CHAPTER

6

WORK, ENERGY AND POWER

6.1 WORK

1. *When is the work said to be done ? Give some examples.*

Work. *Work is said to be done whenever a force acts on a body and the body moves through some distance in the direction of the force.* Thus work is done on a body only if the following two conditions are satisfied :

(*i*) A force acts on the body.

(*ii*) The point of application of the force moves in the direction of the force.

Examples. Work is done when a horse pulls a cart, an engine pulls a train, a man goes up a hill, etc.

6.2 WORK DONE BY A CONSTANT FORCE

How is work done measured when (i) the force acts along the direction of motion of the body and (ii) the force is inclined to the direction of motion of the body ?

(*i*) **Measurement of work done when the force acts along the direction of motion.** As shown in Fig. 6.1, a force $\vec{F}$ displaces a body through a distance s parallel to the line of action of the force or in the direction of force.

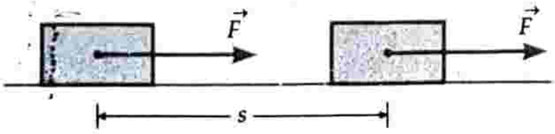


Fig. 6.1 Work done, when force and displacement are in same directions.

Work done = Force × distance moved in the direction of force

$$W = Fs$$

(*ii*) **Measurement of work done when force and displacement are inclined to each other.**

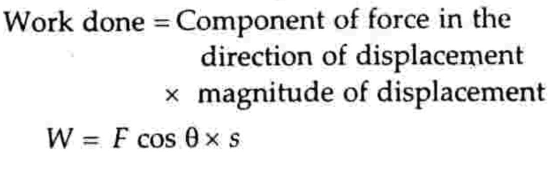


Fig. 6.2 Work done, when force is inclined to displacement.

Work done = Component of force in the direction of displacement × magnitude of displacement

$$W = F\cos\theta \times s$$

or $$W = Fs\cos\theta$$

or $$W = \vec{F}\cdot\vec{s}$$

Thus work done is the dot product of force and displacement vectors. Hence work is a scalar quantity.

6.3 NATURE OF WORK DONE IN DIFFERENT SITUATIONS

3. *What is meant by positive work, negative work and zero works ? Give examples of each type.*

Nature of work done. Although work done is a scalar quantity, yet its value may be positive, negative or zero, as discussed below.

Positive work. *If a force acting on a body has a component in the direction of the displacement, then the work done by the force is positive.* As shown in Fig. 6.3, when θ is acute, $\cos\theta$ is positive.

$\therefore \quad W = Fs\cos\theta$ = a positive value

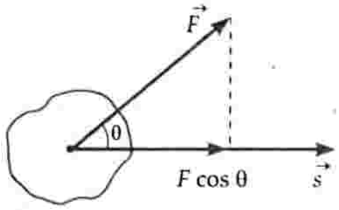


Fig. 6.3 Positive work ($\theta < 90°$).

Examples :

(*i*) When a body falls freely under gravity ($\theta = 0°$), the work done by the gravity is positive.

(*ii*) When a horse pulls a cart, the applied force and displacement are in the same direction, the work done by the horse is positive.

(*iii*) When a gas filled in a cylinder fitted with a movable piston is allowed to expand, the work done by the gas is positive, because the force due to gas pressure and displacement act in the same direction.

(*iv*) When a spring is stretched, both the stretching force and the displacement act in the same direction. So work done is positive.

Negative work. *If a force acting on the body has a component in the opposite direction of displacement, the work done is negative.* As shown in Fig. 6.4, when θ is *obtuse*, $\cos\theta$ is negative.

$\therefore \quad W = Fs\cos\theta$ = a negative value

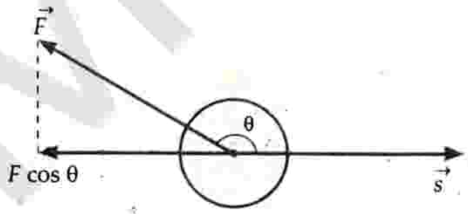


Fig. 6.4 Negative work ($\theta > 90°$)

Examples :

(*i*) When a body slides against a rough horizontal surface, its displacement is opposite to the force of friction. The work done by the friction is negative.

(*ii*) When brakes are applied to a moving vehicle, the work done by the braking force is negative. This is because the braking force and the displacement act in opposite directions.

(*iii*) When a body is lifted, the work done by the gravitational force is negative. This is because the gravitational force acts vertically downwards while the displacement is in the vertically upward direction.

(*iv*) When a positive charge is moved towards another positive charge, the work done by the force of repulsion (between them) is negative because displacement $\vec{s}$ is in the opposition direction of repulsive force $\vec{F}$.

Zero work. *Work done by force is zero if the body gets displaced along a direction perpendicular to the direction of the applied force. Also, the work done is zero if $\vec{F}$ or $\vec{s}$ or both are zero.*

Examples :

(*i*) For a body moving in a circular path, the centripetal force and displacement are perpendicular to each other, as shown in Fig. 6.5. So the *work done by the centripetal force is zero.*

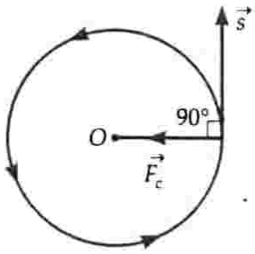


Fig. 6.5 Work done by centripetal force.

(*ii*) When a coolie walks on a horizontal platform with a load on his head, he applies a force on it in the upward direction equal to its weight. The displacement of the load is along the horizontal direction. Thus the angle between $\vec{F}$ and $\vec{s}$ is 90°. So $W = Fs\cos 90° = 0$ *i.e.*, the work done by the coolie on the load is zero.

(*iii*) As shown in Fig. 6.6, the tension in the string of a simple pendulum is always perpendicular to its displacement. So the work done by the tension is zero.

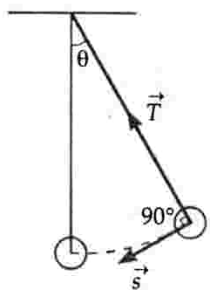


Fig. 6.6 Work done by tension.

(*iv*) The work done in pushing an immovable stone is zero, because displacement of the stone is zero.

6.4 DIMENSIONS AND UNITS OF WORK

4. *Write the dimensional formula of work.*

Dimensions of work.

As work done = Force × distance

$\therefore \quad [W] = MLT^{-2} \times L$

or $\quad [W] = [ML^2T^{-2}]$

This is the dimensional formula of work.

5. *Define the various absolute and gravitational units of work. Write relations between them.*

Units of work. As $W = Fs$

$\therefore$ 1 unit work = 1 unit force × 1 unit distance

Thus *unit work is the amount of work done when a unit force displaces a body through a unit distance in the direction of the force.*

A. Absolute units of work. *Work done is said to be one absolute unit if an absolute unit of force displaces a body through a unit distance in the direction of the force.*

(*i*) **Joule.** It is the absolute unit of work in SI, named after British physicist *James Prescott Joule* (1811 – 1869). *One joule of work is said to be done when a force of one newton displaces a body through a distance of one metre in its own direction.*

1 joule = 1 newton × 1 metre or 1 J = 1 Nm

(*ii*) **Erg.** It is the absolute unit of work in CGS system. *One erg of work is said to be done if a force of one dyne displaces a body through a distance of one centimetre in its own direction.*

1 erg = 1 dyne × 1 cm

Relation between joule and erg.

1 joule = 1 newton × 1 metre

$= 10^5$ dyne × 10^2 cm $= 10^7$ dyne cm

or $\quad$ 1 joule $= 10^7$ erg

B. Gravitational units of work. *Work is said to be done one gravitational unit if a gravitational unit of force displaces a body through a unit distance in the direction of the force.*

(*i*) **Kilogram metre.** It is the gravitational unit of work in SI. *One kilogram metre of work is said to be done when a force of one kilogram weight displaces a body through one metre in its own direction.*

1 kg m = 1 kg wt × 1 m = 9.8 N × 1 m

or $\quad$ 1 kg m = 9.8 J

(*ii*) **Gram centimetre.** It is gravitational unit of force in CGS system. *One gram centimetre of work is said to be done when a force of one gram weight displaces a body through one centimetre in its own direction.*

1 g cm = 1 g wt × 1 cm = 980 dyne × 1 cm

or $\quad$ **1 g cm = 980 erg**

6.5 WORK DONE IN TERMS OF RECTANGULAR COMPONENTS

6. *Write an expression for work done in terms of rectangular components of force and displacement.*

Work done in terms of rectangular components. In terms of rectangular components, the force and displacement vectors can be written as

$$\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$$

and

$$\vec{s} = s_x \hat{i} + s_y \hat{j} + s_z \hat{k}$$

$$\therefore \quad W = \vec{F} \cdot \vec{s}$$

$$= (F_x \hat{i} + F_y \hat{j} + F_z \hat{k}) \cdot (s_x \hat{i} + s_y \hat{j} + s_z \hat{k})$$

or

$$W = F_x s_x + F_y s_y + F_z s_z .$$

Examples based on Work Done by a Constant Force

FORMULAE USED

1. $W = \vec{F} \cdot \vec{s} = Fs \cos \theta$
2. If a body of mass m is raised through height h, then $W = mgh$
3. If a body moves up a plane inclined at an angle θ with a constant speed, then $W = mg \sin \theta \times s$

UNITS USED

In SI, force F is in newton, distance s in metre and work done W in joule. In CGS system, force F is in dyne, distance s in cm and work done W in erg.

CONVERSION USED

$1 \text{ J} = 10^7$ erg

EXAMPLE 1. *A gardener pushes a lawn roller through a distance of 20 m. If he applies a force of 20 kg wt in a direction inclined at 60° to the ground, find the work done by him. Take $g = 9.8 \text{ ms}^{-2}$.*

Solution. Here $F = 20$ kg wt $= 20 \times 9.8$ N,

$s = 20$ m, $\quad \theta = 60°$

$W = Fs \cos \theta = 20 \times 9.8 \times 20 \times \cos 60°$

$= 20 \times 9.8 \times 20 \times 0.5 =$ **1960 J.**

EXAMPLE 2. *A person is holding a bucket by applying a force of 10 N. He moves a horizontal distance of 5 m and then climbs up a vertical distance of 10 m. Find the total work done by him.*

Solution. For horizontal motion, the angle between force and displacement is 90°.

Here $\quad F = 10$ N, $s = 5$ m, $\theta = 90°$

Work done,

$$W_1 = Fs\cos\theta = 10 \times 5 \times \cos 90° = 0$$

For vertical motion, the angle between force and displacement is 0°.

Here $F = 10$ N, $s = 10$ m, $\theta = 0°$

Work done,

$$W_2 = 10 \times 10 \times \cos 0° = 100 \text{ J}$$

Total work done $= W_1 + W_2 = 0 + 100 =$ **100 J.**

Example 3. *A cyclist comes to a skidding stop in 10 m. During this process, the force on the cycle due to the road is 200 N and is directly opposed to the motion. (a) How much work does the road do on the cycle ? (b) How much work does the cycle do on the road ?* **[NCERT]**

Solution. Work done on the cycle by the road is the work done by the stopping force of friction on the cycle due to the road.

(*a*) The stopping force and the displacement make an angle of 180° with each other. Thus, work done by the road, or the work done by the stopping force is

$$W_r = Fs\cos\theta$$
$$= 200 \times 10 \times \cos 180° = \mathbf{-2000\ J.}$$

It is this negative work that brings the cycle to a halt.

(*b*) From Newton's Third law, an equal and opposite force acts on the road due to the cycle. Its magnitude is 200 N. However, the road undergoes no displacement.

∴ Work done by the cycle on the road = **zero.**

Example 4. *A force* $\vec{F} = \hat{i} + 5\hat{j} + 7\hat{k}$ *acts on a particle and displaces it through* $\vec{s} = 6\hat{i} + 9\hat{k}$. *Calculate the work done if the force is in newton and displacement in metre.* **[Delhi 2K]**

Solution. $W = \vec{F}\cdot\vec{s}$

$$= (\hat{i} + 5\hat{j} + 7\hat{k})\cdot(6\hat{i} + 0\hat{j} + 9\hat{k})$$
$$= 1 \times 6 + 5 \times 0 + 7 \times 9 = \mathbf{69\ J.}$$

Example 5. *A force* $\vec{F} = -K(y\hat{i} + x\hat{j})$, *where K is a positive constant, acts on a particle moving in the XY-plane. Starting from the origin, the particle is taken along the positive X-axis to a point (a, 0) and then parallel to the y-axis to the point (a, a). Calculate the total work done by the force on the particle.* **[IIT 98]**

Solution. Position vector of point (a, o),

$$\vec{r}_1 = a\hat{i} + o\hat{j}$$

Position vector of point (a, a),

$$\vec{r}_2 = a\hat{i} + a\hat{j}$$

Displacement vector,

$$\vec{r} = \vec{r}_2 - \vec{r}_1 = (a\hat{i} + a\hat{j}) - (a\hat{i} + o\hat{j}) = a\hat{j}$$

Also, $\vec{F} = -K(y\hat{i} + x\hat{j})$

Work done,

$$W = \vec{F}\cdot\vec{r} = -K(y\hat{i} + x\hat{j})\cdot a\hat{j} = -Kax.$$

As $x = a$

So $W = -\mathbf{Ka^2}.$

Example 6. *A uniform chain of length 2 m is kept on a table such that a length of 60 cm hangs freely from the edge of the table. The total mass of the chain is 4 kg. What is the work done in pulling the entire chain on the table ? Take* $g = 10\ ms^{-2}$. **[AIEEE 04]**

Solution. Mass of length 2 m of the chain

$$= 4 \text{ kg}$$

Mass of length 60 cm or 0.60 m of the chain

$$= \frac{4 \times 0.60}{2} = 1.2 \text{ kg}$$

Weight of the hanging part of the chain

$$= 1.2 \times 10 = 12 \text{ N}$$

As the centre of gravity of the hanging part lies at its mid-point, *i.e.*, 30 cm or 0.30 m below the edge of the table, so the work required in pulling the hanging part on the table is

$$W = \text{Force} \times \text{Distance} = 12 \times 0.30 = \mathbf{3.6\ J.}$$

Example 7. *Calculate the work done in raising a stone of mass 5 kg and specific gravity 3, lying at the bed of a lake through a height of 5 m.*

Solution. Specific gravity of stone

$$= \frac{\text{Mass of stone in air}}{\text{Mass of an equal volume of water}}$$

or $$3 = \frac{5 \text{ kg}}{\text{Mass of an equal volume of water}}$$

∴ Mass of an equal volume of water $= \frac{5}{3}$ kg

According to the Archimedes' principle, loss in the weight of stone when immersed in water

$$= \text{weight of water displaced} = \tfrac{5}{3} \text{ kg wt}$$

∴ Apparent weight of stone in the lake

$$= 5 - \frac{5}{3} = \frac{10}{3} \text{ kg wt}$$

Force required to lift the stone in the lake,

$$F = \frac{10}{3} \text{ kg wt} = \frac{10 \times 9.8}{3} \text{ N}$$

Work done in raising the stone through a height of 5 m,

$$W = F \times s = \frac{10 \times 9.8}{3} \times 5 = \mathbf{163.3\ J.}$$

Example 8. A cluster of clouds at a height of 1000 m above the earth burst and enough rain fell to cover an area of 10^6 m^2 with a depth of 2 cm. How much work would have been done in raising water to the height of clouds ?

Take $g = 9.8 \text{ ms}^{-2}$ *and density of water* $= 10^3 \text{ kgm}^{-3}$.

Solution. Area, $A = 10^6 \text{ m}^2$

Depth, $d = 2 \text{ cm} = 2 \times 10^{-2} \text{ m}$

$\therefore$ Volume of water

$$= Ad = 10^6 \times 2 \times 10^{-2} = 2 \times 10^4 \text{ m}^3$$

Mass of water,

$$m = \text{Volume} \times \text{density}$$
$$= 2 \times 10^4 \times 10^3 = 2 \times 10^7 \text{ kg}$$

Force used in raising water to the height of clouds,

$$F = \text{Weight of water} = mg = 2 \times 10^7 \times 9.8 \text{ N}$$

Work done,

$$W = Fs = 2 \times 10^7 \times 9.8 \times 1000 = \mathbf{1.96 \times 10^{11} \text{ J}}.$$

Problems For Practice

1. What is the work done by a person in carrying a suitcase weighing 10 kg f on his head when he travels a distance of 5 m in the (*i*) vertical direction and (*ii*) horizontal direction ? Take $g = 9.8 \text{ ms}^{-2}$. **[Ans.** (*i*) 490 J (*ii*) Zero]
2. Calculate the amount of work done by a labourer who carries *n* bricks, each of mass *m*, to the roof of a house of height *h* by climbing up a ladder. (**Ans.** *n mgh*)
3. A man moves on a straight horizontal road with a block of mass 2 kg in his hand. If he covers a distance of 40 m with an acceleration of 0.5 ms^{-2}, find the work done by the man on the block during the motion. (**Ans.** 40 J)
4. A force $\vec{F} = (2\hat{i} - 6\hat{j})$ N is applied on a body, which is sliding over a floor. If the body is displaced through $(-3\hat{j})$ m, how much work is done by the force ? *[Himachal 09]* (**Ans.** 18 J)
5. Find the work done by force $\vec{F} = 2\hat{i} - 3\hat{j} + \hat{k}$ when its point of application moves from the point $A(1, 2, -3)$ to the point $B(2, 0, -5)$. (**Ans.** 6 units)
6. A particle is acted upon by constant forces $\vec{F_1} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ and $\vec{F_2} = -\hat{i} + 2\hat{j} - 3\hat{k}$, is displaced from the point $A(2, 1, 0)$ to the point $B(-3, -4, 2)$. Find the total work done by these forces. (**Ans.** 2 units)
7. A man weighing 50 kg f supports a body of 25 kg f on his head. What is the work done when he moves a distance of 20 m up an incline of 1 in 10 ? Take $g = 9.8 \text{ ms}^{-2}$. (**Ans.** 1470 J)

Hints

3. $W = Fs\cos 0° = mas \times 1 = 2 \times 0.5 \times 40 = \mathbf{40 \text{ J}}.$

4. $W = \vec{F}\cdot\vec{s} = (2\hat{i} - 6\hat{j})\cdot(-3\hat{j}) = \mathbf{18 \text{ J}}.$

6. Resultant force, $\vec{F} = \vec{F_1} + \vec{F_2}$

$$= (2\hat{i} - 3\hat{j} + 4\hat{k}) + (-\hat{i} + 2\hat{j} - 3\hat{k}) = \hat{i} - \hat{j} + \hat{k}$$

Displacement,

$$\vec{s} = AB = \vec{OB} - \vec{OA} = (-3\hat{i} - 4\hat{j} + 2\hat{k}) - (2\hat{i} + \hat{j})$$
$$= -5\hat{i} - 5\hat{j} + 2\hat{k}$$
$$W = \vec{F}\cdot\vec{s} = (\hat{i} - \hat{j} + \hat{k})\cdot(-5\hat{i} - 5\hat{j} + 2\hat{k})$$
$$= 1 \times (-5) - 1 \times (-5) + 1 \times 2 = \mathbf{2 \text{ units}}.$$

7. Here $m = 50 + 25 = 75$ kg, $\sin\theta = \frac{1}{10}$,

$s = 10$ m, $g = 9.8 \text{ ms}^{-2}$

Force needed to be applied against gravity

$$= mg\sin\theta$$
$$\therefore \quad W = Fs = mg\sin\theta \times s$$
$$= 75 \times 9.8 \times \frac{1}{10} \times 20 = \mathbf{1470 \text{ J}}.$$

6.6 WORK DONE BY A VARIABLE FORCE

7. *Explain how work done by a variable force may be measured.*

Work done by a variable force. Suppose a variable force F acts on a body along the fixed direction, say x-axis. The magnitude of the force F depends on x, as shown by force-displacement graph in Fig. 6.7(*a*). Let us calculate the work done when the body moves from the initial position x_i to the final position x_f under the force F.

The displacement can be divided into a large number of small equal displacements Δx. During a small displacement Δx, the force F can be assumed to be constant. Then the work done is

$$W \simeq F\,\Delta x = \text{Area of rectangle } abcd$$

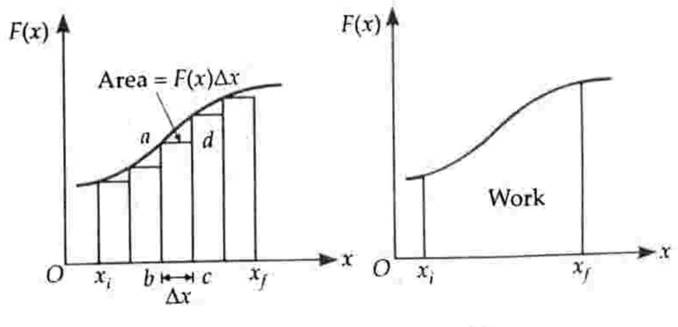


Fig. 6.7 Calculation of work done by a variable force $F(x)$ in moving a body from position x_i to x_f.

Adding areas of all the rectangles in Fig. 6.7(*a*), we get the total work done as

$$W \simeq \sum_{x_i}^{x_f} F\,\Delta x$$

= Sum of areas of all rectangles erected over all the small displacements

In the limit when $\Delta x \to 0$, the number of rectangles tends to be infinite, but the above summation approaches a definite integral whose value is equal to the area under the curve. Thus the total work done is

$$W = \lim_{\Delta x \to 0} \sum_{x_i}^{x_f} F\,\Delta x = \int_{x_i}^{x_f} F\,dx$$

= Area under the force-displacement curve

Hence, for a varying force the work done is equal to the definite integral of the force over the given displacement.

Note When the force varies both in magnitude and direction, the work done is given by

$$W = \int_{s_1}^{s_2} F\,ds\cos\theta = \int_{s_1}^{s_2} \vec{F}\cdot d\vec{s}$$

Examples based on Work done by a Variable Force

FORMULAE USED

1. $W = \sum_i \vec{F}_i \cdot \vec{s}_i$
2. $W = \int_{s_1}^{s_2} \vec{F}\cdot d\vec{s}$
3. W = Area under the force-displacement curve between the initial and final positions of the body.

UNITS USED

In SI, force F is in newton, distance s in metre and work done W in joule.

EXAMPLE 9. *A 2 kg particle starts at the origin and moves along the positive x-axis. The net force acting on it measured at intervals of 1 m is : 27.9, 28.3, 30.9, 34.0, 34.5, 46.9, 48.2, 50.0, 63.5, 13.6, 12.2, 32.7, 46.6 and 27.0 (in newtons). What is the total work done on the particle in this interval ?*

Solution. As the forces and displacements are in same direction, so

$$\begin{aligned} W = \sum_i F_i\,s_i &= 27.9\times1 + 28.3\times1 + 30.9\times1 \\ &+ 34.0\times1 + 46.9\times1 + 48.2\times1 + 50.0\times1 \\ &+ 63.5\times1 + 13.6\times1 + 12.2\times1 + 46.6\times1 + 27.0\times1 \\ &= \mathbf{496.3\ J.} \end{aligned}$$

EXAMPLE 10. *A body moves from point A to B under the action of a force, varying in magnitude as shown in Fig. 6.8. Obtain the work done. Force is expressed in newton and displacement in metre.*

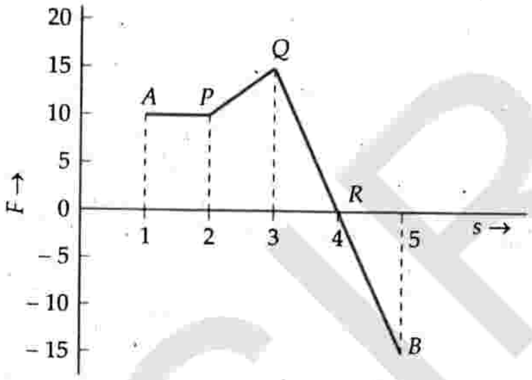


Fig. 6.8

Solution. Work done = Area under *F-s* curve

$$W_{AB} = W_{12} + W_{23} + W_{34} + W_{45}$$

= Area under AP + Area under PQ + Area under QR – Area above RB

$$= 10\times1 + \tfrac{1}{2}(10+15)\times1 + \tfrac{1}{2}\times1\times15 - \tfrac{1}{2}\times1\times15$$

$$= 10 + 12.5 = \mathbf{22.5\ J.}$$

EXAMPLE 11. *A woman pushes a trunk on railway platform which has a rough surface. She supplies a force of 100 N over a distance of 10 m. Thereafter she gets progressively tired and her applied force reduces linearly with distance to 50 N. The total distance by which the trunk has been moved is 20 m. Plot the force applied by the woman and the frictional force, which is 50 N. Calculate the work done by the two forces over 20 m.* **[NCERT]**

Solution. Plots of force F applied by the woman and the opposing frictional force F are shown in Fig. 6.9.

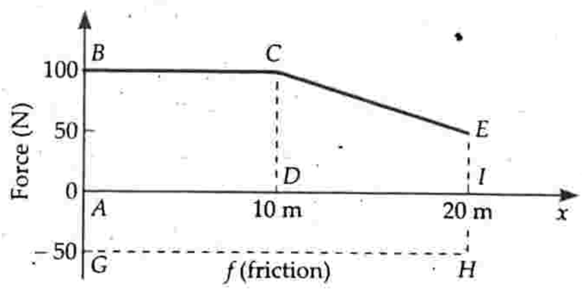


Fig. 6.9

Clearly at $x = 20$ m, $F = 50$ N

As the force of friction f [= 50 N] opposes the applied force F, so it has been shown on the negative side of the force-axis.

Work done by the force F applied by the woman

$$W_F = \text{Area of rectangle } ABCD + \text{Area of trapezium } CEID$$

$$= 100 \times 10 + \frac{1}{2}(100 + 50) \times 10 = 1000 + 750 = \mathbf{1750\ J.}$$

Work done by the frictional force,

$$W_f = \text{Area of rectangle } AGHI$$

$$= (-50) \times 20 = \mathbf{-1000\ J.}$$

EXAMPLE 12. *A particle moves along the X-axis from $x = 0$ to $x = 5$ m under the influence of a force given by $F = 7 - 2x + 3x^2$. Find the work done in the process.* **[CBSE 94]**

Solution. Work done in moving the particle from $x = 0$ to $x = 5$ m will be

$$W = \int_0^5 F dx = \int_0^5 (7 - 2x + 3x^2)\, dx$$

$$= \left[7x - \frac{2x^2}{2} + \frac{3x^3}{3}\right]_0^5 = [7x - x^2 + x^3]_0^5$$

$$= 35 - 25 + 125 = \mathbf{135\ J.}$$

PROBLEMS FOR PRACTICE

1. A force $F = (15 + 0.50x)$ acts on a particle in the X-direction, where F is in newton and x in metre. Find the work done by this force during a displacement from $x = 0$ to $x = 2.0$ m. **(Ans.** 31 J)

2. A force $F = a + bx$ acts on a particle in the X-direction, where a and b are constants. Find the work done by this force during a displacement from $x = 0$ to $x = d$. **(Ans.** $\left(a + \frac{bd}{2}\right)d$)

3. A body moves from a point A to B under the action of a force shown in Fig. 6.10. Force F is in newton and distance x in metre. What is the amount of work done ? **(Ans.** 11.5 J)

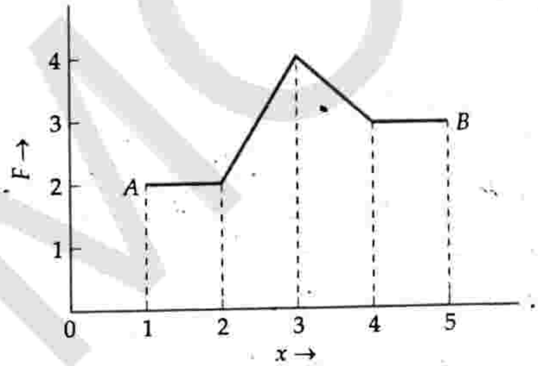


Fig. 6.10

4. The relation between the displacement x and the time t for a body of mass 2 kg moving under the action of a force is given by $x = t^3/3$, where x is in metre and t in second, calculate the work done by the body in first 2 seconds. **(Ans.** 16 J)

5. Fig. 6.11 shows the F-x graph. Here the force F is in newton and distance x in metre. What is the work done ? **(Ans.** 10 J)

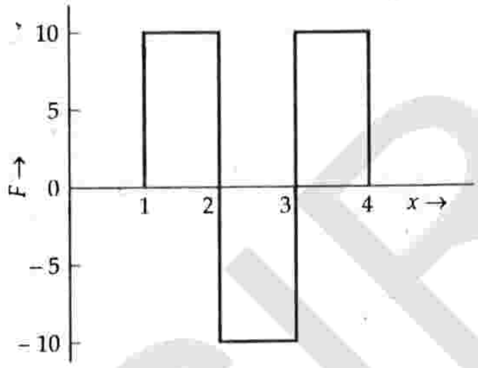


Fig. 6.11

6. Calculate work done in moving the object from $x = 2$ m to $x = 3$ m from the following graph :

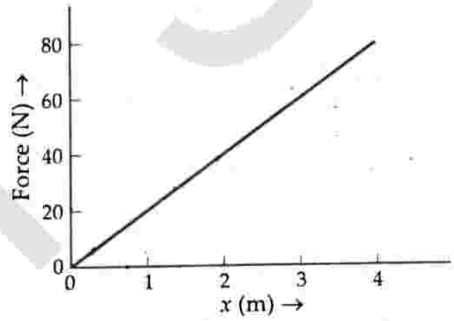


Fig. 6.12

[Central Schools 03] **(Ans.** 50 J)

HINTS

2. $dW = F dx = (a + bx)\, dx$

Total work done in displacement from $x = 0$ to $x = d$ will be

$$W = \int dW = \int_0^d (a + bx)\, dx = \left[ax + \frac{bx^2}{2}\right]_0^d$$

$$= ad + \frac{bd^2}{2} = \left(\boldsymbol{a} + \frac{\boldsymbol{bd}}{\mathbf{2}}\right)\boldsymbol{d}.$$

6. At $x = 2$ m,

$F = 40$ N

At $x = 3$ m,

$F = 60$ N

Work done = Area under $F - x$ graph between $x = 2$ and $x = 3$ m

$$= \frac{1}{2}(40 + 60) \times (3 - 2)$$

$$= \frac{1}{2} \times 100 \times 1 = \mathbf{50\ J.}$$

6.7 ENERGY

8. *Define the term energy. What are its units and dimensions ? Name the different forms of energy.*

Energy. *Energy of a body is defined as its capacity or ability to do work.* The energy of a body is measured by the amount of work the body can perform, therefore

(*i*) like work, energy is a scalar quantity.

(*ii*) the dimensional formula of energy is $[ML^2T^{-2}]$ *i.e.*, same as that of work.

(*iii*) energy is measured in the same units as the work.

The SI unit of energy is joule and the CGS unit is erg.

Energy has several forms : Mechanical energy, sound energy, heat energy, light energy, chemical energy, atomic energy, nuclear energy, electric energy, magnetic energy, solar energy, etc.

Table 6.1 Some other units of work or energy.

S.No.	Unit	Symbol	Value in SI
1.	erg	erg	10^{-7} J
2.	electron volt	eV	1.6×10^{-19} J
3.	calorie	cal	4.186 J
4.	kilowatt hour	kWh	3.6×10^6 J

9. *What is mechanical energy ? What are its two forms ?*

Mechanical energy. *The energy produced by mechanical means is called mechanical energy.* It has two forms :

(*i*) Kinetic energy (*ii*) Potential energy.

6.8 KINETIC ENERGY

10. *What is kinetic energy ? Give some examples.*

Kinetic energy. *The energy possessed by a body by virtue of its motion is called its kinetic energy.* A moving object can do work. The amount of work that a moving object can do before coming to rest is equal to its kinetic energy.

Examples :

(*i*) A moving hammer drives a nail into the wood. Being in motion, it has kinetic energy or ability to do work.

(*ii*) A fast moving stone can break a window pane. The stone has kinetic energy due to its motion and so it can do work.

(*iii*) A bullet fired from a gun can pierce a target due to its kinetic energy.

(*iv*) The kinetic energy of air is used to run wind mills.

(*v*) The kinetic energy of a fast stream of water is used to run water mills.

11. *Derive an expression for the kinetic energy of a body of mass m moving with velocity v .*

Expression for kinetic energy. The kinetic energy of a body can be determined by calculating the amount of work required to bring the body into motion from its state of rest, as shown in Fig. 6.13.

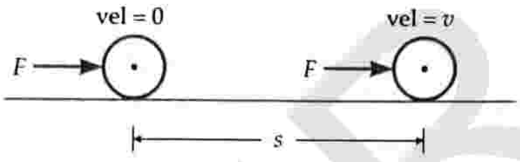


Fig. 6.13 Measurement of K.E.

Let m = mass of the body

$u = 0$ = initial velocity of the body

F = constant force applied on the body

a = acceleration produced in the body in the direction of force

v = final velocity of the body

s = distance covered by the body.

As $\quad v^2 - u^2 = 2as$

$\therefore \quad v^2 - 0^2 = 2as \quad$ or $\quad a = \dfrac{v^2}{2s}$

As the force and displacement are in same direction, so the work done is

$$W = Fs = ma \cdot s = m \cdot \frac{v^2}{2s} \cdot s = \frac{1}{2} mv^2$$

This work done appears as kinetic energy (K) of the body.

$$\therefore \quad K = \frac{1}{2} mv^2$$

Hence, *the kinetic energy of a body is equal to one-half the product of the mass of the body and the square of its velocity.*

12. *Derive an expression for the kinetic energy of a body by calculus method. Deduce its relation with linear momentum.*

Kinetic energy by calculus method. Consider a body of mass m initially at rest. A force $\vec{F}$ applied on the body produces a displacement $\vec{ds}$ in its own direction ($\theta = 0°$). The small work done is

$$dW = \vec{F} \cdot \vec{ds} = Fds \cos 0° = Fds$$

According to Newton's second law of motion,

$$F = ma = m\frac{dv}{dt}$$

$$\therefore \quad dW = Fds = m\frac{dv}{dt} \cdot ds = mv\, dv \quad \left[\because \frac{ds}{dt} = v\right]$$

The total work done to increase its velocity from 0 to v is given by

$$W = \int dW = \int_0^v mv\,dv = m\int_0^v v\,dv = m\left[\frac{v^2}{2}\right]_0^v = \frac{1}{2}mv^2$$

This work done appears as the kinetic energy (K) of the body.

$$\therefore \quad K = \frac{1}{2}mv^2$$

Relation between K.E. and linear momentum. As linear momentum,

$$p = mv$$

therefore $\quad K = \frac{1}{2}mv^2 = \frac{1}{2m}(m^2v^2) = \frac{1}{2m}(mv)^2$

or $\quad K = \frac{p^2}{2m} \quad$ or $\quad p = \sqrt{2mK}$

6.9 WORK-ENERGY THEOREM FOR A CONSTANT FORCE

13. *State the work-energy theorem. Prove it for a constant force.*

Work-energy theorem. *It states that the work done by the net force acting on a body is equal to the change produced in the kinetic energy of the body.*

Proof of W-E theorem for a constant force. Suppose a constant force F acting on a body of mass m produces acceleration a in it. After covering distance s, suppose the velocity of the body changes from u to v. We use the equation of motion,

$$v^2 - u^2 = 2as$$

Multiplying both sides by $\frac{1}{2}m$, we get

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = mas$$

By Newton's second law, $ma = F$, the applied force. Therefore,

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = Fs = W$$

or $\quad K_f - K_i = W$

Change in K.E. of the body = Work done on the body by the net force.

This proves the work energy theorem for a constant force.

6.10 WORK-ENERGY THEOREM FOR A VARIABLE FORCE

14. *Prove the work-energy theorem for a variable force.*

Proof of W-E theorem for a variable force. Suppose a variable force $\vec{F}$ acts on a body of mass m and produces displacement $\vec{ds}$ in its own direction ($\theta = 0°$). The small work done is

$$dW = \vec{F}\cdot\vec{ds} = F\,ds\cos 0° = F\,ds$$

According to Newton's second law of motion,

$$F = ma = m\frac{dv}{dt}$$

$$\therefore \quad dW = m\frac{dv}{dt}\cdot ds$$

$$= mv\,dv \qquad \left[\because \frac{ds}{dt} = v\right]$$

If the applied force increases the velocity from u to v, then the total work done on the body will be

$$W = \int dW = \int_u^v mv\,dv = m\int_u^v v\,dv$$

$$= m\left[\frac{v^2}{2}\right]_u^v = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

or $\quad W = K_f - K_i$

= Change in K.E. of the body.

This proves the work-energy theorem for a variable force.

