point it begins to fall downwards.

Problem 28. Draw position-time graphs for two objects having zero relative velocity.

Solution. As the relative velocity is zero, the two bodies A and B have equal velocities. Hence their position-time graphs are parallel straight lines, equally inclined to the time-axis as shown in Fig. 3.76.

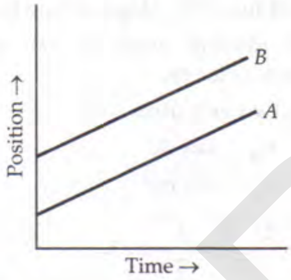


Fig. 3.76

Problem 29. Is it possible to have a constant rate of change of velocity when velocity changes both in magnitude and direction ? If yes, give one example. *[Central Schools 08]*

Solution. Yes, in projectile motion, a body has uniform acceleration in the downward direction while its velocity changes both in magnitude and direction at every point of its trajectory.

Short Answer Conceptual Problems

Problem 1. Can a body be at rest as well as in motion at the same time ? Explain. *[Himachal 06]*

Or

Rest and motion are relative terms. Explain. *[Himachal 09]*

Solution. Yes, object may be at rest relative to one object and at the same time it may be in motion relative to another object. For example, a passenger sitting in a moving train is at rest with respect to his fellow passengers but he is in motion with respect to the objects outside the train. Rest and motion are relative terms.

Problem 2. If the displacement of a body is zero, is the distance covered by it necessarily zero ? Comment with suitable illustration.

Solution. No, it is not necessary that the distance covered by a body is zero when its displacement is zero. Consider a particle moving along a circular track of radius r. After the particle completes one revolution, its displacement is zero while the distance covered is $2\pi r$.

Problem 3. State in the following cases, whether the motion is one, two or three dimensional :

(*i*) **a kite flying on a windy day,**
(*ii*) **a speeding car on a long straight high way,**
(*iii*) **a carrom coin rebounding from the side of the board,**
(*iv*) **an insect crawling on a globe, and**
(*v*) **a planet revolving around its star ?**

Solution.

(*i*) three-dimensional, (*ii*) one-dimensional,
(*iii*) two-dimensional, (*iv*) two-dimensional,
(*v*) two-dimensional.

Problem 4. Which of the two-velocity or acceleration, gives the direction of motion of the body. Justify your answer by an example.

Solution. It is the velocity and not the acceleration, which gives the direction of motion of a body. When a body is projected upward, both its direction of motion and velocity are in upward direction but the acceleration is in the downward direction.

Problem 5. Two straight lines drawn on the same displacement-time graph make angles 30° and 60° with time-axis respectively, as shown in Fig. 3.77. Which line represents greater velocity ? What is the ratio of the two velocities ? *[Delhi 05, 08]*

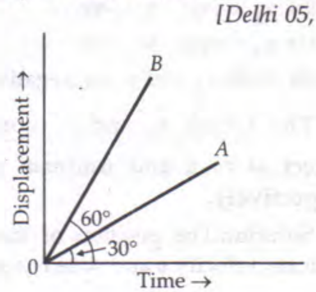


Fig. 3.77

Solution. Slope of displacement-time graph

= Velocity of the object

As slope of line $OB >$ slope of line OA

$\therefore$ The line making angle of 60° with time-axis represents greater velocity.

Ratio of the two velocities

$$= \frac{v_A}{v_B} = \frac{\tan 30^\circ}{\tan 60^\circ}$$

$$= \frac{1/\sqrt{3}}{\sqrt{3}} = \frac{1}{3} = \mathbf{1:3.}$$

Problem 6. If in case of a motion, displacement is directly proportional to the square of the time elapsed, what do you think about its acceleration *i.e.*, constant or variable ? Explain why. *[Chandigarh 02]*

Solution. Given $x \propto t^2$

or $x = ct^2$,

where c is a constant

Velocity, $v = \frac{dx}{dt} = c \times 2t$

Acceleration, $a = \frac{dv}{dt} = 2c = \text{a constant}$

Hence the object is moving with uniform acceleration.

Problem 7. Show that the average velocity of an object over an interval of time is either smaller than or equal to the average speed of the object over the same interval.

Solution. We know that

$$\text{Average velocity} = \frac{\text{Displacement}}{\text{Time interval}}$$

$$\text{Average speed} = \frac{\text{Total path length}}{\text{Time interval}}.$$

As the magnitude of the displacement of an object can be equal to or less than the total length of the path traversed by the body in given time, so average velocity of an object over an interval of time is either equal to or smaller than the average speed of the object over the same interval.

Problem 8. An object is in uniform motion along a straight line. What will be position-time graph for the motion of the object if

(*a*) $x_0 = +\text{ve}, \quad v = +\text{ve}$

(*b*) $x_0 = +\text{ve}, \quad v = -\text{ve}$

(*c*) $x_0 = +\text{ve}, \quad v = -\text{ve}$

(*d*) **both x_0 and v are negative ?**

The letters x_0 and v represent position of the object at $t = 0$ and uniform velocity of the object respectively.

Solution. The position of the object moving with a uniform velocity v and time t is given by $x = x_0 + vt$

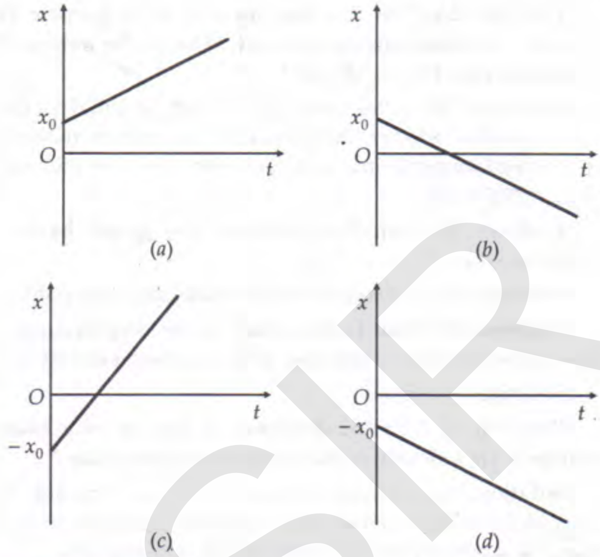


Fig. 3.78

(*a*) If $x_0 = +$ ve and $v = +$ ve, position-time graph will be as shown in Fig. 3.78(*a*).

(*b*) If $x_0 = +$ ve and $v = -$ ve, the position-time graph will be as shown in Fig. 3.78(*b*).

(*c*) If $x_0 = -$ ve and $v = +$ ve, the position-time graph will be as shown in Fig. 3.78(*c*).

(*d*) If both x_0 and v are negative, the position-time graph will be as shown in Fig. 3.78(*d*).

Problem 9. An object has uniformly accelerated motion. The object always slows down before the time, when its velocity becomes zero. Prove this statement graphically, when (*a*) both u and a are positive (*b*) $u = -$ ve and $a = +$ ve (*c*) $u = +$ ve and $a = -$ve and (*d*) both u and a are negative.

Solution.

(*a*) **When both u and a are +ve :** In such a case, the v-t graph will be as shown in Fig. 3.79(*a*). At the time corresponding to point A, the velocity becomes zero. It can be seen that before this time, the velocity is negative but its magnitude decreases with time till it becomes zero at A.

(*b*) **When u is – ve and a is + ve :** In this case, graph will be shown in Fig. 3.92(*b*). At the time corresponding to point A, the velocity becomes zero. It can be seen that before this time the velocity is – ve but its magnitude decreases with time till it becomes zero at A.

(*c*) **When u + ve and a is – ve :** In such a case, graph between v and t will be as shown in Fig. 3.79(*c*). Again at A, velocity is zero. The velocity decreases before the time corresponding to point A.

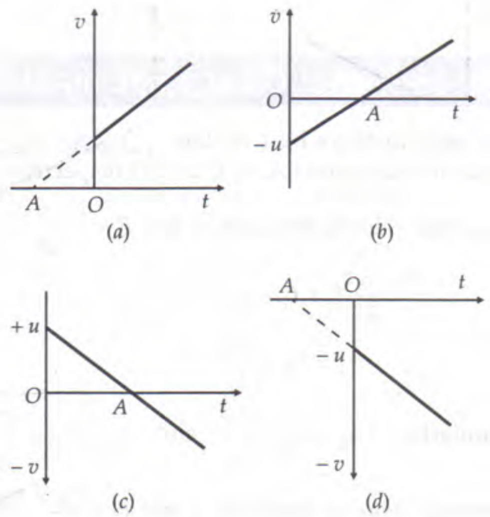


Fig. 3.79

(*d*) **When both *u* and *a* are – ve :** In this case, v-t graph will be as shown in Fig. 3.79(*d*). If we produce graph backwards, it meets the time-axis at point *A*. Before this time, velocity is +ve and decreases till it becomes zero at point *A*.

Problem 10. The distance covered by an object between times t_1 and t_2 is given by the area under the v-t graph between t_1 and t_2. Prove this statement for an object moving with negative acceleration and having a positive velocity at time t_1 and a negative velocity at time t_2

Solution. The velocity-time graph for the given motion of the object is as shown in Fig. 3.80. Let v_1 be the velocity at time t_1 and $-v_2$ at time t_2.

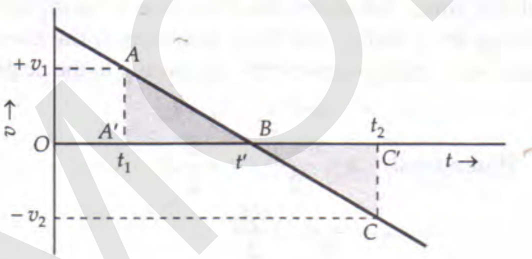


Fig. 3.80

Area under v-t graph between times t_1 and t_2

$$= \text{Area } AA'B + \text{Area } CC'B$$

$$= \frac{1}{2} A'B \times AA' + \frac{1}{2} BC' \times CC'$$

$$= \frac{1}{2}(t' - t_1)(v_1) + \frac{1}{2}(t_2 - t')(-v_2)$$

$$= \frac{1}{2} v_1 (t' - t_1) - \frac{1}{2} v_2 (t_2 - t').$$

Between times t_1 and t', the acceleration of the object is

$$a = \text{Slope of } v\text{-}t \text{ graph } AB = \frac{0 - v_1}{t' - t_1}$$

$$\therefore \quad t' - t_1 = -\frac{v_1}{a}$$

Between times t' and t_2, the acceleration of the object is

$$a = \text{slope of } v\text{-}t \text{ graph } BC = \frac{-v_2 - 0}{t_2 - t'}$$

$$\therefore \quad t_2 - t' = -\frac{v_2}{a}$$

Hence area under v-t graph between times t_1 and t_2

$$= \frac{1}{2} v_1 \left(-\frac{v_1}{a}\right) - \frac{1}{2} v_2 \left(-\frac{v_2}{a}\right)$$

$$= \frac{v_2^2 - v_1^2}{2a} = \frac{2as}{2a} = s$$

= Distance covered by the object between times t_1 and t_2.

Problem 11. Distinguish between distance and displacement. *[Himachal 04, 06, 09C]*

Solution.

Distance	***Displacement***
1. Distance is the length of the actual path traversed by a body, irrespective of its motion.	Displacement is the shortest distance between the initial and final positions of body in a given direction.
2. Distance between two points may be same or different for different paths chosen.	Displacement between two given points is always same.
3. It is a scalar quantity.	It is a vector quantity.
4. Distance covered may be positive or zero.	Displacement covered may be positive, negative or zero.

Problem 12. Distinguish between speed and velocity. *[Himachal 06C, 08, 09C]*

Solution.

Speed	***Velocity***
1. It is the distance travelled by a body per unit time in any direction.	It is the distance travelled by a body per unit time in a fixed direction.
2. It is a scalar quantity.	It is a vector quantity.
3. Speed may be positive or zero but never negative.	Velocity may be positive, negative or zero.

HOTS

Problems on Higher Order Thinking Skills

Problem 1. *The velocity-time relation of an electron starting from rest is given by $v = kt$, where $k = 2\ ms^{-2}$. Calculate the distance traversed in 3 s.*

Solution. Velocity, $v = kt$

Acceleration, $a = \frac{dv}{dt} = \frac{d}{dt}(kt) = k = 2\ \text{ms}^{-2}$

Distance traversed in 3 s,

$$s = ut + \tfrac{1}{2}at^2 = 0 + \tfrac{1}{2} \times 2 \times (3)^2 = \mathbf{9\ m.}$$

Problem 2. *The acceleration experienced by a boat after the engine is cut off, is given by $\frac{dv}{dt} = -kv^3$, where k is a constant. If v_0 is the magnitude of the velocity at cut off, find the magnitude of the velocity at time t after the cut off.* **[CBSE 94]**

Solution. Given : $\frac{dv}{dt} = -kv^3$ or $v^{-3}\ dv = -kdt$

Integrating within the conditions of motion,

$$\int_{v_0}^{v} v^{-3} dv = -k \int_0^t dt$$

or $$\left[\frac{v^{-2}}{-2}\right]_{v_0}^{v} = -k\,[t]_0^t$$

or $$-\frac{1}{2}\left[\frac{1}{v^2} - \frac{1}{v_0^2}\right] = -k\,[t - 0]$$

or $$\frac{1}{v^2} - \frac{1}{v_0^2} = 2\,kt$$

or $$\frac{1}{v^2} = \frac{1}{v_0^2} + 2kt = \frac{1 + 2kt\,v_0^2}{v_0^2}$$

or $$v = \frac{v_0}{\sqrt{1 + 2kt\,v_0^2}}.$$

Problem 3. *If a body moving with uniform acceleration in a straight line describes successive equal distances in time intervals t_1, t_2 and t_3, then show that*

$$\frac{1}{t_1} - \frac{1}{t_2} + \frac{1}{t_3} = \frac{3}{t_1 + t_2 + t_3}.$$

Solution. As shown in Fig. 3.81, let the three successive equal distances be represented by AB, BC and CD.

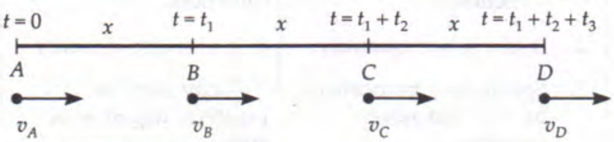


Fig. 3.81

Let each distance be x m. Let v_A, v_B, v_C and v_D be the velocities at points A, B, C and D respectively.

Average velocity between A and $B = \frac{v_A + v_B}{2}$

$\therefore$ $$\frac{v_A + v_B}{2} \times t_1 = x$$

or $$v_A + v_B = \frac{2x}{t_1}$$

Similarly, $$v_B + v_C = \frac{2x}{t_2} \quad \text{and} \quad v_C + v_D = \frac{2x}{t_3}$$

Average velocity between A and $D = \frac{v_A + v_D}{2}$

$\therefore$ $$\frac{v_A + v_D}{2}(t_1 + t_2 + t_3) = x + x + x$$

or $$v_A + v_D = \frac{6x}{t_1 + t_2 + t_3}$$

Hence $$v_A + v_D = (v_A + v_B) - (v_B + v_C) + (v_C + v_D)$$

or $$\frac{6x}{t_1 + t_2 + t_3} = \frac{2x}{t_1} - \frac{2x}{t_2} + \frac{2x}{t_3}$$

or $$\frac{3}{t_1 + t_2 + t_3} = \frac{1}{t_1} - \frac{1}{t_2} + \frac{1}{t_3}.$$

Problem 4. *In a car race, car A takes time t less than car B and passes the finishing point with a velocity v more than the velocity with which car B passes the point. Assuming that the cars start from rest and travel with constant accelerations a_1 and a_2, show that $v = t\sqrt{a_1 a_2}$.*

Solution. Let s be the distance covered by each car. Let the times taken by the two cars to complete the journey be t_1 and t_2, and their velocities at the finishing point be v_1 and v_2 respectively. According to the problem,

$$v_1 - v_2 = v \quad \text{and} \quad t_2 - t_1 = t$$

When $u = 0$, $$s = \frac{0 + v}{2} \times t = \frac{v}{2}.t$$

$\therefore$ $$s = \frac{v_1 t_1}{2} = \frac{v_2 t_2}{2}$$

Also $$s = \tfrac{1}{2}a_1 t_1^2 = \tfrac{1}{2}a_2 t_2^2$$

Hence $$\frac{v}{t} = \frac{v_1 - v_2}{t_2 - t_1} = \frac{\frac{2s}{t_1} - \frac{2s}{t_2}}{t_2 - t_1} = \frac{2s\,(t_2 - t_1)}{t_1 t_2 (t_2 - t_1)} = \frac{2s}{t_1 t_2}$$

$$= \sqrt{\frac{4s^2}{t_1^2 t_2^2}} = \sqrt{\frac{2s}{t_1^2} \cdot \frac{2s}{t_2^2}} = \sqrt{a_1 a_2}$$

or $$v = t\sqrt{a_1 a_2}.$$

Problem 5. *The driver of a train moving at a speed v_1 observes another train at a distance d ahead of him on the same track moving in the same direction with a slower speed v_2. He applies the brakes and gives his train constant deceleration α. Show that if $d > \frac{(v_1 - v_2)^2}{2\alpha}$, there will be no collision and if $d < \frac{(v_1 - v_2)^2}{2\alpha}$, there will be collision.*

Solution. There will be no collision if the driver of the train moving at a speed v_1, reduces the speed of his train to v_2 before the two trains meet *i.e.*, the relative velocity of the two trains must be reduced to zero.

Initial relative velocity $= v_1 - v_2$

If s is the distance covered by first train before its relative velocity becomes zero, then

$$v^2 = u^2 + 2\,as \quad \text{or} \quad 0 = (v_1 - v_2)^2 - 2\alpha s$$

or $$s = \frac{(v_1 - v_2)^2}{2\,\alpha}$$

For no collision, $d > s$ or $d > \frac{(v_1 - v_2)^2}{2\alpha}$.

Problem 6. *A juggler maintains four balls in motion, making each in turn rise to a height of 20 m from his hand. With what velocity does he project them and where will the other three balls be at the instant when the fourth one is just leaving the hand ? Take $g = 10\ ms^{-2}$.*

Solution. *For upward motion of a ball :*

$$v = 0,\ a = -10\ \text{ms}^{-2},\ s = 20\ \text{m},\ u = ?,\ t = ?$$

As $v^2 - u^2 = 2\,as$ $\quad \therefore \quad 0 - u^2 = -2 \times 10 \times 20$

or $\quad u = \mathbf{20\ ms^{-1}}$

Also, $\quad v = u + at \quad \therefore \quad 0 = 20 - 10\,t$

or $\quad t = 2$ s

So the ball returns to the hand of the juggler after 4 s.

To maintain proper distance, the balls must be thrown up at an interval of $\frac{4}{4} = 1$ s.

When the fourth ball is in hand, the third ball has travelled for 1 s, second for 2 s and first for 3 s.

(*i*) *For third ball*, $s = ut + \frac{1}{2} at^2$

$$= 20 \times 1 - \tfrac{1}{2} \times 10 \times (1)^2 = \mathbf{15\ m}$$

Third ball will be 15 m above the ground *going upward.*

(*ii*) *For second ball*, $s = 20 \times 2 - \frac{1}{2} \times 10\,(2)^2 = \mathbf{20\ m}$

Second ball will be 20 m above the ground and will be *at rest*.

(*iii*) *For first ball*, $s = 20 \times 3 - \frac{1}{2} \times 10 \times (3)^2 = \mathbf{15\ m}$

First ball will be 15 m above the ground *going downward.*

Problem 7. *A bullet fired into a fixed target loses half of its velocity after penetrating 3 cm. How much further will it penetrate before coming to rest assuming that it faces constant resistance to motion ?* **[AIEEE 05]**

Solution. ***In first case.*** If u is initial velocity, then $v = u/2, s = 3$ cm

As $\quad v^2 - u^2 = 2as$

$\therefore \quad (u/2)^2 - u^2 = 2as \quad$ or $\quad a = -u^2/8$

In second case : $v = 0,\ a = -u^2/8$

Initial velocity $= u/2$

$\therefore \quad 0^2 - (u/2)^2 = 2(-u^2/8)\,s$

or $\quad s = \mathbf{1\ cm}$

Thus the bullet will penetrate a further distance of 1 cm before coming to rest.