For Your Knowledge

- The work-energy theorem is not independent of Newton's second law. It may be viewed as scalar form of second law.
- The W-E theorem holds in all inertial frames. It can be extended to non-inertial frames provided we include the pseudo force in the calculation of the net force acting on the body under consideration.
- When force and displacement are in same direction, the kinetic energy of the body increases. The increase in K.E. is equal to the work done on the body.
- When force and displacement are oppositely directed, the kinetic energy of the body decreases. The decrease in K.E. is equal to the work done by the body against the retarding force.
- When a body moves along a circular path with uniform speed, there is no change in its kinetic energy. By W-E theorem, the work done by the centripetal force is zero.
- When K.E. increases, the work done is positive and when K.E. decreases, the work done is negative.
- In deriving the W-E theorem, it has been assumed that the work done by the force is effective only in changing the K.E. of the body. However, the work done on a body may also be stored as the P.E. of the body.

Examples based on K.E. and W-E Theorem

FORMULAE USED

1. Kinetic energy, $K = \frac{1}{2} mu^2$
2. According to work-energy theorem,

$$W = K_f - K_i = \frac{1}{2} mv^2 - \frac{1}{2} mu^2$$

UNITS USED

Work done W, kinetic energies K_i and K_f are all in joule.

CONVERSIONS USED

$$1\text{eV} = 1.60 \times 10^{-19}\ \text{J}.$$

EXAMPLE 13. *A body of mass 4 kg initially at rest is subject to a force 16 N. What is the kinetic energy acquired by the body at the end of 10 s ?* **[Himachal 07 ; Delhi 02]**

Solution. Here $m = 4$ kg, $F = 16$ N, $t = 10$ s

Acceleration,

$$a = \frac{F}{m} = \frac{16}{4} = 4\ \text{ms}^{-2}$$

$$\therefore \quad v = u + at = 0 + 4 \times 10 = 40\ \text{ms}^{-1}$$

The kinetic energy acquired by the body,

$$K = \frac{1}{2} mv^2 = \frac{1}{2} \times 4 \times (40)^2 = \mathbf{3200\ J.}$$

EXAMPLE 14. *A toy rocket of mass 0.1 kg has a small fuel of mass 0.02 kg which it burns out in 3 s. Starting from rest on a horizontal smooth track it gets a speed of 20 ms^{-1} after the fuel is burnt out. What is the approximate thrust of the rocket ? What is the energy content per unit mass of the fuel ? (Ignore the small mass variation of the rocket during fuel burning).* **[NCERT]**

Solution. Here $m = 0.1$ kg, $u = 0$, $v = 20$ ms^{-1}, $t = 3$ s

Thrust of the rocket

$$= ma = m\frac{v-u}{t} = 0.1 \times \frac{20-0}{3} = \frac{2}{3}\ \text{N}$$

Kinetic energy gained by the rocket,

$$K = \frac{1}{2} mv^2 = \frac{1}{2} \times 0.1 \times (20)^2 = 20\ \text{J}$$

Energy content per unit mass of the fuel

$$= \frac{\text{Total energy}}{\text{Mass of the fuel}} = \frac{20\ \text{J}}{0.02\ \text{kg}} = \mathbf{1000\ J\ kg^{-1}.}$$

EXAMPLE 15. *A bullet weighing 10 g is fired with a velocity of 800 ms^{-1}. After passing through a mud wall 1 m thick, its velocity decreases to 100 ms^{-1}. Find the average resistance offered by the mud wall.* **[NCERT]**

Solution. Mass of bullet, $m = 10$ g $= 0.01$ kg

Velocity of bullet before passing through mud wall,

$$u = 800\ \text{ms}^{-1}$$

Velocity of bullet after passing through mud wall,

$$v = 100\ \text{ms}^{-1}$$

Distance covered by the bullet,

$$s = 1\ \text{m}$$

Let average resistance offered by the wall = F

According to work-energy theorem,

Work done by resistance offered by mud wall

= Decrease in K.E.

or $$Fs = \frac{1}{2} m(u^2 - v^2)$$

$$\therefore \quad F = \frac{m(u^2 - v^2)}{2s} = \frac{0.01 \times (800^2 - 100^2)}{2 \times 1} = \mathbf{3150\ N.}$$

EXAMPLE 16. *A shot travelling at the rate of 100 ms^{-1} is just able to pierce a plank 4 cm thick. What velocity is required to just pierce a plank 9 cm thick ?*

Solution. Here $v_1 = 100$ ms^{-1}, $s_1 = 4$ cm, $v_2 = ?$, $s_2 = 9$ cm.

K.E. lost by the shot = Work done against plank's resistance

$$\therefore \quad \frac{1}{2} mv_1^2 = F \times s_1 \quad \text{and} \quad \frac{1}{2} mv_2^2 = F \times s_2$$

On dividing,

$$\frac{v_2^2}{v_1^2} = \frac{s_2}{s_1} \quad \text{or} \quad \frac{v_2}{v_1} = \sqrt{\frac{s_2}{s_1}} = \sqrt{\frac{9}{4}} = \frac{3}{2}$$

$$\therefore \quad v_2 = \frac{3}{2} \times v_1 = \frac{3}{2} \times 100 = \mathbf{150\ ms^{-1}.}$$

EXAMPLE 17. *In a ballistics demonstration, a police officer fires a bullet of mass 50.0 g with speed 200 ms^{-1} on soft plywood of thickness 2.00 cm. The bullet emerges with only 10% of its initial kinetic energy. What is the emergent speed of the bullet ?* **[NCERT]**

Solution. Here $m = 50.0$ g $= 0.05$ kg, $u = 200$ ms^{-1}

$$\text{Initial K.E.} = \frac{1}{2} mu^2 = \frac{1}{2} \times 0.05 \times (200)^2 = 1000\ \text{J}$$

$$\text{Final K.E.} = 10\%\ \text{of}\ 1000\ \text{J} = \frac{10 \times 1000}{100} = 100\ \text{J}$$

or $$\frac{1}{2} mv^2 = 100\ \text{J}$$

$$\therefore \quad v = \sqrt{\frac{2 \times 100}{m}} = \sqrt{\frac{2 \times 100}{0.05}} = \mathbf{63.2\ ms^{-1}.}$$

Clearly, the speed reduces nearly by 68% and not by 90% by which the K.E. reduces.

EXAMPLE 18. *It is well known that a raindrop or a small pebble falls under the influence of the downward gravitational force and the opposing resistive force. The latter is known to be proportional to the speed of the drop but is* *otherwise undetermined. Consider a drop of small pebble of mass 1.00 g falling from a cliff of height 1.00 km. It hits the ground with a speed of 50.0 ms^{-1}. What is the work done by the unknown resistive force ?* **[NCERT]**

Solution. We assume that the pebble is initially at rest on the cliff.

$\therefore \quad u = 0, \quad m = 1.00 \text{ g} = 10^{-3}$ kg,

$v = 50 \text{ ms}^{-1}, \quad h = 1.00 \text{ km} = 10^3$ m

The change in K.E. of the pebble is

$$\Delta K = \frac{1}{2} mv^2 - \frac{1}{2} mu^2$$

$$= \frac{1}{2} \times 10^{-3} \times (50)^2 - 0 = 1.25 \text{ J}$$

Assuming that $g = 10 \text{ ms}^{-2}$ is constant, the work done by the gravitational force is

$$W_g = mgh = 10^{-3} \times 10 \times 10^3 = 10.0 \text{ J}$$

If W_r is the work done by the resistive force on the pebble, then from the work-energy theorem,

$$\Delta K = W_g + W_r$$

or $\quad W_r = \Delta K - W_g = 1.25 - 10.0 = \mathbf{-8.75 \text{ J}}.$

EXAMPLE 19. *A block of mass $m = 1$ kg, moving on a horizontal surface with speed $v_i = 2 \text{ ms}^{-1}$ enters a rough patch ranging from $x = 0.10$ m to $x = 2.01$ m. The retarding force F_r on the block in this range is inversely proportional to x over this range.*

$$F_r = \frac{-k}{x} \qquad 0.1 < x < 2.01 \, m$$

$$= 0 \quad \text{for } x < 0.1 \, m \text{ and } x > 2.01 \, m$$

where k = 0.5 J. What is the final kinetic energy and speed v_f of the block as it crosses this patch ? **[NCERT]**

Solution. By work-energy theorem,

$$\Delta K = W_r \quad \text{or} \quad K_f - K_i = \int_{x_i}^{x_f} F_r \, dx$$

$$\therefore \quad K_f = K_i + \int_{0.1}^{2.01} \frac{(-k)}{x} dx$$

$$= \frac{1}{2} mv_i^2 - k \int_{0.1}^{2.01} \frac{1}{x} dx$$

$$= \frac{1}{2} \times 1 \times 2^2 - k [\ln x]_{0.1}^{2.01}$$

$$= 2 - 0.5 \left[\ln \frac{2.01}{0.1} \right] = 2 - 0.5 \ln 20.1$$

$$= 2 - 0.5 \times 2.303 \log 20.1$$

$$= 2 - 0.5 \times 2.303 \times 1.3032 = 2 - 1.5$$

$$= \mathbf{0.5 \text{ J}}.$$

Final speed, $v_f = \sqrt{\frac{2K_f}{m}} = \sqrt{\frac{2 \times 0.5}{1}} = \mathbf{1 \text{ ms}^{-1}}.$

EXAMPLE 20. *Two identical 5 kg blocks are moving with same speed of 2 ms^{-1} towards each other along a frictionless horizontal surface. The two blocks collide, stick together and come to rest. Consider the two blocks as a system. Calculate work done by (i) external forces and (ii) internal forces.* **[CBSE 91]**

Solution. As no external forces are acting on the system, so

$$\vec{F}_{ext} = 0 \qquad \therefore \quad W_{ext} = \vec{F}_{ext} \cdot \vec{s} = \mathbf{0}.$$

According to work-energy theorem,

Total work done = Change in K.E.

= Final K.E. – Initial K.E.

or $\quad W_{ext} + W_{int} = 0 - \left(\frac{1}{2} mv^2 + \frac{1}{2} mv^2\right) = -mv^2$

or $\quad 0 + W_{int} = -5 \times (2)^2 = -20$

$\therefore \quad W_{int} = \mathbf{-20 \text{ J}}.$

The negative sign indicates that internal forces of action and reaction act on the two blocks in a direction opposite to their motion.

EXAMPLE 21. *If the linear momentum of a body increases by 20%, what will be the % increase in the kinetic energy of the body ?* **[AFMC 97]**

Solution. Initial kinetic energy of the body,

$$K = \frac{1}{2} mv^2 = \frac{1}{2} \frac{m^2 v^2}{m} = \frac{p^2}{2m}$$

Increase in momentum = 20% of $p = \frac{20}{100} \times p = \frac{p}{5}$

Final momentum $= p + \frac{p}{5} = \frac{6p}{5}$

Final kinetic energy of the body,

$$K' = \frac{(6p/5)^2}{2m} = \frac{36}{25} \frac{p^2}{2m} = \frac{36}{25} K$$

Increase in kinetic energy

$$= K' - K = \frac{36}{25} K - K = \frac{11}{25} K$$

% Increase in K.E.

$$= \frac{K' - K}{K} \times 100 = \frac{\frac{11}{25} K}{K} \times 100 = \mathbf{44\%}.$$

EXAMPLE 22. *If the kinetic energy of a body increases by 300%, by what % will the linear momentum of the body increase ?* **[Delhi 95, 99]**

Solution. Initial kinetic energy,

$$K = \frac{1}{2} mv^2 = \frac{1}{2} \frac{m^2 v^2}{m} = \frac{p^2}{2m}$$

$\therefore$ Initial momentum, $p = \sqrt{2mK}$

Increase in kinetic energy = 300% of $K = 3 K$

Final kinetic energy,

$$K' = K + 3K = 4K$$

Final momentum,

$$p' = \sqrt{2mK'} = \sqrt{2m \times 4K} = 2\sqrt{2mK} = 2p$$

% Increase in momentum

$$= \frac{p' - p}{p} \times 100 = \frac{2p - p}{p} \times 100 = \mathbf{100\%}.$$

EXAMPLE 23. *A body of mass 0.3 kg is taken up an inclined plane to length 10 m and height 5 m, and then allowed to slide down to the bottom again. The coefficient of friction between the body and the plane is 0.15. What is the*

(*i*) *work done by the gravitational force over the round trip,*
(*ii*) *work done by the applied force over the upward journey,*
(*iii*) *work done by frictional force over the round trip,*
(*iv*) *kinetic energy of the body at the end of the trip ?*

How is the answer to (iv) related to the first three answers ? **[NCERT]**

Solution. Here $m = 0.3$ kg, $l = 10$ m,

$$h = 5 \text{ m}, \quad \mu_k = 0.15$$

$$\sin\theta = \frac{h}{l} = \frac{5}{10} = \frac{1}{2} = 0.5$$

$$\therefore \quad \theta = 30°$$

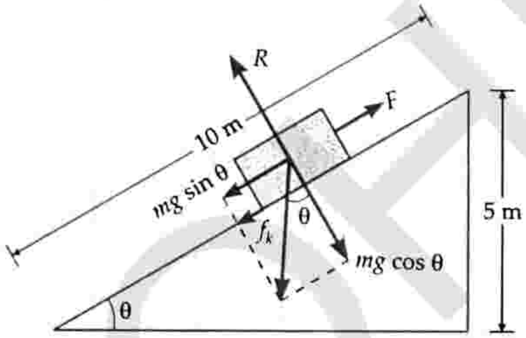


Fig. 6.14

(*i*) Work done by the gravitational force in moving the body up the inclined plane,

$$W = Fs = -mg\sin\theta \times l$$
$$= -0.3 \times 9.8 \times 0.5 \times 10 = -14.7 \text{ J}$$

Work done by the gravitational force in moving the body down the inclined plane,

$$W' = Fs = +mg\sin\theta \times l = +14.7 \text{ J}$$

Work done by the gravitational force over the round trip,

$$W_1 = W + W' = -14.7 + 14.7 = \mathbf{0}.$$

This is in conformity with the fact that the work done by a conservative force (*e.g.*, gravitational force) over one round trip is zero.

(*ii*) Force needed to move the body up the inclined plane,

$$F = mg\sin\theta + f_k = mg\sin\theta + \mu_k R$$
$$= mg\sin\theta + \mu_k mg\cos\theta$$
$$= mg(\sin\theta + \mu_k\cos\theta)$$

Work done by the applied force over the upward journey,

$$W_2 = F \times l = mg(\sin\theta + \mu_k\cos\theta)\, l$$
$$= 0.3 \times 9.8\,(\sin 30° + 0.15\cos 30°) \times 10$$
$$= 0.3 \times 9.8\,(0.5 + 0.15 \times 0.866) \times 10 = \mathbf{18.5\ J}.$$

(*iii*) Work done by the frictional force over the round trip,

$$W_3 = -f_k(l + l) = -2f_k \times l$$
$$= -2\mu_k\, mg\cos\theta \times l$$
$$= -2 \times 0.15 \times 0.3 \times 9.8 \times \cos 30° \times 10 = \mathbf{-7.6\ J}.$$

(*iv*) Kinetic energy of the body at the end of round trip

= Work done by the net force in moving the body down the inclined plane

$$= (mg\sin\theta - \mu_k mg\cos\theta) \times l$$
$$= mg(\sin\theta - \mu_k\cos\theta)\, l$$
$$= 0.3 \times 9.8 \times (0.5 - 0.13) \times 10 = \mathbf{10.9\ J}.$$

From answers to (*i*), (*ii*) and (*iii*), net work done on the body

$$= 0 + 18.5 - 7.6 = 10.9 \text{ J} = \text{K.E. of the body}$$

Thus kinetic energy of the body is equal to the net work done on the body.

PROBLEMS FOR PRACTICE

1. The momentum of a body of mass 5 kg is 500 kg ms^{-1}. Find its K.E. (**Ans.** 2.5×10^4 J)
2. A bullet of mass 20 g is found to pass two points 30 m apart in a time interval of 4 s. Calculate the kinetic energy of the bullet if it moves with constant speed. (**Ans.** 0.5625 J)
3. A body of mass 2 kg is resting on a rough horizontal surface. A force of 20 N is now applied to it for 10 s, parallel to the surface. If the coefficient of kinetic friction between the surfaces in contact is 0.2, calculate : (*a*) Work done by the applied force in 10 s. (*b*) Change in kinetic energy of the object in 10 s. Take $g = 10$ ms^{-2}. **[Delhi 04]** (**Ans.** 8000 J, 6400 J)
4. An electron and a proton are detected in a cosmic ray experiment, the electron with K.E. of 5 keV and the proton with K.E. of 50 keV. Find the ratio of their speeds. Given $m_e = 9.11 \times 10^{-31}$ kg and $m_p = 1.67 \times 10^{-27}$ kg. $\left(\textbf{Ans. } \frac{v_e}{v_p} = 4.28\right)$

5. A neutron of mass 1.67×10^{-27} kg is moving with a speed of 7×10^5 ms^{-1}. Calculate (*i*) its kinetic energy and (*ii*) the average force it will exert in entering a body to a depth of 0.01 cm.
[**Ans.** (*i*) 40.915×10^{-17} J (*ii*) 40.915×10^{-13} N]

6. A body of mass 1 kg is allowed to fall freely under gravity. Find the momentum and kinetic energy of the body 5 seconds after it starts falling. Take $g = 10$ ms^{-2}. (**Ans.** 50 kg ms^{-1}, 1250 J)

7. Two bodies of masses 1 g and 16 g are moving with equal kinetic energies. Find the ratio of the magnitudes of their linear momenta. (**Ans.** 1 : 4)

8. If the momentum of a body is increased by 50%, then what will be the percentage increase in the kinetic energy of the body ? *[Central Schools 10]* (**Ans.** 125%)

9. The kinetic energy of a body decreases by 19%. What is the percentage decrease in its linear momentum ? (**Ans.** 10%)

10. A running man has half the kinetic energy that a boy of half his mass has. The man speeds up by 1.0 ms^{-1} and then has the same energy as the boy. What were the original speeds of the man and the boy ? (**Ans.** 2.414 ms^{-1}, 4.828 ms^{-1})

11. While catching a cricket ball of mass 200 g moving with a velocity of 20 ms^{-1}, the player draws his hands backwards through 20 cm. Find the work done in catching the ball and the average force exerted by the ball on the hand. (**Ans.** 40 J, 200 N)

✘ HINTS

1. $\text{K.E.} = \frac{p^2}{2m} = \frac{(500)^2}{2 \times 5} = \mathbf{2.5 \times 10^4}$ **J.**

2. Speed, $v = \frac{\text{Distance}}{\text{Time}} = \frac{30 \text{ m}}{4 \text{ s}} = 7.5 \text{ ms}^{-1}$

$$\text{K.E.} = \frac{1}{2} \times 0.02 \times (7.5)^2 = \mathbf{0.5625 \text{ J.}}$$

3. Here $m = 2$ kg, $u = 0$, $F = 2$ N, $\mu_k = 0.2$, $t = 10$ s

$$f_k = \mu_k \, mg = 0.2 \times 2 \times 10 = 4 \text{ N}$$

Net force, $F' = F - f_k = 20 - 4 = 16$ N

Acceleration, $a = \frac{F'}{m} = \frac{16}{2} = 8 \text{ ms}^{-2}$

$$s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 8 \times (10)^2 = 400 \text{ m}$$

(*a*) Work done by the applied force,

$$W_1 = Fs = 20 \times 400 = \mathbf{8000 \text{ J.}}$$

(*b*) Change in K.E. = Work done by the net force

$$W_2 = F's = 16 \times 400 = \mathbf{6400 \text{ J.}}$$

5. (*i*) $\text{K.E.} = \frac{1}{2}mv^2 = \frac{1}{2} \times 1.67 \times 10^{-27} \times (7 \times 10^5)^2$

$$= \mathbf{40.915 \times 10^{-17} \text{ J.}}$$

(*ii*) $F \times s = \text{K.E.}$ or $F \times \frac{0.01}{100} = 40.915 \times 10^{-17}$

or $$F = \frac{40.915 \times 100 \times 10^{-17}}{0.01} = \mathbf{40.915 \times 10^{-13} \text{ N.}}$$

7. Kinetic energy, $K = \frac{p^2}{2m}$ $\therefore$ $p = \sqrt{2mK}$ or $p \propto \sqrt{m}$

$$\therefore \quad \frac{p_1}{p_2} = \sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{1}{16}} = \frac{1}{4} = \mathbf{1:4.}$$

8. Proceed as in Example 21.

9. Initial kinetic energy, $K = \frac{p^2}{2m}$

Initial momentum, $p = \sqrt{2mK}$

Decrease in K.E. = 19% of $K = 0.19K$

Final K.E., $K' = K - 0.19\,K = 0.81K$

Final momentum,

$$p' = \sqrt{2mK'} = \sqrt{2m \times 0.81\,K} = 0.9\sqrt{2mK} = 0.9p$$

% Decrease in momentum

$$= \frac{p - p'}{p} \times 100 = \frac{p - 0.9\,p}{p} \times 100 = \mathbf{10\%.}$$

10. Let M be the mass of the man, $M/2$ that of the boy and V and v be their respective velocities. As the K.E. of the man is half the K.E. of the boy, so

$$\frac{1}{2}MV^2 = \frac{1}{2} \cdot \frac{1}{2}\left(\frac{M}{2}\right)v^2 \text{ or } v^2 = 4V^2 \text{ or } v = 2V$$

When the velocity of the man is increased to $(V + 1)$, their kinetic energies become equal.

$$\therefore \quad \frac{1}{2}M(V+1)^2 = \frac{1}{2}\left(\frac{M}{2}\right)v^2 = \frac{1}{4}M \cdot (2V)^2$$

or $V^2 + 2V + 1 = 2V^2$ or $V^2 - 2V - 1 = 0$

$$\therefore \quad V = \sqrt{2} + 1 = \mathbf{2.414 \text{ ms}^{-1}},$$

$$v = 2(\sqrt{2} + 1) = \mathbf{4.828 \text{ ms}^{-1}}.$$

11. $W = \text{Change in K.E.} = \frac{1}{2}mv^2 = \frac{1}{2} \times 0.2 \times 20 \times 20 = \mathbf{40 \text{ J}}$

$$\therefore \quad F = \frac{W}{s} = \frac{40}{0.20} = \mathbf{200 \text{ N.}}$$

6.11 POTENTIAL ENERGY

15. *What is meant by potential energy ? Give some examples.*

Potential energy. *Potential energy is the energy stored in a body or a system by virtue of its position in a field of force or by its configuration.*

Potential energy is also called *mutual energy* or *energy of configuration*. It is measured by the amount of work that a body or system can do in passing from its present position or configuration to some standard position or configuration, called *zero position* or *zero configuration.*

Examples of potential energy due to position :

(*i*) A body lying on the roof of a building has some potential energy. When allowed to fall down, it can do work.

(*ii*) The potential energy of water stored to great heights in dams is used to run turbines for generating hydroelectricity.

Examples of potential energy due to configuration :

(*i*) In a toy car, the wound spring has potential energy. As the spring is released, its potential energy changes into kinetic energy which moves the toy car.

(*ii*) A stretched bow possesses potential energy. As soon as it is released, it shoots the arrow in the forward direction with a large velocity. The potential energy of the stretched bow gets converted into the kinetic energy.

(*iii*) Due to the potential energy of the compressed spring in a loaded gun, the bullet is fired with a large velocity on firing the gun.

16. *Mention some common types of potential energy.*

Different types of potential energies. Three common types of potential energies are as follows :

(*i*) ***Gravitational potential energy.*** It is the potential energy associated with the state of separation of two bodies, which attract one another through the gravitational force.

(*ii*) ***Elastic potential energy.*** It is the potential energy associated with the state of compression or extension of an elastic (spring like) object.

(*iii*) ***Electrostatic potential energy.*** The energy due to the interaction between two electric charges is electrostatic potential energy.

6.12 GRAVITATIONAL POTENTIAL ENERGY

17. *Define gravitational potential energy. Derive an expression for the gravitational P.E. of a body of mass m lying at height h above the earth's surface.*

Gravitational potential energy. *The gravitational potential energy of a body is the energy possessed by the body by virtue of its position above the surface of the earth.*

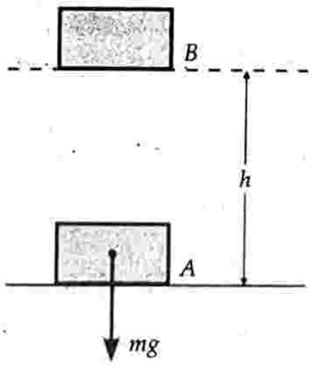


Fig. 6.15 Computation of G.P.E.

Expression for G.P.E. Consider a body of mass m lying on the surface of the earth, as shown in Fig. 6.15. Let g be the acceleration due to gravity at this place.

For heights much smaller than the radius of the earth ($h << R_E$) the value g can be taken constant.

Force needed to lift the body up with zero acceleration,

$$F = \text{Weight of the body} = mg$$

Work done on the body in raising it through height h,

$$W = F \cdot h = mg \cdot h$$

This work done against gravity is stored as the gravitational potential energy (U) of the body.

$$\therefore \quad U = mg\,h$$

At the surface of the earth, $h = 0$

$\therefore$ Gravitational P.E. at the earth's surface = zero.

6.13 CONSERVATIVE AND NON-CONSERVATIVE FORCES

18. *What are conservative forces ? Explain.*

Conservative force. *A force is conservative if the work done by the force in displacing a particle from one point to another is independent of the path followed by the particle and depends only on the end points.*

Suppose a particle moves from point A to point B along either path 1 or path 2, as shown in Fig. 6.16 (*a*). If a conservative force F acts on the particle, then the work done on the particle is same along the two paths.

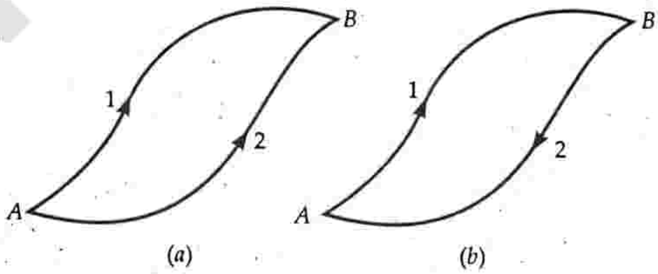


Fig. 6.16 Conservative force.

Mathematically, we can write

$$W_{AB} \text{ (along path 1)} = W_{AB} \text{ (along path 2)} \quad ...(1)$$

Now suppose the particle moves in a round trip, from point A to point B along path 1 and then back to point A along path 2, as shown in Fig. 6.16(*b*). For a conservative force,

Work done on the particle along the path 2 from A to B

= – Work done on the particle along the path 2 from B to A

i.e., $$W_{AB} \text{ (along path 2)} = -W_{BA} \text{ (along path 2)} \quad ...(2)$$

From (1) and (2), we have

$$W_{AB}\text{ (along path 1)} = -W_{BA}\text{ (along path 2)}$$

or $$W_{AB}\text{ (along path 1)} + W_{BA}\text{ (along path 2)} = 0$$

or $$W_{\text{closed path}} = 0$$

Hence *a force is conservative if the work done by the force in moving a particle around any closed path is zero.*

Examples. Gravitational force, electrostatic force and elastic force of a spring are all conservative forces.

19. *Explain qualitatively that how is the gravitational force a conservative force.*

Conservative nature of gravitational force. (*i*) As shown in Fig. 6.17(*a*), suppose a body of mass m is raised to a height h vertically upwards from position A to B. The work done against gravity is

$$W = mg \times AB = mgh$$

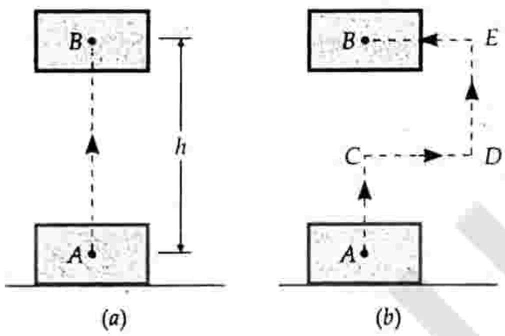


Fig. 6.17 Gravitational force as a conservative force.

As shown in Fig. 6.17(*b*), now suppose the body is taken from position A to B along the path $ACDEB$. During the horizontal paths CD and EB, the force of gravity is perpendicular to the displacement, so work done is zero. Work is done only along vertical paths AC and DE. The total work done is

$$W = W_{AC} + W_{CD} + W_{DE} + W_{EB}$$

$$= mg \times AC + 0 + mg \times DE + 0$$

$$= mg(AC + DE) = mg \times h$$

or $$W = mgh$$

Thus the work done in moving a body against gravity is independent of the path taken and depends only on the initial and final positions of the body. Hence gravitational force is a conservative force.

(*ii*) Suppose a ball is thrown vertically upward. As it rises, the gravitational force does negative work on it, decreasing its kinetic energy. As the ball descends, the gravitational force does positive work on it, increasing its kinetic energy. The ball falls back to the point of projection with the same velocity and K.E.

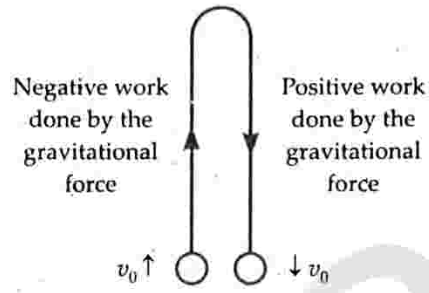


Fig. 16.18 Conservative nature of gravitational force.

with which it was thrown up. The net work done by the gravitational force on the ball during the round trip is zero. This again shows that the gravitational force is a conservative force.

20. *What is a non-conservative force ? Give examples.*

Non-conservative force. *If the amount of work done in moving an object against a force from one point to another depends on the path along which the body moves, then such a force is called a non-conservative force.* The work done in moving an object against a non-conservative force along a closed path is not zero.

Examples. Forces of friction and viscosity are non-conservative forces.

21. *Define potential energy in context with a conservative force. How can these two quantities be determined from each other ? Write the necessary relations.*

Potential energy in relation to conservative force. *The potential energy is the energy associated with the configuration of a system in which a conservative force acts.* When the conservative force $F(x)$ (for simplicity, in one dimension) does work W on a particle within the system, the change in potential energy ΔU of the system is equal to the negative of the work done by the conservative force, *i.e.*,

$$\Delta U = -W$$

But $$W = \int_{x_i}^{x_f} F(x)\,dx$$

$$\therefore \quad \Delta U = -\int_{x_i}^{x_f} F(x)\,dx$$

Differentiating the above equation, we get

$$\frac{dU(x)}{dx} = -F(x)$$

or $$F(x) = -\frac{dU(x)}{dx}$$

Hence *potential energy U may be defined as a function whose negative gradient gives the force. Conversely, we may define conservative force as a force which is equal to the negative gradient of the potential energy U.*

22. *Show analytically that gravitational force is a conservative force.*

Gravitational force as a conservative force (quantitative approach). The gravitational potential energy of a body of mass m lying at height h above the ground is given by the function,

$$U(h) = mgh$$

The negative gradient of the potential energy U is given by

$$-\frac{dU(h)}{dh} = -\frac{d}{dh}(mgh) = -mg = F$$

i.e., the negative gradient of the potential energy is equal to the gravitational force. This proves the conservative nature of the gravitational force. The negative sign in the above equation indicates that the gravitational force acts in the downward direction.

Moreover, if a body of mass m is released from rest, from the top of a smooth (frictionless) inclined plane of height h, it gains speed at the bottom given by

$$v^2 - 0^2 = 2gh \quad \text{or} \quad v = \sqrt{2gh}$$

K.E. acquired by the body at the bottom of the inclined plane

$$= \frac{1}{2}mv^2 = \frac{1}{2}m \times 2gh = mgh$$

= Work done by the gravitational force

Clearly, the work done by the gravitational force does not depend on the angle of inclination of the inclined plane or the path of the falling body. It depends only on the initial and final positions. This again proves that the gravitational force is a conservative force.

6.14 PRINCIPLE OF CONSERVATION OF MECHANICAL ENERGY

23. *State and prove the principle of conservation of mechanical energy.*

Principle of conservation of mechanical energy. *This principle states that if only the conservative forces are doing work on a body, then its total mechanical energy (K.E. + P.E.) remains constant.*

Proof. Suppose that a body undergoes displacement Δx under the action of a conservative force $F(x)$. Then from work-energy theorem, the change in K.E. is

$$\Delta K = F(x)\,\Delta x$$

As the force is conservative the change in potential energy is given by

$$\Delta U = \text{Negative of the work done} = -F(x)\,\Delta x$$

Combining the above two equations, we get

$$\Delta K = -\Delta U$$

or $\quad \Delta K + \Delta U = 0 \quad$ or $\quad \Delta(K + U) = 0$

or $\quad K + U = \text{constant}$

or $\quad K_i + U_i = K_f + U_f$

Although, individually the kinetic energy K and potential energy U may change from one state of the system to another, but their sum or the total mechanical energy of the system remains constant under the conservative force.

24. *Mention the various properties of the conservative forces.*

Properties of the conservative forces :

(*i*) A force F is conservative if it can be defined from the scalar potential energy function $U(x)$ by the relation,

$$F(x) = -\frac{dU(x)}{dx}$$

(*ii*) The work done by a conservative force on an object is path independent and depends only on the end points.

$$W = \int_{x_i}^{x_f} F(x)\,dx = K_f - K_i = U_i - U_f$$

(*iii*) The work done by the conservative force is zero if the object moving around any closed path returns to its initial position.

$$W_{\text{closed path}} = \oint F(x)\,dx = 0$$

(*iv*) If only the conservative forces are acting on body, then its total mechanical energy is conserved.

6.15 CONSERVATION OF MECHANICAL ENERGY IN A FREELY FALLING BODY

25. *Show that the total mechanical energy of a freely falling body remains constant throughout its fall.*

Conservation of mechanical energy in case of a freely falling body. Consider a body of mass m lying at position A at a height h above the ground. As the body falls, its kinetic energy increases at the expense of potential energy.

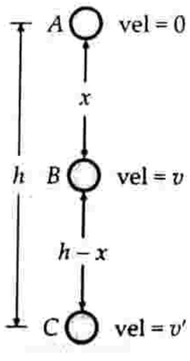


Fig. 6.19 Conservation of energy during a free fall.

At point A. The body is at rest.

K.E. of the body,

$$K_A = 0$$

P.E. of the body,

$$U_A = mgh$$

Total mechanical energy,

$$E_A = K_A + U_A = mgh$$

At point *B*. Suppose the body falls freely through height x and reaches the point B with velocity v. Then

$$v^2 - 0^2 = 2gx \qquad [\text{Using } v^2 - u^2 = 2as]$$

or $$v^2 = 2gx$$

$$\therefore \quad K_B = \frac{1}{2}mv^2 = \frac{1}{2}m \times 2gx = mgx$$

$$U_B = mg(h - x)$$

$$E_B = K_B + U_B = mgx + mg(h - x) = mg\,h$$

At point *C*. Suppose the body finally reaches a point C on the ground with velocity v'. Then considering motion from A to C,

$$v'^2 - 0^2 = 2gh \quad \text{or} \quad v'^2 = 2gh$$

$$\therefore \quad K_C = \frac{1}{2}mv'^2 = \frac{1}{2}m \times 2gh = mg\,h$$

$$U_C = mg \times 0 = 0$$

$$E_C = K_C + U_C = mg\,h$$

Clearly, as the body falls, its P.E. decreases and K.E. increases by an equal amount. However, its total mechanical energy remains constant (= mgh) at all points. Thus total mechanical energy is conserved during free fall of a body. Fig. 6.20 shows the variation of K.E. and P.E. and the constancy of total energy with height.

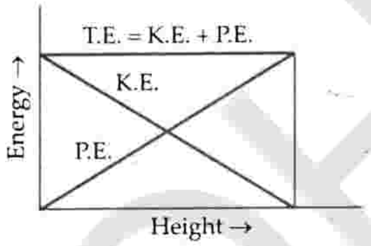


Fig. 6.20 Plots of K.E. and P.E. during free fall of a body.

Examples based on P.E. and Conservation of Energy

FORMULAE USED

1. Gravitational P.E., $U = mgh$
2. For a conservative force, $F = -\dfrac{dU}{dx}$
3. $\Delta U = U_f - U_i = -W = -\int_{x_i}^{x_f} F\,dx$
4. When work is done only by conservative forces only, mechanical energy is conserved.

   $K + U = \text{constant.}$

UNITS USED

Force is in newton and work done W, kinetic energy K and potential energy U are in joule.

EXAMPLE 24. *A vehicle of mass 15 quintal climbs up a hill 200 m high. It then moves on a level road with speed of 30 ms⁻¹. Calculate the potential energy gained by it and its total mechanical energy while running on the top of the hill.*

Solution. Here $m = 15$ quintal $= 1500$ kg,

$$g = 9.8 \text{ ms}^{-2}, \; h = 200 \text{ m}$$

P.E. gained,

$$U = mgh = 1500 \times 9.8 \times 200 = \mathbf{2.94 \times 10^6\ J}$$

When the vehicle runs on a level road with speed of 30 ms^{-1}, its K.E. is

$$K = \frac{1}{2}mv^2 = \frac{1}{2} \times 1500 \times (30)^2 = 0.675 \times 10^6 \text{ J}$$

Total mechanical energy,

$$E = K + U = (0.675 + 2.94) \times 10^6 = \mathbf{3.615 \times 10^6\ J}$$

EXAMPLE 25. *Calculate the velocity of the bob of a simple pendulum at its mean position if it is able to rise to a vertical height of 10 cm. Take $g = 9.8 \text{ ms}^{-2}$.*

Solution. By conservation of energy,

K.E. of the bob at the mean position

= P.E. of the bob at the highest position

or $$\frac{1}{2}mv^2 = mgh$$

$$\therefore \quad v = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 0.10} = \sqrt{1.96} = \mathbf{1.4\ ms^{-1}}.$$

EXAMPLE 26. *A girl of mass 40 kg sits in a swing formed by a rope of 6 m length. A person pulls the swing to a side so that the rope makes an angle of 60° with the vertical. What is the gain in potential energy of the girl ?*

Solution. Here, $m = 40$ kg, $OA = OB = 6$ m

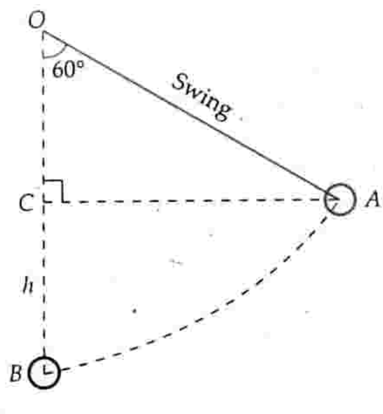


Fig. 6.21

From right ΔOCA,

$$\frac{OC}{OA} = \cos 60°$$

$$OC = OA \cos 60° = 6 \times \frac{1}{2} = 3 \text{ m}$$

Height through which the girl is raised,

$$h = CB = OB - OC = 6 - 3 = 3 \text{ m}$$

$\therefore$ P.E. gained by the girl

$$= mgh = 40 \times 9.8 \times 3 = \mathbf{1176\ J.}$$

EXAMPLE 27. *How high must a body be lifted to gain an amount of potential energy equal to the kinetic energy it has when moving at speed 20 ms^{-1} ? The value of acceleration due to gravity at a place is $g = 9.8$ ms^{-2}.* **[Delhi 97, 05]**

Solution. Here $mg\,h = \frac{1}{2} mv^2$

$$\therefore \quad h = \frac{v^2}{2g} = \frac{20 \times 20}{2 \times 9.8} = \mathbf{20.2\ m.}$$

EXAMPLE 28. *The string of a pendulum is 2.0 m long. The bob is pulled sideways so that the string becomes horizontal and then the bob is released. What is the speed with which the bob arrives at the lowest point ? Assume that 10% of the initial energy is dissipated against air resistance, $g = 10$ ms^{-2}.*

Solution. Here $\frac{1}{2} mv^2 = 90\%$ of $mg\,h$

or $$\frac{1}{2} mv^2 = \frac{90}{100} \times mg\,h$$

$$\therefore \quad v = \sqrt{\frac{2 \times 90 \times g\,h}{100}} = \sqrt{\frac{2 \times 90 \times 10 \times 2}{100}} = \mathbf{6\ ms^{-1}}.$$

EXAMPLE 29. *A ball bounces to 80% of its original height. What fraction of its mechanical energy is lost in each bounce ? Where does this energy go ?* **[Delhi 97]**

Solution. Suppose the ball is dropped from height h.

Initial P.E. = $mg\,h$

P.E. after first bounce = $mg \times 80\%$ of $h = 0.80\ mgh$

P.E. lost in each bounce $= 0.20\ mgh$

Fraction of P.E. lost in each bounce

$$= \frac{0.20\ mg\,h}{mg\,h} = 0.20$$

This energy is lost in the form of heat, sound, etc.

EXAMPLE 30. *A ball at rest is dropped from a height of 12 m. It loses 25% of its kinetic energy in striking the ground, find the height to which it bounces. How do you account for the loss in kinetic energy ?*

Solution. K.E. gained by the ball in falling down

= P.E. lost by the ball in falling down = mgh

On bouncing upwards, the ball loses 25% of its kinetic energy and the remaining 75% changes back into potential energy. If the ball bounces to height h', then

$$mgh' = 75\% \text{ of } mgh$$

$$\therefore \quad mgh' = \frac{75}{100} \times mgh$$

or $$h' = 0.75\,h = 0.75 \times 12 = \mathbf{9\ m.}$$

The loss in kinetic energy of the ball occurs because a part of it changes into sound and heat.

EXAMPLE 31. *Fig. 6.22 shows a frictionless hemispherical bowl of radius R. A ball of mass m is pushed down the wall from a point A. It just rises up to the edge of the bowl. Calculate the speed with which the ball is pushed down along the wall.*

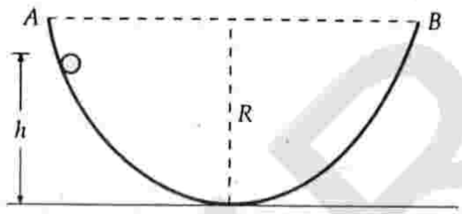


Fig. 6.22

Solution. Let the ball be pushed down along the wall with a speed v. According to law of conservation of energy,

Total energy at A = Total energy at B

or $$\frac{1}{2} mv^2 + mgh = 0 + mgR$$

or $$v^2 = 2g\,(R - h)$$

or $$v = \sqrt{2g\,(R - h)}.$$

EXAMPLE 32. *A body of mass $M = 9.8$ kg with a small disc of mass $m = 0.2$ kg placed on its horizontal surface ab, rests on a smooth horizontal plane, as shown in Fig. 6.23. The disc can move freely along the smooth groove abc of mass M. To what height (relative to the initial position) will the disc rise after separating from the body of mass M when initial velocity $v = 5$ ms^{-1} is given to it in the horizontal direction ?*

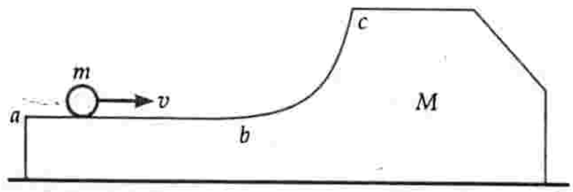


Fig. 6.23

Solution. Let V be the velocity with which the two bodies together move. By law of conservation of momentum,

$$(M + m)\,V = mv \quad \text{or} \quad V = \frac{mv}{M + m}$$

But $m = 0.2$ kg, $M = 9.8$ kg, $v = 5$ ms^{-1}

$$\therefore \quad V = \frac{0.2 \times 5}{9.8 + 0.2} = 0.1\ \text{ms}^{-1}$$

K.E. lost by the disc

$$= \frac{1}{2} mv^2 - \frac{1}{2}(M + m)\,V^2$$

$$= \frac{1}{2} \times 0.2 \times 25 - \frac{1}{2}(9.8 + 0.2) \times 0.01$$

$$= 2.45\ \text{J}$$

Suppose the disc rises to height h after separating from the body of mass M. Then

P.E. gained by the disc

$$= mgh = 0.2 \times 9.8 \times h \text{ joule}$$

By conservation of energy,

P.E gained = K.E. lost

$$\therefore \quad 0.2 \times 9.8 \times h = 2.45$$

or $$h = \frac{2.45}{0.2 \times 9.8} = \mathbf{1.25\ m.}$$

EXAMPLE 33. *A bob of mass m is suspended by a light string of length L. It is imparted a horizontal velocity v_0 at the lowest point A such that it completes a semi-circular trajectory in the vertical plane with the string becoming slack only on reaching the topmost point, C. This is shown in Fig. 6.24. Obtain an expression for (i) v_0; (ii) the speeds at points B and C; (iii) the ratio of the kinetic energies (K_B / K_C) at B and C. Comment on the nature of the trajectory of the bob after it reaches the point C.* **[NCERT]**

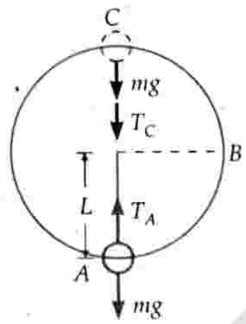


Fig. 6.24

Solution. (*i*) Two external forces act on the bob : gravity and tension (T) in the string. At the lowest point A, the potential energy of the system can be taken zero. So at point A,

Total mechanical energy = Kinetic energy

$$E = \frac{1}{2} mv_0^2 \quad ...(1)$$

If T_A is the tension in the string at point A, then from Newton's second law,

$$T_A - mg = \frac{mv_0^2}{L} \quad ...(2)$$

At the highest point C, the string slackens, so the tension T_C becomes zero. If v_C is the speed at point C, then by conservation of energy,

$$E = K + U \quad \text{or} \quad E = \frac{1}{2} mv_C^2 + 2\ mgL \quad ...(3)$$

From Newton's second law,

$$mg = \frac{mv_C^2}{L} \quad ...(4)$$

or $$mv_C^2 = mgL \quad ...(5)$$

Using (5) in (3),

$$E = \frac{1}{2} mgL + 2\ mgL = \frac{5}{2} mgL \quad ...(6)$$

From equations (1) and (6), we get

$$\frac{5}{2} mgL = \frac{m}{2} v_0^2 \quad \text{or} \quad v_0 = \sqrt{5\ gL} \quad ...(7)$$

(*ii*) From equation (4), we have

$$v_C = \sqrt{gL}$$

The total energy at B is

$$E = \frac{1}{2} mv_B^2 + mgL \quad ...(8)$$

From equations (1) and (8), we get

$$\frac{1}{2} mv_B^2 + mgL = \frac{1}{2} mv_0^2$$

$$\frac{1}{2} mv_B^2 + mgL = \frac{1}{2} m \times 5\ gL \quad \text{[using (7)]}$$

$$\therefore \quad v_B = \sqrt{3\ gL}$$

(*iii*) The ratio of the kinetic energies at B and C is

$$\frac{K_B}{K_C} = \frac{\frac{1}{2} mv_B^2}{\frac{1}{2} mv_C^2} = \frac{3}{1} = \mathbf{3 : 1.}$$

EXAMPLE 34. *A ball falls under gravity from a height of 10 m with an initial downward velocity u. It collides with the ground, loses 50% of its energy in collision and then rises back to the same height. Find the initial velocity u.* **[IIT]**

Solution. If m is the mass of the ball, then its total initial energy at height h

$$= \frac{1}{2} mu^2 + mgh$$

Energy after collision

$$= 50\% \text{ of } \left(\frac{1}{2} mu^2 + mgh\right) = \frac{1}{2}\left(\frac{1}{2} mu^2 + mgh\right)$$

As the ball rebounds to height h, so

$$\frac{1}{2}\left(\frac{1}{2} mu^2 + mgh\right) = mgh \quad \text{or} \quad \frac{1}{4} mu^2 = \frac{1}{2} mgh$$

or $$u = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 10} = \mathbf{14\ ms^{-1}.}$$

PROBLEMS FOR PRACTICE

1. A stone of mass 0.4 kg is thrown vertically up with a speed of 9.8 ms^{-1}. Find the potential and kinetic energies after half second. (**Ans.** 14.386 J, 4.802 J)
2. A ball is thrown vertically up with a velocity of 20 ms^{-1}. At what height, will its K.E. be half its original value ? (**Ans.** 10.20 m)
3. 230 joules were spent in lifting a 10 kg weight to a height of 2 m. Calculate the acceleration with which it was raised. Take $g = 10\ ms^{-2}$. (**Ans.** 1.5 ms^{-2})
4. Calculate the work done in lifting a 300 N weight to a height of 10 m with an acceleration 0.5 ms^{-2}. Take $g = 10\ ms^{-2}$. (**Ans.** 3150 J)
5. A bullet of mass 10 g travels horizontally with speed of 100 ms^{-1} and is absorbed by a wooden block of mass 990 g suspended by a string. Find the vertical height through which the block rises. Take $g = 10\ ms^{-2}$. (**Ans.** 5 cm)

6. A simple pendulum of length 1 m has a wooden bob of mass 1 kg. It is struck by a bullet of mass 10^{-2} kg moving with a speed of 2×10^2 ms^{-1}. The bullet gets embedded into the bob. Obtain the height to which the bob rises before swinging back. Take $g = 10$ ms^{-2} **[Chandigarh 04]** (**Ans.** 0.2 m)

7. A 3.0 kg block, as shown in Fig. 6.25, has a speed of 2 ms^{-1} at A and 6 ms^{-1} at B. If the distance from A to B along the curve is 12 m, how large a frictional force acts on it ? Assuming the same friction, how far from B will it stop ? (**Ans.** 3.35 N, 24.5 m)

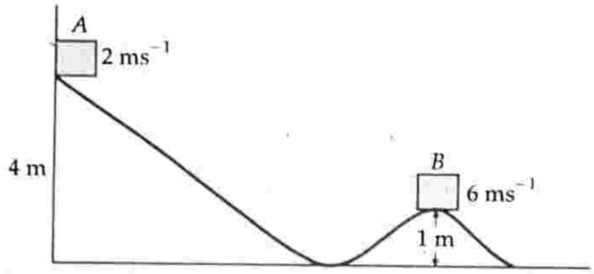


Fig. 6.25

HINTS

1. Here $m = 0.4$ kg, $u = 9.8$ ms^{-1}, $a = -g = -9.8$ ms^{-2}, $t = 1/2$ s

(i) $s = ut + \frac{1}{2}at^2 = 9.8 \times \frac{1}{2} - \frac{1}{2} \times 9.8 \times \frac{1}{2} \times \frac{1}{2} = 3.67$ m

$\therefore$ P.E. $= mgh = 0.4 \times 9.8 \times 3.67 = \mathbf{14.386\ J.}$

(ii) $v = u + at = 9.8 - 9.8 \times \frac{1}{2} = 4.9$ ms^{-1}

$\therefore$ K.E. $= \frac{1}{2}mv^2 = \frac{1}{2} \times 0.4 \times 4.9 \times 4.9 = \mathbf{4.802\ J.}$

2. Work done against gravitation = Change in K.E.

$$mgh = \frac{1}{2}mv^2 - \frac{1}{2} \times \frac{1}{2}mv^2 = \frac{1}{4}mv^2$$

$$\therefore \quad h = \frac{v^2}{4g} = \frac{20 \times 20}{4 \times 9.8} = \mathbf{10.20\ m.}$$

3. Here $W = mgh + mah = m(g + a)h$

$\therefore \quad 230 = 10 \times (10 + a) \times 2$ or $10 + a = 11.5$

or $a = \mathbf{1.5\ ms^{-2}}$.

4. Here $m = \frac{W}{g} = \frac{300\text{ N}}{10\text{ ms}^{-2}} = 30$ kg

$$W = m(g + a)h = 30 \times (10 + 0.5) \times 10 = \mathbf{3150\ J.}$$

6. Momentum of the bullet

$= 10^{-2} \times 2 \times 10^2 = 2$ kg ms^{-1}

Let the combined velocity of the bob + bullet $= v$

Momentum of the bob + bullet

$= (10^{-2} + 1)\,v = 1.01\,v$

By conservation of momentum, $1.01\,v = 2$ kg ms^{-1}

or $v = \frac{2}{1.01} = 1.98$ ms^{-1}

By conservation of energy,

$$\frac{1}{2}(M + m)v^2 = (M + m)gh$$

or $h = \frac{v^2}{2g} = \frac{(1.98)^2}{2 \times 10} = 0.196\text{ m} \approx \mathbf{0.2\ m.}$

7. Change in total energy of block

= Work done against friction

$$\therefore \quad mg(h - x) + \frac{1}{2}m(v_A^2 - v_B^2) = fs$$

$$3 \times 9.8 \times (4 - 1) + \frac{1}{2} \times 3 \times (2^2 - 6^2) = f \times 12$$

$$9 \times 9.8 - 3 \times 16 = f \times 12$$

or $f = \frac{40.2}{12} = \mathbf{3.35\ N.}$

Total energy at $B = 3 \times 9.8 \times 1 + \frac{1}{2} \times 3 \times 6^2$

$= 29.4 + 54 = 83.4$

This energy is used in doing work against friction.

$\therefore \quad f \times s' = 83.4$ or $s' = \frac{83.4}{f} = \frac{83.4}{3.35} = \mathbf{24.5\ m.}$

6.16 POTENTIAL ENERGY OF A SPRING

26. *Show that the elastic force of a spring is a conservative force. Hence write an expression for the potential energy of an elastic stretched spring.*

Potential energy of a spring. Consider an elastic spring of negligibly small mass with its one end attached to a rigid support. Its other end is attached to a block of mass m which can slide over a smooth horizontal surface. The position $x = 0$ is the equilibrium position, as shown in Fig. 6.26(*a*). When the spring is stretched [Fig. 6.26(*b*)] or compressed [Fig. 6.26(*c*)] by pulling or pushing the block, a spring force F_s begins to act in the spring towards the equilibrium position.

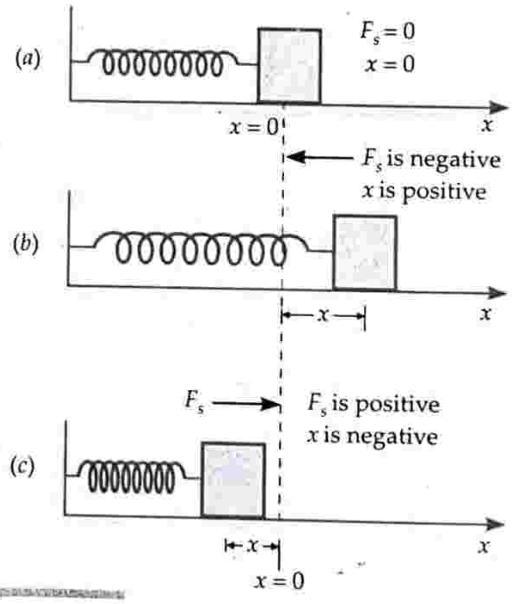


Fig. 6.26 Equilibrium, stretched and compressed states of an elastic spring.

According to Hooke's law, the spring force F_s is proportional to the displacement of the block from the equilibrium position, *i.e.*,

$$F_s \propto x \qquad \text{or} \qquad F_s = -kx$$

The proportionality constant k is called *spring constant*. Its SI unit is Nm^{-1}. The spring is stiff if k is large and soft if k is small. The negative sign shows F_s acts in the opposite direction of x.

The work done by the spring force for the small extension dx is

$$dW_s = F_s\, dx = -kx dx$$

If the block is moved from an initial displacement x_i to the final displacement x_f, the work done by the spring force is

$$W_s = \int dW_s = -\int_{x_i}^{x_f} kx dx = -k\left[\frac{x^2}{2}\right]_{x_i}^{x_f}$$

or $$W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

If the block is pulled from x_i and allowed to return to x_i, then

$$W_s = -\int_{x_i}^{x_f} kx dx = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_i^2 = 0$$

The above discussion shows that

(*i*) The spring force is position dependent as is clear in Hooke's law : $F_s = -kx$,

(*ii*) The work done by the spring force depends on initial and final positions, and

(*iii*) The work done by the spring force in a cyclic process is zero.

Thus the spring force is a *conservative force*.

In order to pull the block outwards with a slow constant speed (quasi-static motion), an external force F equal and opposite to F_s has to be applied. The work done by the external force will be equal to the increase in P.E. of the spring and is given by

$$\Delta U = W = \frac{1}{2}k\,x_f^2 - \frac{1}{2}kx_i^2$$

If we take the potential energy $U(x)$ of the spring to be zero when the block is in equilibrium position, the P.E. of the spring for an extension x will be

$$U(x) - 0 = \frac{1}{2}kx^2 - 0 \quad \text{or} \quad U(x) = \frac{1}{2}kx^2$$

27. *Draw a plot of spring force F_s and displacement x. Hence find an expression for the P.E. of an elastic stretched spring.*

P.E. of an elastic stretched spring by graphical method. Fig. 6.27 shows the plot of spring force F_s versus displacement x of a block attached to the free end of the spring.

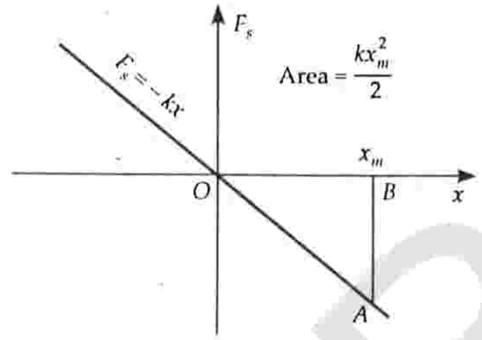


Fig. 6.27 Plot of F_s versus x.

According to Hooke's law, the spring force for an extension x_m is

$$F_s = -k\,x_m$$

The work done by the spring force for an extension x_m is

$$W_s = \text{Area of } \Delta\, OBA = \frac{1}{2}\, AB \times OB$$

$$= \frac{1}{2}F_s \times x_m = \frac{1}{2}(-k\,x_m) \times x_m = -\frac{1}{2}kx_m^2$$

In order to stretch the spring slowly, an external force F equal to and opposite to F_s has to be applied. So work done by the external force F is

$$W = -W_s = +\frac{1}{2}kx_m^2$$

This work done is stored as the P.E. of the spring.

$$\therefore \qquad U = \frac{1}{2}kx_m^2$$

28. *Show that the total energy of the stretched spring remains conserved when it is released. Find the expression for the maximum speed.*

Conservation of energy in an elastic spring. If we stretch a spring to a distance x_m, its P.E. is $\frac{1}{2}kx_m^2$. When it is released, it begins to move under the spring force till it reaches the equilibrium position $x = 0$, where it has maximum velocity. All the P.E. is converted into K.E. Due to inertia of motion, the body overshoots the $x = 0$ position. Its velocity decreases until it momentarily stops at position $x = -x_m$, where all the K.E. is converted into P.E. The spring force again pulls the body towards the position $x = 0$. Thus the body keeps on oscillating. The total mechanical energy remains constant.

At the extreme positions. Here $x = \pm x_m$ and velocity $v = 0$.

$$K = \frac{1}{2}mv^2 = 0$$

$$U = \frac{1}{2}kx_m^2 = \text{a maximum value}$$

At any intermediate position x. For x between $-x_m$ to $+x_m$, the energy is partly kinetic and partly potential.

Total energy = K.E. + P.E.

$$\frac{1}{2}kx_m^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$\therefore \quad K = \frac{1}{2}mv^2 = \frac{1}{2}k(x_m^2 - x^2)$$

Velocity, $v = \sqrt{\frac{k}{m}(x_m^2 - x^2)}$

At the equilibrium position. Here $x = 0$.

$$\therefore \quad U = \frac{1}{2}k(0)^2 = 0$$

$$K = \frac{1}{2}mv_m^2 = \frac{1}{2}kx_m^2$$

Maximum speed,

$$v_m = \sqrt{\frac{k}{m}}\,x_m.$$

The variations of K.E., P.E. and total energy with displacement x are shown in Fig. 6.28. As both K.E. and P.E. depend on x^2, their graphs are parabolic. Total mechanical energy $E = K + U$ remains constant, so its graph is a straight line parallel to displacement axis.

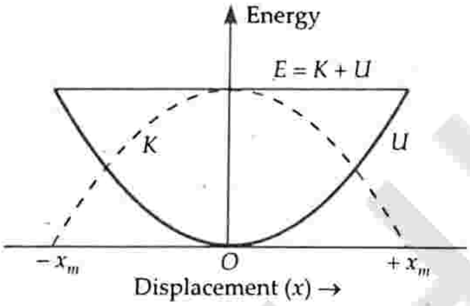


Fig. 6.28 Conservation of energy in an elastic spring.

For Your Knowledge

- The notion of potential energy applies to only those forces where the work done against the force gets *stored up* as energy by virtue of position or configuration of the body. When external constraints are removed, this energy appears as kinetic energy.
- The potential energy of a body subjected to a conservative force is uncertain upto a certain limit. This is because the point of zero potential energy is a matter of choice.
- For the gravitational P.E., the zero of potential energy is chosen to be the ground.
- For the spring potential energy $\frac{1}{2}kx^2$, the zero of the potential energy is the equilibrium position of the oscillating mass.
- Every mechanical force is not associated with a potential energy. The work done by friction over a closed path is not zero because no potential energy can be associated with friction.

Examples based on Potential Energy of a Spring

Formulae Used

1. According to Hooke's law, $F = -kx$
2. Force constant, $k = \frac{F}{x}$
3. Work done on a spring or P.E. of a spring stretched through distance x, $W = U = \frac{1}{2}kx^2$

Units Used

Force F is in newton, distance x in metre, potential energy U in joule and force constant k in Nm^{-1}.

Example 35. *Two springs have force constants k_1 and k_2 ($k_1 > k_2$). On which spring is more work done, if (i) they are stretched by the same force and (ii) they are stretched by the same amount ?*

Solution. (*i*) Suppose the two springs get stretched by distances x_1 and x_2 by the same force F. Then

$$F = k_1 x_1 = k_2 x_2$$

$$\frac{W_1}{W_2} = \frac{\frac{1}{2}k_1 x_1^2}{\frac{1}{2}k_2 x_2^2} = \frac{k_1 x_1 \cdot x_1}{k_2 x_2 \cdot x_2} = \frac{F \cdot x_1}{F \cdot x_2} = \frac{x_1}{x_2} = \frac{k_2}{k_1}$$

As $k_1 > k_2 \quad \therefore W_1 < W_2$ or $\mathbf{W_2 > W_1}$.

(*ii*) Suppose the two springs are stretched by the same distance x. Then

$$\frac{W_1}{W_2} = \frac{\frac{1}{2}k_1 x^2}{\frac{1}{2}k_2 x^2} = \frac{k_1}{k_2}$$

As $k_1 > k_2 \quad \therefore \mathbf{W_1 > W_2}$.

Example 36. *The length of a steel wire increases by 0.5 cm when it is loaded with a weight of 5 kg. Calculate (i) force constant of the wire and (ii) work done in stretching the wire.*

Solution. If a force F applied to a wire increases its length by x, then accordingly to Hooke's law,

$$F = kx$$

where k is force constant.

Given $\quad F = mg = 5 \times 10 = 50$ N,

$$x = 0.5 \text{ cm} = 0.5 \times 10^{-2} \text{ m}$$

$$\therefore \quad k = \frac{F}{x} = \frac{50}{0.5 \times 10^{-2}} = \mathbf{1.0 \times 10^4\ Nm^{-1}}.$$

(*ii*) Work done in stretching the wire,

$$W = \frac{1}{2}kx^2 = \frac{1}{2} \times 1.0 \times 10^4 \times (0.5 \times 10^{-2})^2 = \mathbf{0.125\ J}.$$

Example 37. *The potential energy of a spring when stretched through a distance x is 10 J. What is the amount of work done on the same spring to stretch it through an additional distance x ?* **[AFMC 91]**

Solution. P.E. of the spring when stretched through a distance x,

$$U = \frac{1}{2}kx^2 = 10 \text{ J}$$

When x becomes $2x$, the potential energy will be

$$U' = \frac{1}{2}k(2x)^2 = 4 \times \frac{1}{2}kx^2 = 4 \times 10 = 40 \text{ J}$$

$\therefore$ Work done $= U' - U = 40 - 10 =$ **30 J.**

EXAMPLE 38. *To simulate car accidents, auto manufacturers study the collisions of moving cars with mounted springs of different spring constants. Consider a typical simulation with a car of mass 1000 kg moving with a speed 18.0 kmh^{-1} on a smooth road and colliding with a horizontally mounted spring of spring constant 6.25×10^3 Nm^{-1}. What is the maximum compression of the spring ?* **[NCERT]**

Solution. Here $m = 1000$ kg, $k = 6.25 \times 10^3$ Nm^{-1},

$$v = 18 \text{ kmh}^{-1} = \frac{18 \times 5}{18} \text{ ms}^{-1} = 5 \text{ ms}^{-1}$$

At the maximum compression x_m, the kinetic energy of the car is converted entirely into the potential energy of the spring. Therefore,

Gain in P.E. of the spring = Loss in K.E. of the car

or $\quad \frac{1}{2}kx_m^2 = \frac{1}{2}mv^2$

or $\quad x_m^2 = \frac{mv^2}{k} = \frac{1000 \times 5 \times 5}{6.25 \times 10^3} = 4$

$\therefore \quad x_m =$ **2.0 m.**

EXAMPLE 39. *Consider Example 38 taking the coefficient of friction, μ, to be 0.5 and calculate the maximum compression of the spring.* **[NCERT]**

Solution. In presence of friction, both the spring force and the frictional force act so as to oppose the compression of the spring, as shown in Fig. 6.29.

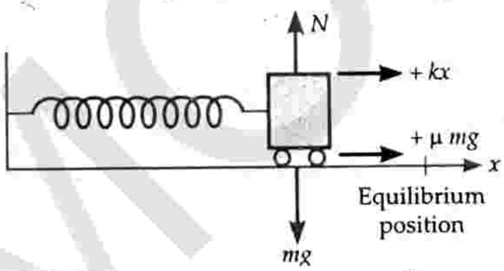


Fig. 6.29

The change in K.E. of the car is

$$\Delta K = K_f - K_i = 0 - \frac{1}{2}mv^2$$

The work done by the two opposing forces is

$$W = -\frac{1}{2}kx_m^2 - \mu\, mgx_m$$

By work-energy theorem,

$$W = \Delta K$$

$\therefore \quad \frac{1}{2}kx_m^2 + \mu\, mg\, x_m = \frac{1}{2}mv^2$

or $\quad kx_m^2 + 2\mu mgx_m - mv^2 = 0$

or $\quad 6.25 \times 10^3 x_m^2 + 2 \times 0.5 \times 1000 \times 10\, x_m - 1000\,(5)^2 = 0$

or $\quad 5x_m^2 + 8x_m - 20 = 0$

$\therefore \quad x_m = \frac{-8 \pm \sqrt{64 + 400}}{10}$

As x_m is positive, so

$$x_m = \frac{-8 + 21.54}{10} = \mathbf{1.354\ m.}$$

As expected, this value is less than the value obtained in the above example.

EXAMPLE 40. *The spring shown in Fig. 6.30 has a force constant of 24 Nm^{-1}. The mass of the block attached to the spring is 4 kg. Initially the block is at rest and spring is unstretched. The horizontal surface is frictionless. If a constant horizontal force of 10 N is applied on the block, then what is the speed of the block when it has been moved through a distance of 0.5 m ?*

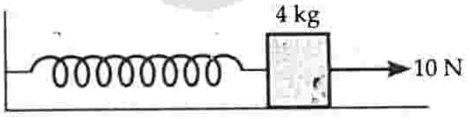


Fig. 6.30

Solution. Here $k = 24$ Nm^{-1}, $m = 4$ kg,

$x = 0.5$ m, $\quad F = 24$ N

By the law of conservation of energy,

Work done on the spring

= Gain in K.E. + Gain in P.E.

or $\quad Fx = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$

or $\quad 10 \times 0.5 = \frac{1}{2} \times 4 \times v^2 + \frac{1}{2} \times 24 \times (0.5)^2$

or $\quad 5 = 2v^2 + 3 \quad$ or $\quad v^2 = 1$

$\therefore \quad v =$ **1 ms^{-1}.**

EXAMPLE 41. *A ball of mass m is dropped from a height h on a platform fixed at the top of a vertical spring, as shown in Fig. 6.31. The platform is depressed by a distance x. What is the spring constant k ?*

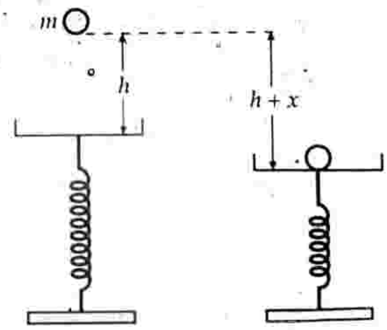


Fig. 6.31

Solution. The ball falls through a total distance of $(h + x)$

$\therefore$ P.E. lost by the ball $= mg\,(h + x)$

Work done on the spring $= \frac{1}{2}kx^2$

By conservation of energy,

$$\frac{1}{2}kx^2 = mg\,(h + x)$$

$$\therefore \quad k = \frac{2\,mg\,(h + x)}{x^2}.$$

Example 42. A block of mass 2 kg initially at rest is dropped from a height of 1 m into a vertical spring having force constant 490 Nm^{-1}. Calculate the maximum distance through which the spring will be compressed.

Solution. Here $m = 2$ kg, $h = 1$ m, $k = 490\ Nm^{-1}$

As shown in Fig. 6.31, let the spring be compressed through distance x. Then the block falls through a height $h + x$.

Get in P.E. of the spring = Loss in P.E. of the block

$$\frac{1}{2}kx^2 = mg\,(h + x)$$

or $$\frac{1}{2} \times 490 \times x^2 = 2 \times 9.8 \times (1 + x)$$

or $$12.5\,x^2 - x - 1 = 0$$

$$\therefore \quad x = \frac{1 \pm \sqrt{1 + 4 \times 12.5}}{2 \times 12.5} = \frac{1 \pm \sqrt{51}}{25} = \mathbf{0.3256\ m.}$$

Example 43. Two blocks A and B are connected to each other as shown in Fig. 6.32. The string and spring is massless and pulley frictionless. Block B slides over the horizontal top surface of stationary block C and the block A slides along the vertical side of C both with same uniform speed. The coefficient of friction between the blocks is 0.2 and the spring constant of spring is 1960 Nm^{-1}. If mass of block A is 2 kg, calculate (i) the mass of block B and (ii) energy stored in spring. **[IIT 82]**

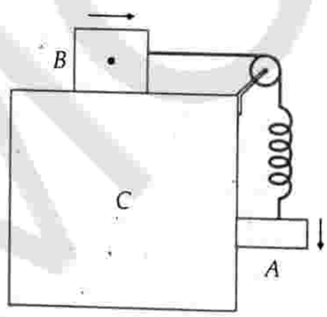


Fig. 6.32

Solution. Various forces acting on the blocks A and B are shown in Fig. 6.33.

Let mass of block $B = m$

Tension in the string $= T$

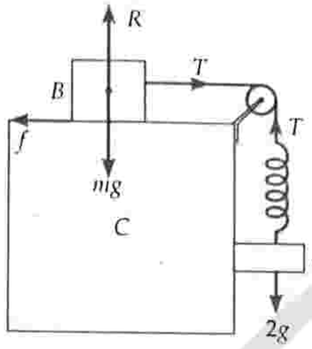


Fig. 6.33

For block A : $\quad T = 2g \quad [\because m = 2\text{ kg}]$

For block B : $\quad T = f = \mu R = \mu mg = 0.2 \times mg$

$$\therefore \quad 0.2 \times mg = 2g \quad \text{or} \quad m = \frac{2}{0.2} = \mathbf{10\ kg}$$

Also $\quad T = 2\,g = 2 \times 9.8 = 19.6$ N

Let x be the extension of the spring due to the tension T. Then $T = kx$

or $$x = \frac{T}{k} = \frac{19.6\text{ N}}{1960\ Nm^{-1}} = 0.01\text{ m}$$

Energy stored in the spring

$$U = \frac{1}{2}kx^2 = \frac{1}{2} \times 1960 \times (0.01)^2 = \mathbf{0.098\ J.}$$

Problems For Practice

1. A spring gun has a spring constant of 18 N cm^{-1}. The spring is compressed 12 cm by a ball of mass 15 g. How much is the potential energy of the spring ? If the trigger is pulled, what will the velocity of the ball be ? (**Ans.** 57.6 J, 87.6 ms^{-1})
2. A solid of mass 2 kg moving with a velocity of 10 ms^{-1} strikes an ideal weightless spring and produces a compression of 25 cm in it. Calculate the force constant of the spring. (**Ans.** 3200 Nm^{-1})
3. A 16 kg block moving on a frictionless horizontal surface with a velocity of 5 ms^{-1} compresses an ideal spring and comes to rest. If the force constant of the spring be 100 Nm^{-1}, then how much is the spring compressed ? (**Ans.** 2.0 m)
4. A block of mass 2 kg is dropped from a height of 40 cm on a spring whose force-constant is 1960 Nm^{-1}. What will be the maximum distance x through which the spring is compressed ? (**Ans.** 10 cm)
5. A block of mass m, initially at rest, is dropped from a height h onto a spring whose force constant is k. Find the maximum distance x through which the spring will be compressed.

$$\left[\textbf{Ans. } x = \frac{1}{2}\left(\frac{2mg}{k} \pm \sqrt{\left(\frac{2mg}{k}\right)^2 + \frac{8\,mgh}{k}}\right)\right]$$

6. An object is attached to a vertical spring and slowly lowered to its equilibrium position. This stretches the spring by a distance d. If the same object is attached to the same vertical spring but permitted to fall freely, through what distance does it stretch the spring ? **(Ans.** $2\,d$**)**

7. A massless platform is kept on a light elastic spring. When a sand particle of mass 0.1 kg is dropped on the pan from a height of 0.24 m, the particle strikes the pan, and the spring compresses by 0.01 m. From what height should the particle be dropped to cause a compression of 0.04 m ? **(Ans.** 3.96 m**)**

HINTS

1. Here $k = 80 \text{ Ncm}^{-1} = 80 \times 100 \text{ Nm}^{-1}$,

$$x = 12 \text{ cm} = 0.12 \text{ m}, \; m = 15 \text{ g} = 15 \times 10^{-3} \text{ kg}$$

Potential energy,

$$U = \frac{1}{2} kx^2 = \frac{1}{2} \times 80 \times 100 \times (0.12)^2 = \mathbf{57.6 \ J.}$$

Also, K.E. of ball = P.E. of the spring

or $\frac{1}{2} mv^2 = U = 57.6 \text{ J}$ or $v^2 = \frac{2 \times 57.6}{15 \times 10^{-3}} = 7680$

or $v = \mathbf{87.6 \ ms^{-1}}$.