Problem 8. *A car, starting from rest, accelerates at the rate f through a distance s, then continues at constant speed for some time t and then decelerates at the rate $f/2$ to come to rest. If the total distance is 5 s, then prove that $s = \frac{1}{2} ft^2$.* **[AIEEE 05]**

Solution. ***For accelerated motion.*** $u = 0,\ a = f,\ s = s$

As $\quad v^2 - u^2 = 2as$

$\therefore \quad v_1^2 - 0^2 = 2\,f\,s \quad$ or $\quad v_1 = \sqrt{2\,f\,s}$

For uniform motion : $u = v_1 = \sqrt{2\,f\,s},\ t = t$

Distance travelled, $\quad s_2 = ut = \sqrt{2\,f\,s}\;t$

For decelerated motion : $u = \sqrt{2fs},\ a = -f/2,\ v = 0$

As $v^2 - u^2 = 2as,$

$\therefore \quad 0^2 - (\sqrt{2\,fs})^2 = 2 \times (-f/2)\,s_3$

Distance travelled, $\quad s_3 = 2s$

Given $\quad s + s_2 + s_3 = 5s$

or $\quad s + \sqrt{2\,f\,s}\,t + 2s = 5s$

or $\quad \sqrt{2\,f\,s}\,t = 2s$

or $\quad s = \mathbf{\frac{1}{2}\,ft^2}.$

Problem 9. *A car moving with a speed of 50 kmh^{-1} can be stopped by brakes after atleast 6 m. What will be the minimum stopping distance, if the same car is moving at a speed of 100 kmh^{-1} ?* **[AIEEE 03 ; Delhi 10]**

Solution. ***In first case :***

$$u = 50\ \text{km h}^{-1} = 50 \times \frac{5}{18} = \frac{125}{9}\ \text{ms}^{-1};\ v = 0,\ s = 6\ \text{m}$$

As $\quad v^2 - u^2 = 2as$

$\therefore \quad 0^2 - \left(\frac{125}{9}\right)^2 = 2a \times 6$

or $\quad a = -\frac{125 \times 125}{81 \times 2 \times 6} \approx -16\ \text{ms}^{-2}$

In second case :

$$u = 100 \text{ kmh}^{-1} = \frac{250}{9} \text{ ms}^{-1}$$

$$v = 0, \quad a = -16 \text{ ms}^{-2}$$

$$\therefore \quad 0^2 - \left(\frac{250}{9}\right)^2 = 2 \times (-16) \times s$$

or $$s = \frac{250 \times 250}{81 \times 32} = \mathbf{24.1\ m.}$$

Problem 10. *The relation between t and distance x is $t = ax^2 + bx$ where a and b are constants. Express the instantaneous acceleration in terms of instantaneous velocity.* **[AIEEE 05]**

Solution. Given : $t = ax^2 + bx$

$$\therefore \quad \frac{dt}{dx} = 2ax + b$$

Velocity, $$v = \frac{dx}{dt} = (2ax + b)^{-1}$$

Acceleration $$= \frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt} = v\frac{dv}{dx} \qquad \left[\because \frac{dx}{dt} = v\right]$$

$$= v\frac{d}{dx}(2ax + b)^{-1}$$

$$= v(-1)(2ax + b)^{-2} \cdot 2a$$

$$= -2a(2ax + b)^{-3} = \mathbf{-2av^3}.$$

Problem 11. *A ball is released from the top of a tower of height h metres. It takes T seconds to reach the ground. What is the position of the ball in T / 3 seconds ?* **[AIEEE 04]**

Solution. As $s = ut + \frac{1}{2}gt^2$

$$\therefore \quad h = 0 \times T + \tfrac{1}{2}gT^2$$

or $$T = \sqrt{\frac{2h}{g}}$$

Distance covered in time $T/3$,

$$h' = 0 \times \frac{T}{3} + \frac{1}{2}g\left(\frac{T}{3}\right)^2$$

$$= \frac{gT^2}{18} = \frac{g}{18} \times \frac{2h}{g} = \frac{h}{9}$$

Position of the ball after time $T/3$,

$$= h - \frac{h}{9} = \frac{\mathbf{8h}}{\mathbf{9}}, \text{ above the ground.}$$

Problem 12. *Points P, Q and R are in a vertical line such that PQ = QR. A ball at P is allowed to fall freely. What is the ratio of the times of descent through PQ and QR ?* **[Central Schools 08]**

Solution. Let t_1 and t_2 be the times of descent through PQ and QR respectively.

Let $PQ = QR = h.$

Then $h = \frac{1}{2}gt_1^2$ and $2h = \frac{1}{2}g(t_1 + t_2)^2$

$$\therefore \quad \frac{1}{2} = \frac{t_1^2}{(t_1 + t_2)^2} \quad \text{or} \quad \frac{1}{\sqrt{2}} = \frac{t_1}{t_1 + t_2}$$

Hence $t_1 : t_2 = \mathbf{1 : (\sqrt{2} - 1)}.$

Guidelines to NCERT Exercises

3.1. *In which of the following examples of motion can the body be considered approximately a point object :*

(*i*) *a railway carriage moving without jerks between two stations.*

(*ii*) *a monkey sitting on the top of a man cycling smoothly on a circular track.*

(*iii*) *a spinning cricket ball that turns sharply on hitting the ground, and*

(*iv*) *tumbling beaker that has slipped off the edge of a table ?*

Ans.

(*i*) The carriage can be considered as the point object because the distance between two stations is much larger than the size of the carriage.

(*ii*) The monkey can be considered as a point object because its size is much smaller than the distance covered by the cyclist.

(*iii*) The spinning ball cannot be considered as point object because its size is quite appreciable as compared to the distance through which it turns on hitting the ground.

(*iv*) The tumbling beaker slipping off the edge of a table cannot be considered a point object because its size is not negligibly smaller than the height of the table.

3.2. *The position-time (x-t) graphs for two children A and B returning from their school O to their homes P and Q respectively are shown in Fig. 3.82. Choose the correct entries in the brackets below :*

(*a*) *A / B lives closer to the school than B / A.*

(*b*) *A / B starts from the school earlier than B / A.*

(*c*) *A / B walks faster than B / A.*

(*d*) *A and B reach home at the (same/different) time.*

(*e*) *A / B overtakes B / A on the road (once/twice.)*

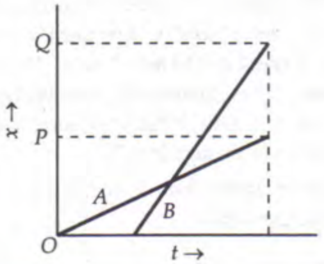


Fig. 3.82

Ans.

(*a*) As $OP < OQ$, **A lives closer to the school than B.**

(*b*) When $x = 0$, $t = 0$ for A; while t has some finite value when B starts moving. So **A starts from the school earlier than B**

(*c*) Speed = Slope of x - t graph.

Slope of x - t graph for B > Slope of x - t graph for A.

∴ **B** walks faster than **A**

(*d*) Corresponding to the positions P and Q, time t is same on time-axis.

∴ **A** and **B** reach home at the same time.

(*e*) The x - t graphs for A and B intersect each other only once. Since B starts from the school afterwards, so **B overtakes A on the road once.**

3.3. *A woman starts from her home at 9.00 A.M. walks with a speed of 5 kmh^{-1} on a straight road upto her office 2.5 km away, stays at the office upto 5 P.M. and returns home by an auto with a speed of 25 kmh^{-1}. Choose suitable scales and plot the x-t graph of her motion.*

Ans. ***For the journey from home to office :***

The time at which woman leaves for her office is 9 A.M.

As she travels with a speed of 5 kmh^{-1} and the distance of office is 2.5 km, hence time taken by her to reach office,

$$t = \frac{\text{distance}}{\text{speed}} = \frac{2.5 \text{ km}}{5 \text{ kmh}^{-1}} = \frac{1}{2} \text{ h}$$

Hence the time at which she reaches office is 9.30 A.M.

Between 9.30 A.M. to 5.00 P.M., she stays in her office *i.e.* at a distance of 2.5 km from her home.

For the return journey from office to home :

The time at which she leaves her office = 5 P.M.

Now she travels a distance of 2.5 km with a speed of 25 kmh^{-1}, hence the time taken

$$t' = \frac{2.5 \text{ km}}{25 \text{ kmh}^{-1}} = \frac{1}{10} \text{ h} = 6 \text{ minutes}$$

The time at which she reaches her home = 5.06 P.M.

The x-t graph of the woman's motion is shown in Fig. 3.83. Here 9 A.M. is regarded as the origin for time-axis and home as the origin for the position-axis.

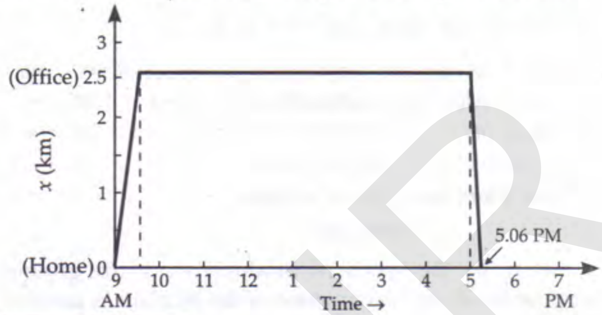


Fig. 3.83

3.4. *A drunkard walking in a narrow lane takes 5 steps forward and 3 steps backward, followed again by 5 steps forward and 3 steps backward, and so on. Each step is 1 m long and requires 1 s. Plot the x-t graph of his motion. Determine graphically and otherwise how long the drunkard takes to fall in a pit 13 m away from the start.*

Ans. (*a*) **Graphical method.** Taking the starting point as origin, the positions of the drunkard at various instants of time are given in the following table.

t (s)	0	5	8	13	16	21	24	29	32	37
x (m)	0	5	2	7	4	9	6	11	8	13

The position-time (x-t) graph for the motion of the drunkard is shown in Fig. 3.84. As is obvious from graph that the drunkard would take 37 s to fall in a pit 13 m away from the starting point.

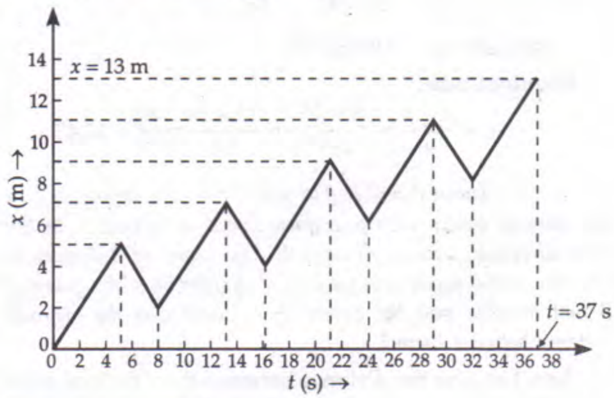


Fig. 3.84

(*b*) **Analytical method.** In each forward motion of 5 steps and backward motion of 3 steps, net distance covered = 5 – 3 = 2 m and time taken = 5 + 3 = 8 s.

∴ Time required to cover a distance of 8 m

$$= \frac{8}{2} \times 8 = 32 \text{ s}$$

Remaining distance of the pit = 13 – 8 = 5 m

In next 5 s, as he moves 5 steps forward, he falls into the pit.

∴ Total time taken = 32 + 5 = **37 s.**

3.5. *A jet airplane travelling at the speed of 500 kmh^{-1} ejects its products of combustion at the speed of 1500 kmh^{-1} relative to the jet plane. What is the speed of the latter with respect to an observer on the ground ?*

Ans. Here speed of jet airplane,

$$v_1 = 500 \text{ kmh}^{-1}$$

Let v_2 be the speed of products w.r.t. the ground. Suppose the direction of motion of the jet plane is positive. Then the relative velocity of products w.r.t. jet plane is

$$v_2 - v_1 = -1500$$

or $$v_2 = v_1 - 1500 = 500 - 1500 = -1000 \text{ kmh}^{-1}$$

Negative sign shows that the direction of products of combustion is opposite to that of the jet plane.

∴ Speed of products of combustion w.r.t. ground

$$= \mathbf{1000 \text{ kmh}^{-1}}.$$

3.6. *A car moving along a straight highway with speed of 126 kmh^{-1} is brought to a stop within a distance of 200 m. What is the retardation of the car (assumed uniform), and how long does it take for the car to stop ?*

Ans. Here $u = 126 \text{ kmh}^{-1} = 126 \times \frac{5}{18} = 35 \text{ ms}^{-1}$,

$$v = 0, \quad s = 200 \text{ m}$$

As $$v^2 - u^2 = 2as$$

∴ $$0^2 - 35^2 = 2a \times 200$$

or $$a = -\frac{35 \times 35}{2 \times 200} = -\frac{49}{16} = -3.06 \text{ ms}^{-2}$$

∴ Retardation = **3.06 ms^{-2}**.

Required time,

$$t = \frac{v-u}{a} = \frac{0-35}{-49/16} = \frac{35 \times 16}{49} = \frac{80}{7} = \mathbf{11.43 \text{ s.}}$$

3.7. *Two trains A and B of length 400 m each are moving on two parallel tracks with a uniform speed of 72 kmh^{-1} in the same direction, with A ahead of B. The driver of B decides to overtake A and accelerates by 1 ms^{-2}. If after 50 s, the guard of B just brushes past the driver of A, what was the original distance between them ?*

Ans. Let x be the distance between the driver of train A and the guard of train B. Initially, both trains are moving in the same direction with the same speed of 72 km h^{-1}. So relative velocity of B w.r.t. $A = v_B - v_A = 0$. Hence the train B needs to cover a distance with

$$a = 1 \text{ ms}^{-2}, \; t = 50 \text{ s}, \; u = 0$$

As $$s = ut + \frac{1}{2}at^2$$

∴ $$x = 0 \times 50 + \frac{1}{2} \times 1 \times (50)^2 = \mathbf{1250 \text{ m.}}$$

3.8. *On a two-lane road, car A is travelling with a speed of 36 kmh^{-1}. Two cars B and C approach car A in opposite directions with a speed of 54 kmh^{-1} each. At a certain instant, when the distance AB is equal to AC, both being 1 km, B decides to overtake A before C does. What minimum acceleration of car B is required to avoid an accident ?*

Ans. At the instant when B decides to overtake A, the speeds of three cars are

$$v_A = 36 \text{ kmh}^{-1} = 36 \times \frac{5}{18} = +10 \text{ ms}^{-1}$$

$$v_B = +54 \text{ kmh}^{-1} = +54 \times \frac{5}{18} = +15 \text{ ms}^{-1}$$

$$v_C = -54 \text{ kmh}^{-1} = -15 \text{ ms}^{-1}$$

Relative velocity of C w.r.t. A,

$$v_{CA} = v_C - v_A = -15 - 10 = -25 \text{ ms}^{-1}$$

∴ Time that C requires to just cross A

$$= \frac{1 \text{ km}}{v_{CA}} = \frac{1000 \text{ m}}{25 \text{ ms}^{-1}} = 40 \text{ s}$$

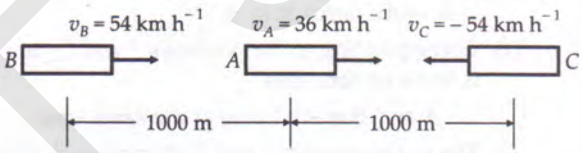


Fig. 3.85

In order to avoid the accident, B must overtake A in a time less than 40 s. So, for car B we have

Relative velocity of car B w.r.t. A,

$$v_{BA} = v_B - v_A = 15 - 10 = 5 \text{ ms}^{-1}$$

∴ $$s = 1 \text{ km} = 1000 \text{ m}, \; u = 5 \text{ ms}^{-1}, t = 40 \text{ s}$$

As $$s = ut + \frac{1}{2}at^2$$

∴ $$1000 = 5 \times 40 + \frac{1}{2}a \times (40)^2$$

or $$1000 = 200 + 800\,a$$

or $$a = \mathbf{1 \text{ ms}^{-2}}$$

Thus 1 ms^{-2} is the minimum acceleration that car B requires to avoid an accident.

3.9. *Two towns A and B are connected by a regular bus service with a bus leaving in either direction every T min. A man cycling with a speed of 20 kmh^{-1} in the direction A to B notices that a bus goes past him every 18 min in the direction of his motion, and every 6 min in the opposite direction. What is the period T of the bus service and with what speed (assumed constant) do the buses ply on the road ?*

Ans. Let speed of each bus = v kmh^{-1}

For buses going from town A to B :

Relative speed of a bus in the direction of motion of the man $= (v - 20)$ kmh^{-1}

Buses plying in this direction go past the cyclist after every 18 min.

$\therefore$ Distance covered

$$= (v - 20)\frac{18}{60} \text{ km}$$

Since a bus leaves the town after every T min, so the above distance covered

$$= v \times \frac{T}{60} \text{ km}$$

$$\therefore \quad (v - 20)\frac{18}{60} = v \times \frac{T}{60} \qquad ...(i)$$

For buses going from town B to A :

Relative speed of bus in the direction opposite to the motion of the man

$$= (v + 20) \text{ kmh}^{-1}$$

Buses going in this direction go past the cyclist after every 6 min, therefore

$$(v + 20)\frac{6}{60} = v \times \frac{T}{60} \qquad ...(ii)$$

Dividing (*i*) by (*ii*), we get

$$\frac{(v - 20)\,18}{(v + 20)\,6} = 1$$

or $\quad 3v - 60 = v + 20$

or $\quad v = \mathbf{40 \text{ kmh}^{-1}}.$

From equation (*ii*),

$$(40 + 20)\frac{6}{60} = \frac{40 \times T}{60}$$

or $\quad T = \frac{60 \times 6}{40} = \mathbf{9 \text{ min.}}$

3.10. *A player throws a ball upwards with an initial speed of 29.4 ms^{-1}.*

(*i*) *What is the direction of acceleration during the upward motion of the ball ?*

(*ii*) *What are the velocity and acceleration of the ball at the highest point of its motion ?*

(*iii*) *Choose the $x = 0$ and $t = 0$ to be the location and time of the ball at its highest point, vertically downward direction to be the positive direction of X-axis, and give the signs of position, velocity and acceleration of the ball during its upward, and downward motion.*

(*iv*) *To what height does the ball rise and after how long does the ball return to the player's hands ?*

(Take $g = 9.8$ ms^{-2}, and neglect air resistance).

Solution. (*i*) The ball moves under the effect of gravity. The direction of acceleration due to gravity is always *vertically downwards.*

(*ii*) At the highest point, velocity of the ball = **0.**

Acceleration = Acceleration due to gravity 'g'

$$= \mathbf{9.8 \text{ ms}^{-2}} \qquad \text{(vertically downwards)}$$

(*iii*) When the highest point is chosen as the location for $x = 0$ and $t = 0$ and vertically downward direction to be the positive direction of X-axis :

During upward motion. Position is positive, velocity is negative and acceleration is positive.

During downward motion. Position is positive, velocity is positive and acceleration is positive.

(*iv*) *For upward motion.*

$$u = -29.4 \text{ ms}^{-1}, \; g = +9.8 \text{ ms}^{-2}, \; v = 0$$

If s is the height to which the ball rises, then

$$v^2 - u^2 = 2as$$

or $\quad 0^2 - (-29.4)^2 = 2 \times 9.8 \times s$

or $\quad s = -\frac{(29.4)^2}{2 \times 9.8} = \mathbf{-44.1 \text{ m}}$

Negative sign shows that the distance is covered in upward direction.

If the ball reaches the highest point in time t, then

$$v = u + at$$

or $\quad 0 = -29.4 + 9.8t$

or $\quad t = \frac{29.4}{9.8} = 3 \text{ s}$

As $\quad$ time of ascent = time of descent

$\therefore \quad$ Total time taken = 3 + 3 = **6 s.**

3.11. *Read each statement below carefully and state with reasons and examples, if it is true or false. A particle in one-dimensional motion :*

(*a*) *with zero speed at an instant may have non-zero acceleration at the instant,*

(*b*) *with zero speed may have non-zero velocity,*

(*c*) *with constant speed must have zero acceleration,*

(*d*) *with positive value of acceleration must be speeding up.*

Ans.

(*a*) **True.** When a body begins to fall freely under gravity, its speed is zero but it has non-zero acceleration of 9.8 ms^{-2}.

(*b*) **False.** Speed is the magnitude of velocity and the magnitude of non-zero velocity cannot be zero.

(*c*) **True.** When a particle moves with a constant speed in the same direction, neither the magnitude nor the direction of velocity changes and so acceleration is zero. In case a particle rebounds instantly with the same speed, its acceleration will be infinite which is physically not possible.

(*d*) **False.** If the initial velocity of a body is negative, then even in case of positive acceleration, the body speeds down. A body speeds up when the acceleration acts in the direction of motion.

3.12. *A ball is dropped from a height of 90 m on a floor. At each collision with the floor, the ball loses one-tenth of its speed. Plot the speed-time graph of its motion between t = 0 to 12 s.*

Ans. (*i*) Time taken by the ball to fall through a height of 90 m is obtained as follows :

$$x = v(0)\,t + \frac{1}{2}gt^2$$

$$90 = 0 + \frac{1}{2}\times 9.8\,t^2$$

or $$t = \sqrt{\frac{2\times 90}{9.8}} = \frac{30}{7}\,\text{s} \simeq 4.3\,\text{s}$$

Now $v(t) = v(0) + gt$

$\therefore$ $$v(4.3) = 0 + 9.8\times\frac{30}{7} = 42\ \text{ms}^{-1}$$

From time $t = 0$ to $t = 4.3\,\text{s}$,

$$v(t) = gt = 9.8\,t \quad \text{or} \quad v(t)\propto t$$

In this duration speed increases linearly with time t from 0 to 42 ms^{-1} during the downward motion of the ball and this speed–time variation has been shown by straight line OA in Fig. 3.86.

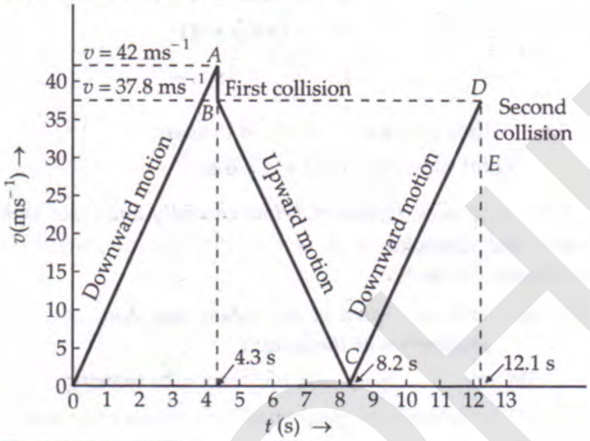


Fig. 3.86

(*ii*) At first collision with the floor, speed lost by ball

$$= \frac{1}{10}\times 42 = 4.2\ \text{ms}^{-1}.$$

Thus, the ball rebounds with a speed of $42 - 4.2 = 37.8\ \text{ms}^{-1}$. For the further upward motion, the speed at any instant t is given by

$$v(t) = v(0) - gt = 37.8 - 9.8\times t$$

Now the speed decreases linearly with time and becomes zero after time

$$t = \frac{37.8}{9.8} \simeq 3.9\,\text{s}$$

Thus, the ball reaches the highest point again after time $t = 4.3 + 3.9 = 8.2\,\text{s}$ from the start. Straight line BC represents the speed-time graph for this upward motion.

(*iii*) At highest point, speed of ball is zero. It again starts falling. At any instant t, its speed is given by

$$v(t) = 0 + 9.8\,t$$

Again the speed of the ball increases linearly with time t from 0 to 37.8 ms^{-1} (initial speed of the previous upward motion) in the next time-interval of 3.9 s. Total time taken from the start = 4.3 + 3.9 + 3.9 = 12.1 s. This part of motion has been shown by straight line CD.

Here, we have assumed a negligible time of collision between the ball and the floor.

3.13. *Explain clearly, with examples, the distinction between :*

(*a*) *magnitude of displacement (sometimes called distance) over an interval of time, and the total length of path covered by a particle over the same interval ;*

(*b*) *magnitude of average velocity over an interval of time, and the average speed over the same interval. [Average speed of a particle over an interval of time is defined as the total path length divided by the time interval].*

(*c*) *Show in both (a) and (b) that the second quantity is either greater than or equal to the first. When is the equality sign true ? [For simplicity, consider one-dimensional motion only].*

Solution.

(*a*) Suppose a body moves from point A and to point B along a straight path and then returns back to the point A along the same path.

As the body returns back to its initial position A, so magnitude of displacement = 0.

Distance covered

= Total length of the path covered

$= AB + BA = AB + AB = 2AB.$

(*b*) In the above example, suppose the body takes time t to complete the whole journey. Then

Magnitude of average velocity

$$= \frac{\text{Magnitude of displacement}}{\text{Time taken}} = \frac{0}{t} = 0.$$

$$\text{Average speed} = \frac{2AB}{t}.$$

(*c*) In example (*a*), distance covered > magnitude of displacement.

In example (*b*), average speed > magnitude of average velocity.

The sign of equality will hold when the body moves along a straight line path in a fixed direction.

3.14. *A man walks on a straight road from his home to a market 2.5 km away with a speed of* $5\ \text{km h}^{-1}$. *Finding the market closed, he instantly turns and walks back home with a speed of* $7.5\ \text{km h}^{-1}$. *What is the*

(*a*) *magnitude of average velocity, and*

(*b*) *average speed of the man over the interval of time*

(*i*) *0 to 30 min,* (*ii*) *0 to 50 min,* (*iii*) *0 to 40 min ?*

Ans. Case (*i*) : *0 to 30 min*

$$\text{Speed} = 5\ \text{kmh}^{-1}$$

Distance covered in 30 min

$$= 5\ \text{kmh}^{-1} \times \frac{30}{60}\ \text{h} = 2.5\ \text{km}$$

Displacement covered = 2.5 km

(*a*) Average velocity

$$= \frac{\text{Displacement}}{\text{Time}} = \frac{2.5\ \text{km}}{30/60\ \text{h}} = \mathbf{5\ kmh^{-1}}.$$

(*b*) Average speed

$$= \frac{\text{Distance}}{\text{Time}} = \frac{2.5\ \text{km}}{30/60\ \text{h}} = \mathbf{5\ kmh^{-1}}.$$

Case (*ii*) : *0 to 50 min*

Displacement covered in first 30 min in going to market

$$= 5\ \text{kmh}^{-1} \times \frac{30}{60}\ \text{h} = 2.5\ \text{km}$$

Displacement covered in next 20 min in coming back to home

$$= 7.5\ \text{kmh}^{-1} \times \frac{20}{60}\ \text{h} = 2.5\ \text{km}$$

Net displacement = 2.5 − 2.5 = 0

Total distance covered = 2.5 + 2.5 = 5 km

(*a*) Average velocity

$$= \frac{\text{Net displacement}}{\text{Time taken}} = \frac{0}{50/60\ \text{h}} = \mathbf{0}.$$

(*b*) Average speed

$$= \frac{\text{Total distance}}{\text{Time taken}} = \frac{5\ \text{km}}{50/60\ \text{h}} = \mathbf{6\ kmh^{-1}}.$$

Case (*iii*) : *0 to 40 min*

Displacement covered in first 30 min in going to market = 2.5 km

Displacement covered in next 10 min in coming back to home

$$= 7.5\ \text{kmh}^{-1} \times \frac{10}{60}\ \text{h} = 1.25\ \text{km}$$

Net displacement = 2.5 − 1.25 = 1.25 km

Total distance travelled = 2.5 + 1.25 = 3.75 km

(*a*) Average velocity

$$= \frac{\text{Net displacement}}{\text{Time taken}}$$

$$= \frac{1.25\ \text{km}}{40/60\ \text{h}} = \mathbf{1.875\ kmh^{-1}}.$$

(*b*) Average speed

$$= \frac{\text{Total distance}}{\text{Time taken}} = \frac{3.75\ \text{km}}{40/60\ \text{h}} = \mathbf{5.625\ kmh^{-1}}.$$

3.15. *The instantaneous speed is always equal to the magnitude of instantaneous velocity. Why ?*

Ans. Instantaneous speed,

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

Instantaneous velocity,

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{x}}{\Delta t}$$

For an arbitrary small interval of time (Δt), the magnitude of displacement $|\Delta \vec{x}|$ is equal to the length of the path Δx. So instantaneous speed is always equal to the magnitude of instantaneous velocity.

3.16. *Look at the graphs (a) to (d) [Fig. 3.87] carefully and state, with reasons, which of these cannot possibly represent one-dimensional motion of a particle.*

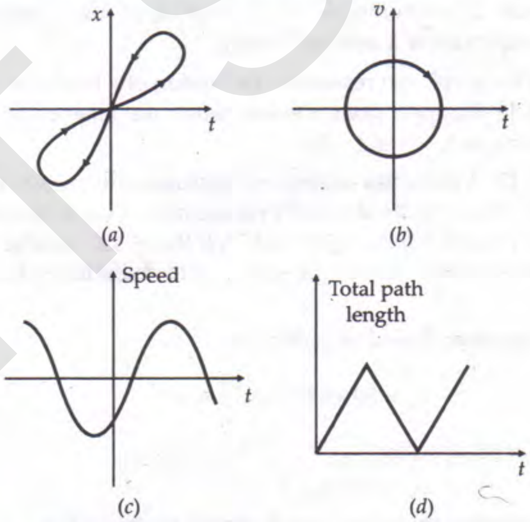


Fig. 3.87

Ans. All the four graphs are impossible.

(*a*) If we draw a line parallel to the position-axis, it intersects the *x-t* graph at two points. This means that the particle occupies two different positions at the same time which is not possible.

(*b*) If we draw a line parallel to the velocity-axis, it meets the circle in two points. This means the particle has two velocities (positive and negative) in opposite directions at the same time. This is not possible.

(*c*) The graph indicates that speed is negative in some time intervals. But speed cannot be negative.

(*d*) The graph indicates that the total path length decreases after a certain time. But total path length can never decrease with time.

3.17. *Fig. 3.88 shows the x-t plot of one-dimensional motion of a particle. Is it correct to say from the graph that the particle moves in a straight line for $t < 0$ and on a parabolic path for $t > 0$? If not, suggest a suitable physical context for this graph.*

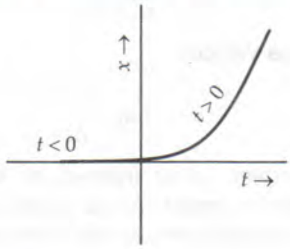


Fig. 3.88

Ans. No, it is wrong to say that the particle moves in a straight line for $t < 0$ and on a parabolic path for $t > 0$, because a position-time $(x - t)$ graph does not represent the trajectory of a moving particle.

This graph can represent the motion of a freely falling particle dropped from a tower when we take its initial position as $x = 0$, at $t = 0$.

3.18. *A police van moving on a highway with a speed of 30 kmh^{-1} fires a bullet at a thief's car speeding away in the same direction with a speed of 192 kmh^{-1}. If the muzzle speed of the bullet is 150 ms^{-1}, with what speed does the bullet hit the thief's car ?*

Solution. Speed of police van,

$$v_p = 30 \text{ kmh}^{-1} = \frac{25}{3} \text{ ms}^{-1}$$

Speed of bullet,

$$v_b = 150 \text{ ms}^{-1}$$

Speed of the police van is shared by the bullet.

$\therefore$ Relative speed of bullet w.r.t. ground

$$= v_b + v_p$$

$$= 150 + \frac{25}{3}$$

$$= \frac{475}{3} \text{ ms}^{-1}$$

Speed of thief's car,

$$v_t = 192 \text{ kmh}^{-1} = \frac{160}{3} \text{ ms}^{-1}$$

Relative speed of bullet w.r.t. thief's car

$$= (v_b + v_p) - v_t$$

$$= = \frac{475}{3} - \frac{160}{3}$$

$$= 105 \text{ ms}^{-1}.$$

Hence the speed of the bullet with which it hits the thief's car = **105 ms^{-1}**.

3.19. *Suggest a suitable physical situation for each of the following graphs [Fig. 3.89] :*

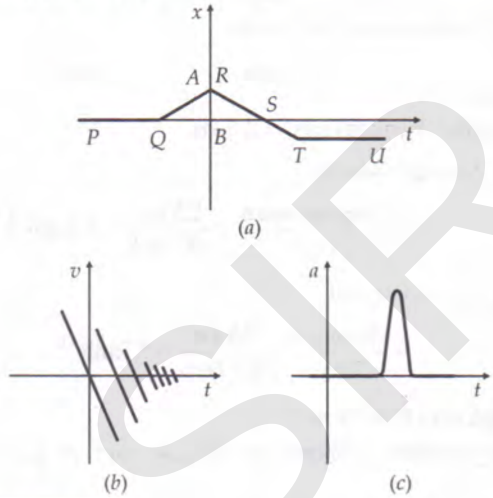


Fig. 3.89

Ans.

(*a*) A ball lies at rest on a smooth floor, as indicated by straight line *PQ* of the *x-t* graph. It is kicked towards a wall. Slope of line *QR* gives speed of kicking. The ball rebounds from the wall with a reduced speed (given by the slope of line *RS*). At *S*, the sign of position coordinate changes sign, it indicates that the ball moves to the opposite wall which stops it. The line *TU* indicates the final rest position of the ball.

(*b*) The velocity-time graph represents the motion of a ball thrown up with some initial velocity, it hits the ground and gets rebounded with a reduced speed. It goes on hitting the ground and after each hit its speed decreases unit it becomes zero and the ball comes to rest finally.

(*c*) The acceleration-time graph represents the motion of a uniformly moving cricket ball turned back by hitting it with a bat for a very short time interval.

3.20. *Figure 3.90 gives the x-t plot of a particle executing one-dimensional simple harmonic motion. Give the signs of position, velocity and acceleration variables of the particle at $t = 0.3$ s, 1.2s, -1.2s.*

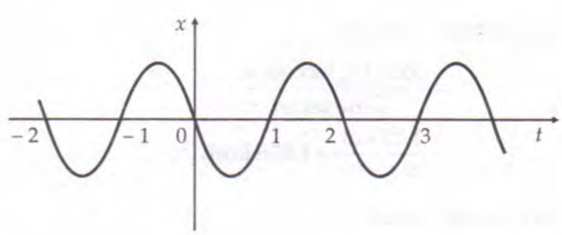


Fig. 3.90

Ans. The acceleration of a particle executing S.H.M. is given by

$$a = -\omega^2 x$$

where ω (angular frequency) is a constant.

At time $t = 0.3$ s

As is obvious from the graph, $x < 0$

As slope of x-t graph is negative, so $v < 0$

As $a = -\omega^2 x$, so $a > 0$.

At time $t = 1.2$ s

As is obvious from the graph, $x > 0$

As slope of $x - t$ graph is positive, so $v > 0$

As $a = -\omega^2 x$, so $a < 0$.

At time $t = -1.2$ s

As is obvious from the graph, $x < 0$

As slope of $x - t$ graph is positive, so $v > 0$

As $a = -\omega^2 x$, so $a > 0$.

3.21. *Figure 3.91 gives the x-t plot of a particle in one-dimensional motion. Three different equal intervals of time are shown. In which interval is the average speed greatest, and in which is it the least ? Give the sign of average velocity for each interval.*

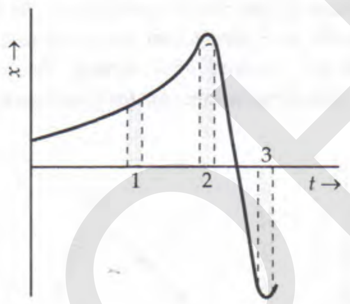


Fig. 3.91

Solution. Slope of x-t graph in a small time interval = Average speed in that interval

As the slope of x-t is greatest in interval 3 and least in interval 1, so **the average speed is greatest in interval 3 and least in interval 1.**

As the slope of x-t is positive in intervals 1 and 2 and negative in interval 3, **so average velocity is positive in intervals 1 and 2 and negative in interval 3.**

3.22. *Figure 3.92 gives a speed-time graph of a particle in motion along a constant direction. Three equal intervals of time are shown. (a) In which interval is the average acceleration greatest in magnitude ? (b) In which interval is the average speed greatest ? (c) Choosing the positive direction as the constant direction of motion, give the signs of v and a in the three intervals. (d) What are the accelerations at the points A, B, C and D ?*

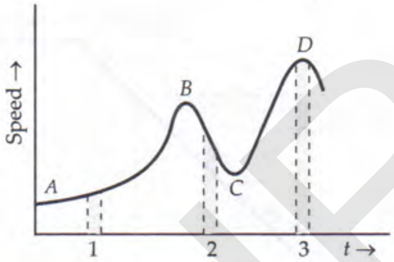


Fig. 3.92

Ans.

(*a*) As the change in speed is greatest in interval 2, so **magnitude of average acceleration is greatest in interval 2.**

(*b*) Obviously from the graph, **average speed is greatest in interval 3**.

(*c*) v is positive in all three intervals. a is positive in the intervals 1 and 3 as speed is increasing in these intervals. But a is negative in interval 2 as speed is decreasing in this interval.

(*d*) At the points *A*, *B*, *C* and *D*, the v-t graph is parallel to the time-axis or has a zero slope, so $a = 0$ at all these points.

3.23. *A three-wheeler starts from rest, accelerates uniformly with 1 ms^{-2} on a straight road for 10 s, and then moves with uniform velocity. Plot the distance covered by the vehicle during the nth second (n = 1, 2, 3...) versus n. What do you expect this plot to be during accelerated motion : a straight line or a parabola ?*

Ans. Distance travelled in nth second,

$$s_{n\,th} = u + \frac{a}{2}(2n - 1)$$

As $u = 0,\ a = 1\ \text{ms}^{-2}$

$$\therefore \quad s_{n\,th} = 0 + \frac{1}{2}(2n - 1) = \frac{1}{2}(2n - 1)\ \text{m}$$

Thus the distances travelled by the three-wheeler at the end of each second are given by

n(s)	1	2	3	4	5	6	7	8	9	10
$s_{n\,th}$ (m)	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	8.5	9.5

Now, velocity of the three-wheeler at the end of 10th s is given by

$$v = u + at = 0 + 1 \times 10 = 10\ \text{ms}^{-1}.$$

Upto $n = 10$ s, the motion is accelerated and the graph between s_{nth} and n is a straight line *AB* inclined to time-axis as shown in Fig. 3.93. After 10th second, the three-wheeler moves with uniform velocity of 10 ms^{-1}, so graph is straight line BC parallel to time-axis.

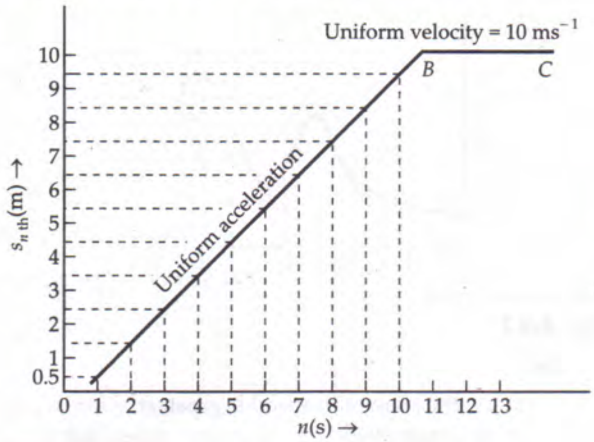


Fig. 3.93

3.24. *A boy standing on a stationary lift (open from above) throws a ball upwards with the maximum initial speed he can, equal to 49 ms^{-1}. (i) How much time does the ball take to return to his hands ? (ii) If the lift starts moving up with a uniform speed of 5 ms^{-1}, and the boy again throws the ball up with the maximum speed he can, how long does the ball take to return to his hands ?*

Ans. (*i*) *When the lift is stationary :* For upward motion of the ball, we have

$$u = 49\ \text{ms}^{-1},\quad g = -9.8\ \text{ms}^{-2},\quad v = 0,\quad t = ?$$

As $\quad v = u + at$

$\therefore \quad 0 = 49 - 9.8\,t \quad$ or $\quad t = \frac{49}{9.8} = 5\,\text{s}.$

As time of ascent = time of descent

$\therefore$ Total time taken = 5 + 5 = **10 s.**

(*ii*) *When the lift moves up with uniform speed :* The uniform speed of the lift does not change the relative velocity of the ball w.r.t. the boy *i.e.* it still remains 49 ms^{-1}.

Hence total time after which the ball returns = **10 s**.

3.25. *On a long horizontally moving belt, a child runs to and fro with a speed 9 kmh^{-1} (with respect to the belt) between his father and mother located 50 m apart on the moving belt. The belt moves with a speed of 4 kmh^{-1}. For an observer on a stationary platform outside, what is the*

(*i*) *speed of the child running in the direction of motion of the belt,*

(*ii*) *speed of the child running opposite to the direction of motion of the belt, and*

(*iii*) *time taken by the child in (i) and (ii) ?*

Which of the answers alter if motion is viewed by one of the parents ?

Ans. (*i*) Speed of the child running in the direction of motion of the belt

$$= (9 + 4)\ \text{kmh}^{-1} = \mathbf{13\ kmh^{-1}}.$$

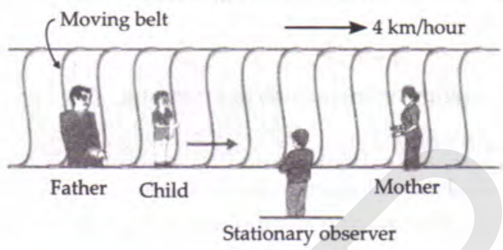


Fig. 3.94

(*ii*) Speed of the child running opposite to the direction of motion of the belt

$$= (9 - 4)\ \text{kmh}^{-1} = \mathbf{5\ kmh^{-1}}.$$

(*iii*) Speed of the child w.r.t. either parent

$$= 9\ \text{kmh}^{-1} = 2.5\ \text{ms}^{-1}$$

Distance to be covered = 50 m

$$\text{Time taken} = \frac{50}{2.5} = \mathbf{20\ s.}$$

If the motion is viewed by one of the parents, answers to (*i*) and (*ii*) are altered but answer to (*iii*) remains unchanged.

3.26. *Two stones are thrown up simultaneously from the edge of a cliff 200 m high with initial speeds of 15 ms^{-1} and 30 ms^{-1}. Verify that the following graph correctly represents the time variation of the relative position of the second stone with respect to the first. Neglect air resistance and assume that the stones do not rebound after hitting the ground. Take $g = 10\ ms^{-2}$. Give the equations for the linear and curved parts of the plot.*

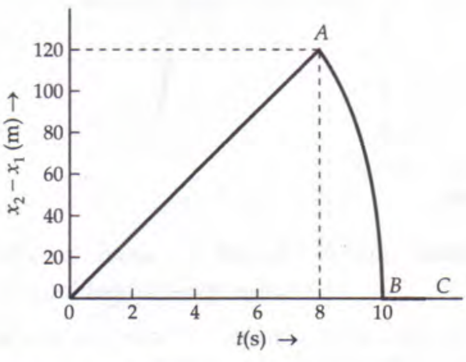


Fig. 3.95

Ans. $\quad x(t) = x(0) + v(0)\,t + \frac{1}{2}gt^2$

If we take origin for position measurement on the ground, then the positions of the two stones at any instant t will be

$$x_1 = 200 + 15\,t - \tfrac{1}{2} \times 10\,t^2 \qquad \text{...(1)}$$

$$x_2 = 200 + 30\,t - \tfrac{1}{2} \times 10\,t^2 \qquad \text{...(2)}$$

When the first stone hits the ground,

$$x_1 = 0$$

or $\quad 200 + 15t - 5t^2 = 0$

or $\quad 5t^2 - 15t - 200 = 0$

or $\quad t^2 - 3t + 40 = 0$

$$\therefore \quad t = \frac{3 \pm \sqrt{9 + 160}}{2} = \frac{3 \pm 13}{2} = 8\,\text{s} \text{ or } -5\,\text{s}$$

As time cannot be negative, so $t = 8$ s

i.e. the first stone hits the ground after 8 s.

From (1) and (2), the relative position of second stone *w.r.t.* first is given by

$$x_2 - x_1 = 15t$$

As there is a linear relationship between $x_2 - x_1$ and t, so the graph is straight line OA upto $t = 8$ s. After $t = 8$ s, only the second stone is in motion. So the graph is parabolic (AB) in accordance with quadratic equation,

$$x_2 = 200 + 30t - 5t^2$$

The second stone will hit the ground, when $x_2 = 0$

or $\quad 200 + 30t - 5t^2 = 0$

On solving, $\quad t = 10$ s

After $t = 10$ s, the separation between the balls is zero, which explains the part BC of the graph.