3. Here $\frac{1}{2} kx^2 = \frac{1}{2} mv^2$

$$\therefore \quad x = \sqrt{\frac{m}{k}} \cdot v = \sqrt{\frac{16}{100}} \times 5 = \mathbf{2.0 \ m.}$$

4. Here $mg\,(h + x) = \frac{1}{2} kx^2$.

5. Here $mg\,(h + x) = \frac{1}{2} kx^2$ or $x^2 - \frac{2mg}{k} x - \frac{2mgh}{k} = 0$

On solving the quadratic equation, we get

$$x = \frac{1}{2}\left[\frac{2mg}{k} \pm \sqrt{\left(\frac{2mg}{k}\right)^2 + \frac{8mgh}{k}}\right]$$

6. If mg is weight of the object, then in equilibrium position,

$$k\,d = mg \quad \text{or} \quad k = \frac{mg}{d}$$

When the object falls freely, its gravitational P.E. is converted into elastic potential energy of the spring.

$$\therefore \quad mgx = \frac{1}{2} kx^2$$

or $$mg\,x = \frac{1}{2}\left(\frac{mg}{d}\right) x^2$$

or $$x = \mathbf{2\,d}.$$

7. Here $m = 0.1 \text{ kg}, h = 0.24 \text{ m}, x = 0.01 \text{ m}$,

$$h' = ?, \quad x' = 0.04 \text{ m}$$

As the particle is dropped from a height h, it compresses the spring through distance x.

$\therefore$ Total loss in P.E. of the particle

= Total gain in P.E. of the spring.

or $$mg\,(h + x) = \frac{1}{2} kx^2 \quad ...(i)$$

and $$mg\,(h' + x') = \frac{1}{2} kx'^2 \quad ...(ii)$$

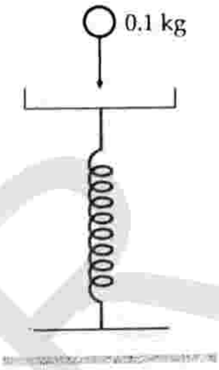


Fig. 6.34

Dividing (*ii*) by (*i*), we get

$$\frac{h' + x'}{h + x} = \left(\frac{x'}{x}\right)^2$$

or $$\frac{h' + 0.04}{0.24 + 0.01} = \left(\frac{0.04}{0.01}\right)^2$$

On solving, $h' = \mathbf{3.96 \ m.}$

6.17 DIFFERENT FORMS OF ENERGY

29. *Describe the various forms of energy.*

Different forms of energy. Energy can manifest itself in many forms. Some of these forms are as follows :

(*i*) ***Mechanical energy.*** The sum of the kinetic and potential energies in bulk is called mechanical energy. Kinetic energy is due to motion while the potential energy is due to position or configuration.

(*ii*) ***Internal energy.*** The molecules of a body vibrate with respect to one another. These molecules have kinetic energy due to their motion and potential energy due to their attractions and repulsions. The sum of the kinetic and potential energies of all the molecules is called the internal energy of the body. As the temperature of a body increases, its internal energy also increases.

(*iii*) ***Heat or thermal energy.*** A body possesses heat energy due to the random motion of its molecules. Heat energy is also related to the internal energy of the body. In winter, we generate heat by rubbing our hands against each other.

(*iv*) ***Chemical energy.*** A stable chemical compound has lesser energy than its constituent atoms, the difference being in the arrangement and motion of electrons in the compound. This difference is called chemical energy. A chemical reaction is just the rearrangement of atoms. If the total energy of the reactants is more than the products, heat is released and the reaction is *exothermic*. If the reverse is true, heat is absorbed and the reaction is *endothermic*. When 1 kg of coal is burnt, it releases 3×10^7 J of energy.

(*iv*) ***Electrical energy.*** Electric charges and currents attract or repel each other, *i.e.*, they exert forces on each other. Work has to be done to move charges with respect to one another. The energy associated with this work is called electrical or electromagnetic energy.

(*v*) ***Nuclear energy.*** Neutrons and protons attract each other very strongly at distances of order 10^{-15} m, and bind together to form nuclei. The associated energy is called nuclear energy. Nuclear energy is released in the process of nuclear fission and nuclear fusion. In each of these processes, a part of the mass (called mass defect) is converted into energy in accordance with the Einstein's mass-energy relation.

6.18 EINSTEIN'S MASS-ENERGY EQUIVALENCE

30. *What is Einstein's mass-energy equivalence ? Mention some of its practical applications.*

Einstein's mass-energy equivalence. In 1905, Albert Einstein discovered that mass can be converted into energy and vice versa. He showed that mass and energy are equivalent and related by the relation

$$E = mc^2$$

where *c*, the speed of light in vacuum is approximately 3×10^8 ms^{-1}. According to Einstein's mass-energy relation, if mass *m* disappears, an energy $E(= mc^2)$ appears in some form. Conversely, when energy *E* disappears, a mass $m(= E/c^2)$ appears.

Applications of mass-energy equivalence :

(*i*) ***Annihilation of matter.*** When an electron $({}_{-1}^{0}e)$ and a positron $({}_{1}^{0}e)$ come close to each other, they annihilate (destroy) each other forming two γ-rays (electromagnetic radiation) of total energy given by Einstein's mass-energy relation.

(*ii*) ***Pair production.*** When a γ-ray photon of energy 1.02 MeV passes close to a massive nucleus, it materialises into a pair of particles–an electron and a positron. Thus energy gets converted into matter.

(*iii*) ***Energy generation in the sun and stars.*** The energy generated in the sun and stars is due to the conversion of mass into energy.

6.19 PRINCIPLE OF CONSERVATION OF ENERGY

31. *State and explain the principle of conservation of energy.*

Principle of conservation of energy. When we throw a ball up, the chemical energy stored in our body gets transferred to the ball as its kinetic energy. As the ball moves up, it loses K.E. and gains P.E. At the highest point, all of its K.E. changes into P.E. As the ball falls on the ground, its P.E. changes into heat and sound. Although energy is being transformed from one form to another at every stage, yet its total amount remains the same.

This is the **principle of conservation of energy** which can be stated in a number of ways :

(*i*) *Energy can neither be created, nor destroyed. It may be transformed from one form to another.*

(*ii*) *The total energy of an isolated system remains constant.*

(*iii*) As the entire universe may be regarded as an isolated system, *the total energy of the universe is constant*. If one part of the universe loses energy, another part must gain an equal amount of energy.

For Your Knowledge

- In the principle of conservation of energy, we include mass into total energy, because mass can be converted into energy.
- The principle of conservation of energy cannot be proved mathematically, but is an empirical principle. The deductions made on the basis of this principle are found to be true.

Examples based on Mass-Energy Equivalence

FORMULAE USED

According to Einstein, energy equivalent of mass *m* is $E = mc^2$, where *c* = speed of light in free space $= 3 \times 10^8$ ms^{-1}

UNITS USED

Mass *m* is in kg and energy *E* in joule.

CONVERSIONS USED

$$1\,\text{eV} = 1.6 \times 10^{-19}\,\text{J},$$
$$1\,\text{MeV} = 10^6\,\text{eV} = 1.6 \times 10^{-13}\,\text{J}$$
$$1\,\text{amu} = 931\,\text{MeV}$$

EXAMPLE 44. *Express :*

(*a*) *The energy required to break one bond* (10^{-20} *J) in DNA in eV.*

(*b*) *The kinetic energy of an air molecule* (10^{-21} *J) in eV.*

(*c*) *The daily intake of a human adult* (10^7 *J) in kilocalories.*

Solution. (*a*) 1 eV = 1.6×10^{-19} J

Energy required to break one bond in DNA

$$= 10^{-20}\,\text{J} = \frac{10^{-20}}{1.6 \times 10^{-19}}\,\text{eV} \simeq \mathbf{0.06\ eV.}$$

(*b*) 1 eV = 1.6×10^{-19} J

Kinetic energy of an air molecule

$$= 10^{-21}\,\text{J} = \frac{10^{-21}}{1.6 \times 10^{-19}} \simeq \mathbf{0.062\ eV.}$$

(*c*) 1 kcal = 4186 J

The average daily human consumption

$$= 10^{-21}\,\text{J} = \frac{10^7}{4186}\,\text{kcal} = 2389\,\text{kcal} \simeq \mathbf{2400\ kcal.}$$

EXAMPLE 45. How much mass is converted into energy per day in Tarapur nuclear power plant operated at 10^7 kW ?

Solution. Power,

$$P = 10^7 \text{ kW} = 10^{10} \text{ W} = 10^{10} \text{ Js}^{-1}$$

Time, $t = 1 \text{ day} = 24 \times 60 \times 60 \text{ s}$

Energy produced per day,

$$E = Pt = 10^{10} \times 24 \times 60 \times 60 = 864 \times 10^{12} \text{ J}$$

As $E = mc^2$

$$\therefore \quad m = \frac{E}{c^2} = \frac{864 \times 10^{12}}{(3 \times 10^8)^2}$$

$$= 9.6 \times 10^{-3} \text{ kg} = \mathbf{9.6 \text{ g}}.$$

EXAMPLE 46. If 1000 kg of water is heated from 0°C to 100°C, calculate the increase in the mass of water.

Solution. Here $m = 1000 \text{ kg} = 10^6 \text{ g}$,

$$\theta = 100 - 0 = 100° \text{ C}$$

Specific heat of water, $s = 1 \text{ cal g}^{-1} {}^\circ\text{C}^{-1}$

Heat gained by water

$$= ms\,\theta = 10^6 \times 1 \times 100 = 10^8 \text{ cal} = 4.2 \times 10^8 \text{ J}$$

[∵ 1 cal = 4.2 J]

Increase in mass,

$$\Delta m = \frac{4.2 \times 10^8}{c^2} = \frac{4.2 \times 10^8}{(3 \times 10^8)^2} = \mathbf{0.466 \times 10^{-8} \text{ kg}}.$$

EXAMPLE 47. Calculate the energy in MeV equivalent to the rest mass of an electron. Given that the rest mass of an electron, $m_0 = 9.1 \times 10^{-31}$ kg, 1 MeV $= 1.6 \times 10^{-13}$ J and speed of light, $c = 3 \times 10^8$ ms^{-1}.

Solution. $E = m_0 c^2 = 9.1 \times 10^{-31} \times (3 \times 10^8)^2$

$$= 81.9 \times 10^{-15} \text{ J}$$

$$= \frac{81.1 \times 10^{-15}}{1.6 \times 10^{-13}} = \mathbf{0.512 \text{ MeV}}.$$

EXAMPLE 48. Estimate the amount of energy released in the following nuclear fusion reaction :

$${}_1^2H + {}_1^2H \rightarrow {}_2^3He + {}_0^1n$$

Given mass of ${}_1^2H = 2.0141$ amu, mass of ${}_2^3He = 3.0160$ amu, mass of ${}_0^1n = 1.0087$ amu and 1 amu $= 1.661 \times 10^{-27}$ kg. Express your answer in units of MeV.

Solution. ${}_1^2\text{H} + {}_1^2\text{H} \rightarrow {}_2^3\text{He} + {}_0^1n$

Total initial mass (${}_1^2\text{H} + {}_1^2\text{H}$)

$$= 2.0141 + 2.0141 = 4.0282 \text{ amu}$$

Total final mass (${}_2^3\text{He} + {}_0^1n$)

$$= 3.0160 + 1.0087 = 4.0247 \text{ amu}$$

Decrease in mass,

$$\Delta m = 4.0282 - 4.0247 = 0.0035 \text{ amu}$$

$$= 0.0035 \times 1.661 \times 10^{-27} \text{ kg}$$

∴ Energy released

$$= \Delta m \times c^2 = 0.0035 \times 1.661 \times 10^{-27} \times (3 \times 10^8)^2$$

$$= 5.232 \times 10^{-13} \text{ J} = \frac{5.232 \times 10^{-13}}{1.6 \times 10^{-13}}$$

$$= \mathbf{3.27 \text{ MeV}}.$$

EXAMPLE 49. When slow neutrons are incident on a target containing ${}_{92}^{235}U$, a possible fission reaction is

$${}_{92}^{235}U + {}_0^1n \rightarrow {}_{56}^{141}Ba + {}_{36}^{92}Kr + 3\,{}_0^1n$$

Estimate the amount of energy released using the following data :

Given, mass of ${}_{92}^{235}U = 235.04$ amu, mass of ${}_0^1n = 1.0087$ amu, mass of ${}_{56}^{141}Ba = 140.91$ amu, mass of ${}_{36}^{92}Kr = 91.926$ amu and energy equivalent to 1 amu = 931 MeV.

Solution. ${}_{92}^{235}\text{U} + {}_0^1n \rightarrow {}_{56}^{141}\text{Ba} + {}_{36}^{92}\text{Kr} + 3\,{}_0^1n$

Total initial mass (${}_{92}^{235}\text{U} + {}_0^1n$)

$$= 235.04 + 1.0087 = 236.0487 \text{ amu}$$

Total final mass (${}_{56}^{141}\text{Ba} + {}_{36}^{92}\text{Kr} + 3\,{}_0^1n$)

$$= 140.91 + 91.926 + 3 \times 1.0087$$

$$= 235.8621 \text{ amu}$$

Decrease in mass,

$$\Delta m = 236.0487 - 235.8621 = 0.1866 \text{ amu}$$

Energy released

$$= \Delta m \times 931 = 0.1866 \times 931$$

$$= \mathbf{173.725 \text{ MeV}}.$$

PROBLEMS FOR PRACTICE

1. About 4×10^9 kg of matter is converted into energy in the sun each second. What is the power output of the sun ? (**Ans.** 3.6×10^{27} W)
2. Show that energy equivalent to atomic mass unit equals nearly 933 MeV of energy. Given 1 atomic mass unit = 1.66×10^{-27} kg.
3. 500 kg of water is heated from 20° to 100°C. Calculate the increase in the mass of water. Given specific heat of water = 4.2×10^3 J kg^{-1} °C^{-1}. (**Ans.** 1.87×10^{-9} kg)
4. 1 mg of uranium is completely destroyed in an atomic bomb. How much energy is liberated ? **[Himachal 03]** (**Ans.** 9×10^{10} J)
5. An electron-positron pair annihilates at rest to produce γ-rays. Calculate the energy produced in MeV if the rest mass of electron is 9.1×10^{-31} kg. (**Ans.** 1.02 MeV)

HINTS

1. Here $m = 4\times10^9$ kg, $c = 3\times10^8\ ms^{-1}$, $t = 1s$

$$E = mc^2 = 4\times10^9\times(3\times10^8)^2 = 3.6\times10^{27}\ J$$

$$P = \frac{E}{t} = \frac{3.6\times10^{27}\ J}{1s} = \mathbf{3.6\times10^{27}\ W.}$$

2. Energy equivalent to 1 amu = mc^2

$$= 1.66\times10^{-27}\times(3\times10^8)^2\ J = 1.66\times9\times10^{-11}\ J$$

$$= \frac{1.66\times9\times10^{-11}}{1.6\times10^{-13}}\ MeV\ [\because\ 1\ MeV = 1.6\times10^{-13}\ J]$$

$$= 9.3375\times10^2\ MeV = \mathbf{933.75\ MeV.}$$

5. Rest mass of an electron or a positron = 9.1×10^{-31} kg

During the annihilation of electron-positron pair, the minimum amount of energy is released when the mass of electron-positron pair at rest is converted into energy.

$\therefore$ Minimum energy released

$$= 2\times9.1\times10^{-31}\times(3\times10^8)^2\ J$$

$$= \frac{2\times9.1\times9\times10^{-15}}{1.6\times10^{-13}} = \mathbf{1.02\ MeV.}$$

6.20 POWER

32. *Define the term power. Is it a scalar or vector quantity ? Give its dimensions and units.*

Power. *Power is defined as the rate of doing work.* If an agent does work W in time t, then its average power is given by

$$P_{av} = \frac{W}{t}$$

The shorter is the time taken by a person or a machine in performing a particular task, the larger is the power of that person or machine.

Power is a scalar quantity, because it is the ratio of two scalar quantities work (W) and time (t).

Dimensions of power.

$$[P] = \frac{[W]}{[t]} = \frac{[ML^2T^{-2}]}{[T]} = [ML^2T^{-3}]$$

Units of power. The SI unit of power is **watt** (W). *The power of an agent is one watt if it does work at the rate of 1 joule per second.*

$$1\ \text{watt} = \frac{1\ \text{joule}}{1\ \text{second}} \quad \text{or} \quad 1\ W = 1\ Js^{-1}$$

The bigger units of power are kilowatt (kW) and horse power (hp).

1 kilowatt = 1000 watt or $1\ kW = 10^3\ W$

1 horse power = 746 watt or 1 hp = 746 W.

33. *Define instantaneous power. Express it as the scalar product of force and velocity vectors.*

Instantaneous power. The power of an agent may not be constant during a time interval. *The instantaneous power is defined as the limiting value of the average power as the time interval approaches zero.* If ΔW work is done in a small time interval Δt, then the instantaneous power is given by

$$P = \lim_{\Delta t\to0}\frac{\Delta W}{\Delta t} = \frac{dW}{dt}$$

Power as dot product. The work done by a force $\vec{F}$ for a small displacement $\vec{dr}$ is given by

$$dW = \vec{F}\cdot\vec{dr}$$

So the instantaneous power can be expressed as

$$P = \frac{dW}{dt} = \vec{F}\cdot\frac{\vec{dr}}{dt}$$

But $\frac{\vec{dr}}{dt} = \vec{v}$, the instantaneous velocity

$$\therefore \quad P = \vec{F}\cdot\vec{v}$$

Thus the power of an agent at any instant is equal to the dot product of its force and velocity vectors at that instant.

34. *Of which physical quantity is the unit kilowatt hour ? Define one kilowatt hour. Express it in joules.*

Kilowatt hour (kWh). *Kilowatt hour* (kWh) or *Board of Trade* (B.O.T.) unit is the commercial unit of electrical energy. *One kilowatt hour is the electrical energy consumed by an appliance of 1000 watt in 1 hour.*

Relation between kWh and joule.

$$1\ kWh = 1\ kW\times1\ h = 1000\ W\times1\ h$$
$$= 1000\ Js^{-1}\times3600\ s$$

or $$1\ kWh = 3.6\times10^6\ J.$$

Examples based on Power

FORMULAE USED

1. Power = $\frac{\text{Work}}{\text{Time}}$ or $P = \frac{W}{t}$
2. Also $P = \vec{F}\cdot\vec{v}$ When $\theta = 0°$, $P = Fv$

UNITS USED

Work W is in joule, force F in newton, time t in second, velocity v in ms^{-1}, power P in watt.

CONVERSIONS USED

1 kilowatt = 1000 watt or 1 kW = 1000 W

1 horsepower = 746 watt or 1 hp = 746 W.

EXAMPLE 50. *A man weighing 60 kg climbs up a staircase carrying a load of 20 kg on his head. The stair case has 20 steps each of height 0.2 m. If he takes 10 s to climb, find his power.*

Solution. Here $m = 60 + 20 = 80$ kg,

$h = 20 \times 0.2 = 4$ m

$g = 9.8\ ms^{-2},\ t = 10$ s

$$P = \frac{W}{t} = \frac{mgh}{t} = \frac{80 \times 9.8 \times 4}{10} = \frac{3136}{10} = \mathbf{313.6\ W.}$$

EXAMPLE 51. *A car of mass 2000 kg is lifted up a distance of 30 m by a crane in 1 min. A second crane does the same job in 2 min. Do the cranes consume the same or different amounts of fuel ? What is the power supplied by each crane ? Neglect power dissipation against friction.* **[NCERT]**

Solution. Here $m = 2000$ kg, $s = 30$ m,

$t_1 = 1$ min $= 60$ s, $t_2 = 2$ min $= 120$ s

Work done by each crane,

$$W = Fs = mgs = 2000 \times 9.8 \times 30 = 5.88 \times 10^5\ J$$

As both the cranes do same amount of work, so both consume same amount of fuel.

Power supplied by first crane,

$$P_1 = \frac{W}{t_1} = \frac{5.88 \times 10^5}{60} = \mathbf{9800\ W.}$$

Power supplied by second crane,

$$P_2 = \frac{W}{t_2} = \frac{5.88 \times 10^5}{120} = \mathbf{4900\ W.}$$

EXAMPLE 52. *The human heart discharges 75 ml of blood at each beat against a pressure of 0.1 m of Hg. Calculate the power of heart assuming that pulse frequency is 80 beats per minute. Density of Hg $= 13.6 \times 10^3\ kgm^{-3}$.*

Solution. Volume of blood discharged per beat,

$$V = 75\ ml = 75 \times 10^{-6}\ m^{-3}$$

Pressure, $P = 0.1$ m of Hg

$$= 0.1 \times 13.6 \times 10^3 \times 9.8\ Nm^{-2}$$

$[\because\ P = h\rho g]$

Work done per beat $= PV$

Work done in 80 beats $= 80 \times PV$

Time, $t = 1$ min $= 60$ s

$$\text{Power} = \frac{\text{Work}}{\text{Time}} = \frac{80 \times PV}{t}$$

$$= \frac{80 \times 0.1 \times 13.6 \times 10^3 \times 9.8 \times 75 \times 10^{-6}}{60}$$

$$= \mathbf{1.33\ W.}$$

EXAMPLE 53. *An electric motor is used to lift an elevator and its load (total mass = 1500 kg) to a height of 20 m. The time taken for the job is 20 s. What is the work done ? What is the rate at which work is done ? If the efficiency of the motor is 75%, at which rate is the energy supplied to the motor ?*

Solution. Here $m = 1500$ kg, $h = 20$ m,

$\eta = 75\%,\ t = 20$ s

Work done,

$$W = mgh = 1500 \times 9.8 \times 20 = \mathbf{2.94 \times 10^5\ J}$$

Rate of doing work

$$= \frac{W}{t} = \frac{2.94 \times 10^5}{20} = \mathbf{1.47 \times 10^4\ W.}$$

As $\eta = \dfrac{\text{Output power}}{\text{Input power}}$

$$\therefore \quad \frac{75}{100} = \frac{1.47 \times 10^4}{\text{Input power}}$$

Input power or the rate at which energy is supplied

$$= \frac{1.47 \times 10^4 \times 100}{75} = \mathbf{1.96 \times 10^4\ W.}$$

EXAMPLE 54. *Calculate the horse power of a man who can chew ice at the rate of 30 g per minute. Given 1 hp = 746 W and J = 4.2 J cal^{-1}.*

Solution. Mass of ice chewed by man,

$$m = 30\ g$$

Latent heat of ice, $L = 80\ cal\ g^{-1}$

Heat required to melt ice,

$$H = mL = 30 \times 80\ cal$$

Work done, $W = JH = 4.2 \times 30 \times 80$ J

Time taken, $t = 1$ min $= 60$ s

$$\text{Power,} \quad P = \frac{W}{t} = \frac{4.2 \times 30 \times 80}{60} = 168\ W$$

$$= \frac{168}{746} = \mathbf{0.225\ hp.}$$

EXAMPLE 55. *A machine gun fires 60 bullets per minute with a velocity of 700 ms^{-1}. If each bullet has a mass of 50 g, find the power developed by the gun.*

Solution. Mass of 60 bullets $= 60 \times 50 = 3000$ g $= 3$ kg

$v = 700\ ms^{-1},\ t = 1$ min $= 60$ s

$$\text{Power} = \frac{W}{t} = \frac{K.E.}{t} = \frac{1}{2} \cdot \frac{mv^2}{t} = \frac{3 \times (700)^2}{2 \times 60}$$

$$= \mathbf{12250\ W.}$$

EXAMPLE 56. *An elevator which can carry a maximum load of 1800 kg (elevator + passengers) is moving up with a constant speed of 2 ms^{-1}. The frictional force opposing the motion is 4000 N. Determine the minimum power delivered by the motor to the elevator in watts as well as in horse power.* **[NCERT ; Delhi 03C]**

Solution. The downward force on the elevator is

$$F = mg + F_f = 1800 \times 10 + 4000 = 22000 \text{ N}$$

The motor must supply enough power to balance this force. Hence,

$$P = Fv = 22000 \times 2 = 44000 \text{ W}$$

$$= \frac{44000}{746} \text{ hp} \simeq \mathbf{59 \text{ hp}}.$$

EXAMPLE 57. *A well 20 m deep and 3 m in diameter contains water to a depth of 14 metre. How long will a 5 hp engine take to empty it ?*

Solution. Radius of the well, $r = \frac{3}{2}$ m

Area of cross-section of the well

$$= \pi r^2 = \frac{22}{7} \times \left(\frac{3}{2}\right)^2 = \frac{99}{14} \text{ m}^2$$

Volume of water in the well

= Area of cross-section × depth

$$= \frac{99}{14} \times 14 = 99 \text{ m}^3$$

∴ Mass of water in the well

= Volume × density $= 99 \times 10^3$ kg

As the well is emptied, the height through which water has to be raised by the engine changes from (20 – 14) m in the beginning to (20 – 0) m at the end.

$$\therefore \text{ Average height raised} = \frac{6 + 20}{2} = 13 \text{ m}$$

Work required to empty the well,

$$W = mgh = 99 \times 10^3 \times 9.8 \times 13 = 12612600 \text{ J}$$

Power, $P = 5 \text{ hp} = 5 \times 746$ W

$$\text{Required time, } t = \frac{W}{P} = \frac{12612600}{5 \times 746} = \mathbf{3381.6 \text{ s}}.$$

EXAMPLE 58. *The turbine pits at the Niagra falls are 50 m deep. The average horse power developed is 500. If the efficiency of the generator is 85%, how much water passes through the turbines per minute ? Take $g = 10 \text{ ms}^{-2}$.*

Solution. Useful power developed = 5000 hp

Efficiency = 85%

∴ Total power generated

$$= \frac{100}{85} \times 5000 \text{ hp} = \frac{100 \times 5000 \times 746}{85} \text{ W}$$

Total work done by the falling water in 1 min or 60 s,

$$W = Pt = \frac{100 \times 5000 \times 746}{85} \times 60 = 26.94 \times 10^7 \text{ J}$$

Now $mgh = W$

$$\therefore \quad m = \frac{W}{gh} = \frac{26.94 \times 10^7}{10 \times 50} = \mathbf{5.39 \times 10^5 \text{ kg}}.$$

EXAMPLE 59. *A man cycles up a hill, whose slope is 1 in 20 with a velocity of 6.4 kmh^{-1} along the hill. The weight of the man and the cycle is 98 kg. What work per minute is he doing ? What is his horse power ?*

Solution. Refer to Fig. 6.35. If the inclination of the hill with the horizontal is θ, then

$$\sin\theta = \frac{1}{20}$$

$$v = 6.4 \text{ kmh}^{-1} = \frac{6.4 \times 5}{18} \text{ ms}^{-1} = \frac{16}{9} \text{ ms}^{-1}$$

$$m = 98 \text{ kg}, \quad t = 1 \text{ min} = 60 \text{ s}$$

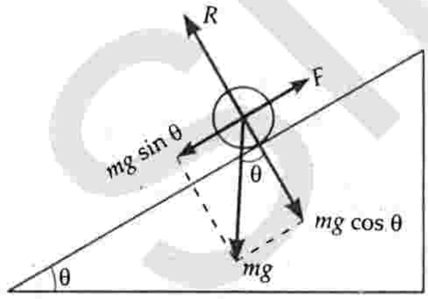


Fig. 6.35

As the velocity of the cyclist is uniform, so the only force he has to exert is against gravity. It is given by

$$F = mg\sin\theta$$

Power of the man,

$$P = Fv = mg\sin\theta \times v$$

$$= 98 \times 9.8 \times \frac{1}{20} \times \frac{16}{9} \text{ W}$$

$$= \frac{98 \times 9.6 \times 16}{746 \times 20 \times 9} \text{ hp} = \mathbf{0.144 \text{ hp}}$$

Work done per minute,

$$W = Pt = \frac{98 \times 9.8 \times 16}{20 \times 9} \times 60 = \mathbf{5122.1 \text{ J}}.$$

PROBLEMS FOR PRACTICE

1. A lift is designed to carry a load of 4000 kg through 10 floors of a building averaging 6 m per floor in 10 seconds. Calculate the horse power of the lift. (**Ans.** 315.28 hp)
2. A machine can take out 1000 kg of mud per hour from a depth of 100 m. If efficiency of the machine is 0.9, calculate its power. (**Ans.** 302.47 W)
3. One coolie takes 1 min to raise a box through a height 2 m. Another takes 30 s for the same job and does the same amount of work. Which one of these two has a greater power and which one uses greater energy ?
(**Ans.** Second coolie has double power than first, both spend same amount of energy)

4. An engine of 4.9 kW power is used to pump water from a well 50 m deep. Calculate the quantity of water in kilolitres which it can pump out in one hour. (**Ans.** 36.0 kilo litre)

5. Water is pumped out of a well 10 m deep by means of a pump rated at 10 kW. Find the efficiency of the motor if 4200 kg of water is pumped out every minute. Take $g = 10\ ms^{-2}$. (**Ans.** 70%)

6. A 30 m deep well is having water upto 15 m. An engine evacuates it in one hour. Calculate the power of the engine if the diameter of the well is 4 m. (**Ans.** 11.55 kW)

7. The human heart forces 4000 cm^3 of blood per minute through the arteries under pressure of 130 mm. The density of blood is 1.03 g cm^{-3}. What is the horse power of the heart ? (**Ans.** 1.17×10^{-4} hp)

8. A car of mass 1000 kg accelerates uniformly from rest to a velocity of 54 kh h^{-1} in 5 seconds. Calculate (*i*) its acceleration (*ii*) its gain in K.E. (*iii*) average power of the engine during this period, neglect friction. **[Chandigarh 03]**
[**Ans.** (*i*) 3 ms^{-2} (*ii*) 1.125×10^5 J (*iii*) 22500 W]

✖ HINTS

1. $$P = \frac{W}{t} = \frac{mgh}{t} = \frac{4000 \times 9.8 \times 10 \times 6}{10}\ W$$
$$= \frac{4000 \times 9.8 \times 6}{746} = \mathbf{315.28\ hp.}$$

2. $$\text{Useful power} = \frac{W}{t} = \frac{mgh}{t} = \frac{1000 \times 9.8 \times 100}{3600}$$
$$= 272.22\ W$$
$$\eta = \frac{\text{Useful power}}{\text{Actual power}} \quad \text{or} \quad 0.9 = \frac{272.22}{\text{Actual power}}$$
$$\therefore \quad \text{Actual power} = \frac{272.22 \times 10}{9} = \mathbf{302.47\ W.}$$

3. As both the coolies do the same amount of work, so they spend the same amount of energy. Also
$$W = P_1 t_1 = P_2 t_2$$
$$\text{or} \quad \frac{P_2}{P_1} = \frac{t_1}{t_2} = \frac{1\ \text{min}}{20\ s} = 2 \quad \text{or} \quad P_2 = 2P_1$$

4. $W = Pt = mgh$
$$\therefore\ 4.9 \times 10^3 \times 3600 = m \times 9.8 \times 50$$
$$\text{or} \qquad m = 36{,}000\ \text{kg}$$
∴ Volume of water pumped out per hour
= 36,000 litre = **36.0 kilo litre.**

5. Here $m = 4200$ kg, $h = 10$ m, $g = 10\ ms^{-2}$, $t = 1\ \text{min} = 60$ s

Output power
$$= \frac{W}{t} = \frac{mgh}{t} = \frac{4200 \times 10 \times 10}{60} = 7000\ W = 7\ kW$$
Efficiency,
$$\eta = \frac{\text{Output power}}{\text{Input power}} = \frac{7}{10} = 0.7 = \mathbf{70\%.}$$

6. Mass of water, $m = \text{Volume} \times \text{density} = \pi r^2\, l \times \rho$
$$= \pi (2)^2 \times 15 \times 1000\ \text{kg}$$
$$\text{Mean} \quad h = \frac{30 + 15}{2} = 22.5\ m$$
$$P = \frac{W}{t} = \frac{mgh}{t}$$
$$= \frac{\pi \times (2)^2 \times 15 \times 1000 \times 9.8 \times 22.5}{3600\ s}\ J$$
$$= 11549.99\ W \approx \mathbf{11.55\ kW.}$$

7. Pressure $= 130$ mm of Hg $= 13$ cm of Hg
$$= h\rho g = 13 \times 1.03 \times 980\ \text{dyne cm}^{-2}$$
Volume $= 4000\ cm^3$; Time, $t = 1$ min $= 60$ s
$$\text{Power,} \quad P = \frac{W}{t} = \frac{\text{Pressure} \times \text{volume}}{t}$$
$$= \frac{13 \times 1.03 \times 980 \times 4000}{60}$$
$$= 874813.33\ \text{erg s}^{-1}$$
$$= 874813.33 \times 10^{-7}\ Js^{-1}\ \text{or W}$$
$$= \frac{874813.33 \times 10^{-7}}{746}\ \text{hp} = \mathbf{1.17 \times 10^4\ hp.}$$

8. Here $m = 1000$ kg, $u = 0$, $v = 54\ kmh^{-1} = 15\ ms^{-1}$, $t = 5$ s

(*i*) Acceleration, $a = \frac{v - u}{t} = \frac{15 - 0}{5} = \mathbf{3\ ms^{-2}}$.

(*ii*) Gain in K.E. $= \frac{1}{2} m (v^2 - u^2) = \frac{1}{2} \times 1000 (15^2 - 0)$
$$= \mathbf{1.125 \times 10^5\ J.}$$

(*iii*) Power, $P = \frac{W}{t} = \frac{1.125 \times 10^5}{5} = \mathbf{22500\ W.}$

6.21 COLLISIONS

35. *Define the terms : collision, elastic collision, inelastic collision, perfectly inelastic collision, super-elastic collision, head-on collision and oblique collision. Give important characteristics and examples of different types of collisions.*

Collision. *A collision is said to occur between two bodies, either if they physically collide against each other or if the path of one is affected by the force exerted by the other.* For a collision to take place, the actual physical contact is not necessary. In Rutherford's scattering experiment, α-particles get scattered due to the electrostatic repulsion between the α-particle and the nucleus from a distance. The α-particle is said to have suffered collision with the nucleus.

The collisions between particles are of following types :

1. **Elastic collision.** *If there is no loss of kinetic energy during a collision, it is called an elastic collision.*

Characteristics of elastic collisions :

(*i*) The momentum is conserved.

(*ii*) Total energy is conserved.

(*iii*) The kinetic energy is conserved.

(*iv*) Forces involved during the collision are conservative.

(*v*) The mechanical energy is not converted into heat, light, sound, etc.

Examples. Collision between subatomic particles, collision between glass balls, etc.

2. **Inelastic collision.** *If there is a loss of kinetic energy during a collision, it is called an inelastic collision.*

Characteristics of inelastic collisions :

(*i*) The momentum is conserved.

(*ii*) Total energy is conserved.

(*iii*) The *kinetic energy is not conserved.*

(*iv*) Some or all of the forces involved are non-conservative.

(*v*) A part of the mechanical energy is converted into heat, light, sound, etc.

Examples. Collision between two vehicles, collision between a ball and floor.

3. **Perfectly inelastic collision.** *If two bodies stick together after the collision and move as a single body with a common velocity, then the collision is said to be perfectly inelastic collision.* In such collisions, momentum is conserved, but the loss of kinetic energy is maximum.

Examples. Mud thrown on a wall and sticking to it, a man jumping into a moving trolley, a bullet fired into a wooden block and remaining embedded in it, etc.

4. **Superelastic or explosive collision.** In such a collision, *there is an increase in kinetic energy.* This occurs if there is a release of potential energy on an impact.

Examples. Bursting of a cracker when it hits the floor forcefully, the collision of a trolley with another may release a compressed spring and thereby releasing the energy stored in the spring.

5. **Head-on or one-dimensional collision.** *It is the collision in which the colliding bodies move along the same straight line path before and after the collision.*

Example. Collision between two railway compartments.