3.27. *The speed-time graph of a particle moving along a fixed direction is shown in Fig. 3.96. Obtain the distance travelled by the particle between (i) $t = 0$ to 10 s (ii) $t = 2$ to 6 s. What is the average speed of the particle in intervals in (i) and (ii) ?*

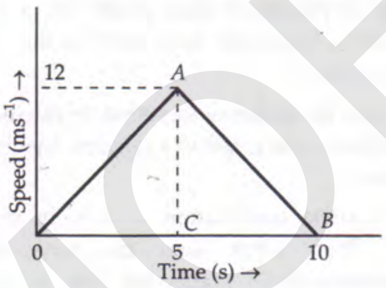


Fig. 3.96

Ans. (*i*) Distance travelled by the particle between $t = 0$ to 10 s is given by

$$s = \text{Area of } \Delta\, OAB = \frac{1}{2}\, OB \times AC$$

$$= \frac{1}{2} \times 10 \times 12 = 60 \text{ m}$$

Average speed

$$= \frac{\text{Total distance covered}}{\text{Total time taken}} = \frac{60}{10} = \mathbf{6\ ms^{-1}}.$$

(*ii*) Acceleration of the particle during journey OA is given by

$$v = u + at \quad \text{or} \quad 12 = 0 + a \times 5$$

or $\quad a = +2.4 \text{ ms}^{-2}$

Similarly, acceleration of the particle during journey AB is given by

$$v = u + at \quad \text{or} \quad 0 = 12 + a \times 5$$

or $\quad a = -2.4 \text{ ms}^{-2}$

Velocity of the particle after 2 s from start will be

$$v = u + at = 0 + 2.4 \times 2 = 4.8 \text{ ms}^{-1}$$

$\therefore$ Distance covered by the particle between $t = 2$ to 5 s (in 3 s) is given by

$$s_1 = ut + \frac{1}{2}at^2$$

$$= 4.8 \times 3 + \frac{1}{2} \times 2.4 \times 3^2 = 25.2 \text{ m}$$

Distance covered by the particle in $t = 5$ to 6 s (in 1 s) is given by

$$s_2 = ut + \frac{1}{2}at^2$$

$$= 12 \times 1 + \frac{1}{2} \times (-2.4) \times 1^2 = 10.8 \text{ m}$$

Total distance travelled in $t = 2$ to 6 s,

$$s = s_1 + s_2 = 25.2 + 10.8 = 36 \text{ m}$$

Average speed in the interval $t = 2$ to 6 s

$$= \frac{\text{Total distance covered}}{\text{Total time taken}} = \frac{36}{4} = \mathbf{9\ ms^{-1}}.$$

3.28. *The velocity-time graph of particle in one-dimensional motion is shown in Fig. 3.97.*

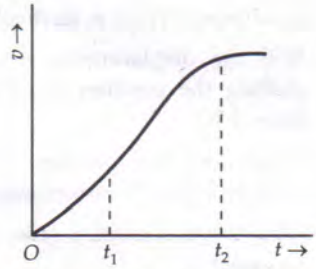


Fig. 3.97

Which of the following formulae are correct for describing the motion of the particle over the time interval t_1 to t_2 :

(*a*) $x(t_2) = x(t_1) + v(t_1) \times (t_2 - t_1) + \frac{1}{2}a(t_2 - t_1)^2$

(*b*) $v(t_2) = v(t_1) + a(t_2 - t_1)$

(*c*) $v_{av} = \{x(t_2) - x(t_1)\}/(t_2 - t_1)$

(*d*) $a_{av} = \{v(t_2) - v(t_1)\}/(t_2 - t_1)$

(*e*) $x(t_2) = x(t_1) + v_{av}(t_2 - t_1) + (\frac{1}{2})a_{av}(t_2 - t_1)^2$.

(*f*) $x(t_2) - x(t_1) =$ *area under the v-t curve bounded by the t-axis and the dotted line shown.*

Ans. (*a*) It is not correct because in the time interval between t_1 and t_2, a is not constant.

(*b*) This relation is also not correct for the same reason as in (*a*).

(*c*) This relation is correct.

(*d*) This relation is also correct.

(*e*) This relation is not correct because average acceleration cannot be used in this relation.

(*f*) This relation is correct.

Text Based Exercises

Type A : Very Short Answer Questions — 1 Mark Each

1. Are rest and motion absolute or relative terms ?
2. Can an object be at rest as well as in motion at the same time ?
3. Is it true that a body is at rest in a frame within which it has been fixed ?
4. Under what condition can an object in motion be considered a point object ?
5. Give an example of a physical phenomenon in which earth cannot be regarded as a point mass.
6. Under what condition will the distance and displacement of moving object have the same magnitude ? **[Chandigarh 08]**
7. A bullet fired vertically upwards falls at the same place after some time. What is the displacement of the bullet ?
8. A particle is moving along a circular track of radius r. What is the distance traversed by particle in half revolution ? What is its displacement ?
9. Will the displacement of an object change on shifting the position of origin of the coordinate system ? **[Himachal 06C]**
10. What does the speedometer of a car measure-average speed or instantaneous speed ?
11. What is the numerical ratio of velocity to speed of an object ?
12. A ball hits a wall with a velocity of $30\ \text{ms}^{-1}$ and rebounces with the same velocity. What is the change in its velocity ?
13. Why does time occur twice in the unit of acceleration ?
14. Give an example which shows that a positive acceleration can be associated with a slowing down object.
15. Give an example which shows that a negative accele ration can be associated with a speeding up object.
16. Is the acceleration of a car greater than when the accelerator is pushed to the floor or when brake pedal is pushed hard ?
17. The v-t graphs of two objects make angles of 30° and 60° with the time-axis. Find the ratio of their accelerations.
18. Is it possible that your cycle has a northward velocity but southward acceleration ? If yes, how ?
19. If the instantaneous velocity of a particle is zero, will its instantaneous acceleration be necessarily zero ?
20. A woman standing on the edge of a cliff throws a ball straight up with a speed of $8\ \text{kmh}^{-1}$ and then throws another ball straight down with a speed of $8\ \text{kmh}^{-1}$ from the same position. What is the ratio of the speeds with which the balls hit the ground ?
21. A body travels, with uniform acceleration a_1 for time t_1 and with uniform acceleration a_2 for time t_2. What is the average acceleration ?
22. What is the nature of position-time graph for a uniform motion ? **[Chandigarh 03]**
23. What does the slope of position-time graph indicate ? **[Himachal 07]**
24. What is the nature of velocity-time graph for uniform motion ?
25. If the displacement-time graph for a particle is parallel to displacement axis, what should be the velocity of the particle ?
26. If the displacement-time graph for a particle is parallel to time-axis, how much is the velocity of the particle ?
27. How can the distance travelled be calculated from the velocity-time graph in a uniform one-dimensional motion ?
28. Suppose the acceleration of a body varies with time. Then what does the area under its acceleration-time graph for any time interval represent ?
29. What is the area under the velocity-time curve in the case of a body projected vertically upwards from the ground after reaching the ground ?
30. Can a particle with zero acceleration speed up ?
31. Is the formula : $s = vt - \frac{1}{2}at^2$ correct, when the body is moving with uniform acceleration ?
32. A body projected up reaches a point P of its path at the end of 4 seconds and the highest point at the end of 12 seconds. After how many seconds from the start will it reach P again ?
33. Can a body subjected to a uniform acceleration always move in a straight line ?

34. Suggest a suitable physical situation for the graph shown in Fig. 3.89(b) on page 3.50.

35. A uniformly moving cricket ball is turned back by hitting it with a bat for a very short time-interval. Suggest acceleration-time graph for the situation.

36. The position coordinate of a moving particle is given by $x = 6 + 18t + 9t^2$ (x in metres and t in seconds). What is its velocity at $t = 2$ sec. ? **[Chandigarh 03]**

37. A player throws a ball upwards with an initial speed of 29.4 m s^{-1}. What are the velocity and acceleration of the ball at the highest point of its motion ? **[Delhi 05 ; Central Schools 12]**

38. Under what condition will the distance and displacement of a moving object have the same magnitude ? **[Chandigarh 08]**

39. State the condition when the magnitude of velocity and speed of an object are equal. **[Delhi 08]**

40. What does the slope of velocity-time graph represent ? **[Himachal 07 ; Delhi 10]**

41. What does the area under velocity-time graph represent ? **[Himachal 07]**

42. What does the area under acceleration-time graph represent ?

43. The displacement-time graphs for the two particles A and B are straight lines inclined at angles of 30° and 45° with the time-axis. What is the ratio of the velocities $v_A : v_B$? **[Delhi 12]**

44. Is the time variation of position, shown in the figure below, observed in nature ? **[Central Schools 12]**

Answers

1. Yes, both rest and motion are relative terms.

2. Yes. A body may be at rest relative to one object and at the same time it may be in motion relative to another object.

3. Yes.

4. An object can be considered a point object if its size is much smaller than the distance travelled by it.

5. Solar or Lunar eclipse.

6. When the object moves along a straight line in the same fixed direction.

7. Zero.

8. Distance travelled $= \pi r$.
Displacement covered $= 2r$.

9. No, the displacement of the object will remain unaltered even on shifting the position of the origin.

10. The speedometer measures the instantaneous speed of the car.

11. Less than or equal to one.

12. Change in velocity $= v - u = -30 - 30 = \mathbf{-60\ ms^{-1}}$.

13. Acceleration

$$= \frac{\text{Change in velocity}}{\text{Time taken}}$$

$$= \frac{\text{Displacement / Time taken}}{\text{Time taken}}$$

$$= \frac{\text{Displacement}}{\text{Time taken}^2}$$

Hence time occurs twice in the unit of acceleration.

14. An object with positive acceleration is slowing down if its initial velocity is negative.

15. An object in simple harmonic motion speeds up while moving from an extreme position to the mean position but its acceleration is negative.

16. Acceleration is greater in the second case, because car suddenly comes to halt, the rate of change of velocity is large.

17. $$\frac{a_1}{a_2} = \frac{\text{Slope of } v\text{–}t \text{ graph of first object}}{\text{Slope of } v\text{–}t \text{ graph of second object}}$$

$$= \frac{\tan 30°}{\tan 60°} = \frac{1/\sqrt{3}}{\sqrt{3}} = \frac{1}{3} = \mathbf{1 : 3.}$$

18. Suppose brakes are applied to a cycle moving northward. At that instant, it has a northward velocity and a southward acceleration.

19. No. When a stone is thrown vertically upwards, its velocity is zero at the highest point but it has a non-zero acceleration of 9.8 ms^{-2} at the same instant.

20. Both the balls will hit the ground with the same speed, so ratio of their speeds = 1: 1. This is because the first fall will cross the point of projection with the same speed of 8 kmh^{-1} while moving downward.

21. Average acceleration $= \dfrac{a_1 t_1 + a_2 t_2}{t_1 + t_2}$.

22. For uniform motion, the position-time graph is a straight line inclined to time-axis.

23. The slope of position-time graph gives velocity of the object.
24. For uniform motion, velocity-time graph is a straight line parallel to time-axis.
25. Infinity.
26. Zero.
27. The distance travelled by an object in any time interval can be determined by finding area between the velocity-time graph and time-axis for the given time interval.
28. Area under acceleration time graph for any time interval
= Change of velocity of the body in that interval.
29. Zero.
30. Not possible.
31. Yes.
$$s = \left(\frac{u+v}{2}\right) \times t = \left(\frac{v - at + v}{2}\right) \times t = vt - \frac{1}{2}at^2.$$
32. $12 + (12 - 4) = 20$ seconds.
33. No. The path of a projectile is a parabola even when it has a uniform acceleration.
34. Refer to the answer of Exercise 3.19(*b*) on page 3.50.
35. The given velocity-time graph shown in Fig. 3.89(*b*) represents the motion of a ball thrown up with some initial velocity and rebounding from the floor with reduced speed after each hit.
36. Given $x = 6 + 18t + 9t^2$
$$v = \frac{dx}{dt} = 18 + 18t$$
At $t = 2$ s,
$$v = 18 + 18 \times 2 = \mathbf{54\ ms^{-1}}.$$
37. At the highest point, the velocity of the ball is zero and its acceleration is equal to acceleration due to gravity acting in the downward direction.
38. When the body moves along a straight line path.
39. When the body moves along a straight line path.
40. Acceleration.
41. Displacement.
42. Change in velocity in the given time interval.
43. $\dfrac{v_A}{v_B} = \dfrac{\tan 30°}{\tan 45°} = \dfrac{1/\sqrt{3}}{1} = \mathbf{1 : \sqrt{3}}$
44. No, a body cannot occupy two different positions at the same instant of time.

Type B : Short Answer Questions

2 or 3 Marks Each

1. Define the following terms and write the SI units :
(*a*) Displacement, and
(*b*) Instantaneous velocity. **[Delhi 03C]**
2. Distinguish between average velocity and instantaneous velocity. If the velocity does not change from instant to instant, will the average velocities be different for the different intervals ?
3. Differentiate between average speed and instantaneous speed of an object. **[Delhi 04]**
4. What are the characteristics of displacement ? **[Himachal 09]**
5. What are the characteristics of uniform motion ? **[Himachal 09]**
6. Derive the equations of motion given below :
(*i*) $v = u + at$ (*ii*) $s = ut + \frac{1}{2}at^2$
where symbols have their usual meanings. **[Himachal 09, 09C ; Central Schools 07]**
7. Deduce the following relation : $v^2 - u^2 = 2as$,
where symbols have their usual meaning. **[Central Schools 04]**
8. Using integration technique, prove that
$v^2 - u^2 = 2as$. **[Himachal 04, 09]**
9. Prove that
$x = x_0 + v_0t + \frac{1}{2}at^2$. **[Delhi 1998 ; Himachal 03, 04]**
10. Use integration technique to prove that the distance travelled in *n*th second,
$s_{nth} = u + \frac{a}{2}(2n - 1)$ **[Himachal 03, 04, 09, 09C]**
11. Show that the slope of displacement-time graph is equal to the velocity of uniform motion. **[Himachal 04, 07C]**
12. Draw velocity-time graph of uniform motion and prove that the area under the velocity-time graph of a particle gives the displacement of the particle in a given time. **[Himachal 04, 09]**
13. Show that area under the velocity-time graph of an object moving with constant acceleration in a straight line in certain time interval is equal to the distance covered by the object in that interval. **[Himachal 06, 06C, 09C]**

14. Derive the three kinematic equations for uniformly accelerated motion graphically. **[Delhi 06]**

15. Derive the relation graphically :

$$s = ut + \frac{1}{2}at^2$$

where symbols have their usual meanings. **[Himachal 07 ; Chandigarh 07]**

16. Using velocity-time graph, prove that

$$v^2 - u^2 = 2as$$

where symbols have their usual meanings. **[Himachal 07]**

17. Explain with example the distinction between the magnitude of average velocity over an interval of time and the average speed over the same period. Show that the average speed is either greater than or equal to the average velocity. When is the equality sign true ? **[Delhi 03C]**

18. Draw the following graphs for an object projected upward with a velocity v_0, which comes back to the same point after some time :

(*i*) Acceleration versus time graph.

(*ii*) Speed versus time graph.

(*iii*) Velocity versus time graph. **[Central Schools 03 ; Delhi 12]**

19. Acceleration-time graph of a moving object is shown in figure. Draw the velocity-time graph and displacement-time graph corresponding to this type of motion.

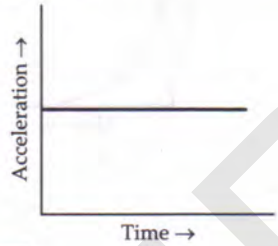


Fig. 3.98

20. Draw the following graphs (expected nature only) between distance and time of an object in case of

(*i*) For a body at rest

(*ii*) For a body moving with uniform velocity

(*iii*) For a body moving with constant acceleration. **[Central Schools 09]**

21. Draw the following graphs (expected nature only) representing motion of an object under free fall. Neglect air resistance.

(*i*) Variation of position with respect to time.

(*ii*) Variation of velocity with respect to time.

(*iii*) Variation of acceleration with respect to time. **[Central Schools 08]**

Answers

1. Refer to points 13 and 23 of Glimpses.

2. For average velocity and instantaneous velocity, refer answer to Q. 11 on page 3.5. No. The average velocity will be same for different time intervals if the instantaneous velocity does not change from instant to instant.

3. Refer to points 17 and 18 of Glimpses.

4. Refer answer to Q.7 on page 3.3.

5. Refer answer to Q. 15 on page 3.5.

6. Refer answer to Q. 21(*i*) and (*ii*) on page 3.11.

7. Refer answer to Q. 21 (*iii*) on page 3.11.

8. Refer answer to Q. 21(*iii*) on page 3.11.

9. Refer answer to Q. 20(*ii*) on page 3.10.

10. Refer answer to Q. 21(*iv*) on page 3.12.

11. Refer answer to Q. 24 on page 3.22.

12. Refer answer to Q. 26 on page 3.23.

13. Refer answer to Q. 28 on page 3.23.

14. Refer answer to Q. 29 on page 3.24.

15. Refer answer to Q. 29 on page 3.24.

16. Refer answer to Q. 29 on page 3.24.

17. Refer answer to Exercise 3.13 on page 3.48.

18. (*a*) See Fig. 3.52 on page 3.27.

(*b*) See Fig. 3.50 on page 3.27.

(*c*) See Fig. 3.48 on page 3.27.

19. The object is moving with a constant acceleration.

(*i*) For velocity-time graph, see Fig. 3.43 on page 3.26.

(*ii*) For displacement-time graph, see Fig. 3.38 on page 3.25.

20. (*i*) See Fig. 3.29 on page 3.25

(*ii*) See Fig. 3.30 on page 3.25.

(*iii*) See Fig. 3.31 on page 3.25.

21. The object falls with uniform acceleration equal to 'g'.

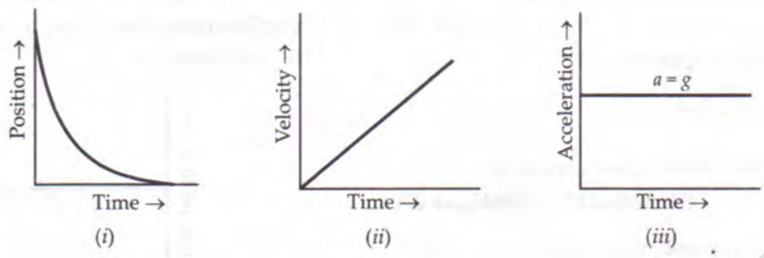


Fig. 3.99

Type C : Long Answer Questions — 5 Marks Each

1. (a) With the help of a simple case of an object moving with a constant velocity, show that the area under velocity-time curve represents the displacement over a given time interval.
   (b) Establish the relation $x = v_0 t + \frac{1}{2} at^2$ graphically.
   (c) A car moving with a speed of 126 km h^{-1} is brought to a stop within a distance of 200 m. Calculate the retardation of the car and the time required to stop it. **[Delhi 03]**
2. Derive an equation for the distance covered by a uniformly accelerated body in nth second of its motion. A body travels half its total path in the last second of its fall from rest, calculate the time of its fall. **[Central Schools 05]**
3. Draw velocity-time graph of uniformly accelerated motion in one dimension. From the velocity-time graph of uniform accelerated motion, deduce the equations of motion in distance and time. **[Chandigarh 04]**
4. Define relative velocity of one object w.r.t. another object. Draw position-time graphs for two objects moving along a straight line ; when their relative velocity is (i) zero and (ii) positive. **[Himachal 06 ; Central Schools 12]**

Answers

1. (a) Refer answer to Q. 26 on page 3.23.
   (b) Refer answer to Q. 29 (ii) on page 3.24.
   (c) Refer answer to Exercise 3.6 on page 3.46.
2. For derivation, refer answer to Q. 21 (iv) on page 3.12.

   Here $u = 0$,

   $$\therefore \quad h = 0 + \frac{1}{2} g t^2 \quad \text{...(i)}$$

   and
   $$\frac{h}{2} = 0 + \frac{1}{2} g (2t - 1) \quad \text{...(ii)}$$

   Dividing (i) by (ii), we get

   $$2 = \frac{t^2}{2t - 1}$$

   or $$t^2 - 4t + 2 = 0$$

   or $$t = \frac{4 \pm \sqrt{16 - 8}}{2} = \frac{4 \pm 2\sqrt{2}}{2}$$

   or $$t = 2 \pm \sqrt{2}$$

   or $t =$ **3.41 s** or 0.59 s (neglected)
3. Refer answer to Q. 29 on page 3.24.
4. Refer answer to Q.35 on page 3.33. See Fig. 3.72 and Fig. 3.73 on page 3.34.

Competition Section

Motion in a Straight Line

GLIMPSES

1. **Mechanics.** It is the branch of physics that *deals with the conditions of rest or motion of the material objects around us.*
2. **Statics.** It is the branch of mechanics that *deals with the study of objects at rest or in equilibrium.*
3. **Kinematics.** It is the branch of mechanics that *deals with the study of motion of objects without considering the cause of motion.*
4. **Dynamics.** It is the branch of mechanics that *deals with the study of motion of objects taking into consideration the cause of their motion.*
5. **Rest.** An object is at rest if it does not change its position w.r.t. its surroundings with the passage of time.
6. **Motion.** An object is in motion if it changes its position w.r.t. its surroundings with the passage of time.
7. **Rest and motion are relative terms.** No body can exist in a state of absolute rest or of absolute motion.
8. **Point object.** If the position of an object changes by distances much greater than its own size in a reasonable duration of time, then the object may be regarded as a point object.
9. **One dimensional motion.** The motion of an object is said to be one dimensional motion if only one out of the three coordinates specifying the position of the object changes with time. In such a motion, an object moves along a straight line path.
10. **Two dimensional motion.** The motion of an object is said to be two dimensional motion if two out of the three coordinates specifying the position of the object change with time. In such a motion, the object moves in a plane.
11. **Three dimensional motion.** The motion of an object is said to be three dimensional motion if all the three coordinates specifying the position of the object change with time. In such a motion, the object moves in space.
12. **Distance or path length.** It is the length of the actual path traversed by a body between its initial and final positions. It is a scalar quantity. Its SI unit is metre. It is always positive or zero.
13. **Displacement.** It is defined as the change in the position of an object in a fixed direction. It is given by the vector drawn from the initial position to the final position of the object. It is a vector quantity. It can be positive, negative or zero. Its SI unit is metre.

    The magnitude of displacement is less than or equal to the actual distance travelled by the object in the given time interval.
14. **Speed.** It is rate of change of position of a body in any direction.

    $$\text{Speed} = \frac{\text{Distance travelled}}{\text{Time taken}}$$

    Speed is a scalar quantity. Its SI unit is ms^{-1} and its dimensional formula is $[M^{\circ}L^{1}T^{-1}]$.
15. **Uniform speed.** An object is said to be moving with uniform speed if it covers equal distances in equal intervals of time, however small these time intervals may be.
16. **Variable speed.** An object is said to be moving with variable speed if it covers unequal distances in equal intervals of time.
17. **Average speed.** It is equal to the total distance travelled by the object divided by the total time taken to cover that distance.

    $$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

18. **Instantaneous speed.** The speed of an object at any particular instant of time or at a particular point of its path is called the instantaneous speed of the object.

$$\text{Instantaneous speed, } v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

19. **Velocity.** It is the rate of change of position of an object in a particular direction. It is equal to the displacement covered by a body per unit time.

$$\text{Velocity} = \frac{\text{Displacement}}{\text{Time}}$$

Velocity is a vector quantity. Its SI unit is ms^{-1} and dimensional formula is $[M^{\circ}L^{1}T^{-1}]$.

20. **Uniform velocity.** A body is said to be moving with uniform velocity if it covers equal displacements in equal intervals of time, however small these time intervals may be.

21. **Variable velocity.** A body is said to be moving with variable velocity if either its speed changes or direction of motion changes or both change with time.

22. **Average velocity.** It is equal to the net displacement covered divided by the total time taken.

$$v_{av} = \frac{\text{Net displacement}}{\text{Total time taken}} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

23. **Instantaneous velocity.** The velocity of an object at a particular instant of time or at a particular point of its path is called its instantaneous velocity.

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{x}}{\Delta t} = \frac{d\vec{x}}{dt}$$

24. **Uniform motion.** An object is said to be in uniform motion if it covers equal distances in equal intervals of time, however small these intervals may be, in the same fixed direction.

25. **Displacement in uniform motion.** If at time $t = 0$, the displacement is x_0, then

$$x = x_0 + vt \quad \text{or} \quad x - x_0 = vt \quad \text{or} \quad s = vt.$$

26. **Non-uniform motion.** A body is said to be in non-uniform motion if its velocity changes with time.

27. **Acceleration.** The rate of change of velocity of an object is called its acceleration.

$$\text{Acceleration} = \frac{\text{Change in velocity}}{\text{Time taken}}$$

Acceleration is a vector quantity. Its SI unit is ms^{-2} and its dimensional formula is $[M^{\circ}L^{1}T^{-2}]$.

28. **Uniform acceleration.** If the velocity of an object changes by equal amounts in equal intervals of time, however small these intervals may be, then the object is said to move with uniform or constant acceleration.

29. **Variable acceleration.** If the velocity of an object changes by unequal amounts in equal intervals of time, then the object is said to be in variable acceleration.

30. **Average acceleration.** The average acceleration of an object between two points on the path of its motion is defined as ratio of the change in velocity to the total time interval in which that change has taken place.

$$a_{av} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

31. **Instantaneous acceleration.** The acceleration of an object at a given instant of time or at a given point of its motion is called its instantaneous acceleration.

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{x}}{dt^2}$$

32. **Positive acceleration.** If the velocity of an object increases with time, its acceleration is positive.

33. **Negative acceleration.** If the velocity of an object decreases with time, its acceleration is negative. Negative acceleration is also called *retardation* or *deceleration*.

34. **Equations of motion for constant acceleration**

(*a*) *In conventional form.* Let u be the initial velocity of a particle, a the uniform acceleration, v its velocity after time t and s is the distance travelled in time t, then the following equations hold good :

(*i*) $v = u + at$ (*ii*) $s = \frac{u+v}{2} \times t$

(*iii*) $s = ut + \frac{1}{2}at^2$ (*iv*) $v^2 - u^2 = 2as$

(*v*) $s = vt - \frac{1}{2}at^2$ (*vi*) $s_{n\,th} = u + \frac{a}{2}(2n - 1)$

where $s_{n\,th}$ = the distance travelled in n^{th} second.

(*b*) *In cartesian form.* Suppose a particle moves with uniform acceleration a along X-axis. Let x_0 and x be its position co-ordinates and v_0 and v be its velocities at times $t = 0$ and t respectively. Then the following equations hold good :

(*i*) $v = v_0 + at$

(*ii*) $x = x_0 + v_0 t + \frac{1}{2}at^2$ or $s = v_0 t + \frac{1}{2}at^2$

(*iii*) $v^2 = v_0^2 + 2a(x - x_0) = v_0^2 + 2as$

(*iv*) $s_{n\,th} = v_0 + \frac{a}{2}(2n - 1)$

35. **Free fall.** In the absence of air resistance, all bodies fall with the same acceleration near the surface of the earth. This motion of a body falling towards the earth from a small height is called free fall. The acceleration with which a body falls is called acceleration due to gravity and is denoted by g. Near the surface of the earth, $g = 9.8\ ms^{-2}$.

36. Motion under gravity. For a freely falling body, the following equations of motion hold good :

(i) $v = u + gt$ (ii) $s = ut + \frac{1}{2}gt^2$

(iii) $v^2 - u^2 = 2gs$

When a body falls freely under the action of gravity, its velocity increases and the value of g is taken *positive*.

When a body is thrown vertically upward, its velocity decreases and the value of g is taken *negative*.

For a body thrown vertically upward with initial velocity u, we have

(i) Maximum height reached, $h = \frac{u^2}{2g}$

(ii) Time of ascent = Time of descent $= \frac{u}{g}$

(iii) Total time of flight to come back to the point of projection $= \frac{2u}{g}$

(iv) Velocity of fall at the point of projection $= u$

(v) Velocity attained by a body dropped through height h, $v = \sqrt{2gh}$.

37. Position-time graph. It is the graph between the time t and position x of a particle relative to a fixed origin. Its slope at any point gives the instantaneous velocity at that point.

(i) For a stationary object, the position-time graph is a straight line parallel to the time-axis.

(ii) For a body in uniform motion, the position-time graph is a straight line inclined to the time-axis.

(iii) For uniformly accelerated motion, the position-time graph is a parabola.

38. Velocity-time graph. It is a graph of time versus velocity. Its slope at any point gives the acceleration at the corresponding instant. Distance covered in time t equals area under the velocity-time graph bounded by the time-axis.

(i) For uniform motion, the velocity-time graph is a straight line parallel to the time-axis.

(ii) For uniform acceleration, the velocity-time graph is a straight line inclined to the time-axis.

(iii) For variable acceleration, the velocity-time graph is a curve.

39. Relative velocity. The relative velocity of an object B with respect to object A when both are in motion is the rate of change of position of object B with respect to object A.

Relative velocity of object B w.r.t. object A,

$$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$$

Relative velocity of object A w.r.t. object B,

$$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$$

When both the objects A and B move in the same direction,

$$v_{AB} = v_A - v_B$$

When the object B moves in the opposite direction of A,

$$v_{AB} = v_A + v_B$$

When v_A and v_B are inclined to each other at angle θ,

$$v_{AB} = \sqrt{v_A^2 + v_B^2 + 2v_Av_B\cos(180^\circ - \theta)}$$

$$= \sqrt{v_A^2 + v_B^2 - 2v_Av_B\cos\theta}$$

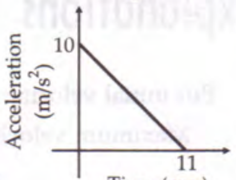
If v_{AB} makes angle β with v_A, then

$$\tan\beta = \frac{v_B\sin\theta}{v_A - v_B\cos\theta}.$$

IIT Entrance Exam

MULTIPLE CHOICE QUESTIONS WITH ONE CORRECT ANSWER

1. A body starts from rest at time $t = 0$, the acceleration-time graph is shown in figure. The maximum velocity attained by the body will be



(a) 110 m/s   (b) 55 m/s

(c) 650 m/s   (d) 550 m/s

[IIT 04]

2. A small block slides, without friction, down an inclined plane starting from rest. Let s_n be the distance travelled from $t = (n-1)$ to $t = (n)$.

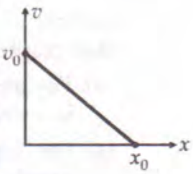
Then $\frac{s_n}{s_{n+1}}$ is

(a) $\frac{2n-1}{2n}$ (b) $\frac{2n+1}{2n-1}$

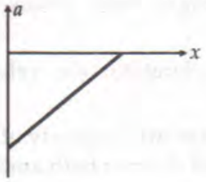
(c) $\frac{2n-1}{2n+1}$ (d) $\frac{2n}{2n+1}$

[IIT 04]

3. The velocity-displacement graph of a particle moving along a straight line is shown. The most suitable acceleration-displacement graph will be [IIT 05]



(a)



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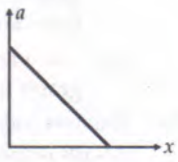

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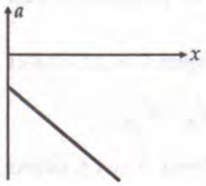
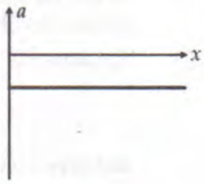
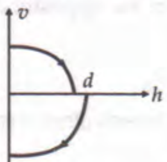


(d)

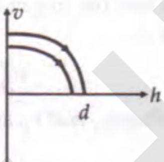


4. A ball is dropped vertically from a height d above the ground. It hits the ground and bounces up vertically to a height $d/2$. Neglecting subsequent motion and air resistance, its velocity v varies with the height h above the ground as [IIT 2000]

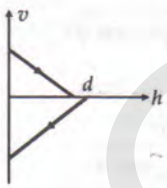
(a)



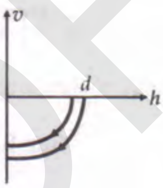
(b)



(c)



(d)



MULTIPLE CHOICE QUESTIONS WITH ONE OR MORE THAN ONE CORRECT ANSWER

5. A particle is moving eastwards with a velocity of 5 m/s. In 10 s the velocity changes to 5 m/s northwards. The average acceleration in this time is

(a) zero

(b) $\frac{1}{\sqrt{2}}$ m/s^2 towards north-west

(c) $\frac{1}{\sqrt{2}}$ m/s^2 towards north-east

(d) $\frac{1}{2}$ m/s^2 towards north-west [IIT 82]

6. A particle of mass m moves on the x-axis as follows : it starts from rest at $t=0$ from the point $x=0$, and comes to rest at $t=1$ at the point $x=1$. No other information is available about its motion at intermediate times $(0<t<1)$. If α denotes the instantaneous acceleration of the particle, then

(a) α cannot remain positive for all t in the interval $0 \le t \le 1$.

(b) $|\alpha|$ cannot exceed 2 at any point in its path.

(c) $|\alpha|$ must be ≥ 4 at some point or points in its path.

(d) α must change sign during the motion, but no other assertion can be made with the information given. [IIT 93]

REASONING TYPE

7. **Statement 1.** For an observer looking out through the window of a fast moving train, the nearby objects appear to move in the opposite direction to the train, while the distant objects appear to be stationary.

Statement 2. If the observer and the object are moving at velocities $\vec{V}_1$ and $\vec{V}_2$ respectively with reference to a laboratory frame, the velocity of the object with respect to the observer is $\vec{V}_2 - \vec{V}_1$.

(a) Statement 1 is true ; Statement 2 is true ; Statement 2 is a correct explanation for Statement 1.

(b) Statement 1 is true ; Statement 2 is true ; Statement 2 is not a correct explanation for Statement 1.

(c) Statement 1 is true ; Statement 2 is false.

(d) Statement 1 is false ; Statement 2 is true. [IIT 08]

Answers and Explanations

1. (b) Change in velocity

= Area under acceleration-time graph

$= \frac{1}{2} \times 10 \times 11 = 55$ m/s

But initial velocity = 0

$\therefore$ Maximum velocity attained

= 55 m/s.

2. (*c*) Here $s_n = u + \frac{a}{2}(2n-1)$

As $u = 0$, $s_n = \frac{a}{2}(2n-1)$

$$s_{n+1} = \frac{a}{2}[2(n+1)-1] = \frac{a}{2}(2n+1)$$

$$\therefore \quad \frac{s_n}{s_{n+1}} = \frac{\mathbf{2n-1}}{\mathbf{2n+1}}.$$

3. (*a*) From the given velocity-displacement graph,

$$\text{Slope} = -\frac{v_0}{x_0}, \text{ intercept on } y\text{-axis} = v_0$$

Thus the equation for this graph is

$$v = -\frac{v_0}{x_0}.x + v_0$$

$$\therefore \quad \frac{dv}{dt} = -\frac{v_0}{x_0}.\frac{dx}{dt}$$

or $$a = -\frac{v_0}{x_0}.v$$

$$= -\frac{v_0}{x_0}\left(-\frac{v_0}{x_0}x + v_0\right)$$

or $$a = -\frac{v_0^2}{x_0^2}.x - \frac{v_0^2}{x_0}$$

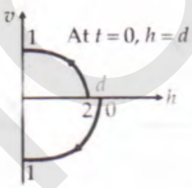
Clearly, the *a*-*x* graph must have a positive slope (v_0^2/x_0^2) and negative intercept $(-v_0^2/x_0)$ on *y*-axis.

Hence the correct option is (*a*).

4. (*a*) For the uniformly accelerated/decelerated motion,

$$v^2 = u^2 \pm 2gh$$

Thus the *v*-*h* graph is a parabola.



Initially, velocity is downwards $(-ve)$. After collision, it reverses the direction with a smaller magnitude. The velocity is upwards (+ ve). Graph (*a*) satisfies all these conditions.

Moreover, at time $t = 0$, $h = d$.

$0 \to 1$, v increases downwards

At $1 \to v$, reverses its direction

$1 \to 2$, v decreases upwards

Afterwards, the next collision occurs.

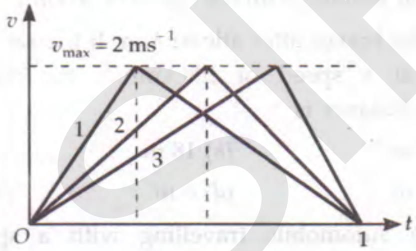
5. (*b*) Refer to the solution of Example 56 on page 4.36.

6. (*a*), (*c*) and (*d*)

The body is at rest at $t = 0$ and $t = 1$. Initially α is positive, the body acquires some velocity. Then α is negative, the body comes to rest.

Hence α cannot remain positive for all *t* in the interval $0 < t < 1$.

Options (*a*) and (*d*) are correct.



$$s = \text{Area under } v\text{-}t \text{ graph.}$$

or $$s = \frac{1}{2} \times t \times v_{max}$$

$$\therefore \quad v_{max} = \frac{2s}{t}$$

$$= \frac{2 \times 1\text{m}}{1\text{s}}$$

$$= 2\ \text{ms}^{-1}$$

In case 1 :

first $\alpha > 4\ \text{ms}^{-2}$, then $\alpha < -4\ \text{ms}^{-2}$

In case 2 :

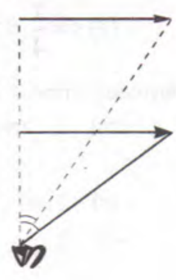
first $\alpha = 4\ \text{ms}^{-2}$, then $\alpha < -4\ \text{ms}^{-2}$

In case 3 :

first $\alpha < 4\ \text{ms}^{-2}$, then $\alpha < -4\ \text{ms}^{-2}$

Hence $|\alpha| \geq 4$ at some point or points in the path. Option (*b*) is wrong and option (*c*) is correct.

7. (*b*) Both statements are true but statement 2 is not correct explanation for statement 1. Distance appeared to move depends upon angle subtended on the eye as illustrated in the figure.



AIEEE

1. A bullet fired into a fixed target loses half of its velocity after penetrating 3 cm. How much further will it penetrate before coming to rest assuming that it faces constant resistance in motion ?

(*a*) 1.5 cm (*b*) 1.0 cm
(*c*) 3.0 cm (*d*) 2.0 cm [AIEEE 05]

2. A car moving with the speed of $50\,\text{kmh}^{-1}$ can be stopped by brakes after atleast 6 m. If the same car is moving at a speed of $100\,\text{kmh}^{-1}$, the minimum stopping distance is

(*a*) 12 m (*b*) 18 m
(*c*) 24 m (*d*) 6 m [AIEEE 03]

3. An automobile travelling with a speed of $60\ \text{kmh}^{-1}$ can brake to stop within a distance of 20 m. If the car is going twice as fast *i.e.*, $120\ \text{kmh}^{-1}$, the stopping distance will be

(*a*) 20 m (*b*) 40 m
(*c*) 60 m (*d*) 80 m [AIEEE 04]

4. Speeds of two identical cars are u and $4u$ at a specific instant. The ratio of the respective distances at which the two cars are stopped from that instant is

(*a*) 1 : 1 (*b*) 1 : 4
(*c*) 1 : 8 (*d*) 1 : 16 [AIEEE 02]

5. If a body loses half of its velocity on penetrating 3 cm in a wooden block, then how much will it penetrate more before coming to rest ?

(*a*) 1 cm (*b*) 2 cm
(*c*) 3 cm (*d*) 4 cm [AIEEE 02]

6. A car, starting from rest, accelerates at the rate f through a distance s, then continues at constant speed for time t and then decelerates at the rate $f/2$ to come to rest. If the total distance traversed is $5s$, then

(*a*) $s = ft$ (*b*) $s = \frac{1}{6}ft^2$
(*c*) $s = \frac{1}{2}ft^2$ (*d*) $s = \frac{1}{4}ft^2$ [AIEEE 05]

7. The relation between time t and distance x is $t = ax^2 + bx$, where a and b are constants. The acceleration is

(*a*) $-2\,abv^2$ (*b*) $-2\,bv^3$
(*c*) $-2\,av^3$ (*d*) $-2\,av^2$ [AIEEE 05]

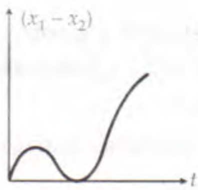
8. A particle located at $x = 0$ at time $t = 0$, starts moving along the positive X-direction with a velocity v that varies with time as

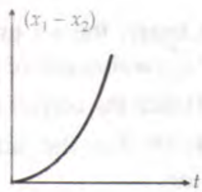
(*a*) $t^{1/2}$ (*b*) t^3
(*c*) t^2 (*d*) t [AIEEE 06]

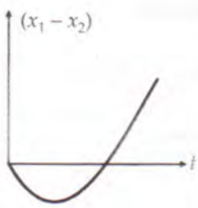
9. The velocity of a particle is $v = v_0 + gt + ft^2$. If its position is $x = 0$ at $t = 0$, then its displacement after time $(t = 1)$ is

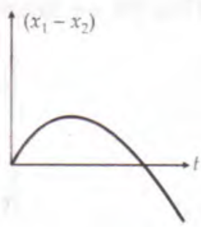
(*a*) $v_0 + \frac{g}{2} + f$ (*b*) $v_0 + 2g + 3f$
(*c*) $v_0 + \frac{g}{2} + \frac{f}{3}$ (*d*) $v_0 + g + f$ [AIEEE 07]

10. A body is at rest at $x = 0$. At $t = 0$, it starts moving in the positive x-direction with a constant acceleration. At the same instant another body passes through $x = 0$ moving in the positive x-direction with a constant speed. The position of the first body is given by $x_1(t)$ after time t and that of second body by $x_2(t)$ after the same time interval. Which of the following graphs correctly describes $(x_1 - x_2)$ as a function of time t ?

(*a*)


(*b*)


(*c*)


(*d*)


[AIEEE 08]

11. From a building two balls A and B are thrown such that A is thrown upwards and B downwards (both vertically). If v_A and v_B are their respective velocities on reaching the ground, then

(*a*) $v_B > v_A$ (*b*) $v_A = v_B$
(*c*) $v_A > v_B$
(*d*) their velocities depend on their masses. [AIEEE 02]

12. A ball is released from the top of a tower of height h metres. It takes T seconds to reach the ground. What is the position of the ball in $T/3$ seconds ?

(*a*) $h/9$ metres from the ground

(*b*) $7h/9$ metres from the ground

(*c*) $8h/9$ metres from the ground

(*d*) $17h/18$ metres from the ground [AIEEE 04]

13. A parachutist after bailing out falls 50 m without friction. When parachute opens, it decelerates at $2\ \text{ms}^{-2}$. He reaches the ground with a speed of $3\ \text{ms}^{-2}$. At what height, did he bail out ?

(*a*) 293 m (*b*) 111 m

(*c*) 91 m (*d*) 182 m [AIEEE 05]

14. An object, moving with a speed of 6.25 m/s, is decelerated at a rate given by

$$\frac{dv}{dt} = -2.5\sqrt{v}$$

where v is the instantaneous speed. The time taken by the object, to come to rest, would be

(*a*) 1 s (*b*) 2 s

(*c*) 4 s (*d*) 8 s [AIEEE 2011]

Answers and Explanations

1. (*b*) Refer to the solution of Problem 7 on page 3.43.

2. (*c*) Refer to the solution of Problem 9 on page 3.43.

3. (*d*) In first case,

Velocity, $v = \frac{60 \times 1000}{3600} = \frac{50}{3}\ \text{ms}^{-1}$

$\therefore$ Retardation, $a = \frac{v^2}{2s} = \frac{\frac{50}{3} \times \frac{50}{3}}{2 \times 20} = \frac{250}{36}\ \text{ms}^{-2}$

In second case,

Velocity, $v = \frac{120 \times 1000}{3600} = \frac{100}{3}\ \text{ms}^{-1}$

$\therefore \quad s = \frac{v^2}{2a} = \frac{\frac{100}{3} \times \frac{100}{3}}{2} \times \frac{36}{200} = \mathbf{80\ m.}$

4. (*d*) **For first car :** $u = u,\ a = -a,\ v = 0$

As $\quad v^2 - u^2 = 2as$

$\therefore \quad v^2 - u^2 = 2(-a)s_1$ or $s_1 = \frac{u^2}{2a}$

For second car : $\quad u = 4u,\ a = -a,\ v = 0$

$\therefore \quad s_2 = \frac{(4u)^2}{2a} = \frac{8u^2}{a}$

Hence, $\quad \frac{s_1}{s_2} = \frac{u^2}{2a} \cdot \frac{a}{8u^2} = \frac{1}{16} = \mathbf{1 : 16.}$

5. (*a*) **In first case :** $v = \frac{u}{2}$, $s = 3$ cm.