6. **Oblique or two-dimensional collision.** *If two bodies do not move along the same straight line path but lie in the same plane before and after the collision, the collision is said to be oblique or two-dimensional collision.*

Example. Collision between two carrom coins.

36. *Show that total linear momentum is conserved in all collisions.*

Conservation of linear momentum in a collision. Suppose two bodies 1 and 2 collide against each other. They exert mutual impulsive forces on each other during the collision time Δt. The changes produced in the momenta of the two bodies will be

$$\Delta \vec{p}_1 = \vec{F}_{12}\,\Delta t \quad \text{and} \quad \Delta \vec{p}_2 = \vec{F}_{21}\,\Delta t$$

where $\vec{F}_{12}$ = the force exerted on body 1 by body 2

and $\vec{F}_{21}$ = the force exerted on body 2 by body 1

According to Newton's third law,

$$\vec{F}_{12} = -\vec{F}_{21}$$

$$\therefore \quad \vec{F}_{12}\,\Delta t = -\vec{F}_{21}\,\Delta t$$

or $$\vec{F}_{12}\,\Delta t + \vec{F}_{21}\,\Delta t = 0$$

or $$\Delta \vec{p}_1 + \Delta \vec{p}_2 = 0$$

or $$\Delta(\vec{p}_1 + \vec{p}_2) = 0$$

or $$\vec{p}_1 + \vec{p}_2 = \text{constant}$$

Thus *linear momentum is conserved during a collision,* even though the forces vary in a complex manner during a collision.

For Your Knowledge

- Total linear momentum is conserved at each instant of every collision.
- Total energy is conserved in all collisions.
- The total kinetic energy may or may not be conserved during a collision.
- Even for an elastic collision, the kinetic energy conservation holds after the collision is over and does not hold at every instant of the collision.
- When two bodies collide ; they get deformed and may be momentarily at rest with respect to each other.
- The impact and deformation during a collision may convert part of the initial kinetic energy into heat and sound.

6.22 ELASTIC COLLISION IN ONE DIMENSION

37. *Prove that in an elastic one-dimensional collision between two bodies, the relative velocity of approach before collision is equal to the relative velocity of separation after the collision. Hence derive expressions* *for the velocities of the two bodies in terms of their initial velocities before collision. Discuss the special cases also.*

Elastic collision in one dimension. As shown in Fig. 6.36, consider two perfectly elastic bodies A and B of masses m_1 and m_2 moving along the same straight line with velocities u_1 and u_2 respectively. Let $u_1 > u_2$. After some time, the two bodies collide head-on and continue moving in the same direction with velocities v_1 and v_2 respectively. The two bodies will separate after the collision if $v_2 > v_1$.

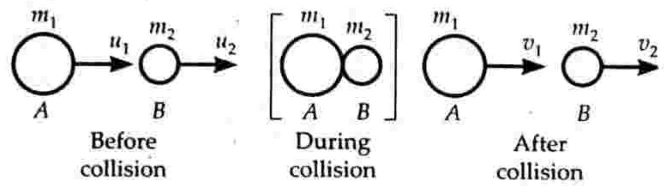


Fig. 6.36 Elastic collision in one dimension.

As linear momentum is conserved in any collision, so

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \quad ...(1)$$

or
$$m_1u_1 - m_1v_1 = m_2v_2 - m_2u_2$$

or
$$m_1(u_1 - v_1) = m_2(v_2 - u_2) \quad ...(2)$$

Since K.E. is also conserved in an elastic collision, so

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

or
$$m_1u_1^2 - m_1v_1^2 = m_2v_2^2 - m_2u_2^2$$

or
$$m_1(u_1 + v_1)(u_1 - v_1) = m_2(v_2 + u_2)(v_2 - u_2) \quad ...(3)$$

Dividing (3) by (2), we get

$$u_1 + v_1 = v_2 + u_2$$

or
$$u_1 - u_2 = v_2 - v_1 \quad ...(4)$$

or Relative velocity of A w.r.t. B before collision

= Relative velocity of B w.r.t. A after collision

or Relative velocity of approach

= Relative velocity of separation

Thus, *in an elastic one-dimensional collision, the relative velocity of approach before collision is equal to the relative velocity of separation after the collision.*

Velocities of the bodies after the collision. From equation (4), we get

$$v_2 = u_1 - u_2 + v_1$$

Putting this value of v_2 in equation (1), we get

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2(u_1 - u_2 + v_1)$$
$$= m_1v_1 + m_2u_1 - m_2u_2 + m_2v_1$$

or $(m_1 - m_2)u_1 + 2m_2u_2 = (m_1 + m_2)v_1$

or
$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2 \quad ...(5)$$

Interchanging the subscripts 1 and 2 in the above equation, we get

$$v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 + \left(\frac{2m_1}{m_1 + m_2}\right)u_1 \quad ...(6)$$

Equations (5) and (6) give the final velocities of the colliding bodies in terms of their initial velocities.

Special cases :

(*i*) **When two bodies of equal masses collide.** Let

$$m_1 = m_2 = m \quad \text{(say)}.$$

From equation (5),

$$v_1 = \frac{2mu_2}{2m} = u_2$$

= velocity of body of mass m_2 before collision

From equation (6),

$$v_2 = \frac{2mu_1}{2m} = u_1$$

= velocity of body of mass m_1 before collision

Hence *when two bodies of equal masses suffer one dimensional elastic collision, their velocities get exchanged after the collision.*

(*ii*) **When a body collides against a stationary body of equal mass.** Here $m_1 = m_2 = m$(say) and $u_2 = 0$.

From equation (5), $\quad v_1 = 0$

From equation (6), $\quad v_2 = u_1$

Hence *when an elastic body collides against another elastic body of equal mass, initially at rest, after the collision the first body comes to rest while second body moves with the initial velocity of the first.*

(*iii*) **When a light body collides against a massive stationary body.** Here $m_1 << m_2$ and $u_2 = 0$. Neglecting m_1 in equation (5), we get

$$v_1 = -\frac{m_2u_1}{m_2} = -u_1$$

From (6), $v_2 \simeq 0$.

Hence *when a light body collides against a massive body at rest, the light body rebounds after the collision with an equal and opposite velocity while the massive body practically remains at rest.* A light ball on striking a wall rebounds almost with the same speed and the wall remains at rest.

(*iv*) **When a massive body collides against a light stationary body.** Here $m_1 >> m_2$ and $u_2 = 0$.

Neglecting m_2 in equation (5), we get

$$v_1 = \frac{m_1 u_1}{m_1} = u_1$$

and $$v_2 = \frac{2 m_1 u_1}{m_1} = 2u_1$$

Hence *when a massive body collides against a light body at rest, the velocity of the massive body remains almost unchanged while the light body starts moving with twice the velocity of the massive body.*

6.23 PERFECTLY INELASTIC COLLISION IN ONE DIMENSION

38. *What is a perfectly inelastic collision ? Show that kinetic energy is invariably lost in such a collision.*

Perfectly inelastic collision. *When the two colliding bodies stick together and move as a single body with a common velocity after the collision, the collision is perfectly inelastic.*

As shown in Fig. 6.37, a body of mass m_1 moving with velocity u_1 collides head-on with another body of mass m_2 at rest. After the collision, the two bodies move together with a common velocity v.

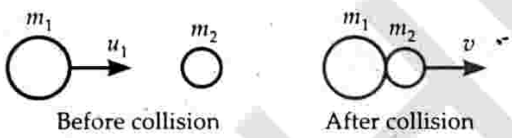


Fig. 6.37

As the linear momentum is conserved, so

$$m_1 u_1 + m_2 \times 0 = (m_1 + m_2) v$$

or $$v = \frac{m_1}{m_1 + m_2} u_1$$

The loss in kinetic energy on collision is

$$\Delta K = K_i - K_f = \frac{1}{2} m_1 u_1^2 - \frac{1}{2}(m_1 + m_2) v^2$$

$$= \frac{1}{2} m_1 u_1^2 - \frac{1}{2}(m_1 + m_2)\left[\frac{m_1}{m_1 + m_2} u_1\right]^2$$

$$= \frac{1}{2} m_1 u_1^2 - \frac{1}{2} \frac{m_1^2}{m_1 + m_2} u_1^2$$

$$= \frac{1}{2} m_1 u_1^2 \left[1 - \frac{m_1}{m_1 + m_2}\right]$$

or $$\Delta K = \frac{1}{2} \frac{m_1 m_2}{m_1 + m_2} u_1^2 .$$

This is a positive quantity. The kinetic energy is lost mainly in the form of heat and sound.

Moreover,

$$\frac{K_f}{K_i} = \frac{\frac{1}{2}(m_1 + m_2) v^2}{\frac{1}{2} m_1 u_1^2} = \frac{m_1 + m_2}{m_1} \cdot \frac{v^2}{u_1^2}$$

$$= \frac{m_1 + m_2}{m_1}\left(\frac{m_1}{m_1 + m_2}\right)^2$$

or $$\frac{K_f}{K_i} = \frac{m_1}{m_1 + m_2}$$

which is < 1. This again shows that the kinetic energy after the collision is less than the kinetic energy before the collision.

If the target is massive, *i.e.*, $m_2 >> m_1$, then

$$\frac{K_f}{K_i} \simeq 0 \quad i.e., \quad K_f \simeq 0$$

Hence *when a light moving body collides against any massive body at rest and sticks to it, practically all of its kinetic energy is lost.*

6.24 ELASTIC COLLISION IN TWO DIMENSIONS

39. *Discuss elastic collision in two dimensions. What are the conditions of (i) glancing collision (ii) head-on collision ? (iii) Show that two identical particles move at right angles to each other after elastic collision in two dimensions.*

Elastic collision in two dimensions. As shown in Fig. 6.38, suppose a particle of mass m_1 moving along X-axis with velocity u_1 collides with another particle of mass m_2 at rest. After the collision, let the two particles move with velocities v_1 and v_2, making angles θ_1 and θ_2 with X-axis.

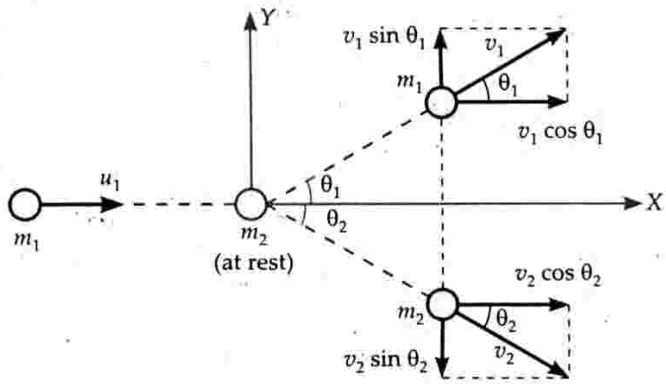


Fig. 6.38 Elastic collision in two dimensions.

After the collision, the rectangular components of the momentum of m_1 are

(*i*) $m_1 v_1 \cos \theta_1$, along +ve X-axis

(*ii*) $m_1 v_1 \sin \theta_1$, along +ve Y-axis

After the collision, the rectangular components of the momentum of m_2 are

(i) $m_2 v_2 \cos\theta_2$, along +ve X-axis

(ii) $m_2 v_2 \sin\theta_2$, along –ve Y-axis

Applying the principle of conservation of momentum along X-axis,

$$m_1 u_1 = m_1 v_1 \cos\theta_1 + m_2 v_2 \cos\theta_2 \quad ...(1)$$

The initial momentum of m_1 or m_2 along Y-axis is zero.

Applying the principle of conservation of momentum along Y-axis,

$$0 = m_1 v_1 \sin\theta_1 - m_2 v_2 \sin\theta_2 \quad ...(2)$$

As the K.E. is conserved in an elastic collision, so

$$\frac{1}{2} m_1 u_1^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \quad ...(3)$$

The four unknown quantities v_1, v_2, θ_1 and θ_2 cannot be calculated using the three equations (1), (2) and (3). By measuring one of the four unknowns, say θ_1, experimentally ; the values of other three unknowns can be solved.

Special cases :

(i) **Glancing collision.** For such collisions, $\theta_1 \simeq 0°$ and $\theta_2 \simeq 90°$.

From equations (1) and (2), we get

$$u_1 = v_1 \quad \text{and} \quad v_2 = 0$$

K.E. of the target particle $= \frac{1}{2} m_2 v_2^2 = 0$

Hence in a glancing collision, the incident particle does not lose any kinetic energy and is scattered almost undeflected.

(ii) **Head-on collision.** In such a collision, the target particle moves in the direction of the incident particle, *i.e.*, $\theta_2 = 0°$. Then equations (1) and (2) take forms :

$$m_1 u_1 = m_1 v_1 \cos\theta_1 + m_2 v_2 \quad \text{and} \quad 0 = m_1 v_1 \sin\theta_1$$

Equation (3) for the kinetic energy remains unchanged.

(iii) **Elastic collision of two identical particles.** As the two particles are identical, so $m_1 = m_2 = m$ (say). By conservation of K.E. for elastic collision,

$$\frac{1}{2} m u_1^2 = \frac{1}{2} m v_1^2 + \frac{1}{2} m v_2^2 \quad \text{or} \quad u_1^2 = v_1^2 + v_2^2$$

By conservation of linear momentum,

$$m\vec{u}_1 = m\vec{v}_1 + m\vec{v}_2 \quad \text{or} \quad \vec{u}_1 = \vec{v}_1 + \vec{v}_2$$

$$\therefore \quad \vec{u}_1 \cdot \vec{u}_1 = (\vec{v}_1 + \vec{v}_2) \cdot (\vec{v}_1 + \vec{v}_2)$$

$$= \vec{v}_1 \cdot \vec{v}_1 + \vec{v}_1 \cdot \vec{v}_2 + \vec{v}_2 \cdot \vec{v}_1 + \vec{v}_2 \cdot \vec{v}_2$$

or $$u_1^2 = v_1^2 + v_2^2 + 2\vec{v}_1 \cdot \vec{v}_2$$

or $$u_1^2 = u_1^2 + 2\vec{v}_1 \cdot \vec{v}_2 \quad [\because v_1^2 + v_2^2 = u_1^2]$$

or $$\vec{v}_1 \cdot \vec{v}_2 = 0.$$

This shows that the angle between $\vec{v}_1$ and $\vec{v}_2$ is 90°. Hence *two identical particles move at right angles to each other after elastic collision in two dimensions.*

6.25 COEFFICIENT OF RESTITUTION

40. *What is the coefficient of restitution ? What is its significance ?*

Coefficient of restitution or coefficient of resilience. Most of the real collisions are neither perfectly elastic nor perfectly plastic. They are *partially elastic collisions,* in which the K.E. reduces and so the speed of separation is less than the speed of approach.

The coefficient of restitution gives a measure of the degree of restitution of a collision and is defined as the ratio of the magnitude of relative velocity of separation after collision to the magnitude of relative velocity of approach before collision. It is given by

$$e = \frac{|v_1 - v_2|}{|u_1 - u_2|} = -\frac{v_1 - v_2}{u_1 - u_2}$$

The value of e depends on the materials of the colliding bodies. For two glass balls, $e = 0.95$ and for the lead balls, $e = 0.20$.

The coefficient of restitution can be used to distinguish between the different types of collisions as follows :

(i) *For a perfectly elastic collision,* $e = 1$ *i.e.*, relative velocity of separation is equal to the relative velocity of approach.

(ii) *For an inelastic collision,* $0 < e < 1$ *i.e.*, the relative velocity of separation is less than relative velocity of approach.

(iii) *For a perfectly inelastic collision,* $e = 0$ *i.e.*, the relative velocity of separation is zero. The two bodies move together with a common velocity.

(iv) *For a superelastic collision.* $e > 1$ *i.e.*, the kinetic energy increases.

Table 6.2 *Different types of collisions*

Collision	***Kinetic energy***	***Coefficient of restitution***	***Main domain***
Elastic	Conserved	$e = 1$	Between atomic particles
Inelastic	Not conserved	$0 < e < 1$	Between ordinary objects
Perfectly inelastic	Max. loss of K.E.	$e = 0$	During shooting
Super elastic	K.E. increases	$e > 1$	In explosions

For Your Knowledge

- At each instant of the collision, the total kinetic energy and total linear momentum are both conserved in elastic as well as inelastic collisions.
- In an elastic collision, the kinetic energy conservation holds only after the collision is over. It does not hold during the short duration of actual collision.
- At the time of collision, the two colliding objects are deformed and may be momentarily at rest with respect to each other.
- When two equal masses suffer a glancing collision with one of them at rest, after the collision, the two masses move at right angles to each other.

Examples based on Collisions

FORMULAE USED

1. Linear momentum is conserved both in elastic and inelastic collisions.
$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$
2. Kinetic energy is conserved in elastic collisions.
$$\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$
3. In one-dimensional elastic collision, velocities after the collision are given by
$$v_1 = \frac{m_1 - m_2}{m_1 + m_2} \cdot u_1 + \frac{2m_2}{m_1 + m_2} \cdot u_2$$
$$v_2 = \frac{2m_1}{m_1 + m_2} \cdot u_1 + \frac{m_2 - m_1}{m_1 + m_2} \cdot u_2$$
4. Coefficient of restitution for a collision is given by
$$e = -\frac{v_1 - v_2}{u_1 - u_2} = \frac{|v_1 - v_2|}{|u_1 - u_2|}$$
5. For a ball rebounding from a floor, $e = \frac{v}{u}$
6. For an elastic collision (involving no loss of K.E.), $e = 1$
7. For an inelastic collision (involving loss of K.E.), $e < 1$

UNITS USED

Masses m_1, m_2 are in kg, velocities u_1, u_2, v_1, v_2 are in ms^{-1}, linear momenta in kg ms^{-1}, kinetic energy in joule and coefficient of restitution 'e' has no units.

EXAMPLE 60. *Two bodies of masses 5 kg and 3 kg moving in the same direction along the same straight line with velocities 5 ms^{-1} and 3 ms^{-1} respectively suffer one-dimensional elastic collision. Find their velocities after the collision.*

Solution. Here $m_1 = 5$ kg, $u_1 = 5$ ms^{-1},
$m_2 = 3$ kg, $u_2 = 3$ ms^{-1}

$$\therefore\ v_1 = \frac{m_1 - m_2}{m_1 + m_2} \cdot u_1 + \frac{2m_2}{m_1 + m_2} \cdot u_2$$
$$= \frac{5-3}{5+3} \times 5 + \frac{2 \times 3}{5+3} \times 3$$
$$= \frac{2}{8} \times 5 + \frac{6}{8} \times 3 = \frac{5}{4} + \frac{9}{4} = \frac{14}{4} = \mathbf{3.5\ ms^{-1}}.$$
$$v_2 = \frac{2m_1}{m_1 + m_2} \cdot u_1 + \frac{m_2 - m_1}{m_1 + m_2} \cdot u_2$$
$$= \frac{2 \times 5}{5+3} \times 5 + \frac{3-5}{5+3} \times 3$$
$$= \frac{50}{8} - \frac{6}{8} = \frac{44}{8} = \mathbf{5.5\ ms^{-1}}.$$

EXAMPLE 61. *A 10 kg ball and 20 kg ball approach each other with velocities 20 ms^{-1} and 10 ms^{-1} respectively. What are their velocities after collision if the collision is perfectly elastic ?*

Solution. Here $m_1 = 10$ kg, $m_2 = 20$ kg,
$u_1 = 20$ ms^{-1}, $u_2 = -10$ ms^{-1}

$$v_1 = \frac{m_1 - m_2}{m_1 + m_2} \cdot u_1 + \frac{2m_2}{m_1 + m_2} \cdot u_2$$
$$= \frac{10-20}{10+20} \times 20 + \frac{2 \times 20}{10+20} \times (-10)$$
$$= -\frac{20}{3} - \frac{40}{3} = -\frac{60}{3} = \mathbf{-20\ ms^{-1}}.$$
$$v_2 = \frac{2m_1}{m_1 + m_2} \cdot u_1 + \frac{m_2 - m_1}{m_1 + m_2} \cdot u_2$$
$$= \frac{2 \times 10}{10+20} \times 20 + \frac{20-10}{10+20} \times (-10)$$
$$= \frac{40}{3} - \frac{10}{3} = \frac{30}{3} = \mathbf{10\ ms^{-1}}.$$

EXAMPLE 62. *Two ball bearings of mass m each moving in opposite directions with equal speeds v collide head on with each other. Predict the outcome of the collision, assuming it to be perfectly elastic.* **[NCERT]**

Solution. Here $m_1 = m_2 = m$ (say), $u_1 = v$, $u_2 = -v$

As the collision is perfectly elastic, velocities after the collision will be

$$v_1 = \frac{m_1 - m_2}{m_1 + m_2} \cdot u_1 + \frac{2m_2}{m_1 + m_2} \cdot u_2$$
$$= \frac{m - m}{m + m} \cdot v + \frac{2m}{m + m}(-v)$$
$$= 0 - v = -v.$$

$$v_2 = \frac{2m_1}{m_1+m_2} \cdot u_1 + \frac{m_2 - m_1}{m_1+m_2} \cdot u_2$$

$$= \frac{2m}{m+m} \cdot v + \frac{m-m}{m+m} \cdot (-v)$$

$$= v + 0 = v.$$

Thus the two balls bounce back with equal speeds after the collision.

EXAMPLE 63. *A railway carriage of mass 9000 kg moving with a speed of 36 kmh^{-1} collides with a stationary carriage of the same mass. After the collision, the carriages get coupled and move together. What is their common speed after collision ? What type of collision is this ?*

Solution. Here

$m_1 = 9000$ kg, $u_1 = 36$ kmh^{-1} $= 10$ ms^{-1}

$m_2 = 9000$ kg, $u_2 = 0$, $v_1 = v_2 = v = ?$

By conservation of momentum,

$$m_1u_1 + m_2u_2 = (m_1 + m_1)\, v$$

$$9000 \times 10 + 0 = (9000 + 9000) v$$

or $$v = \frac{90000}{18000} = \mathbf{5\ ms^{-1}}.$$

Total K.E. before collision

$$= \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2$$

$$= \frac{1}{2} \times 9000 \times 10 \times 10 + 0 = 450000 \text{ J}$$

Total K.E. after collision $= \frac{1}{2}(m_1 + m_2)\, v^2$

$$= \frac{1}{2} \times 2 \times 9000 \times 5^2 = 225000 \text{ J}.$$

Thus total K.E. after collision < Total K.E. before collision.

Hence the collision is inelastic.

EXAMPLE 64. *What percentage of kinetic energy of a moving particle is transferred to a stationary particle, when moving particle strikes with a stationary particle of mass (i) 9 times in mass (ii) equal in mass and (iii) 1/19th of its mass ?*

Solution. For the moving particle, $m_1 = m$ (say),

initial velocity $= u_1$

For the stationary particle, $m_2 = xm$ (say),

initial velocity $= u_2 = 0$

As $$v_2 = \frac{2m_1}{m_1+m_2} \cdot u_1 + \frac{m_2 - m_1}{m_1+m_2} \cdot u_2$$

$\therefore$ $$v_2 = \frac{2m}{m+xm} \cdot u_1 + 0 = \frac{2u_1}{1+x}$$

K.E. of the moving particle before collision,

$$K_1 = \frac{1}{2} m_1 u_1^2 = \frac{1}{2} mu_1^2$$

K.E. of the stationary particle after collision,

$$K_2 = \frac{1}{2} m_2 v_2^2 = \frac{1}{2} \cdot mx \cdot \frac{4u_1^2}{(1+x)^2}$$

$$= \frac{4x}{(1+x)^2} \cdot \frac{1}{2} mu_1^2 = \frac{4x}{(1+x)^2} \cdot K_1$$

% of K.E. transferred

$$= \frac{K_2}{K_1} \times 100 = \frac{4xK_1}{(1+x)^2} \times \frac{100}{K_1}$$

$$= \frac{4x}{(1+x)^2} \times 100\ \%$$

(*i*) When moving particle strikes with a stationary particle 9 times in mass, $x = 9$

% of K.E. transferred $= \frac{4 \times 9}{(1+9)^2} \times 100 = \mathbf{36\%}.$

(*ii*) When moving particle strikes with a stationary particle of equal mass, $x = 1$

% of K.E. transferred $= \frac{4 \times 1}{(1+1)^2} \times 100 = \mathbf{100\%}.$

(*iii*) When moving particle strikes a stationary particle $\frac{1}{19}$ th of its mass, $x = \frac{1}{19}$.

% of K.E. transferred $= \frac{4 \times (1/19)}{(1+1/19)^2} \times 100 = \mathbf{19\%}.$

EXAMPLE 65. ***Slowing down of neutrons.*** *In a nuclear reactor a neutron of high speed (typically 10^7 ms^{-1}) must be slowed to 10^3 ms^{-1} so that it can have a high probability of interacting with isotope ${}^{235}_{92}U$ and causing it to fission. Show that a neutron can lose most of its kinetic energy in an elastic collision with a light nucleus like deuterium or carbon which has a mass of only a few times the neutron mass. The material making up the light nuclei, usually heavy water (D_2O) or graphite is called a moderator.*

Or **[NCERT]**

A body of mass M at rest is struck by a moving body of mass m. Prove that fraction of the initial K.E. of the mass m transferred to the struck body is $4\,mM/(m+M)^2$ in an elastic collision.

Solution. Here m_1 = mass of neutron $= m$

m_2 = mass of target nucleus $= M$

$u_1 = u$ and $u_2 = 0$

Now $$v_2 = \frac{2m_1}{m_1+m_2} \cdot u_1 + \frac{m_2 - m_1}{m_1+m_2} \cdot u_2$$

$$= \frac{2m}{m+M} \cdot u + 0 = \frac{2mu}{m+M}$$

Initial K.E. of mass m,

$$K_1 = \frac{1}{2} m_1 u_1^2 = \frac{1}{2} mu^2$$

Final K.E. of mass M,

$$K_2 = \frac{1}{2} m_2 v_2^2 = \frac{1}{2} M\left(\frac{2mu}{m+M}\right)^2 = \frac{2\,Mm^2 u^2}{(m+M)^2}$$

Fraction of the Initial K.E. transferred,

$$f = \frac{K_2}{K_1} = \frac{2\,Mm^2 u^2}{(m+M)^2} \times \frac{2}{mu^2} = \frac{4\,mM}{(m+M)^2}.$$

(*i*) *For deuterium*, $M = 2m$, therefore

$$f = \frac{4m \times 2m}{(m+2m)^2} = \frac{8}{9} \simeq 0.9$$

About 90% of the neutron's energy is transferred to deuterium.

(*ii*) *For carbon*, $M = 12m$, therefore

$$f = \frac{4m \times 12\,m}{(m+12m)^2} = 0.284$$

About 28.4 % of the neutron's energy is transferred to carbon.

Example 66. *A ball is dropped to the ground from a height of 2 m. The coefficient of restitution is 0.6. To what height will the ball rebound ?*

Solution. As the ball falls to the ground, its potential energy mgh_1 changes into kinetic energy $\frac{1}{2} mv_1^2$.

$$\therefore \quad mgh_1 = \frac{1}{2} mv_1^2 \qquad ...(i)$$

After rebounding, its kinetic energy $\frac{1}{2} mv_2^2$ changes into potential energy mgh_2.

$$\therefore \quad mgh_2 = \frac{1}{2} mv_2^2 \qquad ...(ii)$$

Dividing (*ii*) by (*i*), we get $\frac{h_2}{h_1} = \left(\frac{v_2}{v_1}\right)^2$

But $h_1 = 2$ m, $e = \frac{v_2}{v_1} = 0.6$

$$\therefore \quad \frac{h_2}{2\text{ m}} = (0.6)^2 = 0.36 \quad \text{or} \quad h_2 = \mathbf{0.72\ m.}$$

Example 67. *A ball is dropped vertically from a height of 3.6 m. It rebounds from a horizontal surface to a height of 1.6 m. Find the coefficient of restitution of the material of the ball.*

Solution. Here $h_1 = 3.6$ m, $h_2 = 1.6$ m

Velocity of the ball with which it reaches the horizontal surface,

$$u = \sqrt{2gh_1} = \sqrt{2 \times 9.8 \times 3.6} = 8.4 \text{ ms}^{-1}$$

Velocity of the ball with which it rebounds,

$$v = \sqrt{2gh_2} = \sqrt{2 \times 9.8 \times 1.6} = 5.6 \text{ ms}^{-1}$$

Coefficient of restitution, $e = \frac{v}{u} = \frac{5.6}{8.4} = \mathbf{0.667.}$

Example 68. *A ball is dropped from a height h. It rebounds from the ground a number of times. Given that the coefficient of restitution is e, to what height does it go after the n^{th} rebounding ?*

Solution. Let v_0 be the velocity with which the ball strikes the ground first time and v_n the velocity after n^{th} rebounding. Then the coefficient of restitution will be

$$e = \frac{v_1}{v_0} = \frac{v_2}{v_1} = \frac{v_3}{v_2} = ... = \frac{v_n}{v_{n-1}}$$

$$\therefore \quad e^n = \frac{v_1}{v_0} \times \frac{v_2}{v_1} \times \frac{v_3}{v_2} \times .... \times \frac{v_n}{v_{n-1}} = \frac{v_n}{v_0}$$

But $v_0 = \sqrt{2gh}$ and $v_n = \sqrt{2gH}$

where H is the height to which the ball rises after nth rebounding.

Hence $e^n = \frac{v_n}{v_0} = \sqrt{\frac{2gH}{2gh}} = \sqrt{\frac{H}{h}}$ or $H = \mathbf{he^{2n}}$.

Example 69. *A sphere of mass m moving with a velocity u hits another stationary sphere of same mass. If e is the coefficient of restitution, what is the ratio of the velocities of two spheres after the collision ?*

Solution. Here $u_1 = u$, $u_2 = 0$

$$\therefore \quad e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{v_2 - v_1}{u - 0}$$

or $$v_2 - v_1 = e\,u \qquad ...(i)$$

By the law of conservation of momentum,

$$mu + m \times 0 = mv_1 + mv_2$$

or $$v_1 + v_2 = u \qquad ...(ii)$$

Adding (*i*) and (*ii*),

$$2v_2 = u + eu = u(1+e)$$

or $$v_2 = \frac{u(1+e)}{2}$$

Again, from (*ii*),

$$v_1 = u - v_2 = u - \frac{u(1+e)}{2} = \frac{u(1-e)}{2}$$

Hence $\frac{v_2}{v_1} = \frac{1+e}{1-e}$.

Example 70. *A ball moving with a speed of 9 ms^{-1} strikes an identical ball such that after the collision the direction of each ball makes an angle 30° with the original line of motion. Find the speeds of the two balls after the collision. Is the kinetic energy conserved in the collision process ?*

Solution. Let m be the mass of each ball and v_1 and v_2 be their final velocities after collision.

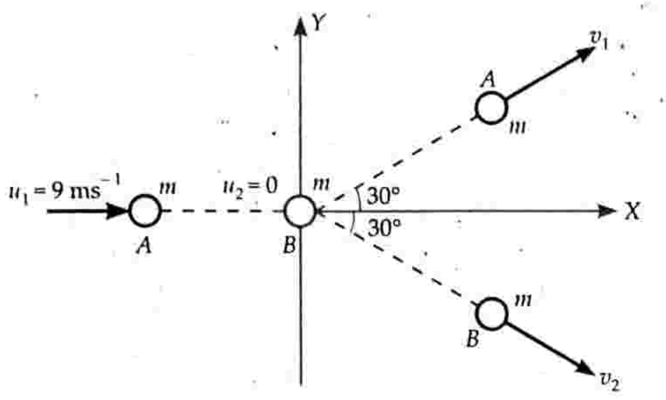


Fig. 6.39

Here $u_1 = 9\ ms^{-1}$, $u_2 = 0$

Initial momentum = Final momentum along the original line of motion

$$\therefore \quad m \times 9 + m \times 0 = m\,v_1 \cos 30° + m\,v_2 \cos 30°$$

or $$m \times 9 = m \frac{\sqrt{3}}{2}(v_1 + v_2)$$

or $$v_1 + v_2 = \frac{18}{\sqrt{3}} = \frac{6 \times 3}{\sqrt{3}} = 6\sqrt{3} \quad ...(1)$$

By conservation of momentum along a direction perpendicular to the original line, we have

$$m \times 0 + m \times 0 = mv_1 \sin 30° - mv_2 \sin 30°$$

or $$0 = m(v_1 - v_2) \times \frac{1}{2}$$

or $$v_1 - v_2 = 0$$

or $$v_1 = v_2 \quad ...(2)$$

From (1) and (2), we have

$$v_1 = v_2 = \mathbf{3\sqrt{3}\ ms^{-1}}.$$

Total K.E. before collision

$$= \frac{1}{2} \times m \times (9)^2 + 0 = \frac{81}{2}m = 40.5m$$

Total K.E. after collision

$$= \frac{1}{2} \times m \times (3\sqrt{3})^2 + \frac{1}{2} \times m \times (3 \times \sqrt{3})^2$$

$$= 27m$$

i.e. Total K.E. after collision < Total K.E. before collision

Hence K.E. is not conserved in the collision process.

EXAMPLE 71. *A ball moving on a horizontal frictionless plane hits an identical ball at rest with a velocity of* $0.5\ ms^{-1}$. *If the collision is elastic, calculate the speed imparted to the target ball if the speed of the projectile after the collision is* $30\ cm\,s^{-1}$. *Show that the two balls will move at right angles to each other after the collision.*

Solution. The situation is shown in Fig. 6.40. Let m be the mass of each ball.

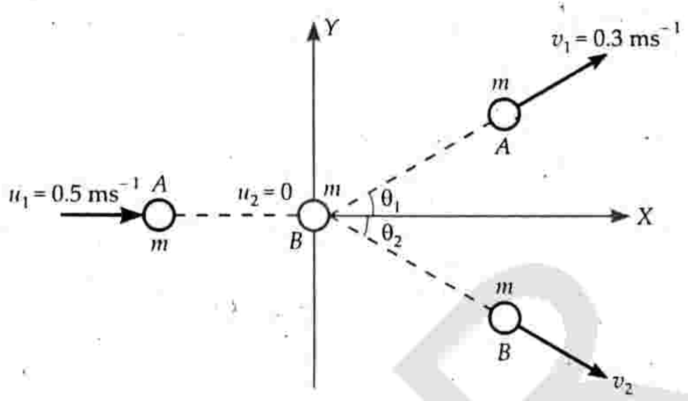


Fig. 6.40

As the collision is elastic, so K.E. is conserved.

$$\frac{1}{2}mu_1^2 + \frac{1}{2}mu_2^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$$

or $$u_1^2 + u_2^2 = v_1^2 + v_2^2$$

or $$(0.5)^2 + 0 = (0.3)^2 + v_2^2$$

or $$v_2 = \mathbf{0.4\ ms^{-1}}.$$

Applying law of conservation of momentum along X-axis, we get

$$0.5\,m + 0 = 0.3\,m \cos\theta_1 + 0.4\,m\cos\theta_2$$

or $$5 = 3\cos\theta_1 + 4\cos\theta_2$$

or $$3\cos\theta_1 = 5 - 4\cos\theta_2 \quad ...(1)$$

Again, applying law of conservation of momentum along Y-axis, we get

$$0.3\,m\sin\theta_1 = 0.4\,m\sin\theta_2$$

or $$3\sin\theta_1 = 4\sin\theta_2 \quad ...(2)$$

Squaring and adding equations (1) and (2), we get

$$9(\cos^2\theta_1 + \sin^2\theta_1) = (5 - 4\cos\theta_2)^2 + (4\sin\theta_2)^2$$

or $$9 = 16(\cos^2\theta_2 + \sin^2\theta_2) + 25 - 40\cos\theta_2$$

or $$\cos\theta_2 = \frac{4}{5}$$

From (1), $$\cos\theta_1 = \frac{1}{3}\left[5 - 4 \times \frac{4}{5}\right] = \frac{3}{5}$$

Hence $$\sin\theta_1 = \frac{4}{5} \quad \text{and} \quad \sin\theta_2 = \frac{3}{5}$$

Now $$\sin(\theta_1 + \theta_2) = \sin\theta_1\cos\theta_2 + \cos\theta_1\sin\theta_2$$

$$= \frac{4}{5} \times \frac{4}{5} + \frac{3}{5} \times \frac{3}{5} = 1$$

$$\therefore \quad \theta_1 + \theta_2 = 90°$$

Hence after the collision, the two balls will move off at right angles to each other.

EXAMPLE 72. *Consider the collision depicted in Fig. 6.40 to be between two billiard balls with equal masses* $m_1 = m_2$. *The first ball is called the cue while the second ball is called the target. The billiard player wants to "sink" the target ball in a* *corner pocket, which is at an angle $\theta_2 = 37°$. Assume that the collision is elastic and that friction and rotational motion are not important. Obtain θ_1.* **[NCERT]**

Solution. By conservation of momentum, we have

$$m\vec{u}_1 + 0 = m\vec{v}_1 + m\vec{v}_2$$

or $$\vec{u}_1 = \vec{v}_1 + \vec{v}_2 \quad ...(i)$$

By conservation of energy, we have

$$\frac{1}{2}mu_1^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$$

or $$u_1^2 = v_1^2 + v_2^2 \quad ...(ii)$$

From (*i*), we have

$$\vec{u}_1 \cdot \vec{u}_1 = (\vec{v}_1 + \vec{v}_2)\cdot(\vec{v}_1 + \vec{v}_2)$$
$$= \vec{v}_1 \cdot \vec{v}_1 + \vec{v}_1 \cdot \vec{v}_2 + \vec{v}_2 \cdot \vec{v}_1 + \vec{v}_2 \cdot \vec{v}_2$$

or $$u_1^2 = v_1^2 + v_2^2 + 2\vec{v}_1 \cdot \vec{v}_2$$

or $$u_1^2 = u_1^2 + 2\vec{v}_1 \cdot \vec{v}_2 \quad \text{[using (ii)]}$$

or $$\vec{v}_1 \cdot \vec{v}_2 = 0$$

Thus the angle between $\vec{v}_1$ and $\vec{v}_2$ is 90°.

or $$\theta_1 + \theta_2 = 90°$$

$$\therefore \quad \theta_1 = 90° - \theta_2 = 90 - 37° = \mathbf{53°}.$$

Example 73. *A nucleus of radium ($^{226}_{88}Ra$) decays to $^{222}_{86}Rn$ by the emission of α-particle (^4_2He) of energy 4.8 MeV. If mass of $^{222}_{86}Rn = 222.0$ amu and mass of $^4_2He = 4.003$ amu, then calculate the recoil energy of the daughter nucleus $^{222}_{86}Rn$.* **[NCERT]**

Solution. The nuclear decay may be represented as follows :

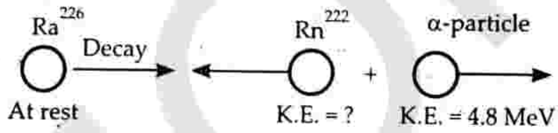


Fig. 6.41

The kinetic energy of a particle is given by

$$K = \frac{p^2}{2m} \quad \therefore \quad p = \sqrt{2mK}$$

As momentum is conserved in the absence of an external force, so

$$mK = \text{constant}$$

or $$m_{Rn}K_{Rn} = m_\alpha K_\alpha$$

or $$K_{Rn} = \frac{m_\alpha K_\alpha}{m_{Rn}}$$
$$= \frac{4.003 \times 4.8}{222} = \mathbf{0.0866\ MeV.}$$

Example 74. *The nucleus Fe^{57} emits a γ-ray of energy 14.4 keV. If the mass of the nucleus is 56.935 amu, calculate the recoil energy of the nucleus. Take 1 amu = 1.66×10^{-27} kg.* **[NCERT]**

Solution. The nuclear decay may be represented as follows :

$$Fe^{57} \rightarrow Fe^{57} + h\nu \quad (\gamma\text{-ray photon})$$

Excited state — Ground state

According to de-Broglie hypothesis, momentum of a photon of energy E is

$$p = \frac{E}{c} = \frac{14.4 \text{ keV}}{c} = \frac{14.4 \times 1.6 \times 10^{-16} \text{ J}}{3 \times 10^8 \text{ ms}^{-1}}$$
$$= 7.68 \times 10^{-24} \text{ kg ms}^{-1}.$$

By conservation of momentum, the momentum of daughter nucleus,

$$p = \text{momentum of } \gamma\text{-ray photon}$$
$$= 7.68 \times 10^{-24} \text{ kg ms}^{-1}$$

The recoil energy of the nucleus will be

$$K = \frac{p^2}{2m} = \frac{(7.68 \times 10^{-24})^2}{2 \times 56.935 \times 1.66 \times 10^{-27}}$$
$$= 0.312 \times 10^{-21} \text{ J} = \frac{0.312 \times 10^{-21}}{1.6 \times 10^{-16}} \text{ keV}$$
$$= \mathbf{1.95 \times 10^{-6}\ keV.}$$

Problems For Practice

1. A vehicle of mass 30 quintals moving with a speed of 18 km h^{-1} collides with another vehicle of mass 90 quintals moving with a speed of 14.4 km h^{-1} in the opposite direction. What will be the velocity of each after the collision ?
(**Ans.** 30.6 km h^{-1}, 1.8 km h^{-1})
2. A ball of 0.1 kg makes an elastic head on collision with a ball of unknown mass that is initially at rest. If the 0.1 kg ball rebounds at one-third of its original speed, what is the mass of the other ball ?
(**Ans.** 0.2 kg)
3. A body of mass m strikes a stationary body of mass M and undergoes an elastic collision. After collision m has a speed one-third of its initial speed. What is the ratio M/m ? (**Ans.** 1 : 2)
4. Two particles of masses 0.5 kg and 0.25 kg moving with velocities 4.0 ms^{-1} and -3.0 ms^{-1} collide head on in a perfectly inelastic collision. Find (*i*) the velocity of the composite particle after the collision and (*ii*) the kinetic energy lost in the collision.
[**Ans.** (*i*) 1.7 ms^{-1} (*ii*) 4.1 J]

5. What percentage of the K.E. of a moving particle is transferred to a stationary particle when it strikes the stationary particle of four times its mass ? **(Ans.** 64%**)**

6. A neutron moving with a speed of $10^6\ \text{ms}^{-1}$ suffers a head-on collision with a nucleus of mass number 80. What is the fraction of energy retained by the nucleus ? **(Ans.** 79/81**)**

7. What percentage of kinetic energy of a moving particle is transferred to a stationary particle, when moving particle strikes with a stationary particle of mass (*i*) 19 times its mass (*ii*) equal in mass and (*iii*) 1/9th of its mass ? **[Ans.** (*i*) 19% (*ii*) 100% (*ii*) 36%**]**

8. Show that when a moving body collides with stationary body of mass m or $1/m$ times its mass, then the moving body transfers $\frac{4m}{(1+m)^2}$ part of its kinetic energy to the stationary body.

9. A ball is dropped from a height of 3 m. What is the height upto which the ball will rebound ? The coefficient of restitution is 0.5. **(Ans.** 0.75 m**)**

10. A ball is dropped from a height h on to a floor. If the coefficient of restitution is e, calculate the height to which the ball first rebounds ? **(Ans.** $h\,e^2$**)**

✗ HINTS

2. Here $m_1 = 0.1\ \text{kg}, \quad m_2 = ?$

$$u_1 = u\,(\text{say}), \quad v_1 = -\frac{u}{3}, \quad u_2 = 0$$

Now $$v_1 = \frac{m_1 - m_2}{m_1 + m_2}\cdot u_1 + \frac{2m_2}{m_1 + m_2}\cdot u_2$$

$\therefore$ $$-\frac{u}{3} = \frac{0.1 - m_2}{0.1 + m_2}\cdot u + 0$$

or $$-\frac{1}{3} = \frac{0.1 - m_2}{0.1 + m_2}$$

or $$-0.1 - m_2 = 0.3 - 3m_2 \quad \text{or} \quad 2m_2 = 0.4$$

$\therefore$ $$m_2 = \mathbf{0.2\ kg.}$$

8. **Case 1.** Mass of moving body $= m_1$

initial velocity $= u_1$

Mass of stationary body, $m_2 = mm_1$

initial vel., $u_2 = 0$;

Velocity of the stationary body after collision will be

$$v_2 = \frac{2m_1}{m_1 + m_2}\cdot u_1 + \frac{m_2 - m_1}{m_1 + m_2}\cdot u_2$$

$$= \frac{2m_1}{m_1 + mm_1}\cdot u_1 + 0 = \frac{2}{1+m}\cdot u_1$$

Fraction of the K.E. transferred to the stationary body

$$\frac{\frac{1}{2}m_2 v_2^2}{\frac{1}{2}m_1 u_1^2} = \frac{mm_1\left(\frac{2}{1+m}\cdot u_1\right)^2}{m_1 u_1^2} = \frac{4m}{(1+m)^2}$$

Case 2. Here $m_2 = \frac{m_1}{m}$

$$v_2 = \frac{2m_1}{m_1 + \frac{m_1}{m}}\cdot u_1 + 0 = \frac{2m_1}{m_1\left(1 + \frac{1}{m}\right)}\cdot u_1$$

$$= \frac{2m}{1+m}\cdot u_1$$

Fraction of the K.E. transferred to the stationary body

$$\frac{\frac{1}{2}m_2 u_2^2}{\frac{1}{2}m_1 u_1^2} = \frac{\frac{m_1}{m}\left(\frac{2m}{1+m}u_1\right)^2}{m_1 u_1^2} = \frac{4m}{(1+m)^2}.$$

$\therefore$ In each case, K.E. of the stationary body after collision

$$= \frac{4m}{(1+m)^2} \times \text{K.E. of the moving body before collision.}$$

10. The velocity with which the ball strikes the floor is

$$u = \sqrt{2gh}$$

If the ball rebounds with velocity v, then

$$e = \frac{v}{u} \quad \text{or} \quad v = eu = e\sqrt{2g\,h}$$

If the ball rebounds to height h', then

$$0^2 - v^2 = 2(-g)\,h'$$

or $$h' = \frac{v^2}{2g} = \frac{1}{2g}\times e^2\,.\,2gh = \mathbf{he^2}.$$

Very Short Answer Conceptual Problems

Problem 1(*a*). If a force acts perpendicular to the direction of motion of a body, what is the amount of work done ?

Solution. The work done is zero because there is no displacement in the direction of force.

Problem 1(*b*). A body is moving at constant speed over a frictionless surface. What is the work done by the weight ?

Solution. Work done by the weight is zero, since the force and the displacement are at right angles to each other.

Problem 2. The earth moving around the sun in a circular orbit is acted upon by a force and hence work must be done on the earth by the force. Do you agree by this statement ?

Solution. The statement is wrong. The gravitational force is a conservative force. So the work done by the gravitational force over every complete orbit of the earth is zero.

Problem 3. A body is moving along a circular path. How much work is done by the centripetal force ?

Or *[Himachal 05C, 08 ; Delhi 06]*

Why is the work done by centripetal force zero ?

[Central Schools 12]

Solution. For a body moving along a circular path, the centripetal force acts along the radius while the displacement is tangential *i.e.*, $\theta = 90°$, therefore $W = Fs \cos 90° = 0$.

Problem 4. A body of mass m is moving in a circle of radius r with a constant speed v. The force on the body is $\frac{mv^2}{r}$ and is directed towards the centre. What is the work done by this force in moving the body over half the circumference of the circle ?

Solution. Work done by the force is zero, because the direction of displacement of the body at every point is perpendicular to the direction of force acting on the body.

Problem 5. Is it possible that a body be in accelerated motion under a force acting on the body, yet no work is being done by the force ? Explain your answer giving a suitable example.

Solution. Yes, it is possible, when the force is perpendicular to the direction of motion. The moon revolves round the earth under the centripetal force of attraction of the earth, but earth does no work on the moon.

Problem 6. How much work is done by a coolie walking on a horizontal platform with a load on his head ?

Solution. Zero. In order to balance the load on his head, the coolie applies a force on it in the upward direction equal to its weight. His displacement is along the horizontal direction. Thus the angle between force $\vec{F}$ and displacement $\vec{s}$ is 90°. Therefore work done, $W = Fs \cos \theta = Fs \cos 90° = 0$.

Problem 7. A porter moving vertically up the stairs with a suitcase on his head does work. Why ?

Solution. The porter lifts the suitcase vertically to the upstairs. Force has to be applied on the suitcase against the force of gravitation. Hence the porter does work.

Problem 8. Does the work done in raising a suitcase on to a platform depend upon how fast it is raised up ?

[Himachal 08]

Solution. No. The work done depends on the force of gravity and the height through which the suitcase is raised. It does not depend on the time rate with which the suitcase is raised.

Problem 9. In a tug of war one team is slowly giving way to the other. What work is being done and by whom ?

Solution. The work is done by the winning team and is equal to the product of resultant force applied by the two teams and displacement that the losing team suffers.

Problem 10. A man rowing boat upstream is at rest with respect to the shore. Is he doing work ?

Solution. The boat is at rest with respect to the shore but it is moving upstream with respect to water. The man is doing work relative to the stream because he is applying force to produce relative motion between the boat and the stream. But he does no work relative to the shore as displacement relative to the shore is zero.

Problem 11. Mountain roads rarely go straight up but wind up gradually. Why ?

Solution. If the roads go straight up, the angle of slope θ would be large. In that case frictional force ($f = \mu\, mg \cos \theta$) would be less and the vehicles may slip. Moreover, while going up a large slope, a greater power would be required (as it will take less time).

Problem 12. What sort of energy is associated with a bird flying in air ?

Solution. A flying bird possesses both potential and kinetic energies because it is at a certain height above the ground and moving with certain velocity.

Problem 13. When an arrow is shot, where from the arrow will acquire its kinetic energy ?

Solution. A stretched bow possesses potential energy on account of a change in its shape. To shoot an arrow, the bow is released. The potential energy of the bow is converted into the kinetic energy of the arrow.

Problem 14. Can a body have energy without momentum ?

Solution. Yes, there is an internal energy in a body due to the thermal agitation of the particles of the body, while the vector sum of the momenta of the moving particles may be zero.

Problem 15. Can a body have momentum without energy ?

Solution. No, if a body has momentum, it must be in motion and consequently possess kinetic energy.

Problem 16. A light body and a heavy body have the same momentum. Which one will have greater kinetic energy ? *Or* *[Himachal 02, 04, 05, 07C, 08]*

Two bodies of unequal masses have same linear momentum, which one has greater kinetic energy ?

[Delhi 08]

Solution. Kinetic energy, $K = \frac{1}{2} mv^2 = \frac{(mv)^2}{2m} = \frac{p^2}{2\,m}$

For constant p, $K \propto \frac{1}{m}$

Thus the lighter body has more kinetic energy than the heavier body.

Problem 17. A light body and a heavy body have the same kinetic energy. Which one will have the greater momentum ? *[Himachal 02, 05]*

Solution. Kinetic energy,

$$K = \frac{1}{2} mv^2 = \frac{(mv)^2}{2m} = \frac{p^2}{2\,m}$$

or $$p = \sqrt{2mK} \quad i.e., \quad p \propto \sqrt{m}$$

Thus the heavier body has a greater momentum than the lighter one.

Problem 18. How does the kinetic energy of a body change if its momentum is doubled ? *[Himachal 04, 07C, 08]*

Solution. $$K = \frac{1}{2} mv^2 = \frac{(mv)^2}{2\,m} = \frac{p^2}{2\,m} \quad i.e., \quad K \propto p^2$$

When the momentum of a body is doubled, its kinetic energy becomes four times the initial kinetic energy.

Problem 19. Two bodies of masses m_1 and m_2 have the same linear momentum. What is the ratio of their kinetic energies ?

Solution. If v_1 and v_2 are the velocities of two bodies having masses m_1 and m_2 respectively, then

$$m_1 v_1 = m_2 v_2 \quad \text{or} \quad m_1^2\, v_1^2 = m_2^2\, v_2^2$$

or $$\frac{m_1 v_1^2}{m_2 v_2^2} = \frac{m_2}{m_1} \quad \text{or} \quad \frac{(m_1 v_1^2/2)}{(m_2 v_2^2/2)} = \frac{m_2}{m_1}$$

or $$\frac{K_1}{K_2} = \frac{m_2}{m_1}$$

Problem 20. Two bodies of masses m_1 and m_2 have equal kinetic energies. What is the ratio of their linear momenta ?

Solution. Given that $\frac{1}{2} m_1\, v_1^2 = \frac{1}{2} m_2\, v_2^2$

or $$\frac{v_1}{v_2} = \sqrt{\frac{m_2}{m_1}}$$

or $$\frac{m_1\, v_1}{m_2\, v_2} = \frac{m_1}{m_2}\sqrt{\frac{m_2}{m_1}} = \sqrt{\frac{m_1}{m_2}}$$

or $$\frac{p_1}{p_2} = \frac{\sqrt{m_1}}{\sqrt{m_2}}$$

Problem 21. Can there be a situation in which $E - U < 0$?

Solution. No. As $E = K + U$ or $K = E - U$. But kinetic energy K cannot be negative. So $E - U$ is never less than zero.

Problem 22. Can the overall energy of a body be negative ?

Solution. Yes. As $E = K + U$, when U is negative and has magnitude greater than that of K, E is negative. For example, the energy of an electron bound to the nucleus of an atom is negative.

Problem 23. Does the potential energy of a spring decrease or increase when it is compressed or stretched ?

Solution. The potential of the spring increases because work is done on it when it is compressed or stretched.

Problem 24. If a block attached to a spring (whose other end is rigidly fixed) is pulled up to distance x_0 and released, the amplitude of its motion cannot exceed $\pm x_0$. Why ?

Solution. If the amplitude of motion exceeds $\pm x_0$, then the potential energy would be greater than $\frac{1}{2} kx_0^2$ at $x > x_0$.

So its K.E. $= \frac{1}{2} kx_0^2 - \frac{1}{2} kx^2$ would be negative. This is impossible. Hence the block cannot go beyond $x = \pm x_0$.

Problem 25. Springs A and B are identical except that A is stiffer than B, *i.e.*, force constant $k_A > k_B$. In which spring is more work expended if they are stretched by the same amount ?

Solution. Work done in stretching a spring of force constant k through a distance x,

$$W = \frac{1}{2} kx^2$$

$$\therefore \quad \frac{W_A}{W_B} = \frac{(1/2)\, k_A\, x^2}{(1/2)\, k_B\, x^2} = \frac{k_A}{k_B}$$

As $k_A > k_B$, therefore, $W_A > W_B$.

Problem 26. Springs A and B are identical except that A is stiffer than B. In which spring is more work expended if they are stretched by the same force ?

Solution. As $F = kx$, so $x = \frac{F}{k}$

For same F, $$W_A = \frac{1}{2} k_A x^2 = \frac{1}{2} k_A \left(\frac{F}{k_A}\right)^2 = \frac{F^2}{2\,k_A}$$

and $$W_B = \frac{F^2}{2\,k_B}$$

$$\therefore \quad \frac{W_A}{W_B} = \frac{k_B}{k_A}$$

As $k_A > k_B$ therefore, $W_A < W_B$.

Problem 27. Which of the two : kilowatt hour or electron volt is a bigger unit of energy and by what factor ?

Solution. Kilowatt hour (kWh) is a bigger unit of energy.

$$\frac{1\,\text{kWh}}{1\,\text{eV}} = \frac{3.6 \times 10^6\ \text{J}}{1.6 \times 10^{-19}\ \text{J}} = 2.25 \times 10^{25}$$

Problem 28. Will water at the foot of the fall be at a different temperature from that at the top ? If yes, explain.

Solution. When water falls from a height, its P.E. is converted into kinetic energy and its velocity is maximum at the foot of the fall. The velocity is suddenly reduced to zero, and hence its kinetic energy gets converted into heat energy raising its temperature.

Problem 29. Is it necessary that work done in the motion of a body over a closed loop is zero for every force in nature ? Why ? *[Central Schools 2003]*

Solution. No. The work done in the motion of body over a closed loop is zero only in case of a conservative force but it is not zero in case of a non-conservative force like friction.

Problem 30. When a constant force is applied to a body moving with constant acceleration, is the power of the force constant ? If not, how would force have to vary with speed for power to be constant ?

Solution. Power, $P = Fv$

As the body is moving with acceleration, v changes and so P also changes, F being constant. For P to be constant, Fv = a constant or ($F \propto 1/v$). Thus as v increases, F should decrease to keep P constant.

Problem 31. What are central forces ? Are they conservative in nature ?

Solution. Force between two objects is called a central force, if it acts between them along the line joining their centres. Electrostatic force between the charges and magnetic force between two poles are central forces and are conservative forces.

Problem 32. When is the exchange of energy maximum during an elastic collision ?

Solution. Energy exchange will be maximum if the two colliding bodies are of equal masses.

Problem 33. Is whole of the kinetic energy lost in any perfectly inelastic collision ?

Solution. No, only that much amount of kinetic energy is lost as is necessary for the conservation of momentum.

Problem 34. Can you associate potential energy with a non-conservative force ?

Solution. No, P.E. can be associated only with conservative force.

Problem 35. Two bodies moving towards each other collide and move away in opposite directions. There is some rise of temperature of the bodies in the process. Explain the reason for the rise of temperature and state what type of collision is it.

Solution. Involved collision of the process is inelastic because the bodies suffer loss of kinetic energy which appears in the form of heat energy raising the temperature of the bodies.

Problem 36. A spark is produced, when two stones are struck against each other. Why ? *[Himachal 01]*

Solution. The work done in striking the two stones against each other gets converted into heat. This appears as a spark.

Short Answer Conceptual Problems

Problem 1. A lorry and a car with the same kinetic energy are brought to rest by the application of the brakes which provide equal retarding force. Which of them will come to rest in a shorter distance ?

Solution. By work-energy theorem,

Loss in K.E. of the vehicle

= Work done against retarding force

= Retarding force × distance travelled

$$\therefore \quad \text{Distance travelled} = \frac{\text{Loss in K.E.}}{\text{Retarding force}}$$

As both the kinetic energy and retarding force are same, so both the lorry and the car would come to rest in the same distance.

Problem 2. A truck and a car are moving with the same kinetic energy on a straight road. Their engines are simultaneously switched off. Which one will stop at a lesser distance ?

Solution. By work-energy theorem,

Loss in K.E. of the vehicle

= Work done against the force of friction × distance

or $$\text{K.E.} = f \times s = \mu R \times s = \mu mg s$$

or $$s = \frac{\text{K.E.}}{\mu mg}$$

For constant K.E., $s \propto \frac{1}{m}$

As the truck has more mass than the car, so it will stop in a lesser distance than the car.

Problem 3. A rocket explodes in mid air. How does this affect (*a*) its total momentum and (*b*) its total kinetic energy ?

Solution. (*a*) Because no external force acts on the rocket, its total momentum remains unchanged.

(*b*) When the rocket explodes, its fragments receive additional kinetic energy from the explosion. The chemical energy of the fuel changes into kinetic energy. As a result of this, the total kinetic energy gets increased.

Problem 4. The velocity of an aeroplane is doubled. (*a*) What will happen to its momentum ? Will the momentum remain conserved ? (*b*) What will happen to its K.E. ? Will the energy remain conserved ?

Solution. (*a*) When the velocity of the aeroplane is doubled, its momentum also gets doubled. However, the combined momentum of aeroplane + air is conserved. As the momentum of the aeroplane increases, the momentum of air also increases by an equal amount in the opposite direction.

(*b*) The kinetic energy becomes four times. The additional energy is obtained by burning of fuel. However, the total energy is still conserved.

Problem 5. A uniform rectangular parallelopiped of mass m having sides l, $2l$ and $4l$ is placed in turn on each of its three sides on a horizontal surface. What is the potential energy of the parallelopiped in each of these positions ? Which position will be the most stable ?

Solution. When the parallelopiped is placed on its small, middle and large side, its centre of gravity will lie at heights $2l$, l and $l/2$ respectively above the horizontal surface. If U_1, U_2 and U_3 are the potential energies in the three cases, then

$$U_1 = mg \times 2l = 2\,mgl$$
$$U_2 = mg \times l = mgl$$
$$U_3 = mg \times \frac{l}{2} = \frac{1}{2}\,mgl$$

When the parallelopiped rests on its large side, its potential energy is minimum. So this position is the position of stable equilibrium.

Problem 6. What happens to the potential energy when (*a*) two protons are brought close together (*b*) one proton and one electron are brought close together ?

(*a*) The potential energy increases when two protons are brought closer because work has to be done against the force of repulsion between them.

(*b*) The potential energy decreases when a proton and an electron are brought closer because work is done by the force of attraction between them (origin of the force being the system itself).

Problem 7. A particle is moving in a circular path of a given radius with number of rotations per second (*i*) constant, (*ii*) increasing, (*iii*) decreasing. What happens to work done in the three cases ?

Solution. Work is equal to the change in kinetic energy of the system. (*i*) In case speed is constant, kinetic energy does not change, hence work done is zero, (*ii*) when speed increases, K.E. increases, work is done by the exerting force, (*iii*) when speed decreases, K.E. decreases, work is done by the body on the force.

Problem 8. "Chemical, gravitational and nuclear energies are nothing but potential energies for different types of forces in nature." Explain the statement clearly with examples.

Solution. A system has potential energy, when various objects are held at certain distance against some force, by virtue of their position or configuration. Chemical potential energy results from the chemical bonding of atoms, gravitational potential energy results when objects are held at some distance against gravitational force while nuclear energy arises from the nuclear force between the nucleons.

Problem 9. In a thermal power station, coal is used for the generation of electricity. Mention how energy changes from one form to another before it is transformed into electrical energy ?

Solution. The heat energy produced due to combustion of coal converts water into steam. The heat energy of steam is converted into mechanical energy when it turns blades of a turbine. The mechanical energy so obtained is converted into electrical energy by the generators.

Problem 10. Explain how fast moving neutrons can be quickly slowed down by passing through water or heavy water.

Solution. Water and heavy water contain protons having approximately the same mass as that of a neutron. When fast moving neutrons collide with protons, the neutrons attain the low velocities of protons while the protons begin to move with the high velocities of neutrons.

Problem 11. Two ball bearings of mass m each moving in the opposite directions with equal speed v collide head on with each other. Predict the outcome of the collision, assuming it to be perfectly elastic.

Solution. Here $m_1 = m_2 = m$, $u_1 = v$, $u_2 = -v$

$$\therefore\; v_1 = \frac{(m_1 - m_2)\,u_1 + 2\,m_2\,u_2}{m_1 + m_2} = \frac{(m - m)\,v + 2m(-v)}{m + m} = -v$$

and

$$v_2 = \frac{(m_2 - m_1)\,u_2 + 2m_1 u_1}{m_1 + m_2} = \frac{(m - m)(-v) + 2mv}{m + m} = v$$

Hence after collision, the two ball bearings will move with same speeds but their directions of motion are reversed.

Problem 12. A body of mass m moving with speed v collides elastically head-on with another body of mass m initially at rest. Show that the moving body will come to a stop as a result of this collision. *[Delhi 04]*

Solution. By conservation of momentum,

$$m \times v + m \times 0 = mv_1 + mv_2$$

or $$v = v_1 + v_2$$

or $$v - v_1 = v_2 \qquad \text{...(1)}$$

By conservation of kinetic energy,

$$\frac{1}{2}\,mv^2 + 0 = \frac{1}{2}\,mv_1^2 + \frac{1}{2}\,mv_2^2$$

or $$v^2 = v_1^2 + v_2^2$$

or $$(v + v_1)(v - v_1) = v_2^2 \qquad \text{...(2)}$$

Dividing (2) by (1), we get

$$v + v_1 = v_2 \qquad \text{...(3)}$$

Solving (1) and (3),

$$v_1 = 0 \text{ and } v_2 = v$$

As a result of the collision, the moving body comes to a stop and the stationary body begins to move with the speed of the first body.

Problem 13. A uniform chain of length L and mass M is lying on a smooth table and one third of its length is hanging vertically down over the edge of the table. If g is acceleration due to gravity, calculate work required to pull the hanging part on the chain. *[IIT 85; MNREC 90]*

Solution. Weight of length L of the chain = Mg

$$\text{Weight of length } \frac{L}{3} \text{ of the chain} = \frac{1}{3} Mg$$

As the centre of gravity of the hanging part lies at its midpoint *i.e.* at a distance equal to $L/6$ below the edge of the table, so the work required to pull the hanging part on the table is

$$W = \text{Force} \times \text{distance} = \frac{1}{3} Mg \times \frac{L}{6} = \frac{MgL}{18}.$$

Problem 14. A body of mass M is moved along a straight line by a machine delivering a constant power P. Find the expression for the distance moved by the body in terms of M, P and t. *[AIEEE 03]*

Solution. Power, $P = Fv = ma \cdot v = m\frac{dv}{dt} \cdot v$

or $$v\,dv = \frac{P}{m}\,dt$$

Integrating both sides, we get

$$\int v\,dt = \int \frac{P}{m}\,dt = \frac{P}{m}\int dt \quad [\because P, m \text{ are constant}]$$

or $$\frac{v^2}{2} = \frac{P}{m} \cdot t$$

or $$v^2 = \frac{2P}{m} \cdot t$$

or $$v = \sqrt{\frac{2P}{m}} \cdot t^{1/2}$$

Thus $v \propto t^{1/2}$

Also $v = \frac{dx}{dt}$ or $dx = v\,dt = \sqrt{\frac{2P}{m}} \cdot t^{1/2}\,dt$

Integrating both sides, we get

$$\int dx = \sqrt{\frac{2P}{m}} \int t^{1/2}\,dt$$

or $$x = \sqrt{\frac{2P}{m}} \cdot \frac{t^{3/2}}{3/2} = \frac{2}{3}\sqrt{\frac{2P}{m}} \cdot t^{3/2}.$$

Thus $x \propto t^{3/2}$.

Problem 15. Why a metal ball rebounds better than a rubber ball ?

Solution. When a rubber ball hits a massive object, say, earth, the ball is distorted. A large amount of heat is generated in the ball by the rubbing of the rubber molecules against each other. This effect is essentially absent in a hard material. So, a metal ball would often lose less energy upon collision than would a rubber ball.

Problem 16. Explain, throwing mud on a wall is an example of perfectly inelastic collision.

Solution. When mud is thrown on a wall, it sticks to the wall. The kinetic energy of the mud is reduced to zero and there is non-conservation of kinetic energy. Hence it is a case of perfectly inelastic collision.

Problem 17. Nuclear fission and fusion reactions are examples of conversion of mass into energy. Can we say that strictly speaking, mass is converted into energy even in an exothermic chemical reaction ?

Solution. Yes, the generation of heat in exothermic reaction involves the conversion of mass into energy. However, the amount of mass involved is much less than that in fission or fusion.

Problem 18. The energy released in fusion reaction of light nuclei is much less than the energy released in a fission reaction of a heavy nucleus. Why is then a hydrogen bomb (based on nuclear fusion) far more powerful than an atomic bomb (based on nuclear fission) ?

Solution. A hydrogen bomb (based on nuclear fusion) is more powerful than atom bomb (based on nuclear fission), because in case of fusion, the energy released per unit mass of the fuel is much larger than that in fission.

Problem 19. What is the minimum amount of energy released in the annihilation of an electron-positron pair ?

Solution. The energy released in the annihilation is minimum when both electron and positron are at rest.

Hence, $$E_{min} = 2\,m_0c^2$$

(where, m_0 = rest mass of electron or positron)

$$E_{min} = 2 \times 9.1 \times 10^{-31} \times (3 \times 10^8)^2 \text{ J}$$

$$= \frac{2 \times 9.1 \times 10^{-31} \times (3 \times 10^8)^2}{1.6 \times 10^{-13}} \text{ MeV}$$

$$= \mathbf{1.02\ MeV.}$$

Problem 20. By convention, the forces which fall off to zero at large distances, the potential energy at infinity is taken to be zero. With this choice, is the potential energy positive for (*a*) electron-positron bound state (*b*) planet-satellite system and (*c*) electron-electron system ?

Solution.

(*a*) *Negative,* because of attraction between electron and positron.

(*b*) *Negative,* because of attraction between planet and satellite.

(*c*) *Positive,* because of repulsion between two electrons.

HOTS

Problems on Higher Order Thinking Skills

Problem 1. *A locomotive of mass m starts moving so that its velocity varies according to the law $v = \alpha\sqrt{s}$, where α is a constant and s is the distance covered. Find the total work done by all the forces acting on the locomotive during the first t seconds after the beginning of motion.*

Solution. Velocity, $v = \alpha\sqrt{s} = \alpha\, s^{1/2}$

Acceleration,

$$a = \frac{dv}{dt} = \frac{1}{2}\alpha\, s^{-1/2} \cdot \frac{ds}{dt} = \frac{1}{4}\alpha\, s^{-1/2} \cdot v$$

$$= \frac{1}{2}\alpha\, s^{-1/2} \cdot \alpha\, s^{1/2} = \frac{1}{2}\alpha^2$$

Force, $F = ma = \frac{1}{2} m\alpha^2$

Distance covered by the locomotive in first t seconds,

$$s = ut + \frac{1}{2}at^2 = 0 \times t + \frac{1}{2} \times \frac{1}{2}\alpha^2 t^2 = \frac{1}{4}\alpha^2 t^2$$

$\therefore$ Work done,

$$W = Fs = \frac{1}{2} m\alpha^2 \times \frac{1}{4}\alpha^2 t^2 = \frac{1}{8} m\alpha^4 t^2.$$

Problem 2. *A 0.5 kg particle moves along the x-axis from $x = 5$ m to $x = 17.2$ m under the influence of a force $F(x)$,*

$$F = \frac{200}{2x + x^3}\ N$$

Estimate (with at least 5% accuracy) the total work done by this force during this displacement.

Solution.

$$W = \int_{x_i}^{x_f} F\, dx = \int_5^{17.2} \frac{200}{2x + x^3}\, dx = \int_5^{17.2} \frac{200}{x(x^2 + 2)}\, dx$$

Put $t = x^2 + 2$, $dt = 2x dx$. Then

$$W = \frac{200}{2}\int \frac{dt}{(t-2)\,t}$$

$$= 100 \int \left[\frac{1}{2(t-2)} - \frac{1}{2t}\right] dt$$

$$= 50\,[\log_e (t-2) - \log_e t] = 50\left[\log_e \frac{t-2}{t}\right]$$

$$= 50 \times 2.303 \left| \log \frac{x^2}{x^2 + 2} \right|_5^{17.5}$$

$$= 50 \times 2.303 \log \frac{295.84}{297.84} \times \frac{27}{25}$$

$$= 50 \times 2.303 \log 1.07275$$

$$= 50 \times 2.303 \times 0.0305 = \mathbf{3.51\ J}.$$

Problem 3. *The potential energy of a 2 kg particle free to move along the x-axis is given by*

$$U(x) = \left(\frac{x}{b}\right)^4 - 5\left(\frac{x}{b}\right)^2\ J$$

where $b = 1$ m. Plot this potential, identifying the extremum points. Identify the regions where particle may be found and its maximum speed given that the total mechanical energy is (i) 36 J ; (ii) – 4 J.