As $\quad v^2 - u^2 = 2as$

$\therefore \left(\frac{u}{2}\right)^2 - u^2 = 2a \times 3 \quad$ or $\quad a = -\frac{u^2}{8}$

In second case : $v = 0,\ u = \frac{u}{2},\ a = -\frac{u^2}{8}$

$\therefore \quad 0^2 - \left(\frac{u}{2}\right)^2 = 2\left(-\frac{u^2}{8}\right)s \quad$ or $\quad s = \mathbf{1\ cm.}$

6. (*c*) Refer to the solution of Problem 8 on page 3.43.

7. (*c*) Refer to the solution of Problem 10 on page 3.44.

8. (*c*) Given : $\quad v = \alpha\sqrt{x}$

or $\quad \frac{dx}{dt} = \alpha x^{1/2}$

or $\quad x^{-1/2}dx = \alpha\, dt$

Integrating both sides,

$$\int x^{-1/2}dx = \alpha \int dt$$

$$\frac{x^{1/2}}{1/2} = \alpha t \quad \text{or} \quad x = \frac{\alpha^2 t^2}{4}$$

Hence, $\quad x \propto t^2$.

9. (*c*) $\quad v = \frac{dx}{dt} = v_0 + gt + ft^2$

$$\therefore \quad \int_0^x dx = \int_0^1 (v_0 + gt + ft^2)dt$$

$$x = \left[v_0 t + \frac{gt^2}{2} + \frac{ft^3}{3}\right]_0^1 = v_0 + \frac{g}{2} + \frac{f}{3}.$$

10. (*c*) For particle 1, $v = 0,\ v_1 = at$.

For particle 2, $v_2 =$ constant.

Initially, both are at $x = 0$.

Relative velocity of 1 w.r.t. $2 = v_1 - v_2$.

At first, v_1 is less than v_2.

The distance travelled by particle 1 increases with time as

$$x_1 = \frac{1}{2}at^2$$

The distance travelled by second particle is proportional to time t.

$$x_2 = v_1 t \quad \therefore \quad x_1 - x_2 = \frac{1}{2}at^2 - v_1 t$$

Hence graph of $(x_1 - x_2)$ versus t is a parabola even after crossing x-axis.

11. (*b*) Suppose the ball A is thrown upward with velocity u and ball B is thrown downward with the same velocity u. After reaching the highest point, the ball A comes back to its position of projection attaining velocity u in the downward direction. As the two balls fall down from the same position of projection with the same velocity, both attain the same velocity on reaching the ground.

12. (*c*) Refer to the solution of problem 11 on page 3.44.

13. (*a*) Velocity attained after falling freely through first 50 m,

$$v = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 50} = \sqrt{980} \text{ m/s}$$

After the parashute opens,

$$u = \sqrt{980} \text{ m/s}, \quad v = 3 \text{ m/s}, \quad a = -2 \text{ m/s}^2$$

As $\quad v^2 - u^2 = 2gh$

$\therefore \quad 9 - 980 = 2 \times 2 \times h \quad$ or $\quad h = \frac{971}{4} \simeq 243$ m

Total height $= 50 + 243 =$ **293 m.**

14. (*b*) Given $\quad \frac{dv}{dt} = -2.5\sqrt{v}$

or $\quad \frac{1}{\sqrt{v}} dv = -2.5\, dt$

Integrating, $\quad \int_{6.25}^{0} v^{-1/2}\, dv = -2.5 \int_0^t dt$

$$[2\sqrt{v}]_{6.25}^{0} = -2.5[t]_0^t$$

$$0 - 2\sqrt{6.25} = -2.5[t - 0]$$

$$-2 \times 2.5 = -2.5t \quad \text{or} \quad t = \mathbf{2\ s}$$

DCE and G.G.S. Indraprastha University Engineering Entrance Exam

(Similar Questions)

1. What is the relation among displacement, time and acceleration in case of a body having uniform acceleration f ?

(*a*) $s = ut + \frac{1}{2}ft^2$ (*b*) $s = (u + f)t$

(*c*) $s = v^2 - 2fs$ (*d*) none of these [DCE 99]

2. The motion of a particle is described by the equation $u = at$. The distance travelled by particle in first 4 s is

(*a*) $4a$ (*b*) $12a$

(*c*) $6a$ (*d*) $8a$ [DCE 2K]

3. The displacement x of a particle varies with time t as $\quad x = ae^{-\alpha t} + be^{\beta t}$

where a, b, α and β are positive constants. The velocity of the particle will

(*a*) go on decreasing with time

(*b*) be independent of α and β

(*c*) drop to zero when $\alpha = \beta$

(*d*) go on increasing with time [DCE 01]

4. A train started from rest from a station and accelerated at 2 ms^{-2} for 10 s. Then, it ran at constant speed for 30 s and thereafter it decelerated at 4 ms^{-2} until it stopped at the next station. The distance between two station is

(*a*) 650 m (*b*) 700 m

(*c*) 750 m (*d*) 800 m [DCE 03]

5. A ball falls from 20 m height on floor and rebounds to 5 m. Time of contact is 0.02 sec. Find acceleration during impact.

(*a*) 1200 m/s^2 (*b*) 1000 m/s^2

(*c*) 2000 m/s^2 (*d*) 1500 m/s^2 [DCE 06]

6. A ball is dropped from top of a tower of 100 m height. Simultaneously another ball was thrown upward from bottom of the tower with a speed of 50 m/s. They will cross each other after ($g = 10$ m/s^2):

(*a*) 1 sec (*b*) 2 sec

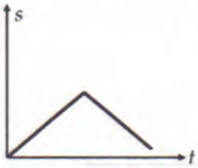
(*c*) 3 sec (*d*) 4 sec [IPUEE 04]

7. A ball thrown upward from the top of a tower with speed v reaches the ground in t_1 second. If this ball is thrown downward from the top of the same tower with speed v, it reaches the ground in t_2 seconds. In what time will the ball reach the ground if it is allowed to fall freely under gravity from the top of the tower ?

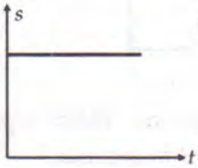
(*a*) $\frac{t_1 + t_2}{2}$ (*b*) $\frac{t_1 - t_2}{2}$

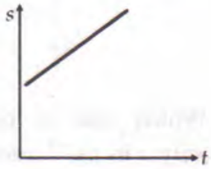
(*c*) $\sqrt{t_1 t_2}$ (*d*) $t_1 + t_2$ [DCE 07]

8. Which one of the following graphs represents uniform motion ? [DCE 99]

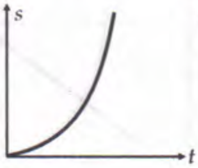
(a) 

(b) 

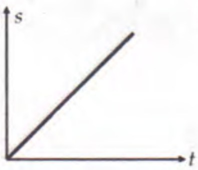
(c) 

(d) 

9. Which graph pertains to uniform acceleration ?

(a) 

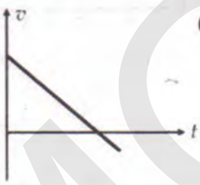
(b) 

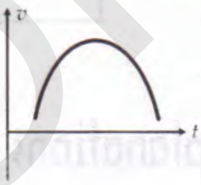
(c) 

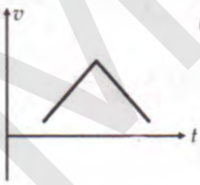
(d) 

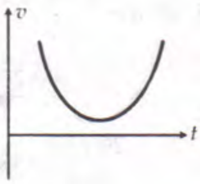
[DCE 2K, 03]

10. A particle is thrown above, then correct v-t graph will be [DCE 07]

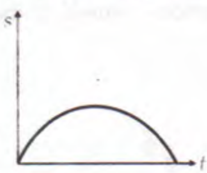
(a) 

(b) 

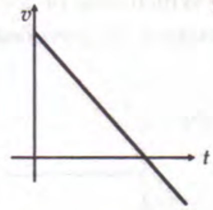
(c) 

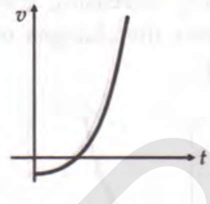
(d) 

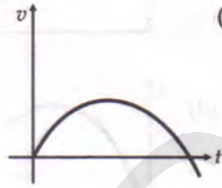
11. The graph of displacement *vs.* time is

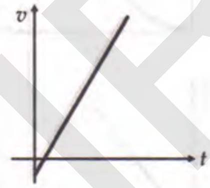


The corresponding velocity-time graph will be

(a) 

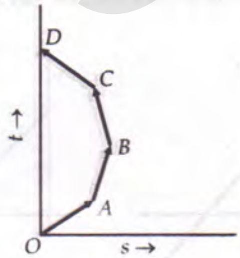
(b) 

(c) 

(d) 

[DCE 01]

12. Which of the following options is correct for the object having a straight line motion represented by the following graph ?



(a) The object moves with constantly increasing velocity from O to A and then it moves with constant velocity.

(b) Velocity of the object increases uniformly

(c) Average velocity is zero

(d) The graph shown is impossible. [DCE 04]

13. The driver of a train travelling at 115 kmh^{-1} sees on the same track, 100 m in front of him, a slow train travelling in the same direction at 25 kmh^{-1}. The least retardation that must be applied to faster train to avoid collision is

(a) $25\,\text{ms}^{-2}$ (b) $50\,\text{ms}^{-2}$

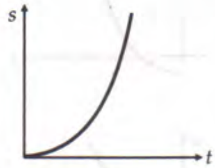
(c) $75\,\text{ms}^{-2}$ (d) $3.125\,\text{ms}^{-2}$ [DCE 07]

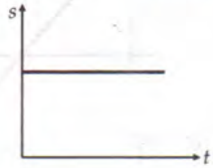
14. The acceleration 'a' of a particle starting from rest varies with time according to relation $a = \alpha t + \beta$. The velocity of the particle after a time 't' will be

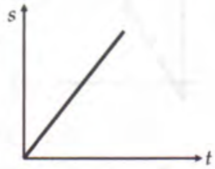
(a) $\dfrac{\alpha t^2}{2} + \beta$ (b) $\dfrac{\alpha t^2}{2} + \beta t$

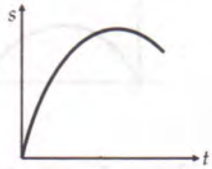
(c) $\alpha t^2 + \dfrac{1}{2}\beta t$ (d) $\dfrac{\alpha t^2 + \beta}{2}$ [DCE 09]

15. A body is travelling in a straight line with a uniformly increasing speed. Which one of the plots represents the changes in distance (s) travelled with time (t) ? [DCE 09]

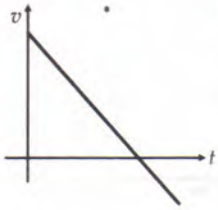
(*a*)


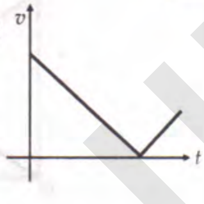
(*b*)


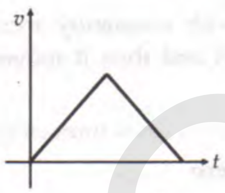
(*c*)


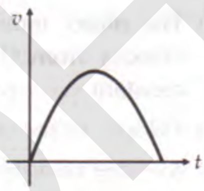
(*d*)


16. A ball is thrown vertically upward. Ignoring the air resistance, which one of the following plots represents the velocity-time plot for the period ball remains in air ? [DCE 09]

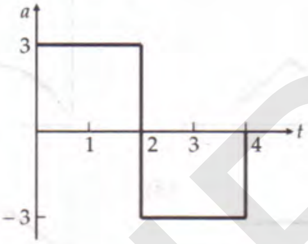
(*a*)


(*b*)


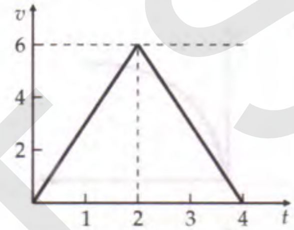
(*c*)


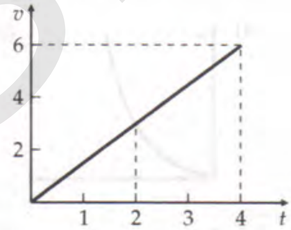
(*d*)


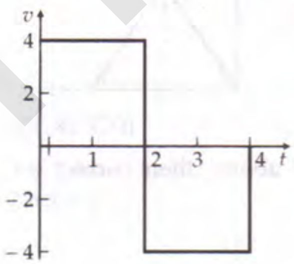
17. A particle starts from rest at $t = 0$ and undergoes an acceleration 'a' in ms^{-2} with time 't' in seconds which is as shown below :

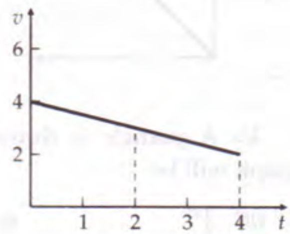


Which one of one following plots represents velocity v in ms^{-1} verses time t in seconds ?

(*a*)


(*b*)


(*c*)


(*d*)


[DCE 09]

Answers and Explanations

1. (*a*) $s = ut + \frac{1}{2} ft^2$

2. (*d*) $u = \frac{dx}{dt} = at$

$$\therefore \int_0^x dx = \int_0^4 at\, dt$$

$$x = a\left[\frac{t^2}{2}\right]_0^4 = \frac{a}{2}[16 - 0] = 8a.$$

3. (*d*) Refer to the solution of Problem 10 on page 3.74 .

4. (*c*) Refer to the solution of Problem 3 on page 3.70.

5. (*d*) $a = \frac{v_2 - (-v_1)}{t} = \frac{v_2 + v_1}{t} = \frac{\sqrt{2gh_2} + \sqrt{2gh_1}}{t}$

$$= \frac{\sqrt{2 \times 10 \times 20} + \sqrt{2 \times 10 \times 5}}{0.02}\ \text{m/s}^2$$

$$= \frac{20 + 10}{0.02} = \mathbf{1500\ m/s^2}.$$

6. (*b*) Suppose the two balls cross each other after time t. Then the distances covered by the two balls will be

$$s_1 = \frac{1}{2} gt^2$$

and $$s_2 = ut - \frac{1}{2} gt^2$$

But $s_1 + s_2 = \frac{1}{2}gt^2 + ut - \frac{1}{2}gt^2 = 100$ m

$\therefore \quad ut = 100$

or $\quad t = \frac{100}{u} = \frac{100}{50} = \mathbf{2\ s}.$

7. (*c*) For upward motion,

$$h = -vt_1 + \frac{1}{2}gt_1^2$$

For downward motion,

$$h = vt_2 + \frac{1}{2}gt_2^2$$

$$\therefore \quad \frac{h}{t_1} + \frac{h}{t_2} = \frac{1}{2}g(t_1 + t_2) \quad \text{or} \quad h = \frac{1}{2}gt_1t_2$$

For freely falling ball,

$$h = \frac{1}{2}gt^2$$

$$\therefore \quad \frac{1}{2}gt^2 = \frac{1}{2}gt_1t_2 \quad \text{or} \quad t = \sqrt{t_1t_2}$$

8. (*d*) Graph (*d*) represents uniform motion because here $s \propto t$.

9. (*a*) For uniform acceleration, displacement-time graph is a parabola symmetric about displacement-axis.

10. (*a*) In upward motion, velocity (+ve) decreases and becomes zero at highest point. In downward motion, velocity (– ve) increases.

11. (*a*) The given displacement-time graph is a parabola. It represents the motion of a body thrown up against gravity and then returning to the ground. As explained in the above problem, its *v*-*t* graph will be (*a*).

12. (*c*) In the given graph, net displacement is zero.

$$\therefore \text{ Average velocity} = \frac{\text{Net displacement}}{\text{Time taken}} = \frac{0}{t} = 0.$$

13. (*d*) Relative velocity $= 115 - 25 = 90\ \text{kmh}^{-1}$

$$= 90 \times \frac{5}{18}\ \text{ms}^{-1} = 25\ \text{ms}^{-1}.$$

As $\quad v^2 - v^2 = 2as$

$$0^2 - 25^2 = 2a \times 100$$

or $\quad a = -\frac{625}{200} = \mathbf{-3.125\ ms^{-2}}.$

14. (*b*) $\quad a = \frac{dv}{dt} = \alpha t + \beta$

$$\therefore \quad \int_0^v dv = \int_0^t (\alpha t + \beta)\, dt$$

or $\quad v = \frac{\alpha t^2}{2} + \beta t.$

15. (*a*) $\quad s = ut + \frac{1}{2}at^2$

for $u = 0$, $\quad s = \frac{1}{2}at^2.$

Hence, *s*-*t* graph is a parabola as given in option (*a*).

16. (*a*) During upward motion the magnitude of (+ve) velocity decreases while during downward motion the magnitude of (–ve) velocity increases.

17. (*a*) Area under *a*-*t* curve = change in velocity

For constant acceleration, $v \propto t$

As a is first +ve and then –ve, so the correct *v*-*t* graph is the one given in option (*a*).

AIIMS Entrance Exam

1. The displacement of a body is given to be proportional to the cube of time elapsed. The magnitude of the acceleration of the body is

(*a*) increasing with time

(*b*) decreasing with time

(*c*) constant but not zero

(*d*) zero [AIIMS 96]

2. The velocity of a bullet is reduced from $200\ \text{ms}^{-1}$ to $100\ \text{ms}^{-1}$ while travelling through a wooden plank of thickness 10 cm. The retardation, assuming it to be uniform, will be

(*a*) $10 \times 10^4\ \text{ms}^{-2}$ (*b*) $12 \times 10^4\ \text{ms}^{-2}$

(*c*) $13.5 \times 10^4\ \text{ms}^{-2}$ (*d*) $15 \times 10^4\ \text{ms}^{-2}$ [AIIMS 01]

3. A particle starts from rest and has an acceleration of $2\ \text{ms}^{-2}$ for 10 s. After that, the particle travels for 30 s with constant speed and then undergoes a retardation of $4\ \text{ms}^{-2}$ and comes back to rest. The total distance covered by the particle is

(*a*) 650 m (*b*) 700 m

(*c*) 750 m (*d*) 800 m [AIIMS 02]

4. The body *A* starts from rest with an acceleration a_1. After 2 s, another body *B* starts from rest with an acceleration a_2. If they travel equal distances in the 5th second after the start of A, then the ratio $a_1 : a_2$ is equal to

(*a*) 5 : 9 (*b*) 5 : 7

(*c*) 9 : 5 (*d*) 9 : 7 [AIIMS 01]

5. Velocity of a body on reaching the point, from which it was projected upwards, is

(*a*) $v = 0$ (*b*) $v = 2u$

(*c*) $v = 0.5u$ (*d*) $v = u$ [AIIMS 99]

6. Three different objects of masses m_1, m_2 and m_3 are allowed to fall from the same point O along three different frictionless paths. The speeds of the three objects, on reaching the ground, will be in the ratio of

(*a*) $m_1 : m_2 : m_3$ (*b*) $m_1 : 2m_2 : 3m_3$

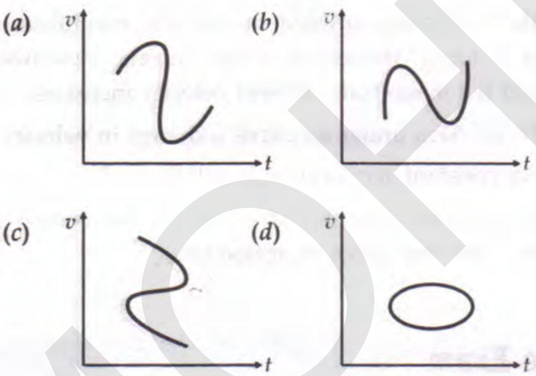
(*c*) $1/m_1 : 1/m_2 : 1/m_3$ (*d*) 1 : 1 : 1 [AIIMS 02]

7. When a ball is thrown up vertically with velocity v_0, it reaches a maximum height of h. If one wishes to triple the maximum height, then the ball should be thrown with velocity

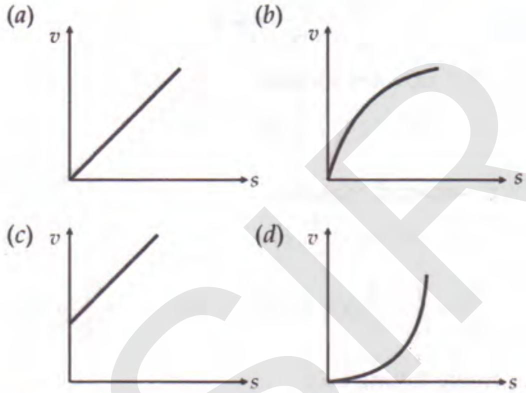
(*a*) $\sqrt{3}v_0$ (*b*) $3v_0$

(*c*) $9v_0$ (*d*) $3v_0/2$ [AIIMS 05]

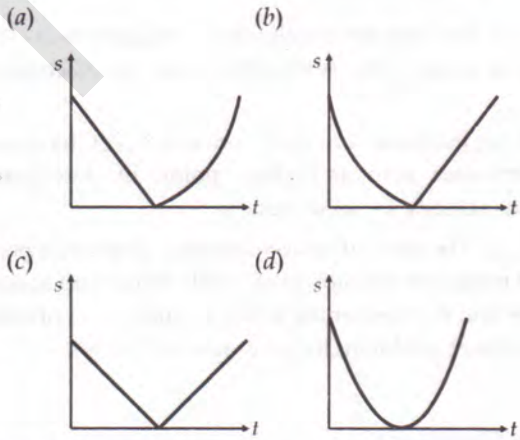
8. Which of the following velocity-time graph shows a realistic situation for a body in motion ? [AIIMS 04, 07]



9. A body starting from rest moves along a straight line with constant acceleration. The variation of speed v with distance s is [AIIMS 03]



10. A ball is thrown vertically upwards. Which of the following plots represents the speed-time graph of the ball during its flight, if the air resistance is not ignored ? [AIIMS 03]



Answers and Explanations

1. (*a*) Refer to the solution of Problem 23 on page 3.38.