Solution. For $b = 1$, $U = x^4 - 5x^2$

$$\therefore \quad \frac{dU}{dx} = 4x^3 - 10x = 2x(2x^2 - 5)$$

For extremum points, $\frac{dU}{dx} = 0$

or $\quad 2x(\sqrt{2}x - \sqrt{5})(\sqrt{2}x + \sqrt{5}) = 0$

$\therefore \quad x = 0, \sqrt{\frac{5}{2}}, -\sqrt{\frac{5}{2}}$ are the extremum points.

x	0	± 1	$\pm\sqrt{5/2}$	± 2	± 3
U	0	– 4	– 6.25	– 4	– 36

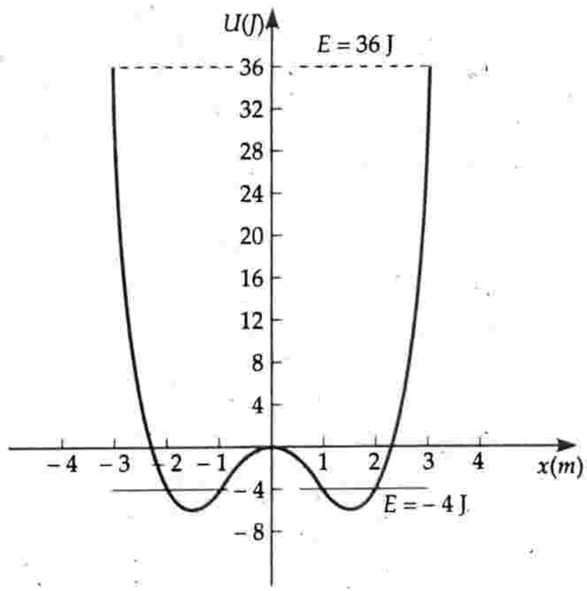


Fig. 6.42 Plot of potential energy U versus x

(*i*) Here $E = K + U = 36$ J. The particle may be found in the region in which its K.E. is positive. This region is $-3\ \text{m} < x < 3$ m because here $U_{max} = 0$ and $k_{min} = -36$ J (at $x = 0$)

At $x = \pm\sqrt{\frac{5}{2}}$, $\quad U_{min} = -6.25$ J

$\therefore \quad K_{max} = E - U_{min} = 36 + 6.25 = 42.25$ J

or $\quad \frac{1}{2} mv_{max}^2 = 42.25$

or $\quad v_{max} = \sqrt{\frac{2 \times 42.25}{m}} = \sqrt{\frac{2 \times 42.25}{2}} = \mathbf{6.5\ ms^{-1}}.$

(*ii*) Here $E = K + U = -4$ J. Again for the positive K.E., the particle may be found in the regions

$$-2 \text{ m} < x < -1 \text{ m and } 1 \text{ m} < x < 2 \text{ m}$$

As $\quad K_{max} = E - U_{min}$

or $\quad \frac{1}{2} mv_{max}^2 = -4 + 6.25 = 2.25$

$$v_{max} = \sqrt{\frac{2 \times 2.25}{2}} = \mathbf{1.5\ ms^{-1}}.$$

Problem 4. *A particle of mass m is moving in a horizontal circle of radius r, under a centripetal force equal to $-(K/r^2)$, where K is constant. What is the total energy of the particle ?* **[IIT ; Delhi 10]**

Solution. As the particle is moving in horizontal circle, so

Centripetal force, $F = \frac{mv^2}{r} = \frac{K}{r^2}$

This gives $\quad mv^2 = \frac{K}{r}$

$\therefore$ K.E. of the particle,

$$K = \frac{1}{2} mv^2 = \frac{K}{2r}$$

As $\quad F = -\frac{dU}{dr}$

$\therefore$ Potential energy,

$$U = -\int_{\infty}^{r} F dr = -\int_{\infty}^{r} \left(-\frac{K}{r^2}\right) dr = K \int_{\infty}^{r} r^{-2} dr = -\frac{K}{r}$$

Total energy $= K + U = \frac{K}{2r} - \frac{K}{r} = -\frac{K}{2r}.$

Problem 5. *A small object of weight mg hangs from a string of length l, as shown in Fig. 6.43. A variable force F, which starts at zero and gradually increases is used to pull the object very slowly (so that equilibrium exists at all the times) until the string makes an angle θ with the vertical. Calculate the work done by the force F.* **[CPMT 93]**

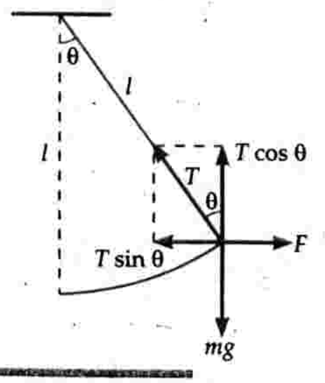


Fig. 6.43

Solution. Various forces acting on the small object are shown in the figure. As the object remains in equilibrium, so

$$F = T \sin \theta \quad \text{and} \quad mg = T \cos \theta$$

$$\therefore \quad \frac{F}{mg} = \frac{T \sin \theta}{T \cos \theta} = \tan \theta$$

or $\quad F = mg \tan \theta$

Also, displacement of the object, $s = l\theta$

$\therefore \quad ds = l\, d\theta$

Work done by the force F,

$$W = \int_0^\theta \vec{F} \cdot d\vec{s} = \int_0^\theta F ds \cos \theta$$

$$= \int_0^\theta mg \tan \theta \times l\, d\theta \times \cos \theta$$

$$= mgl \int_0^\theta \sin \theta\, d\theta$$

$$= mgl [-\cos \theta]_0^\theta = mgl [-\cos \theta + \cos 0]$$

or $\quad W = mgl (1 - \cos \theta).$

Problem 6. *A chain is held on a frictionless table with 1/n th of its length hanging over the edge. If the chain has a length l and a mass m, how much work is required to pull the hanging part back on the table ?* **[MNREC 90]**

Solution. Mass per unit length of the chain $= \frac{m}{l}$

Let length of the hanging part of the chain $= y$

Mass of the hanging part of the chain $= \frac{m}{l} \cdot y$

Force required to be applied = Weight of the hanging part

or $\quad F = \left[\frac{m}{l} y\right] g = \frac{mg}{l} \cdot y$

The work done in pulling the chain through a small distance dy is

$$dW = -\frac{mg}{l} \cdot y\, dy$$

Here negative sign indicates that the weight and displacement are oppositely directed. Total work done in pulling the $1/n$ th length of the chain is

$$W = \int dW = -\frac{mg}{l} \int_{y = \frac{l}{n}}^{y=0} y\, dy$$

$$= -\frac{mg}{l} \left[\frac{y^2}{2}\right]_{l/n}^{0}$$

$$= -\frac{mg}{2l} \left[0 - \frac{l^2}{n^2}\right] = \frac{mgl}{2n^2}.$$

Problem 7. *A person decides to use his bath tub water to generate electric power to run a 40 W bulb. The bath tub is located at a height of 10 m from the ground and it holds 200 litres of water. He instals a water driven wheel generator on the ground. At what rate should the water drain from the bath tub to light the bulb ? How long can he keep the bulb on, if bath tub was full initially ? Efficiency of generator = 90%. Take $g = 9.8\, ms^{-2}$.* **[Roorkee 90]**

Solution. Let V be the volume of water flowing out per second from the tub which is kept at height h from the ground. Then,

Input power $= (V\rho)\, gh$ [$\because$ $V\rho$ = mass of water flowing per second]

Output power = 40 W

As efficiency,

$$\eta = \frac{\text{Output power}}{\text{Input power}}$$

$$\therefore \quad \frac{90}{100} = \frac{40}{V\rho gh}$$

or
$$V = \frac{40 \times 100}{90\, \rho\, gh} = \frac{40 \times 100}{90 \times 1000 \times 9.8 \times 10}$$

$$= 0.453 \times 10^{-3}\ m^3/s = 0.453 \text{ litre/s}$$

$[\because\ 1 \text{ litre} = 10^{-3}\ m^3]$

Time for which the bulb can be kept on

$$= \frac{200 \text{ litre}}{0.453 \text{ litre/s}} = \mathbf{441\ s.}$$

Problem 8. *A bullet of mass 0.01 kg and travelling at a speed of 500 ms^{-1} strikes a block of mass 2 kg which is suspended by a string of length 5 m. The centre of gravity of the block is found to rise a vertical distance of 0.1 m. What is the speed of the bullet after it emerges from the block ? Take $g = 9.8\ ms^{-2}$.* **[Roorkee 90]**

Solution. In Fig. 6.44, $m = 0.01$ kg, $v = 500\ ms^{-1}$,

$M = 2$ kg, $l = 5$ m, $h = 0.1$ m

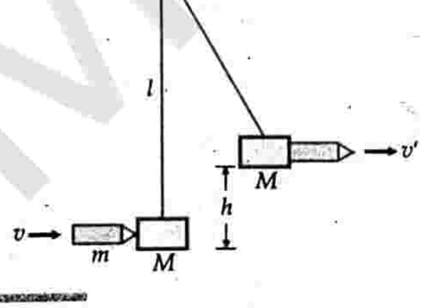


Fig. 6.44

Let V be the velocity gained by the block of mass M. Then by conservation of energy,

K.E. of the block = P.E. of the block

or
$$\frac{1}{2} MV^2 = Mgh$$

or
$$V = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 0.1}$$

$$= 1.4\ ms^{-1}$$

Let v' be the speed of the bullet after it emerges from the block. Then by conservation of momentum, we have

$$mv + M \times 0 = MV + mv'$$

or
$$0.01 \times 500 + 0 = 2 \times 1.4 + 0.01 \times v'$$

or
$$v' = \mathbf{220\ ms^{-1}}.$$

Problem 9. *A pendulum bob of mass 10^{-2} kg is raised to a height of 5×10^{-2} m and then released. At the bottom of its swing, it picks up a mass of 10^{-3} kg. To what height will the combined mass rise ? Take $g = 10\ ms^{-2}$.* **[Roorkee 95]**

Solution. By conservation of energy,

P.E. of the bob at height h

= K.E. of bob at the bottom

$$Mgh = \frac{1}{2} Mv^2$$

or
$$v = \sqrt{2gh} = \sqrt{2 \times 10 \times 5 \times 10^{-2}}$$

$$= 1.0\ ms^{-1}$$

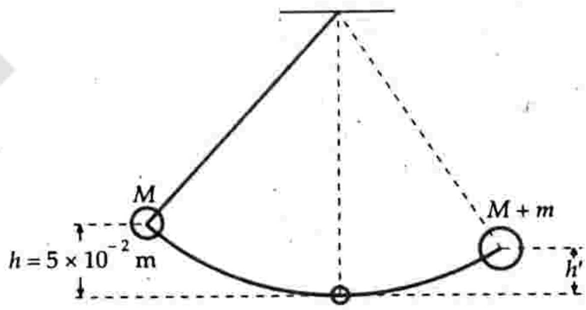


Fig. 6.45

When the bob picks up a mass $m = 10^{-3}$ kg, suppose its velocity changes to v'. By conservation of momentum, we have

$$Mv = (M + m)\, v'$$

$$v' = \frac{Mv}{M + m} = \frac{0.01 \times 1.0}{0.01 + 0.001} = 0.91\ ms^{-1}$$

Let the combined mass moving with velocity v' rise to height h'. Then

$$\frac{1}{2}(M + m)\, v'^2 = (M + m)\, gh'$$

or
$$h' = \frac{1}{2}\frac{v'^2}{g} = \frac{1}{2} \times \frac{(0.91)^2}{10} = \mathbf{0.0414\ m.}$$

Problem 10. *A particle of mass 1 g moving with a velocity $\vec{v}_1 = 3\hat{i} - 2\hat{j}$ experiences a perfectly elastic collision with another particle of mass 2 g and velocity $\vec{v}_2 = 4\hat{j} - 6\hat{k}$. Find the velocity of the particle formed (both $\vec{v}$ and v). ($\vec{v}_1$ and $\vec{v}_2$ are given in SI units).* **[Delhi 06]**

Solution. By conservation of momentum,

$$m_1\vec{v}_1 + m_2\vec{v}_2 = (m_1 + m_2)\vec{v}$$

or $$1(3\hat{i} - 2\hat{j}) + 2(4\hat{j} - 6\hat{k}) = 3\vec{v}$$

or $$3\hat{i} + 6\hat{j} - 12\hat{k} = 3\vec{v}$$

or $$\vec{v} = \hat{i} + 2\hat{j} - 4\hat{k}$$

and $$v = |\vec{v}| = \sqrt{1 + 4 + 16} = \mathbf{4.6\ ms^{-1}}.$$

Problem 11. *A ball, whose kinetic energy is E, is projected at an angle of 45° to the horizontal. What will be the kinetic energy of the ball at the highest point of its flight?* **[AIEEE 02]**

Solution. Suppose the ball is projected with velocity u. Then

$$E = \frac{1}{2}mu^2$$

At the highest point, the velocity of the ball will be

v = Horizontal component of the velocity of projection

$= u\cos 45° = u/\sqrt{2}$

The kinetic energy of the ball at the highest point,

$$E' = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{u}{\sqrt{2}}\right)^2$$

$$= \frac{1}{2} \times \frac{1}{2}mu^2 = \frac{1}{2}E.$$

Problem 12. *A particle of mass m moves in a straight line with retardation proportional to its displacement. Find the expression for loss of kinetic energy for any displacement x.* **[AIEEE 04]**

Solution. Given $a \propto x$ or $a = -kx$ ...(1)

The negative sign is taken for retardation.

Now

$$a = \frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt} = v\frac{dv}{dx} \qquad ...(2)$$

From (1) and (2), we get

$$v\frac{dv}{dx} = -kx \quad \text{or} \quad v\,dv = -kx\,dx \qquad ...(3)$$

Suppose the velocity of the particle changes from u to v during its displacement x. Then integrating (3) within these limits of velocity and displacement, we get

$$\int_u^v v\,dv = -k\int_0^x x\,dx$$

or $$\left[\frac{v^2}{2}\right]_u^v = -k\left[\frac{x^2}{2}\right]_0^x$$

or $$\frac{1}{2}v^2 - \frac{1}{2}u^2 = -\frac{1}{2}kx^2$$

or $$\frac{1}{2}u^2 - \frac{1}{2}v^2 = \frac{1}{2}kx^2$$

∴ Loss in K.E. $= \frac{1}{2}mu^2 - \frac{1}{2}mv^2 = \mathbf{\frac{1}{2}mkx^2}.$

Problem 13. *A body of mass m accelerates uniformly from rest to velocity v_1 in time t_1. Find the expression for the instantaneous power delivered to the body as a function of time.* **[AIEEE 04, 05]**

Solution. If a is the uniform acceleration of the body, then

$$v_1 = 0 + at_1 \quad \text{or} \quad a = v_1/t_1$$

The velocity of the body at instant t,

$$v = 0 + at = \frac{v_1}{t_1} \cdot t$$

Instantaneous power,

$$P = Fv = mav = m \times \frac{v_1}{t_1} \times \frac{v_1}{t_1} \cdot t = \mathbf{\frac{mv_1^2 t}{t_1^2}}.$$

Problem 14. *Show that coefficient of restitution for one dimensional elastic collision is equal to one.* **[Chandigarh 08]**

Solution. As momentum is conserved in any collision, so

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

or $$m_1(u_1 - v_1) = m_2(v_2 - u_2) \qquad ...(i)$$

Also, kinetic energy is conerved in an elastic collision, so

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

or $$m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2) \qquad ...(ii)$$

On dividing (ii) by (i), we get

$$u_1 + v_1 = v_2 + u_2$$

or $$v_2 - v_1 = u_1 - u_2$$

Hence, the coefficient of restitution for one dimenional elastic collision will be

$$e = \frac{v_2 - v_1}{u_1 - u_2} = \mathbf{1}.$$

Guidelines to NCERT Exercises

6.1. *The sign of work done by a force on a body is important to understand. State carefully if the following quantities are positive or negative.*

(*i*) *Work done by a man in lifting a bucket out of a well by means of a rope tied to the bucket.*

(*ii*) *Work done by gravitational force in the above case.*

(*iii*) *Work done by friction on a body sliding down an inclined plane.*

(*iv*) *Work done by an applied force on a body moving on a rough horizontal plane with uniform velocity.*

(*v*) *Work done by the resistive force of air on a vibrating pendulum in bringing it to rest.*

Ans.

(*i*) Work done is *positive,* because the bucket moves in the direction of the force applied by the man.

(*ii*) Work done by gravitational force is *negative* because the bucket moves upwards while the gravitational force acts downwards.

(*iii*) Work done is *negative,* because force of friction acts on the body in the opposite direction of its motion.

(*iv*) Work done is *positive,* because the applied force acts in the direction of motion of the body.

(*v*) Work done is *negative,* because the resistive force of air acts in a direction opposite to the direction of motion of the vibrating pendulum.

6.2. *A body of mass 2 kg initially at rest moves under the action of an applied horizontal force of 7N on a table with coefficient of kinetic friction = 0.1. Compute the*

(*i*) *work done by the applied force in 10 s,*

(*ii*) *work done by the friction in 10 s,*

(*iii*) *work done by the net force on the body in 10 s, and*

(*iv*) *change in kinetic energy of the body in 10 s. Interpret your results.*

Ans. Here $m = 2\text{ kg}$, $u = 0$, $F = 7\text{ N}$,

$$\mu_k = 0.1, \quad t = 10\text{ s}$$

Force of friction,

$$f_k = \mu_k R = \mu_k mg = 0.1 \times 2 \times 9.8 = 1.96\text{ N}$$

Net force with which the body moves,

$$F' = F - f_k = 7 - 1.96 = 5.04\text{ N}$$

Acceleration,

$$a = \frac{F'}{m} = \frac{5.04}{2} = 2.52\text{ ms}^{-2}$$

Distance,

$$s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 2.52 \times (10)^2$$

$$= 126\text{ m}$$

(*i*) Work done by the applied force,

$$W_1 = Fs = 7 \times 126 = \mathbf{882\ J.}$$

(*ii*) Work done by the friction,

$$W_2 = -f_k \times s = -1.96 \times 126 = \mathbf{-246.9\ J.}$$

(*iii*) Work done by the net force,

$$W_3 = F's = 5.04 \times 126 = \mathbf{635\ J.}$$

(*iv*) Final velocity acquired by the body after 10 s,

$$v = u + at = 0 + 2.52 \times 10 = 25.2\text{ ms}^{-1}$$

Change in K.E. of the body

$$= \frac{1}{2}mv^2 - \frac{1}{2}mu^2 = \frac{1}{2} \times 2 \times (25.2)^2 - 0 = \mathbf{635\ J}$$

Thus the change in K.E. of the body is equal to the work done by the net force on the body.

6.3. *Given below (Fig. 6.46) are examples of some potential energy functions in one dimension. The total energy of the particle is indicated by a cross on the ordinate axis. In each case, specify the regions, if any, in which the particle cannot be found*

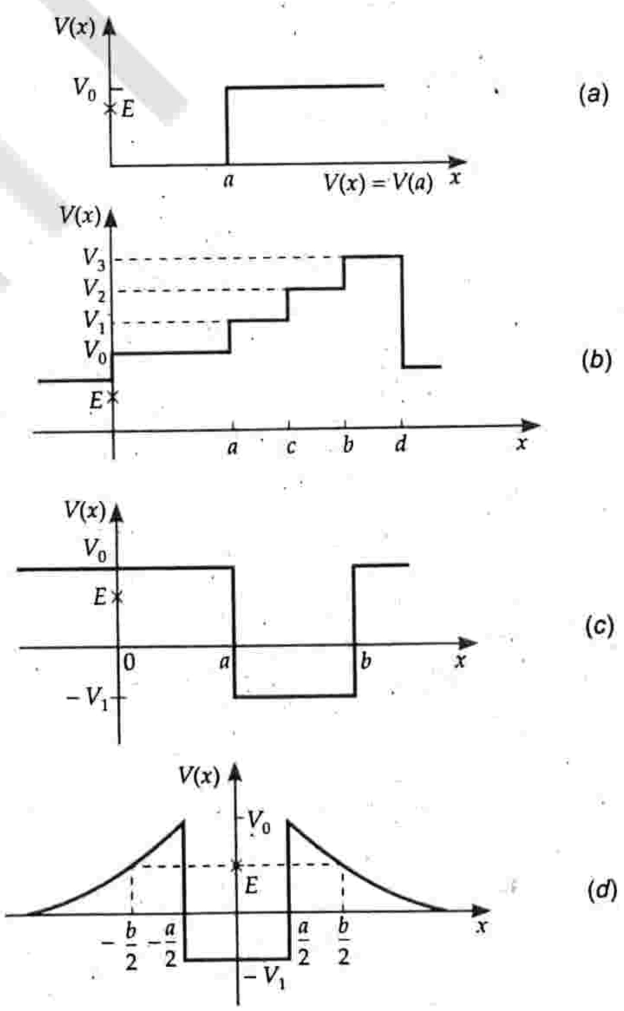


Fig. 6.46

for the given energy. Also, indicate the minimum total energy the particle must have in each case. Think of simple physical contexts for which these potential energy shapes are relevant.

Ans. Total energy, E = K.E. + P.E.

$\therefore$ K.E. = E – P.E.

The particle can exist in such a region in which its K.E. is positive.

(*a*) For $x > a$, P.E. $(V_0) > E$

$\therefore$ K.E. is negative. The particle cannot exist in the region $x > a$. Here $E_{min} = 0$.

(*b*) In every region of the graph, P.E. $(V) > E$.

$\therefore$ K.E. is negative. The particle cannot be found in any region. Here $E_{min} = -V_1$.

(*c*) For $x < a$ and $x > b$, P.E. $(V_0) > E$

$\therefore$ K.E. is negative. The particle cannot be found in the region $x < a$ and $x > b$. Here $E_{min} = -V_1$.

(*d*) For $-\frac{b}{2} < x < -\frac{a}{2}$ and $\frac{a}{2} < x < \frac{b}{2}$, P.E. $(V) > E$.

$\therefore$ K.E. is negative. The particle cannot be present in these regions. Here $E_{min} = -V_1$.

6.4. *The potential energy function for a particle executing linear simple harmonic motion is given by $V(x) = kx^2/2$, where k is the force constant of the oscillator. For $k = 0.5\ Nm^{-1}$, the graph of $V(x)$ versus x is shown in Fig. 6.47. Show that a particle of total energy 1 J moving under this potential must "turn back" when it reaches $x = \pm 2$ m.*

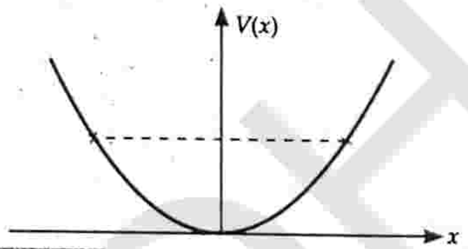


Fig. 6.47

Ans. At any instant, the energy of the oscillator is partly kinetic and partly potential. Its total energy is

$$E = K + V$$

or $$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

An oscillating particle turns back at the point where its instantaneous velocity is zero *i.e.*, the particle will turn back at such a point x where $v = 0$.

$\therefore$ $$E = 0 + \frac{1}{2}kx^2$$

But $E = 1$ J, $k = 0.5\ \text{Nm}^{-1}$

$\therefore$ $$1 = \frac{1}{2} \times 0.5 \times x^2 \quad \text{or} \quad x^2 = 4$$

or $$x = \pm 2 \text{ m.}$$

6.5. *Answer the following :*

(*a*) *The casing of a rocket in flight burns up due to friction. At whose expense is the heat energy required for burning obtained ? The rocket or the atmosphere or both ?* **[Delhi 12]**

(*b*) *Comets move around the sun in highly elliptical orbits. The gravitational force on the comet due to the sun is not normal to the comet's velocity in general. Yet the work done by the gravitational force over every complete orbit of the comet is zero. Why ?*

(*c*) *An artificial satellite orbiting the earth in very thin atmosphere loses its energy gradually due to dissipation against atmospheric resistance, however small. Why then does its speed increase progressively as it comes closer and closer to the earth ?*

(*d*) *In Fig. 6.48 (i) the man walks 2 m carrying a mass of 15 kg on his hands. In Fig. 6.48 (ii), he walks the same distance pulling the rope behind him. The rope goes over a pulley, and a mass of 15 kg hangs at its other end. In which case is the work done greater ?*

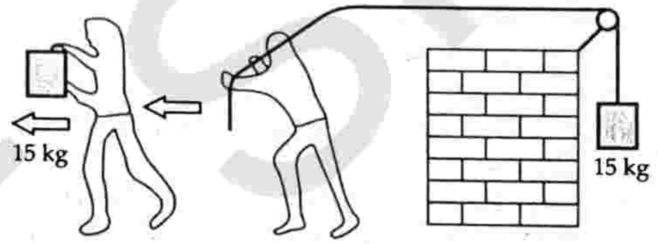


Fig. 6.48

Ans. (*a*) Heat energy required for the burning of the casing of a rocket in flight is obtained from the rocket itself. It is obtained at the expense of the mass of the rocket and its kinetic and potential energies.

(*b*) The gravitational force acting on the comet is a conservative force. The work done by a conservative over any path is equal to the negative of the change in P.E. Over a complete orbit of any shape, there is no change in P.E. of the comet. Hence no work is done by the gravitational force on the comet.

(*c*) As the satellite comes closer to the earth, its potential energy decreases. As the sum of kinetic and potential energy remains constant, the kinetic energy and velocity of the satellite increase. But the total energy of the satellite goes on decreasing due to the loss of energy against friction.

(*d*) In **case (*i*)**, no work is done against gravity because the displacement of 2 m (horizontal) and the weight of 15 kg (acting vertically downwards) are perpendicular to each other. Work is done only against friction.

In **case (*ii*)**, work has to be done against gravity ($= mg\,h = 15 \times 9.8 \times 2 = 294$ J) in addition to the work to be done against friction while moving a distance of 2 m. Thus the *work done in case (ii) is greater* than that in case (*i*).

6.6. *Underline the correct alternative :*

(*a*) *When a conservative force does positive work on a body, the potential energy of the body increases / decreases / remains unaltered.*

(b) *Work done by a body against friction always results in a loss of its kinetic/potential energy.*

(c) *The rate of change of total momentum of a many-particle system is proportional to the external force/sum of the internal forces on the system.*

(d) *In an inelastic collision of two bodies, the quantities which do not change after the collision are the total kinetic energy/total linear momentum/total energy of the system of two bodies.* **[Central Schools 08]**

Ans.

(a) The work done by a conservative force is equal to the negative of the potential energy. *When the work done is positive, the potential energy decreases.*

(b) Friction always opposes motion. A body does work against friction at the expense of its kinetic energy. *Work done by a body against friction results in a loss of its kinetic energy.*

(c) Internal forces in a many-particle system cancel out in pairs and so they cannot change the net momentum of the system. Only the external forces can produce change in momentum. *The rate of change of momentum of a many-particle system is proportional to the external force on the system.*

(d) In an elastic collision, the kinetic energy of the system decreases after the collision but the total energy of the system and its total linear momentum do not change after the inelastic collision.

6.7. *State if each of the following statements is true or false. Give reasons for your answer.*

(a) *In an elastic collision of two bodies, the momentum and energy of each body is conserved.*

(b) *Total energy of a system is always conserved, no matter what internal and external forces on the body are present.*

(c) *Work done in the motion of a body over a closed loop is zero for every force in nature.*

(d) *In an inelastic collision, the final kinetic energy is always less than the initial kinetic energy of the system.*

Ans.

(a) *False.* Total momentum and total energy of the entire system are conserved and not of individual bodies.

(b) *False.* The external forces acting on a body may change its energy.

(c) *False.* In case of a non-conservative force like friction, the work in the motion of a body over a closed loop is not zero.

(d) *True.* In an elastic collision, a part of the initial K.E. of the system always changes into some other form of energy.

6.8. *Answer carefully, with reasons :*

(a) *In an elastic collision of two billiard balls, is the total kinetic energy conserved during the short time of collision of the balls (i.e., when they are in contact) ?* **[Delhi 12]**

(b) *Is the total linear momentum conserved during the short time of an elastic collision of two balls ?*

(c) *What are the answers to (a) and (b) for an inelastic collision ?*

(d) *If the potential energy of two billiard balls depends only on the separation distance between their centres, is the collision elastic or inelastic ? (Note, we are talking here of potential energy corresponding to the force during collision, not gravitational potential energy.)*

Ans.

(a) During the short time of collision when the balls are in contact, the kinetic energy of the balls gets converted into potential energy. In an elastic collision, though the kinetic energy before collision is equal to the kinetic energy after the collision but kinetic energy is not conserved during the short time of collision.

(b) Yes, the total linear momentum is conserved during the short time of an elastic collision of two balls.

(c) In an inelastic collision, the total K.E. is not conserved during collision as well as even after the collision. But the total linear momentum of the two balls is conserved.

(d) The collision is elastic because the forces involved are conservative.

6.9. *A body is initially at rest. It undergoes one-dimensional motion with constant acceleration. The power delivered to it at time t is proportional to*

(i) $t^{1/2}$ (ii) t (iii) $t^{3/2}$ (iv) t^2

Ans. Instantaneous velocity, $v = 0 + at = at$

Power, $P = Fv = mav = ma \times at = ma^2 t$

As m and a are constant, so $P \propto t$

$\therefore$ *Alternative (ii) is correct.*

6.10. *A body is moving unidirectionally under the influence of a source of constant power. Its displacement in time t is proportional to*

(i) $t^{1/2}$ (ii) t (iii) $t^{3/2}$ (iv) t^2

Ans. By work-energy theorem,

$$W = P \times t = \frac{1}{2} mv^2$$

or
$$v^2 = \frac{2\,Pt}{m}$$

$\therefore$
$$v = \frac{ds}{dt} = \left(\frac{2\,Pt}{m}\right)^{1/2}$$

On integration,

$$s = \left(\frac{2P}{m}\right)^{1/2} \frac{2}{3} t^{3/2}$$

$\therefore$
$$s \propto t^{3/2}$$

Hence, *alternative (iii) is correct.*

6.11. *A body constrained to move along the Z-axis of a co-ordinate system is subject to a constant force $\vec{F} = -\hat{i} + 2\hat{j} + 3\hat{k}$ N, where $\hat{i}, \hat{j}, \hat{k}$ are unit vectors along the X-, Y-, and Z-axis of the system respectively. What is the work done by this force in moving the body a distance of 4 m along the Z-axis ?*

Ans. Here, $\vec{F} = -\hat{i} + 2\hat{j} + 3\hat{k}$ N

As the body moves a distance of 4 m along Z-axis, so

$\vec{s} = 4\hat{k}$ m.

$$\therefore \quad W = \vec{F} \cdot \vec{s} = (-\hat{i} + 2\hat{j} + 3\hat{k}) \cdot (4\hat{k})$$
$$= (-\hat{i} + 2\hat{j} + 3\hat{k}) \cdot (0\hat{i} + 0\hat{j} + 4\hat{k})$$
$$= -1 \times 0 + 2 \times 0 + 3 \times 4 = \mathbf{12\ J.}$$

6.12. *An electron and a proton are detected in a cosmic ray experiment, the first with kinetic energy 10 keV, and the second with 100 keV. Which is faster, the electron or the proton ? Obtain the ratio of their speeds.*
(Electron mass $= 9.11 \times 10^{-31}$ kg,
proton mass $= 1.67 \times 10^{-27}$ kg, 1 eV $= 1.60 \times 10^{-19}$ J).
[Delhi 08]

Ans. K.E. of the electron $= \frac{1}{2} m_e v_e^2 = 10$ keV

K.E. of the proton $= \frac{1}{2} m_p v_p^2 = 100$ keV

$$\therefore \quad \frac{\frac{1}{2} m_e v_e^2}{\frac{1}{2} m_p v_p^2} = \frac{10 \text{ keV}}{100 \text{ keV}} = \frac{1}{10}$$

or $$\frac{9.11 \times 10^{-31} \times v_e^2}{1.67 \times 10^{-27} \times v_p^2} = \frac{1}{10}$$

or $$\frac{v_e^2}{v_p^2} = \frac{1670}{9.11} = 183.3$$

or $$\frac{v_e}{v_p} = \mathbf{13.53}$$

Thus the electron moves faster than the proton.

6.13. *A rain drop of radius 2 mm falls from a height of 500 m above the ground. It falls with decreasing acceleration (due to viscous resistance of the air) until at half its original height, it attains its maximum (terminal) speed and moves with uniform speed thereafter. What is the work done by the gravitational force on the drop in the first and second half of its journey ? What is the work done by the resistive force in the entire journey if its speed on reaching the ground is 10 ms^{-1} ?*

Ans. Whether the rain drop falls with decreasing acceleration or with uniform speed, the work done by gravitational force on the drop remains same.

Here $r = 2$ mm $= 2 \times 10^{-3}$ m

Distance moved in each half journey,

$$h = \frac{500}{2} = 250 \text{ m}$$

Density of water,

$$\rho = 10^3 \text{ kgm}^{-3}$$

Mass of rain drop,

$$m = \text{Volume} \times \text{density} = \frac{4}{3}\pi r^3 \rho$$
$$= \frac{4}{3}\pi \times (2 \times 10^{-3})^3 \times 10^3 = \frac{32\pi}{3} \times 10^{-6} \text{ kg}$$

Work done by the gravitational force on the rain drop in each journey,

$$W = F \times s = mg \times h$$
$$= \frac{32\pi}{3} \times 10^{-6} \times 9.8 \times 250 = \mathbf{0.082\ J.}$$

For entire journey,

Work done by gravitational force + Work done by resistive force = Gain in K.E.

$$2 \times 0.082 + W_r = \frac{1}{2} mv^2$$

or $$W_r = \frac{1}{2} \times \frac{32\pi \times 10^{-6} \times (10)^2}{3} - 0.164$$
$$= 0.0017 - 0.164 = \mathbf{-0.1623\ J.}$$

6.14. *A molecule in a gas container hits a horizontal wall with speed 200 ms^{-1} and angle 30° with the normal, and rebounds with the same speed. Is momentum conserved in the collision ? Is the collision elastic or inelastic ?*

Ans. Momentum is always conserved, whether the collision is elastic or inelastic.

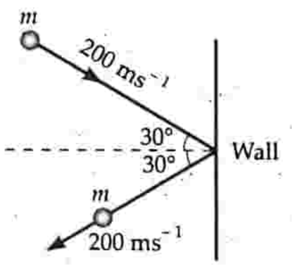


Fig. 6.49

As the wall is heavy, the molecule rebounding with its own speed does not produce any velocity in the wall. Let m be the mass of the molecule and M that of the wall.

K.E. before collision,

$$K_i = \frac{1}{2} m (200)^2 + \frac{1}{2} M (0)^2 = 2 \times 10^4 \text{ m}$$

K.E. after collision,

$$K_f = \frac{1}{2} m (200)^2 + \frac{1}{2} M (0)^2 = 2 \times 10^4 \text{ m}$$

$$\therefore \quad K_f = K_i$$

As the K.E. is conserved, the collision is elastic.

6.15. *A pump on the ground floor of a building can pump up water to fill a tank of volume 30 m^3 in 15 min. If the tank is 40 m above the ground, and the efficiency of the pump is 30%, how much electric power is consumed by the pump ?*

Ans. Mass of water = Volume × density

$= 30 \times 1000 = 3 \times 10^4$ kg

$\therefore$ Output power $= \dfrac{\text{Work done}}{\text{Time}} = \dfrac{mgh}{t}$

$= \dfrac{3 \times 10^4 \times 9.8 \times 40}{15 \times 60} = \dfrac{39200}{3}$ W

As Efficiency $= \dfrac{\text{Output power}}{\text{Input power}} \times 100$

$\therefore$ Input power $= \dfrac{\text{Output power}}{\text{Efficiency}} \times 100$

$= \dfrac{39200}{3 \times 30} \times 100 = 43.6 \times 10^3$ W

$=$ **43.6 kW.**

6.16. *Two identical ball bearings in contact with each other and resting on a frictionless table are hit head-on by another ball bearing of the same mass moving initially with a speed v. If the collision is elastic, which of the situations shown in Fig 6.50, is a possible result after collision ?*

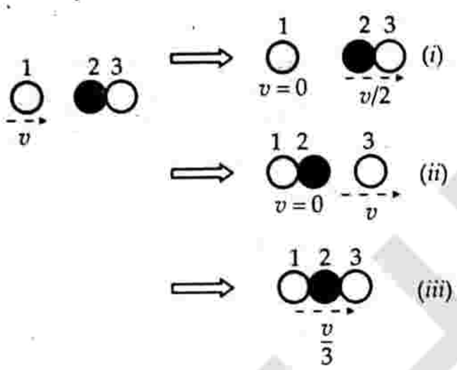


Fig. 6.50

Ans. The system consists of three identical ball bearings marked as 1, 2 and 3. Let m be the mass of each ball bearing.

Total kinetic energy of the system before collision

$$= \frac{1}{2}mv^2 + 0 + 0 = \frac{1}{2}mv^2$$

Case (*i*) K.E. of the system after collision

$$= 0 + \frac{1}{2}(2m)\left(\frac{v}{2}\right)^2 = \frac{1}{4}mv^2$$

Case (*ii*) K.E. of the system after collision

$$= 0 + \frac{1}{2}mv^2 = \frac{1}{2}mv^2$$

Case (*iii*) K.E. of the system after collision

$$= \frac{1}{2}(3m)\left(\frac{v}{3}\right)^2 = \left(\frac{1}{6}\right)mv^2$$

Because in an elastic collision, the kinetic energy of the system remains unchanged. Hence, *case (ii) is the only possible result of the collision.*

6.17. *The bob A of a pendulum released from 30° to the vertical hits another bob B of the same mass at rest on a table as shown in Fig. 6.51. How high does the bob A rise after the collision ? Neglect the size of the bobs and assume the collision to be elastic.*

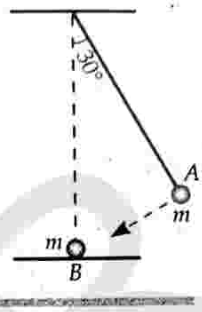


Fig. 6.51

Ans. When the bob A hits bob B on the table, it transfers its entire K.E. to the bob B because the collision is elastic. The bob A comes to rest at the location of B while the bob B begins to move with the velocity of A.

6.18. *The bob of a pendulum is released from a horizontal position A as shown. If the length of the pendulum is 1.5 m, what is the speed with which the bob arrives at the lowermost point B, given that it dissipates 5% of its initial energy against air resistance ?*

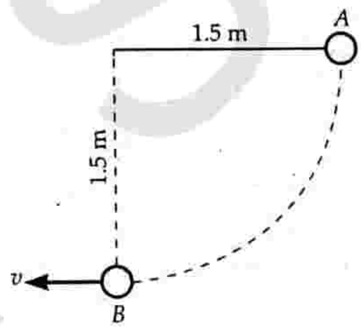


Fig. 6.52

Ans. Here $h = 1.5$ m, $v = ?$

P.E. of the bob at $A = mgh$

K.E. of the bob at $B = \frac{1}{2}mv^2$

As 5% of the P.E. is dissipated against air resistance, so

$$\frac{1}{2}mv^2 = 95\% \text{ of } mgh$$

or $$\frac{1}{2}mv^2 = \frac{95}{100} \times mgh$$

or $$v = \sqrt{\frac{2 \times 95 \times gh}{100}} = \sqrt{\frac{2 \times 95 \times 9.8 \times 1.5}{100}}$$

$$= \sqrt{27.93} = \mathbf{5.3\ ms^{-1}}.$$

6.19. *A trolley of mass 300 kg carrying a sandbag of 25 kg is moving uniformly with a speed of 27 km/h on a frictionless track. After a while, sand starts leaking out of a hole on the trolley's floor at the rate of 0.05 kg s^{-1}. What is the speed of the trolley after the entire sand bag is empty ?*

Ans. As the trolley carrying the sandbag is moving with uniform speed of 27 km/h, so no external force is acting on the trolley + sandbag system. When the sand leaks out, it does not cause any external force to act on the system. Hence the speed of the trolley remains unchanged even after the sandbag becomes empty.