2. (*d*) As $v^2 - u^2 = 2as$

$\therefore \quad 100^2 - 200^2 = 2a \times 0.10$

or $\quad \mathbf{a = -15 \times 10^4\ ms^{-2}}.$

3. (*c*) *For motion with acceleration* : $u = 0$

$a = 2\ \text{ms}^{-2},\ t = 10$ s.

$$\therefore \quad s_1 = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 2 \times (10)^2 = 100 \text{ m}$$

$$v = u + at = 0 + 10 \times 2 = 20 \text{ ms}^{-1}$$

For motion with constant speed :

$$t = 30 \text{ s}$$

$$s_2 = vt = 20 \times 30 = 600 \text{ m}.$$

For motion with retardation :

$$u = 20\,\text{ms}^{-1},\ a = -4\,\text{ms}^{-2},\ v = 0$$

As $v^2 - u^2 = 2as$

$\therefore \quad 0 - 20^2 = 2(-4)\times s_3$

$s_3 = 50$ m

Total distance covered

$s = s_1 + s_2 + s_3$

$= 100 + 600 + 50 = \mathbf{750\ m.}$

4. (*a*) Here $\quad s_{5th}(A) = s_{3rd}(B)$

or $\quad 0 + \frac{a_1}{2}(2\times 5 - 1) = 0 + \frac{a_2}{2}(2\times 3 - 1)$

or $\quad \frac{a_1 \times 9}{2} = \frac{a_2 \times 5}{2}$

or $\quad \frac{a_1}{a_2} = \frac{5}{9} = \mathbf{5:9.}$

5. (*d*) When a body is projected upwards with a velocity *u*, it returns to the point of projection with the same velocity *u*.

6. (*d*) When allowed to fall from the same height, three objects attain the same speed on reaching the ground, independent of their masses or the frictionless paths.

7. (*a*) $0^2 - v_0^2 = 2(-g)h$

or $\quad v_0^2 = 2gh \quad$ *i.e.,* $\quad v_0^2 \propto h$

$\therefore \quad v_0'^2 \propto 3h \quad \therefore \quad \frac{v_0'^2}{v_0^2} = \frac{3h}{h} = 3$

or $\quad v_0' = \sqrt{3}\, v_0$.

8. (*b*) In options (*a*), (*c*) and (*d*), the body possesses more than one velocity at any given time which is not physically possible.

9. (*d*) $v^2 - 0^2 = 2as \quad$ *i.e.,* $\quad v^2 \propto s$

Hence the speed versus distance or *v-s* graph is a parabola symmetric about *v*-axis.

10. (*a*) As the ball goes up, its speed decreases with time and becomes zero at the highest point. As the ball falls down, its speed increases. The ball attains the same speed with which it was thrown up.

CBSE PMT Prelims and Final Exams

1. A car covers the first half of the distance between two places at 40 km/h and another half at 60 km/h. The average speed of the car is

(*a*) 40 km/h (*b*) 48 km/h
(*c*) 50 km/h (*d*) 60 km/h [CBSE PMT 90]

2. A car moves a distance of 200 m. It covers the first half of the distance at speed 40 km/h and the second half of distance at speed *v*. The average speed is 48 km/h. The value of *v* is

(*a*) 56 km/h (*b*) 60 km/h
(*c*) 50 km/h (*d*) 48 km/h [CBSE PMT 91]

3. A bus travelling the first one-third distance at a speed of 10 km/h, the next one-third at 20 km/h and at last one-third at 60 km/h. The average speed of bus is

(*a*) 9 km/h (*b*) 16 km/h
(*c*) 18 km/h (*d*) 48 km/h [CBSE PMT 91]

4. A car moves from *X* to *Y* with a uniform speed v_u and returns to *Y* with a uniform speed v_d. The average speed of this sound trip is

(*a*) $\sqrt{v_u v_d}$ (*b*) $\frac{v_d v_u}{v_d + v_u}$
(*c*) $\frac{v_u + v_d}{2}$ (*d*) $\frac{2 v_d v_u}{v_d + v_u}$ [CBSE PMT 07]

5. A car moves along a straight line, whose equation of motion is given by

$$s = 12t + 3t^2 - 2t^3$$

where *s* is in metres and *t* in seconds. The velocity of the car at the start will be

(*a*) $7\ ms^{-1}$ (*b*) $9\ ms^{-1}$
(*c*) $12\ ms^{-1}$ (*d*) $16\ ms^{-1}$ [CBSE PMT 98]

6. A particle moves along a straight line *OX*. At a time *t* (in seconds) the distance *x* (in metres) of the particle from *O* is given by $x = 40 + 12t - t^3$. How long would the particle travel before coming to rest ?

(*a*) 16 m (*b*) 24 m
(*c*) 40 m (*d*) 56 m [CBSE PMT 06]

7. The position *x* of a particle varies with time *t* as $x = at^2 - bt^3$. The acceleration will be zero at time *t* equal to

(*a*) $\frac{a}{3b}$ (*b*) zero
(*c*) $\frac{2a}{3b}$ (*d*) $\frac{a}{b}$ [CBSE PMT 97]

8. Motion of a particle is given by equation $s = (3t^3 + 7t^2 + 14t + 8)$ m. The value of acceleration of the particle at $t = 1$ sec is

(*a*) $10\ m/s^2$ (*b*) $32\ m/s^2$
(*c*) $23\ m/s^2$ (*d*) $16\ m/s^2$ [CBSE PMT 2K]

9. A particle moves along a straight line such that its displacement at any time t is given by $s=(t^3-6t^2+3t+4)$ metres. The velocity when the acceleration is zero is

(a) 3 m/s (b) 42 m/s
(c) –9 m/s (d) –15 m/s [CBSE PMT 94]

10. The displacement x of a particle varies with time t as $x=ae^{-\alpha t}+be^{\beta t}$, where a, b, α and β are positive constants. The velocity of the particle will

(a) be independent of β
(b) drop to zero when $\alpha=\beta$
(c) go on decreasing with time
(d) go on increasing with time [CBSE PMT 05]

11. The position x of a particle with respect to time t along x-axis is given by

$$x=9t^2-t^3,$$

where x is in metres and t in seconds.

What will be the position of this particle when it achieves maximum speed along the $+x$ direction ?

(a) 54 m (b) 81 m
(c) 24 m (d) 32 m [CBSE PMT 07]

12. The acceleration experienced by a moving motor boat, after its engine is cut off, is given by

$$\frac{dv}{dt}=-kv^3$$

where k is constant. If v_0 is the magnitude of velocity at cut-off, the magnitude of the velocity at a time t after the cut off is

(a) $v_0/2$ (b) v_0
(c) $\dfrac{v_0}{\sqrt{2v_0^2kt+1}}$ (d) v_0e^{-kt} [CBSE PMT 94]

13. A particle moving along x-axis has acceleration f at time t, given by

$$f=f_0\left(1-\frac{t}{T}\right),$$

where f_0 and T are constants.

The particle at $t=0$ has zero velocity. In the time interval between $t=0$ and the instant when $f=0$, the particle's velocity v_x is

(a) $\frac{1}{2}f_0T^2$ (b) f_0T^2
(c) $\frac{1}{2}f_0T$ (d f_0T [CBSE PMT 07]

14. The acceleration of a particle is increasing linearly with time t as bt. The particle starts from origin with an initial velocity v_0. The distance travelled by the particle in time t will be

(a) $v_0t+\frac{1}{3}bt^2$ (b) $v_0t+\frac{1}{2}bt^2$
(c) $v_0t+\frac{1}{6}bt^3$ (d) $v_0t+\frac{1}{3}bt^3$ [CBSE PMT 95]

15. If a car at rest accelerates uniformly to a speed of 144 km/h in 20 sec, it covers a distance of

(a) 1440 cm (b) 2980 cm
(c) 20 m (d) 400 m [CBSE PMT 97]

16. The velocity of train increases uniformly from 20 km/h to 60 km/h in 4 hours. The distance travelled by the train during this period is

(a) 160 km (b) 180 km
(c) 100 km (d) 120 km [CBSE PMT 94]

17. A car moving with a speed of 40 km/h can be stopped by applying brakes after atleast 2 m. If the same car is moving with a speed of 80 km/h, what is the minimum stopping distance ?

(a) 4 m (b) 6 m
(c) 8 m (d) 2 m [CBSE PMT 98]

18. A car is moving along a straight road with a uniform acceleration. It passes through two points P and Q separated by a distance with velocity 30 km/h and 40 km/h respectively. The velocity of the car midway between P and Q is

(a) 33.3 km/h (b) $20\sqrt{2}$ km/h
(c) $25\sqrt{2}$ km/h (d) 35 km/h
[CBSE PMT 90]

19. A car accelerates from rest at a constant rate α for some time after which it decelerates at a constant rate β and comes to rest. If total time elapsed is t, then maximum velocity acquired by car will be

(a) $\dfrac{(\alpha^2-\beta^2)t}{\alpha\beta}$ (b) $\dfrac{(\alpha^2+\beta^2)t}{\alpha\beta}$
(c) $\dfrac{(\alpha+\beta)t}{\alpha\beta}$ (d) $\dfrac{\alpha\beta t}{\alpha+\beta}$ [CBSE PMT 94]

20. If a ball is thrown vertically upwards with speed u, the distance covered during the last t seconds of its ascent is

(a) ut (b) $\frac{1}{2}gt^2$
(c) $ut-\frac{1}{2}gt^2$ (d) $(u+gt)t$
[CBSE PMT 03]

21. What will be the ratio of the distances moved by a freely falling body from rest in 4th and 5th seconds of journey ?

(a) 4 : 5 (b) 7 : 9
(c) 16 : 25 (d) 1 : 1 [CBSE PMT 89, 93]

22. Two bodies A (of mass 1 kg) and B (of mass 3 kg) are dropped from heights of 16 m and 25 m, respectively. The ratio of the time taken by them to reach the ground is

(a) 4/5 (b) 5/4
(c) 12/5 (d) 5/12 [CBSE PMT 06]

23. A ball is thrown vertically upward. It has a speed of 10 m/sec when it has reached one half of its maximum height. How high does the ball rise ? Take $g = 10\,\text{m/s}^2$.

(a) 10 m (b) 5 m
(c) 15 m (d) 20 m [CBSE PMT 05]

24. A body dropped from top of a tower falls through 40 m during the last two seconds of its fall. The height of tower ($g = 10\,\text{ms}^{-2}$) is

(a) 60 m (b) 45 m
(c) 80 m (d) 50 m [CBSE PMT 92]

25. A body dropped from a height h with initial velocity zero, strikes the ground with a velocity 3 m/s. Another body of same mass dropped from the same height h with an initial velocity of 4 m/s. The final velocity of second mass, with which it strikes the ground is

(a) 5 m/s (b) 12 m/s
(c) 3 m/s (d) 4 m/s [CBSE PMT 06]

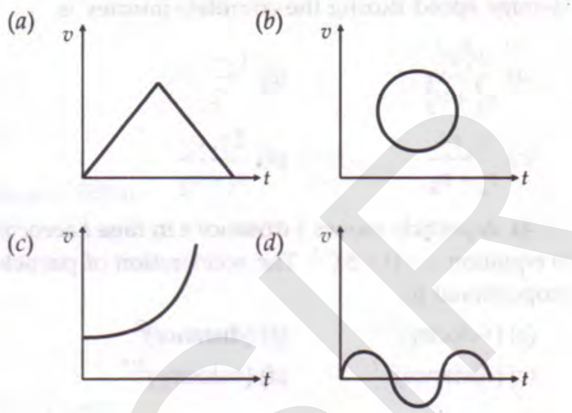
26. The water drop falls at regular intervals from a tap 5 m above the ground. The third drop is leaving the tap at instant the first drop touches the ground. How far above the ground is the second drop at that instant ?

(a) 3.75 m (b) 4.00 m
(c) 1.25 m (d) 2.50 m [CBSE PMT 95]

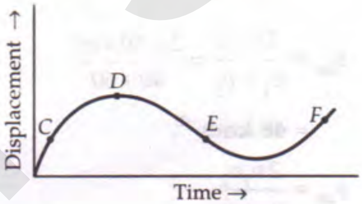
27. A rubber ball is dropped from a height of 5 m on a plane. On bouncing it rises to 1.8 m. The ball loses its velocity on bouncing by a factor of

(a) $\frac{3}{5}$ (b) $\frac{2}{5}$
(c) $\frac{16}{25}$ (d) $\frac{9}{25}$ [CBSE PMT 98]

28. Which of the following curves does not represent motion in one dimension ? [CBSE PMT 92]



29. The displacement-time graph of a moving particle is shown ahead. The instantaneous velocity of the particle is negative at the point



(a) E (b) F
(c) C (d) D [CBSE PMT 94]

30. A train of 150 metre length is going towards north direction at a speed of 10 m/s. A parrot flies at the speed of 5 m/s towards south direction parallel to the railways track. The time taken by the parrot to cross the train is

(a) 12 sec (b) 8 sec
(c) 15 sec (d) 10 sec. [CBSE PMT 88]

31. A particle starts its motion from rest under the action of a constant force. If the distance covered in first 10 seconds is S_1 and that covered in the first 20 seconds is S_2, then

(a) $S_2 = S_1$ (b) $S_2 = 2S_1$
(c) $S_2 = 3S_1$ (d) $S_2 = 4S_1$ [CBSE PMT 09]

32. A bus is moving with a speed of $10\,\text{ms}^{-1}$ on a straight road. A scooterist wishes to overtake the bus in 100 s. If the bus is at a distance of 1 km from the scooterist, with what speed should the scooterist chase the bus ?

(a) $10\,\text{ms}^{-1}$ (b) $20\,\text{ms}^{-1}$
(c) $40\,\text{ms}^{-1}$ (d) $25\,\text{ms}^{-1}$ [CBSE PMT 09]

33. A particle covers half of its total distance with speed v_1 and the rest half distance with speed v_2. Its average speed during the complete journey is

(a) $\dfrac{v_1^2 v_2^2}{v_1^2+v_2^2}$ (b) $\dfrac{v_1+v_2}{2}$

(c) $\dfrac{v_1 v_2}{v_1+v_2}$ (d) $\dfrac{2v_1 v_2}{v_1+v_2}$ [CBSE Final 2011]

34. A particle moves a distance x in time t according to equation $x=(t+5)^{-1}$. The acceleration of particle is proportional to

(a) $(\text{velocity})^{3/2}$ (b) $(\text{distance})^2$

(c) $(\text{distance})^{-2}$ (d) $(\text{velocity})^{2/3}$

[CBSE Pre 2010]

35. A boy standing at the top of a tower of 20 m height drops a stone. Assuming $g=10\,\text{ms}^{-2}$, the velocity with which it hits the ground is

(a) 10.0 m/s (b) 20.0 m/s

(c) 40.0 m/s (d) 5.0 m/s [CBSE Pre 2011]

36. A ball is dropped from a high rise platform at $t=0$ starting from rest. After 6 seconds another ball is thrown downwards from the same platform with a speed v. The two balls meet at $t=18\,\text{s}$. What is the value of v?

(take $g=10\,\text{m/s}^2$)

(a) 75 m/s (b) 55 m/s

(c) 40 m/s (d) 60 m/s [CBSE Pre 2010]

Answers and Explanations

1. (b) $v_{av}=\dfrac{2v_1v_2}{v_1+v_2}=\dfrac{2\times40\times60}{40+60}$

$= \mathbf{48\ kmh^{-1}}.$

2. (b) $v_{av}=\dfrac{2v_1v_2}{v_1+v_2}$

$\therefore \quad 48=\dfrac{2\times40\times v}{40+v}$

or $\quad v=\mathbf{60\ km/h}.$

3. (c) Total distance travelled $= s$

Total time taken,

$$t=\frac{s/3}{10}+\frac{s/3}{20}+\frac{s/3}{60}$$

$$=\frac{10s}{180}=\frac{s}{18}$$

$$v_{av}=\frac{s}{t}=\frac{s}{s/18}$$

$= \mathbf{18\ km/h}.$

4. (d) $v_{av}=\dfrac{s+s}{\dfrac{s}{v_u}+\dfrac{s}{v_d}}=\dfrac{2v_uv_d}{v_d+v_u}.$

5. (c) $s=12t+3t^2-2t^3$

$$v=\frac{ds}{dt}=12+6t-6t^2$$

At $t=0$, $\quad v=\mathbf{12\ ms^{-1}}.$

6. (d) Given : $x=40+12t-t^3$

$$v=\frac{dx}{dt}=0+12-3t^2$$

Velocity becomes zero, when

$$12-3t^2=0$$

or $\quad t=2\text{ s}$

Distance travelled by particle before coming to rest,

$$x=40+12\times2-2^3=\mathbf{56\ m}.$$

7. (c) $x=at^2-bt^3$

$$v=\frac{dx}{dt}=2at-3bt^2.$$

$$\text{Acceleration}=\frac{dv}{dt}=2a-6bt=0$$

or $\quad t=\dfrac{a}{3b}.$

8. (b) $s=3t^3+7t^2+14t+8$

$$v=\frac{ds}{dt}=9t^2+14t+14$$

$$a=18t+14$$

At $t=1$ s,

$$a=18\times1+14=\mathbf{32\ ms^{-2}}.$$

9. (c) $s=t^3-6t^2+3t+4$

$$v=\frac{ds}{dt}=3t^2-12t+3$$

$$a=6t-12=0$$

or $\quad t=2\text{ s}$

At $t=2$ s,

$$v=3\times2^2-12\times2+3=\mathbf{-9\ m/s}.$$

10. (*d*) $x = ae^{-\alpha t} + be^{\beta t}$

$$v = \frac{dx}{dt} = -a\alpha e^{-\alpha t} + b\beta e^{\beta t}$$

Clearly, v is not independent of α and β.

Moreover, it does not become zero for $\alpha = \beta$. Infact, the velocity increases with time because

$$\frac{dv}{dt} = a\alpha^2 e^{-\alpha t} + b\beta^2 e^{\beta t} > 0$$

11. (*a*) $x = 9t^2 - 3t^3$

$$v = \frac{dx}{dt} = 18t - 9t^2$$

For maximum speed, $\frac{dv}{dt} = 0$

or $18t - 9t^2 = 0 \Rightarrow t = 3$ s.

$\therefore$ At maximum speed, position of the particle is

$$x = 9 \times 3^2 - 3^3 = 81 - 27 = \mathbf{54\ m}.$$

12. (*c*) Given : $\frac{dv}{dt} = -kv^3$

or $v^{-3}\,dv = -k\,dt$

On integration,

$$\int v^{-3}\,dv = -k\int dt$$

or $\frac{v^{-2}}{-2} = -kt + C$

or $\frac{1}{2v^2} = kt - C$

where C is constant of integration.

At $t = 0$, $v = v_0$

$\therefore$ $\frac{1}{2v_0^2} = 0 - C$

Hence $\frac{1}{2v^2} = kt + \frac{1}{2v_0^2} = \frac{2v_0^2 kt + 1}{2v_0^2}$

or $v = \frac{v_0}{\sqrt{2v_0^2 kt + 1}}$.

13. (*c*) At time $t = 0$, velocity $v = 0$.

Acceleration, $f = f_0\left(1 - \frac{t}{T}\right)$

When $f = 0$, $f_0\left(1 - \frac{t}{T}\right) = 0$

As f_0 is constant

$$1 - \frac{t}{T} = 0 \quad \text{or} \quad t = T$$

Now acceleration,

$$f = \frac{dv}{dt}$$

$\therefore$ $\int_0^{v_x} dv = \int_0^T f\,dt = \int_0^T f_0\left(1 - \frac{t}{T}\right)dt$

or $[v]_0^{v_x} = \left[f_0 t - \frac{f_0 t^2}{2T}\right]_0^T = f_0 T - \frac{f_0 T^2}{2T}$

$$v_x = \frac{1}{2} f_0 T.$$

14. (*c*) $a = \frac{dv}{dt} = bt$

$\therefore$ $v = \int bt\,dt = \frac{bt^2}{2} + C$

At $t = 0$, $v = v_0$

$\therefore$ $v_0 = C$

Hence $v = \frac{dx}{dt} = \frac{bt^2}{2} + v_0$

or $x = \int\left(\frac{bt^2}{2} + v_0\right)dt$

$$= \frac{bt^3}{6} + v_0 t + C'$$

At $t = 0$, $x = 0$

$\Rightarrow$ $C' = 0$

$\therefore$ $\mathbf{x = v_0 t + \frac{1}{6} bt^3}$.

15. (*d*) $u = 0$, $v = 144\ \text{kmh}^{-1} = 40\ \text{ms}^{-1}$, $t = 20$ s

$$a = \frac{v - u}{t} = \frac{40 - 0}{20} = 2\ \text{ms}^{-1}.$$

$$s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 2 \times (20)^2 = \mathbf{400\ m}.$$

16. (*a*) $a = \frac{v - u}{t} = \frac{60 - 20}{4} = 10\ \text{km/h}^2$

$$s = ut + \frac{1}{2}at^2 = 20 \times 4 + \frac{1}{2} \times 10 \times 4^2$$

$$= \mathbf{160\ km}.$$

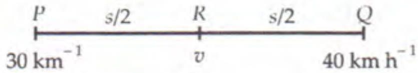
17. (*c*) $v^2 - u^2 = 2as$

$\therefore$ $0^2 - u_1^2 = 2as_1$ and $0^2 - u_2^2 = 2as_2$

or $\frac{s_2}{s_1} = \frac{u_2^2}{u_1^2} = \frac{(80)^2}{(40)^2} = 4$

$$s_2 = 4s_1 = 4 \times 2\ \text{m} = \mathbf{8\ m}.$$

18. (*c*) The situation is shown below :



As $\quad v^2 - u^2 = 2as$

$\therefore \quad (40)^2 - (30)^2 = 2as$

$$a = \frac{1600 - 900}{2s} = \frac{350}{s}$$

Again, $\quad v^2 - (30)^2 = 2a \times \frac{s}{2}$

or $\quad v^2 - (30)^2 = 2 \times \frac{350}{s} \times \frac{s}{2}$

or $\quad v = \mathbf{25\sqrt{2}}$ **km/h.**

19. (*d*) Refer to the solution of Example 21(*i*) on page 3.14.

20. (*b*) Suppose the ball takes time T to reach the maximum height H.

As $\quad v = u + at$

$\therefore \quad 0 = u - gT$

or $\quad T = \frac{u}{g}$

Also, $\quad s = ut + \frac{1}{2}at^2$

$\therefore \quad H = uT - \frac{1}{2}gT^2$

Height attained by the ball in time $(T - t)$,

$$H' = u(T - t) - \frac{1}{2}g(T - t)^2$$

Height covered by the ball in last t seconds is given by

$$h = H - H'$$

$$= uT - \frac{1}{2}gT^2 - \left[u(T - t) - \frac{1}{2}g(T - t)^2\right]$$

or $\quad h = ut + \frac{1}{2}gt^2 - gtT = ut + \frac{1}{2}gt^2 - gt \cdot \frac{u}{g}$

or $\quad h = \mathbf{\frac{1}{2}gt^2}$.

21. (*b*) $\quad s_{nth} = u + \frac{a}{2}(2n - 1)$

Here $u = 0, \; a = g$

$\therefore \quad s_{4th} = \frac{g}{2}(2 \times 4 - 1) = \frac{7g}{2}$

$$s_{5th} = \frac{g}{2}(2 \times 5 - 1) = \frac{9g}{2}$$

$$\frac{s_{4th}}{s_{5th}} = \frac{7}{9} = \mathbf{7 : 9.}$$

22. (*a*) $\quad s = ut + \frac{1}{2}gt^2 = 0 + \frac{1}{2}gt^2$

$\therefore \quad \frac{t_1}{t_2} = \sqrt{\frac{s_1}{s_2}} = \sqrt{\frac{16}{25}} = \mathbf{\frac{4}{5}}$.

23. (*a*) Speed becomes zero at maximum height.

$$v^2 - u^2 = 2as$$

$$0 - (10)^2 = -2 \times 10 \times \frac{h}{2}$$

$\therefore \quad h = \mathbf{10\ m}$.

24. (*b*) Height covered in t seconds,

$$h = \frac{1}{2}gt^2$$

Height covered in $(t - 2)$ seconds,

$$h' = \frac{1}{2}g(t - 2)^2$$

Height covered in last two seconds,

$$h - h' = \frac{1}{2}g[t^2 - (t - 2)^2]$$

$$40 = \frac{1}{2} \times 10(4t - 4)$$

or $\quad t = 3$ s .

$\therefore \quad h = \frac{1}{2} \times 10 \times (3)^2 = \mathbf{45\ m}$.

25. (*a*) We use, $v^2 - u^2 = 2as$

For first body : $\quad 3^2 - 0^2 = 2gh$

For second body : $v^2 - 4^2 = 2gh$

$\therefore \quad v^2 - 4^2 = 3^2$

or $\quad v = \sqrt{25} = \mathbf{5\ m/s}$.

26. (*a*) Time t taken by a drop to reach the ground is given by

$$s = ut + \frac{1}{2}at^2$$

$$5 = 0 + \frac{1}{2} \times 10 \times t^2 \qquad \text{or} \qquad t = 1\ \text{s}$$

When the third drop leaves the tap, the first drop reaches the ground. So a drop leaves the tap at interval of $\frac{1}{2}$s. Distance covered by second drop during this time,

$$s = 0 + \frac{1}{2} \times 10 \times \left(\frac{1}{2}\right)^2 = 1.25\ \text{m}$$

Height of the second drop from the ground

$$= 5 - 1.25 = \mathbf{3.75\ m}.$$

27. (b) When the ball falls down,

$$v^2 - 0^2 = 2 \times 10 \times 5 \quad \text{or} \quad v = 10 \text{ m/s}$$

When the ball rises up,

$$0^2 - u^2 = -2 \times 10 \times 1.8 \quad \text{or} \quad u = 6 \text{ m/s}$$

The factor by which ball loses velocity

$$= \frac{10-6}{10} = \frac{\mathbf{2}}{\mathbf{5}}.$$

28. (b) In one-dimensional motion, a body can have only one value of velocity at any time and not two values of velocity.

29. (a) Instantaneous velocity,

$$v = \frac{ds}{dt}$$

= Slope of the displacement-time curve at any instant.

Slope and hence instantaneous velocity is negative at point E.

30. (d) $v_T = +10 \text{ms}^{-1}, \; v_P = -5 \text{ms}^{-1}$

Relative velocity of parrot w.r.t. train,

$$= v_p - v_T = -5 - 10 = -15 \text{ ms}^{-1}.$$

Time taken by parrot to cross the train

$$= \frac{150 \text{ m}}{15 \text{ms}^{-1}} = \mathbf{10 \text{ s}}.$$

31. (d) $S_1 = \frac{1}{2}at_1^2$ and $S_2 = \frac{1}{2}at_2^2$

$$\therefore \quad \frac{S_2}{S_1} = \left(\frac{t_2}{t_1}\right)^2 = \left(\frac{20}{10}\right)^2 = 4$$

or $\quad S_2 = \mathbf{4S_1}$.

32. (b) Relative velocity of scooter w.r.t. bus,

$$v_{SB} = \frac{1 \text{ km}}{100 \text{ s}} = \frac{1000 \text{ m}}{100 \text{ s}} = 10 \text{ ms}^{-1}$$

But $\quad v_{SB} = v_S - v_B$

$$\therefore \quad v_S = v_{SB} + v_B = 10 + 10 = \mathbf{20 \text{ ms}^{-1}}.$$

33. (d) $v_{av} = \dfrac{\text{Total distance}}{\text{Total time}}$

$$= \frac{d}{\frac{d/2}{v_1} + \frac{d/2}{v_2}} = \frac{\mathbf{2v_1 v_2}}{\mathbf{v_1 + v_2}}$$

34. (a) $x = (t+5)^{-1}$

$$v = \frac{dx}{dt} = \frac{-1}{(t+5)^2}$$

$$a = \frac{dv}{dt} = \frac{2}{(t+5)^3}$$

As $\quad \frac{1}{t+5} \propto v^{1/2}$

$$\therefore \quad a \propto \frac{1}{(t+5)^3} \propto v^{3/2}$$

35. (b) $v = \sqrt{2gh} = \sqrt{2 \times 10 \times 20} = \sqrt{400}$

$$= \mathbf{20 \text{ ms}^{-1}}$$

36. (a) Distance moved in 18 s by first ball

$$= \frac{1}{2} \times 10 \times 18^2$$

$$= 90 \times 18 = 1620 \text{ m}$$

Distance moved in 12 s by second ball

$$= ut + \frac{1}{2}gt^2$$

$$\therefore \quad 1620 = 12v + 5 \times 144$$

$$v = 132 - 60 = \mathbf{75 \text{ ms}^{-1}}$$

Delhi PMT and VMMC Entrance Exam

(Similar Questions)

1. A particle is constrained to move on a straight line path. It returns to the starting point after 10 s. The total distance covered by the particle during this time is 30 m. Which of the following statements about the motion of the particle is false ?

(a) displacement of the particle is zero

(b) displacement of the particle is 30 m

(c) average speed of the particle is 3 m/s

(d) both (a) and (c)

[DPMT 92]

2. A car travels first half of the distance between two places with a speed of 30 km/hr and remaining half with a speed of 50 km/hr. The average speed of the car is

(a) 37.5 km/hr (b) 42 km/hr

(c) 40 km/h (d) 49 km/h [DPMT 07]

3. If the first one-third of a journey is travelled at 20 km/h, next one-third at 40 km/h and the last one-third at 60 km/h, the average speed of whole journey will be

(a) 32.7 km/h (b) 35 km/h

(c) 40 km/h (d) 45 km/h [VMMC 07]

4. A car travelling on a straight track moves with uniform velocity of v_1 for some time and with uniform velocity of v_2 for the next equal time. Average velocity of the car is

(a) $\sqrt{v_1 v_2}$ (b) $\left(\frac{1}{v_1}+\frac{1}{v_2}\right)^{-1}$

(c) $\frac{v_1+v_2}{2}$ (d) $2\left(\frac{1}{v_1}+\frac{1}{v_2}\right)^{-1}$ [DPMT 93, 96]

5. Given $a=2t+5$. Calculate the velocity of the body after five sec if it starts from rest.

(a) 50 m/s (b) 25 m/s

(c) 100 m/s (d) 75 m/s [DPMT 07]

6. The displacement of a particle moving in straight line is given by $x=2t^2+t+5$, where x is expressed in metres and t in seconds. The acceleration at $t=2$ sec is

(a) $4\,m/s^2$ (b) $10\,m/s^2$

(c) $8\,m/s^2$ (d) $15\,m/s^2$ [DPMT 96]

7. The velocity of a particle at an instant is 10 m/s. After 3 s its velocity will become 16 m/s. The velocity at 2 s, before the given instant will be

(a) 6 m/s (b) 4 m/s

(c) 2 m/s (d) 1 m/s [VMMC 06]

8. A body initially at rest is moving with uniform acceleration a. Its velocity after n seconds is v. The displacement of the body in last 2 s is

(a) $\frac{2v(n-1)}{n}$ (b) $\frac{v(n-1)}{n}$

(c) $\frac{v(n+1)}{n}$ (d) $\frac{2v(n+1)}{n}$ [VMMC 06]

9. Velocity of a body on reaching the point from which it was projected upwards, is

(a) $v=0$ (b) $v=0.5u$

(c) $v=2u$ (d) $v=u$ [DPMT 99]

10. If a ball is thrown vertically upwards with 40 m/s, its velocity after two sec will be

(a) 10 m/s (b) 30 m/s

(c) 20 m/s (d) 40 m/s [DPMT 95, 99]

11. A stone released with zero velocity from top of the tower reaches the ground in 4 sec. The height of the tower is about

(a) 20 m (b) 80 m

(c) 40 m (d) 160 m [DPMT 99, 04]

12. A stone is thrown vertically up from the ground. It reaches a maximum height of 50 m in 10 sec. After what time it will reach the ground from maximum height position ?

(a) 1.2 sec (b) 10 sec

(c) 5 sec (d) 25 sec [DPMT 94, 98]

13. A stone falls freely such that the distance covered by it in the last second of its motion is equal to the distance covered by it in the first 5 seconds. It remained in air for

(a) 12 sec (b) 13 sec

(c) 25 sec (d) 26 sec [VMMC 03]

14. A body sliding on a smooth inclined plane requires 4 seconds to reach the bottom, starting from rest at the top. How much time does it take to cover one-fourth the distance starting from rest at the top ?

(a) 1 sec (b) 4 sec

(c) 2 sec (d) 16 sec [DPMT 93]

15. A body falling from rest describes distances s_1, s_2 and s_3 in the first, second and third seconds of its fall, then the ratio $s_1 : s_2 : s_3$ is

(a) 1 : 1 : 1 (b) 1 : 3 : 5

(c) 1 : 2 : 3 (d) 1 : 4 : 9 [DPMT 90]

16. When a ball is thrown vertically upwards, at the maximum height

(a) the velocity is zero and therefore there is no acceleration acting on the particle

(b) the acceleration is present and therefore velocity is not zero

(c) the acceleration depends on the velocity as $a=\frac{dv}{dt}$

(d) the acceleration is independent of the velocity [DPMT 06]

17. A police jeep is chasing with velocity 45 km/h. A thief in another jeep is moving with 155 km/h. Police fires a bullet with a muzzle velocity 180 m/s. The bullet strikes the jeep of the thief with a velocity

(a) $27\,ms^{-1}$ (b) $150\,ms^{-1}$

(c) $250\,ms^{-1}$ (d) $450\,ms^{-1}$ [DPMT 06]

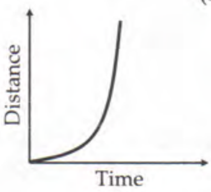
18. Velocity-time curve for a body projected vertically upwards is

(a) ellipse (b) hyperbola

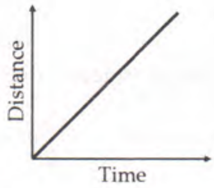
(c) parabola (d) straight line [DPMT 95]

19. Which of the following distance-time graph shows accelerated motion ?

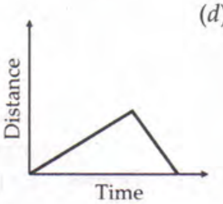
(*a*)



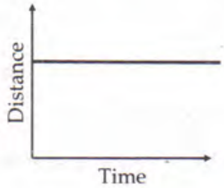
(*b*)



(*c*)



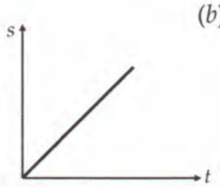
(*d*)



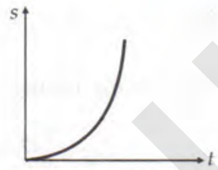
[DPMT 2K]

20. Which of the following graph represents uniformly accelerated motion ? [DPMT 01]

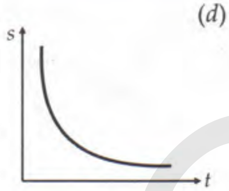
(*a*)



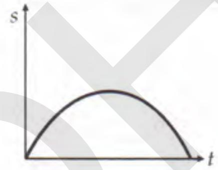
(*b*)



(*c*)



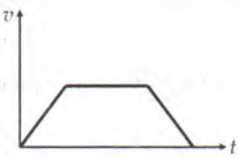
(*d*)



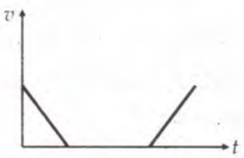
21. Acceleration-time graph of a body is shown.

The corresponding velocity-time graph of the same body is :

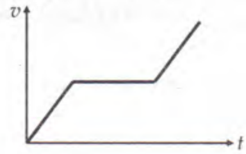
(*a*)



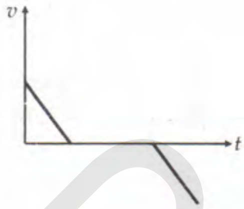
(*b*)



(*c*)

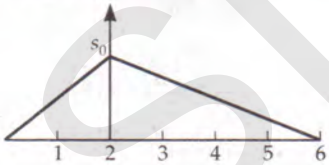


(*d*)



[DPMT 04]

22. What will be ratio of speed in first two seconds to the speed in next 4 seconds ?



(*a*) $\sqrt{2} : 1$ (*b*) $3 : 1$
(*c*) $2 : 1$ (*d*) $1 : 2$ [VMMC 02]

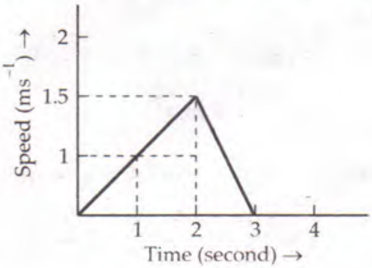
23. You drive a car at a speed of 70 km/h in a straight road for 8.4 km, and then the car runs out of petrol. You walk for 30 min to reach a petrol pump at a distance of 2 km. The average velocity from the beginning of your drive till you reach the petrol pump is

(*a*) 16.8 km/h (*b*) 35 km/h
(*c*) 64 km/h (*d*) 18.6 km/h. [DPMT 09]

24. Tom and Dick are running forward with the same speed. They are following a rubber ball to each other at a constant speed v as seen by the thrower. According to Sam who is standing on the ground the speed of ball is

(*a*) same as v (*b*) greater than v
(*c*) less than v (*d*) none of these. [DPMT 09]

25. The speed-time graph of a particle moving along a solid curve is shown below. The distance traversed by the particle from $t=0$ to $t=3$ is



(*a*) $\frac{9}{2}$ m (*b*) $\frac{9}{4}$ m
(*c*) $\frac{9}{3}$ m (*d*) $\frac{9}{5}$ m [DPMT 2011]

Answers and Explanations

1. (*d*) As the particle returns to the starting point, its displacement is zero.

Average speed $= \frac{30 \text{ m}}{10 \text{ s}} = \mathbf{3\ m/s}$.

2. (*a*) $$v_{av} = \frac{2v_1v_2}{v_1+v_2}$$
$$= \frac{2\times 30\times 50}{30+50} = \mathbf{37.5\ km/h}.$$

3. (*a*) $$v_{av} = \frac{3v_1v_2v_3}{v_1v_2+v_2v_3+v_1v_3}$$
$$= \frac{3\times 20\times 40\times 60}{20\times 40+40\times 60+20\times 60}$$
$$= \mathbf{32.7\ km/h}.$$

4. (*d*) Let total distance $=2x$

Total time taken $= \frac{x}{v_1} + \frac{x}{v_2} = x\left(\frac{1}{v_1}+\frac{1}{v_2}\right)$

Average speed $= \frac{2x}{x\left(\frac{1}{v_1}+\frac{1}{v_2}\right)} = 2\left(\frac{1}{v_1}+\frac{1}{v_2}\right)^{-1}$

5. (*a*) $$a = \frac{dv}{dt} = 2t+5$$
$$\int_0^v dv = \int_0^5 (2t+5)\,dt$$
$$v = [t^2]_0^5 + 5[t]_0^5$$
$$= (25-0)+5(5-0)$$
$$= \mathbf{50\ m/s}.$$

6. (*a*) Given : $x = 2t^2+t+5$
$$v = \frac{dx}{dt} = 4t+1$$
$$a = \frac{dv}{dt} = \mathbf{4\ m/s^2}.$$

7. (*a*) Here $u = 10$ m/s, $t = 3$ s, $v = 16$ m/s.

$\therefore$ $$a = \frac{v-u}{t} = \frac{16-10}{3} = 2 \text{ m/s}^2$$

Velocity at 2 s before the given instant,

$$10 = u + 2\times 2 \qquad [v = u+at]$$
$$u = 10-4 = \mathbf{6\ m/s}.$$

8. (*a*) $$a = \frac{v-u}{t} = \frac{v-0}{n} = \frac{v}{n}$$

Displacement in last 2 s,
$$s_n - s_{n-2} = \frac{1}{2}a^2 - \frac{1}{2}a(n-2)^2$$
$$= 2a(n-1) = 2\times\frac{v}{n}(n-1)$$
$$= \frac{\mathbf{2v(n-1)}}{\mathbf{n}}.$$

9. (*d*) $v = u$.

10. (*c*) $v = u + at = 40 - 10\times 2 = \mathbf{20\ m/s}$.

11. (*b*) $h = \frac{1}{2}gt^2 = \frac{1}{2}\times 10\times (4)^2 = \mathbf{80\ m}$.

12. (*b*) Time of ascent = Time of decent, in the absence of air friction.

13. (*b*) $s_{nth} = u + \frac{g}{2}(2n-1)$

For a freely falling body, $u = 0$

$\therefore$ $$s_{nth} = \frac{g}{2}(2n-1)$$

Distance travelled in first 5 s,
$$s = ut + \frac{1}{2}gt^2 = 0 + \frac{1}{2}g(5)^2 = \frac{25}{2}g$$

Given : $s_{nth} = s$

or $$\frac{g}{2}(2n-1) = \frac{25}{2}g$$

or $$n = \mathbf{13\ s}.$$

14. (*c*) Here : $u = 0$, $s = \frac{1}{2}at^2$ or $s \propto t^2$

$\therefore$ $$\frac{t_2}{t_1} = \sqrt{\frac{s_2}{s_1}} = \sqrt{\frac{s/4}{s}} = \frac{1}{2}$$

or $$t_2 = \frac{1}{2}\times 4 = \mathbf{2\ s}.$$

15. (*d*) $$s \propto t^2$$

$\therefore$ $s_1 : s_2 : s_3 = 1^2 : 2^2 : 3^2 = \mathbf{1 : 4 : 9}$.

16. (*d*) The acceleration is independent of velocity.

17. (*b*) Relative speed of bullet with respect to thief's jeep,
$$= (v_b + v_p) - v_t$$

$$= 180\,\text{ms}^{-1} + 45\,\text{kmh}^{-1} - 155\,\text{kmh}^{-1}$$

$$= 180\,\text{ms}^{-1} + 45 \times \frac{5}{18}\,\text{ms}^{-1} - 155 \times \frac{5}{18}\,\text{ms}^{-1}$$

$$= 180\,\text{ms}^{-1} - 30.5\,\text{ms}^{-1}$$

$$\approx \mathbf{150\ ms^{-1}}.$$

18. (*d*) As acceleration (g) remains constant throughout, the velocity-time graph is a straight line.

19. (*a*) For accelerated motion, distance-time graph is a parabola about distance-axis.

20. (*b*) For uniformly accelerated motion, s-t graph is a parabola symmetric about distance-axis.

21. (*c*) When acceleration a is constant, velocity v increases uniformly with time t and when $a = 0$, velocity v remains constant.

22. (*c*) $$\frac{v_1}{v_2} = \frac{\frac{s_0}{2-0}}{\frac{s_0}{6-2}}$$

$$= \frac{4}{2} = \mathbf{2:1}.$$

23. (*a*) Average speed

$$= \frac{\text{Total distance}}{\text{Total time}}$$

$$= \frac{s_1 + s_2}{t_1 + t_2} = \frac{8.4\text{ km} + 2\text{ km}}{\frac{8.4}{70}\text{h} + \frac{30}{60}\text{h}}$$

$$= \frac{10.4}{0.12 + 0.5} = \frac{10.4}{0.62}$$

$$\simeq \mathbf{16.8\ km/h}.$$

24. (*b*) As they are moving in the same direction, the relative velocity of ball w.r.t. Tom or Dick will be

$$v = v_B - v_{Tom}$$

or $$v = v_B - v_{Dick}$$

$\therefore$ $$v_B = v + v_{Tom} = v + v_{Dick}$$

For Sam, the speed of the ball will be greater than v.

25. (*b*)

Distance traversed = Area under speed-time graph

$$= \frac{1}{2} \times 1.5 \times 3 = \frac{9}{4}\,\mathbf{m}$$