6.20. *A particle of mass 0.5 kg travels in a straight line with velocity $v = ax^{3/2}$ where $a = 5\ m^{-1/2}\ s^{-1}$. What is the work done by the net force during its displacement from $x = 0$ to $x = 2\ m$.*

Ans. Velocity, $v = ax^{3/2}$

$$\text{Acceleration} = \frac{dv}{dt} = \frac{3}{2}ax^{1/2}\frac{dx}{dt} = \frac{3}{2}ax^{1/2}.v$$

$$= \frac{3}{2}ax^{1/2}.ax^{3/2} = \frac{3}{2}a^2x^2.$$

Force, $F = m \times \text{acceleration} = \frac{3}{2}ma^2x^2$

Work done,

$$W = \int_0^2 F\,dx = \frac{3}{2}\int_0^2 ma^2x^2\,dx = \frac{3}{2}ma^2\left[\frac{x^3}{3}\right]_0^2$$

$$= \frac{3 \times 0.5 \times (5)^2}{2 \times 3}[2^3 - 0^3] = \mathbf{50\ J}.$$

6.21. *The blades of a windmill sweep out a circle of area A. (a) If the wind flows at a velocity v perpendicular to the circle, what is the mass of the air passing through it in time t ? (b) What is the kinetic energy of the air ? (c) Assume that the windmill converts 25% of the wind's energy into electrical energy, and that $A = 30\ m^2$, $v = 36\ km/h$ and the density of air is $1.2\ kg\ m^{-3}$. What is the electrical power produced ?*

Ans. (*a*) Volume of the air passing through the windmill in time t

= Area of circle × distance covered by wind in time t

$= A \times vt = Avt$

Mass of the air passing through the windmill in time t,

$$m = \text{Density} \times \text{volume} = \rho Avt.$$

(*b*) Kinetic energy of the air is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\rho\ A\,v\,t \times v^2 = \frac{1}{2}\rho Av^3t.$$

(*c*) K.E. of air converted into electrical energy in time t

$$K' = 25\%\text{ of } K = \frac{25}{100} \times \frac{1}{2}\rho Av^3t = \frac{1}{8}\rho\,Av^3t$$

Electrical power produced

$$= \frac{K'}{t} = \frac{1}{8}\rho\,a\,v^3 = \frac{1}{8} \times 1.2 \times 30 \times (10)^3$$

$$[\because v = 36\ \text{kmh}^{-1} = 10\ \text{ms}^{-1}]$$

$= 4.5 \times 10^3\ \text{W} = \mathbf{4.5\ kW}.$

6.22. *A person trying to lose weight (dieter) lifts a 10 kg mass 0.5 m, 1000 times. Assume that the potential energy lost each time she lowers the mass is dissipated. (a) How much work does she do against the gravitational force ? (b) Fat supplies 3.8×10^7 J of energy per kilogram which is converted to mechanical energy with a 20% efficiency rate. How much fat will the dieter use up ?*

Ans. (*a*) Here $m = 10$ kg, $h = 0.5$ m, $n = 1000$, $g = 9.8\ \text{ms}^{-2}$

Work done against the gravitational force,

$$W = n \times mgh = 1000 \times 10 \times 9.8 \times 0.5$$
$$= \mathbf{49{,}000\ J}.$$

(*b*) Mechanical energy supplied by 1 kg of fat

$$= 20\%\text{ of } 3.8 \times 10^7\ \text{J}$$

$$= \frac{20 \times 3.8 \times 10^7}{100} = 76 \times 10^5\ \text{J}$$

∴ Fat consumed for 76×10^5 J of energy = 1 kg

Fat consumed for 49,000 J of energy

$$= \frac{1 \times 49{,}000}{76 \times 10^5} = \mathbf{6.45 \times 10^{-3}\ kg}.$$

6.23. *A large family uses 8 kW of power. (a) Direct solar energy is incident on the horizontal surface at an average rate of 200 W per square meter. If 20% of this energy can be converted to useful electrical energy, how large an area is needed to supply 8 kW ? (b) Compare this area to that of the roof of a house constructed on a plot of size 20 m × 15 m with a permission to cover upto 70%.* **[Delhi 03]**

Ans. (*a*) Let the area needed to supply 8 kW = A m^2

Energy incident per unit area = 200 W

Energy incident on area $A = 200 \times A$ W

Energy converted into useful electrical energy

$$= 20\%\text{ of } 200 \times A = 40A\ \text{W}$$

But $40\ A\ W = 8\ \text{kW} = 8000$ W

or $$A = \frac{8000}{40} = \mathbf{200\ m^2}.$$

(*b*) Area of the roof of the given house,

$$A' = 70\%\text{ of } 20\ \text{m} \times 15\ \text{m}$$

$$= \frac{70 \times 20 \times 15}{100} = 210\ \text{m}^2$$

$$\text{Required ratio} = \frac{A}{A'} = \frac{200}{210} = \mathbf{20 : 21}.$$

6.24. *A bullet of mass 0.012 kg and horizontal speed $70\ ms^{-1}$ strikes a block of wood of mass 0.4 kg and instantly comes to rest with respect to the block. The block is suspended from the ceiling by means of thin wires. Calculate the height to which the block rises. Also estimate the amount of heat produced in the block.* **[Chandigarh 07]**

Ans. Mass of bullet, $m = 0.012$ kg

Speed of bullet, $v = 70\ \text{ms}^{-1}$

Mass of block, $M = 0.4$ kg

If V is the velocity of the combination after collision, then from the law of conservation of momentum,

$$mv = (M + m)\,V$$

or $$V = \frac{mv}{M + m} = \frac{0.012 \times 70}{0.4 + 0.012}$$

$$= \frac{0.84}{0.412} = 2.04\ \text{ms}^{-1}$$

Let h be the height through which the block rises. Then from the conservation of energy,

P.E. of the combination = K.E. of the combination

$$(M+m)gh = \frac{1}{2}(M+m)V^2$$

or
$$h = \frac{V^2}{2g} = \frac{(2.04)^2}{2 \times 9.8} = 0.212 \text{ m}$$

Amount of heat produced

= Loss in K.E. of the bullet

= Initial K.E. of the bullet

– K.E. of the combination

$$= \frac{1}{2}mv^2 - \frac{1}{2}(M+m)V^2$$

$$= \frac{1}{2} \times 0.012 \times (70)^2 - \frac{1}{2} \times 0.412 \times (2.04)^2$$

$= 29.4 - 0.86 =$ **28.54 J.**

6.25. *Two inclined frictionless tracks, one gradual and the other steep meet at A from where two stones are allowed to slide down from rest, one on each track (Fig. 6.53). Will the stones reach the bottom at the same time ? Will they reach there with the same speed ? Explain. Given $\theta_1 = 30°$, $\theta_2 = 60°$ and $h = 10$ m, what are the speeds and times taken by the two stones ?*

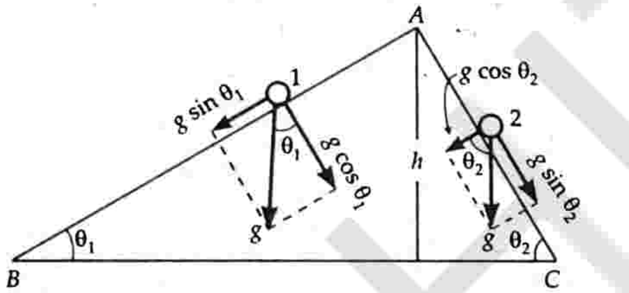


Fig. 6.53

Ans. Let a_1 be the acceleration of the stone 1 down the inclined track AB. Then

$$a_1 = g \sin \theta_1.$$

If the stone 1 takes time t_1 to slide down the track AB, then

$$AB = 0 + \frac{1}{2}a_1 t_1^2 \qquad \left[s = ut + \frac{1}{2}at^2\right]$$

$$\frac{h}{\sin \theta_1} = \frac{1}{2} g \sin \theta_1 \, t_1^2$$

or
$$t_1^2 = \frac{2h}{g \sin^2 \theta_1}$$

or
$$t_1 = \frac{1}{\sin \theta_1}\sqrt{\frac{2h}{g}}$$

Similarly, for stone 2, we can write

$$t_2 = \frac{1}{\sin \theta_2}\sqrt{\frac{2h}{g}}$$

For both the stones, h is same.

As $\theta_1 < \theta_2 \quad \therefore \sin \theta_1 < \sin \theta_2$

Consequently, $t_1 > t_2$

Thus, the stone 2 on the steeper plane AC reaches the bottom earlier than stone 1.

As both the stones are initially at the same height h, so

P.E. at A = K.E. at B or C

$$mgh = \frac{1}{2}mv^2$$

$$v = \sqrt{2gh}$$

i.e., both the stones will reach the bottom with the same speed.

Given : $\theta_1 = 30°$, $\theta_2 = 60°$, $h = 10$ m, $g = 10$ ms^{-2}

$$\therefore \quad t_1 = \frac{1}{\sin \theta_1}\sqrt{\frac{2h}{g}}$$

$$= \frac{1}{\sin 30°}\sqrt{\frac{2 \times 10}{10}} = 2\sqrt{2} \text{ s.}$$

$$t_2 = \frac{1}{\sin \theta_2}\sqrt{\frac{2h}{g}} = \frac{1}{\sin 60°}\sqrt{\frac{2 \times 10}{10}} = 2\sqrt{\frac{2}{3}} \text{ s.}$$

$$v = \sqrt{2gh} = \sqrt{2 \times 10 \times 10} = \sqrt{2} \times 10$$

$= 1.414 \times 10 =$ **14.14 ms^{-1}.**

6.26. *A 1 kg block situated on a rough incline is connected to a spring of spring constant 100 Nm^{-1} as shown in Fig. 6.54(a). The block is released from rest with the spring in the unstretched position. The block moves 10 cm down the incline before coming to rest. Find the coefficient of friction between the block and the incline. Assume that spring has negligible mass and the pulley is frictionless.*

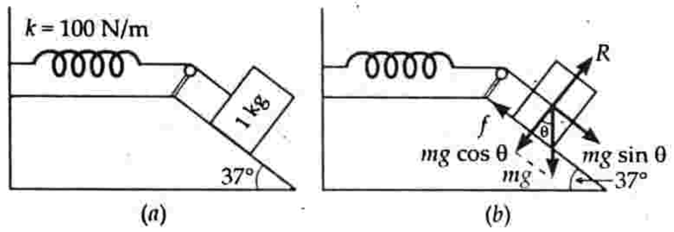


Fig. 6.54

Ans. Here $m = 1$ kg, $k = 100$ Nm^{-1}, $g = 10$ ms^{-2}

Clearly, from Fig. 6.54(b), we have

$$R = mg \cos \theta \; ; \quad f = \mu R = \mu \, mg \cos \theta$$

Net force on the block down the incline

$$= mg \sin \theta - f = mg \sin \theta - \mu \, mg \cos \theta$$

$$= mg(\sin \theta - \mu \cos \theta)$$

Distance moved, $x = 10$ cm $= 0.1$ m

In equilibrium

Work done = P.E. of stretched spring

$$mg(\sin \theta - \mu \cos \theta)x = \frac{1}{2}kx^2$$

or $\quad 2\,mg(\sin\theta - \mu\cos\theta) = kx$

or $\quad 2 \times 1 \times 10(\sin 37^\circ - \mu\cos 37^\circ) = 100 \times 0.1$

or $\quad 20(0.601 - \mu \times 0.798) = 10$

$\therefore \quad \mu = \mathbf{0.126.}$

6.27. *A bolt of mass 0.3 kg falls from the ceiling of an elevator moving down with a uniform speed of 7 ms^{-1}. It hits the floor of the elevator (length of the elevator 3 m) and does not rebound. What is the heat produced by the impact ? Would your answer be different, if the elevator were stationary ?*

Ans. As the elevator is moving down with a uniform speed ($a = 0$), so the value of g remains the same.

Here $m = 0.3$ kg, $h = 3$ m, $g = 9.8\ \text{ms}^{-2}$

P.E. lost by the bolt $= mgh = 0.3 \times 9.8 \times 3 = 8.82$ J

As the bolt does not rebound, the energy is converted into heat.

$\therefore$ Heat produced = **8.82 J**

Even if the elevator were stationary, the same amount of heat would have produced because the value of g is same in all inertial frames of reference.

6.28. *A trolley of mass 200 kg moves with a uniform speed of 36 km/h on a frictionless track. A child of mass 20 kg runs on the trolley from one end to the other (10 m away) with a speed of 4 m s^{-1} relative to the trolley in a direction opposite to the trolley's motion, and jumps out of the trolley. What is the final speed of the trolley ? How much has the trolley moved from the time the child begins to run ?*

Ans. The child gives an impulse to the trolley at the start and then runs with a constant relative velocity of 4 ms^{-1} with respect to the trolley's new velocity.

Total initial momentum,

$$p_i = (m_1 + m_2)\,u_1 = (20 + 200) \times \frac{36 \times 5}{18} = 2200\ \text{kg ms}^{-1}$$

Let new velocity of the trolley $= v_2$

Child's velocity relative to the trolley in opposite direction = 4 ms^{-1}

$\therefore$ Child's actual velocity (relative to ground)

$$= v_2 - 4$$

Total final momentum,

$$p_f = m_1 v_1 + m_2 v_2 = 20(v_2 - 4) + 200\,v_2 = 220\,v_2 - 80$$

By conservation of linear momentum,

$$p_f = p_i$$

$$220\,v_2 - 80 = 2200$$

$$\therefore \quad v_2 = \frac{2280}{220} = \mathbf{10.36\ ms^{-1}}$$

Time taken by the child to cover length of the trolley

$$= \frac{10\ \text{m}}{4\ \text{ms}^{-1}} = 2.5\ \text{s}$$

Distance covered by the trolley in 2.5 s

$$= 10.36 \times 2.5 = \mathbf{25.9\ m.}$$

6.29. *Which of the following potential energy curves in Fig. 6.55 cannot possibly describe the elastic collision of two billiard balls ? Here r is the distance between centres of the balls.*

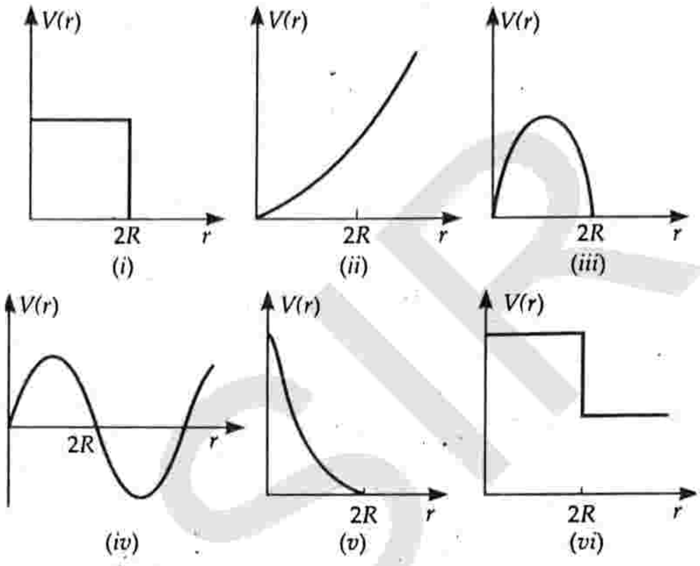


Fig. 6.55

Ans. During the short time of collision, the kinetic energy gets converted into potential energy. But the P.E. of a system of two masses varies inversely as the distance between them, *i.e.*, $V \propto 1/r$. Hence all the potential energy curves except the one shown in Fig. 6.55(v) cannot describe an elastic collision.

6.30. *Consider the decay of a free neutron at rest :*

$$n \rightarrow p + e^-$$

Show that the two-body decay of this type must necessarily give an electron of fixed energy and, therefore, cannot account for the observed continuous energy distribution in the β-decay of neutron or a nucleus (Fig. 6.56).

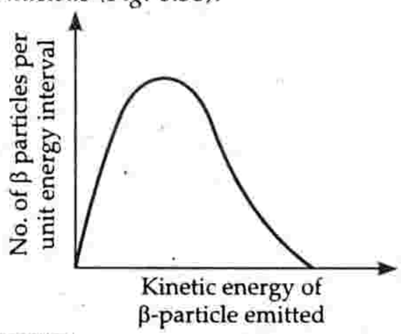


Fig. 6.56

Ans. If the decay of the neutron (inside the nucleus) into proton and electron is according to the given reaction, then the energy released in the decay must be carried by the electrons coming out of the nucleus. By mass-energy conservation, these electrons must have a definite value of energy. However, the given graph shows that the emitted electron can have any value of energy between zero and the maximum value. Hence the given decay mode cannot account for the observed continuous energy spectrum in the β-decay.

Text Based Exercises

Type A : Very Short Answer Questions — 1 Mark Each

1. What do you mean by work done by a force ? **[Manipur 99]**
2. State the factors on which the work done by a force depends.
3. What should be the angle between the directions of force and displacement for maximum and minimum work ?
4. Give one such example in which the work done by the force acting upon a body is zero, although due to the applied force the body is displaced from its initial position.
5. State the conditions under which a force does no work.
6. What is the work done by a person in holding a 15 kg suitcase, while waiting for a bus for 15 minutes ? **[Himachal 05C]**
7. A person continues to push a rock for some time but fails to move it. What is the work done by it ? Explain.
8. What is the work done by the force of tension in the string of simple pendulum ?
9. Is work done a scalar or a vector quantity ?
10. What physical quantity does the area under the force-displacement curve represent ?
11. Moment of a force and work done by a force have same units. What is the difference between them ?
12. Name and define SI unit of work. **[Himachal 03, 05, 07, 08]**
13. How many joules make up one erg ?
14. What do you mean by gravitational unit of work ?
15. State gravitational unit of work in SI.
16. Find the number of joules in the gravitational unit of work in SI.
17. A man runs a distance on a level road. The same man ascends up a hill with the same velocity through the same distance. When does he do more work ?
18. Give an example in which a force does work on a body but fails to change its kinetic energy.
19. Does the work done depend on how fast or slow a body is moved ?
20. Has an object in motion ability to do work ?
21. Does a man standing at rest on a moving car possess kinetic energy ?
22. Does a single force acting an a particle necessarily change its K.E. and momentum ?
23. Can the K.E. of a body be negative ?
24. Can the P.E. of a body be negative ?
25. Out of joule, calorie, kilowatt and electron volt, which one is not the unit of energy ?
26. Is the mechanical energy always conserved ?
27. What kind of energy is stored in the spring of a watch ? **[Himachal 05C]**
28. When an air bubble rises in water, what happens to its potential energy ?
29. What kind of energy transformations takes place at a hydro-electric power house ?
30. A spring is cut into two halves. How is the spring constant of each half affected ?
31. Define power and its SI unit. **[Manipur 99 ; Delhi 09 ; Kerala 01]**
32. Is the unit watt second associated with energy or power ?
33. What are the dimensions of power ? Is it scalar or vector ? **[Delhi 05]**
34. How many watts are there in one horse power ? **[Himachal 05C, 08]**
35. Name the physical quantity which is expressed as force times velocity. Is it a scalar or vector quantity ? **[Delhi 98]**
36. When a planet revolves round the sun, when is its kinetic energy maximum ?
37. What is a conservative force ? **[Himachal 03]**
38. Which physical quantities are conserved in an elastic collision ?
39. What is that physical quantity which is conserved both in elastic and inelastic collisions ?
40. If two bodies stick together after collision, will the collision be elastic or inelastic ?
41. A bullet gets embedded into a block of wood, how does the loss of energy appear ?

42. Give three examples of forces which are conservative in nature. **[Himachal 05C]**
43. Friction is non-conservative force. How ?
44. Name the process in which (*i*) momentum is conserved but K.E. is not conserved and (*ii*) momentum changes but K.E. dose not change.
45. What is Einstein's energy-mass equivalence relationship ? **[Delhi 95]**
46. How much work a man will do in carrying a 10 kg load over his head horizontally on smooth surface for 10 metres ? **[Delhi 96]**
47. An aritificial satellite is at a height of 36,500 km above earth's surface. What is the work done by earth's gravitational force in keeping it in its orbit ? **[Central Schools 08]**
48. Give two examples from daily life where according to physics, work done is zero. **[Central Schools 07]**
49. What is the source of the kinetic energy of the falling rain drops ? **[Central Schools 08 ; Delhi 10]**
50. State work-energy theorem. **[Delhi 08]**
51. What is spring constant of a spring ? Give its S.I. unit. **[Himachal 05C]**
52. Calculate the number of joules in 1 kWh. **[Himachal 08C]**
53. Show that $1\,J = 10^7\,erg$. **[Himachal 08]**
54. How will the momentum of a body change if its kinetic energy is doubled ? **[Himachal 05C, 08, 08C]**

Answers

1. Work is said to be done whenever a force acts on a body and the body moves in the direction of the force.
2. The work done by a force depends upon
   (*i*) the magnitude of force,
   (*ii*) the magnitude of displacement of the body,
   (*iii*) the angle between the force and the displacement.
3. (*i*) For maximum work, $\theta = 0°$.
   (*ii*) For minimum work, $\theta = 90°$.
4. The revolution of a satellite around the earth.
5. A force does no work, when
   (*i*) the displacement is zero, or
   (*ii*) the displacement is perpendicular to the force, or
   (*iii*) the conservative force moves a body over a closed path.
6. No work is done as displacement $\vec{s} = 0$ and so $W = Fs\cos\theta = 0$.
7. The work done by the person is zero. As $W = \vec{F}\cdot\vec{s}$ and the displacement $\vec{s}$ is zero, hence the work done is also zero.
8. Zero, because the tension in the string of a simple pendulum is always perpendicular to its displacement. So the work done by the tension is zero.
9. Scalar quantity, because work can be expressed as the scalar product of force and displacement vectors.
10. The area under the force-displacement curve represents the work done by the force over the given displacement.
11. Moment of force is a vector quantity while work done is a scalar quantity.
12. The SI unit of work is joule (J). One joule of work is said to be done when a force of one newton displaces a body through a distance of one metre in its own direction.
13. As 10^7 erg = 1 joule,
    therefore, 1 erg = 10^{-7} joule.
14. Work done is said to be one gravitational unit if a force of one gravitational unit displaces a body through unit distance in its own direction.
15. In SI, the gravitational unit of work done is one kilogram metre. It is defined as the work done when a force of one kilogram weight displaces a body through a distance of one metre in its own direction.
16. $1\text{ kg m} = 1\text{ kg wt} \times 1\text{ m} = 9.81\text{ N} \times 1\text{ m} = 9.81\text{ J}$.
17. More work is done in the second case.
18. When a body is pulled on a rough horizontal surface with constant velocity, work is done by the applied force on the body but K.E. of the body remains unchanged.
19. No. The work done is independent of time in which the work is completed.
20. Yes, because an object in motion possesses kinetic energy.
21. Yes, because the man forms part of the moving car and so he possesses kinetic energy.
22. When force and displacement are perpendicular to each other, $W = Fs\cos 90° = 0$ and so K.E. remains constant. But force changes the direction of velocity, so momentum changes.

23. No. As both m and v^2 are always positive, so K.E. $\left(= \frac{1}{2} mv^2\right)$ cannot be negative.

24. Yes, the potential energy is negative when forces involved are attractive.

25. Kilowatt, which is the unit of power and not of energy.

26. No. The mechanical energy is conserved only when conservative forces are involved and that too in an isolated system.

27. Potential energy.

28. Potential energy of air bubble decreases.

29. Potential energy of water changes into K.E. and then into electrical energy.

30. Spring constant of each half becomes twice the spring constant of the original spring.

31. Power is the rate of doing work. Its SI unit is watt. If an agency does work at the rate of 1 joule per second, its power is 1 watt.

32. Watt second is the unit of energy.

33. [Power] = $[ML^2T^{-3}]$. Power is a scalar quantity.

34. 1 Horse power = 746 watt.

35. The physical quantity is power. It is a scalar quantity.

36. When the planet is nearest to the sun.

37. A force is said to be conservative, if the amount of work done in moving an object against the force depends only on the initial and final positions of the object.

38. Momentum and kinetic energy.

39. Linear momentum is conserved both in elastic and inelastic collision.

40. Inelastic collision.

41. Kinetic energy of bullet gets converted into heat and sound energy.

42. Gravitational, magnetic and electrostatic forces.

43. Work done against friction in any round path is never zero, hence friction is non-conservative force.

44. (*i*) Inelastic collision, (*ii*) Uniform circular motion.

45. $E = mc^2$.

46. Zero, because force and displacement are perpendicular to each other.

47. Zero.

48. (*i*) Work done by the earth's gravitational force in keeping the moon in its orbit is zero.
(*ii*) Work done by a man carrying a load on his head and walking along horizontal is zero.

49. Potential energy of the falling rain drops changes gradually into their kinetic energy.

50. According to work-energy theorem, the work done by the net force acting on a body is equal to the change produced in the kinetic energy of the body.

51. The spring constant of a string is the restoring force set up in the spring per unit extension. Its SI unit is Nm^{-1}.

52. $1\,kWh = 1\,kW \times 1\,h = 1000\,Js^{-1} \times 3600\,s = 3.6 \times 10^6\,J$.

53. $1\,J = 1\,N \times 1\,m = 10^5\,dyne \times 100\,cm = 10^7\,erg$.

54. $p = \sqrt{2mK}$. When the kinetic energy K is doubled, the momentum p increases by $\sqrt{2}$ times its initial value.

Type B : Short Answer Questions — 2 or 3 Marks Each

1. Define the term work. Calculate the work done by a constant force. Is work done a scalar or a vector quantity ? **[Himachal 06, 08]**

2. Give the expression for work done if angle between force $\vec{F}$ and displacement $\vec{S}$ is θ. Also find the dimensions and S.I. unit of work. **[Central Schools 07]**

3. Define the term work and derive its SI unit. Write an expression for the kinetic energy of a body of mass m moving with a uniform speed v. **[Delhi 03C]**

4. How do we calculate work done by a force ? Write any two conditions under which work done by a force is zero. **[Central Schools 03]**

5. Define the term work. Show that work done is equal to the dot product of force and displacement vectors. **[Himachal 04]**

6. What is meant by positive work, negative work and zero work ? Give one example of each. **[Himachal 04]**

7. How do you calculate the work done by a variable force ? **[Kerala 02]**

8. What is the amount of work done by
(*a*) a weight-lifter in holding a weight of 120 kg on his shoulder for 30 s, and
(*b*) a locomotive against gravity, if it is travelling on a level plane ? **[Delhi 02]**

9. Define the absolute and gravitational units of work in SI and CGS systems.

10. What are conservative and non-conservative forces ? Give one example of each. [Central Schools 05 ; Himachal 08]

11. Define kinetic energy. Derive an expression for the kinetic energy of a body moving with a uniform velocity. [Himachal 04, 07, 09C]

12. Define the term potential energy, and derive its dimensions. Write an expression for the gravitational potential energy of a body of mass m raised to a height h above the earth's surface. [Delhi 03C]

13. Derive an expression for the potential energy of an elastic stretched spring. [Himachal 05 ; Delhi 08]

14. An elastic spring of force constant k is compressed by an amount x. Show that its potential energy is $\frac{1}{2}kx^2$. [Himachal 09 ; Delhi 02 ; Central Schools 08]

15. Draw the graph of equation $F_s = -kx$, where F_s is the spring force and x is the displacement of block from equilibrium position. Using the graph, show that maximum work done by the spring at x_m is $W_s = -kx_m^2 / 2$ (k = spring constant). [Delhi 09]

16. Derive an expression for the potential energy stored in a system of a block attached to a massless spring, when the block is pulled from its equilibrium position. [Himachal 07]

17. Derive an expression for the kinetic energy of a body of mass m moving with velocity v by calculus method.

18. Define power. Prove that $P = \vec{F} \cdot \vec{v}$, where the symbols have their usual meanings. [Himachal 09C]

19. State and prove work-energy theorem. [Chandigarh 02, 09 ; Himachal 07, 08 ; Delhi 04 ; Central Schools 04, 05]

20. Derive the work-energy theorem for a variable force. [Delhi 09]

21. Define potential energy and conservative force. Write two relations between them.

22. By what factor the velocity of a body should be increased so that its kinetic energy is increased by a factor of nine ? Justify your answer. [Delhi 96]

23. Two bodies A and B weighing 5 kg and 6 kg respectively have equal momenta. Which one has more kinetic energy ? [Delhi 96]

24. Define kinetic energy. Give its units and dimensional formula. [Himachal 09C]

25. Define potential energy. Give its units and dimensional formula. [Himachal 09C]

26. What is a conservative force ? Explain its various properties. [Himachal 08C]

27. What are conservative forces ? Show that gravitational force is a conservative force. [Himachal 06, 07]

28. Work done by a force is given by $W = \vec{F} \cdot \vec{S}$ where $\vec{F}$ is the force and $\vec{S}$ is the displacement. Show that :
   (*a*) Work done is also equal to change in K.E.
   (*b*) Work done is also equal to change in potential energy using this expression.
   [Central Schools 08]

29. Draw the variation of potential energy and kinetic energy of a block attached to a spring, which obeys Hooke's law. [Central Schools 08, 09]

30. Draw a graph showing the variation potential energy and kinetic energy with respect to height of a free fall under gravitational force. [Central Schools 08, 09]

31. State and prove the principle of conservation of mechanical energy. [Central Schools 07]

32. Show that the total mechanical energy of a body falling freely under gravity is conserved. Discuss it graphically also. [Central Schools 12 ; Delhi 12]

33. A particle of mass 'm' revolves in a horizontal circle of radius r under the influence of centripetal force $-K/r^2$, where K is a constant. Find total mechanical energy of the particle. [Delhi 95, 10]

34. Define work, power and energy and give their SI units. [Himachal 01, 02, 03, 04, 05C]

35. Draw a plot of spring force versus displacement x. Hence find an expression for the P.E. of an elastic stretched spring.

36. Discuss the conservation of energy in an elastic spring. Hence write an expression for the maximum speed of a body of mass m oscillating at its one end.

37. What is meant by mass-energy equivalence ? Discuss its significance in physics. [Chandigarh 03 ; Himachal 07]

38. What is the meaning of 'Collision' in physics ? Differentiate between elastic and inelastic collision. Give one example each. [Central Schools 03 ; Himachal 07C]

39. Define the terms Elastic collision and Inelastic collision. What is the difference between an inelastic collision and a completely inelastic collision ? [Delhi 03]

40. Define inelastic collision. Write its three important characteristics. [Central Schools 12]

41. Show that in case of one dimensional elastic collision of two bodies, the relative velocity of separation after the collision is equal to the relative velocity of approach before the collision.

[Himachal 06, 09C ; Chandigarh 02 ; Delhi 05 ; Central Schools 09]

42. Prove that bodies of identical masses exchange their velocities after head-on elastic collision.

[Himachal 03, 07, 09C ; Delhi 01]

43. Define elastic and inelastic collision. A lighter body collides with a much more massive body at rest. Prove that the direction of lighter body is reversed and massive body remains at rest. **[Delhi 98]**

44. Discuss elastic collision between two bodies in two dimensions. **[Himachal 04]**

Answers

1. Refer answer to Q.1 and Q.2 on page 6.1.

2. Refer answer to Q.2 on page 6.1 and Q.4 on page 6.3.

3. Refer to points 1, 3 and 7 of Glimpses.

4. Refer answer to Q. 2 on page 6.1.

Work done is zero when $F = 0$ or $s = 0$ or $\theta = 90°$.

5. Refer answer to Q. 2 on page 6.1.

6. Refer answer to Q. 3 on page 6.2.

7. Refer answer to Q. 7 on page 6.5.

8. (*a*) Here $s = 0$, so $W = Fs\cos\theta = 0$.

(*b*) Here $\theta = 90°$, so $W = Fs\cos\theta = 0$.

9. Refer answer to Q. 5 on page 6.3.

10. Refer answer to Q. 18 on page 6.14 and Q. 20 on page 6.15.

11. Refer answer to Q. 11 on page 6.8.

12. Refer answer to Q. 15 on page 6.13 and Q. 17 on page 6.14.

13. Refer answer to Q.27 on page 6.21.

14. Refer answer to Q. 27 on page 6.21.

15. Refer answer to Q.27 on page 6.21.

16. Refer answer to Q.27 on page 6.21.

17. Refer answer to Q. 12 on page 6.8.

18. Refer answer to Q. 32, 33 on page 6.28.

19. Refer answer to Q. 13 on page 6.9.

20. Refer answer to Q.14 on page 6.9.

21. Refer answer to Q. 21 on page 6.15.

22. $\frac{1}{2} mv'^2 = 9 \times \frac{1}{2} mv^2$

$$v' = 3v$$

Thus the velocity should be increased by a factor of three.

23. $\frac{K_A}{K_B} = \frac{m_B}{m_A} = \frac{6}{5}$

Thus body A has more kinetic energy.

24. Refer to point 7 of Glimpses. Its SI unit is joule and dimensional formula is $[ML^2T^{-2}]$.

25. Refer to point 9 of Glimpses. Its SI unit is joule and dimensional formula is $[ML^2T^{-2}]$.

26. Refer answer to Q. 18 on page 6.14 and Q. 24 on page 6.16.

27. Refer answer to Q. 22 on page 6.16.

28. (*a*) Refer answer to Q.13 on page 6.9

(*b*) Refer answer to Q.17 on page 6.14.

29. See Fig. 6.28 on page 6.22.

30. See Fig. 6.20 on page 6.17.

31. Refer answer to Q. 23 on page 6.16.

32. Refer answer to Q. 25 on page 6.16.

33. Refer to the solution of Problem 4 on page 6.48.

34. Refer to points 1 and 6 of Glimpses.

35. Refer answer to Q. 27 on page 6.21.

36. Refer answer to Q. 28 on page 6.21.

37. Refer answer to Q. 30 on page 6.26.

38. Refer answer to Q. 35 on page 6.31.

39. Refer answer to Q. 35 on page 6.31.

40. Refer answer to Q. 35 on page 6.31.

41. Refer answer to Q. 37 on page 6.32.

42. Refer to solution of Example 62 on page 6.36.

43. Refer answer to Q. 37(*iii*) on page 6.33.

44. Refer answer to Q.39 on page 6.34.

Type C : Long Answer Questions **5 Marks Each**

1. Obtain mathematically and graphically the work done by a variable force. **[Himachal 06]**
2. Define potential energy and kinetic energy. Derive expressions for potential and kinetic energies. **[Himachal 02, 08]**
3. State work-energy theorem. Prove it for a variable force.
4. State the law of conservation of mechanical energy. Show that the total mechanical energy of a body falling freely under gravity is conserved. Show it graphically. **[Meghalaya 2K ; Himachal 02, 03, 09C ; Delhi 10]**
5. Show that the elastic force of a spring is a conservative force. Hence write an expression for the potential energy of an elastic stretch of spring.
6. Define elastic collision and discuss it for two bodies in one dimension. Calculate the velocities of bodies after collision. Discuss special cases also. **[Chandigarh 02 ; Himachal 01, 02, 04]**
7. A large mass 'M' moving with a velocity 'v' collides head-on with a very small mass 'm' at rest. If the collision is elastic, obtain an expression for the energy lost by the large mass M (Take $M + m \approx M$). **[Delhi 97]**

Answers

1. Refer answer to Q.7 on page 6.5.
2. Refer answer to Q. 11 on page 6.8 and Q. 17 on page 6.14.
3. Refer answer to Q. 14 on page 6.9.
4. Refer answer to Q. 25 on page 6.16.
5. Refer answer to Q. 26 on page 6.20.
6. Refer answer to Q. 37 on page 6.33.
7. Here $m_1 = M,\ u_1 = v,\ m_2 = m,\ u_2 = 0$

   Velocity of M after collision,

$$v_1 = \frac{m_1 - m_2}{m_1 + m_2} u_1 + \frac{2m_2}{m_1 + m_2} \cdot u_2$$

$$= \frac{M - m}{M + m} \cdot v + \frac{2m}{M + m} \cdot 0$$

Kinetic energy lost by mass M

$$= \frac{1}{2} Mv^2 - \frac{1}{2} M \left(\frac{M - m}{M + m} v \right)^2$$

$$= \frac{1}{2} Mv^2 \left[1 - \left(\frac{M - m}{M + m} \right)^2 \right]$$

$$= \frac{1}{2} Mv^2 \cdot \frac{4Mm}{(M + m)^2}$$

$$= \frac{1}{2} Mv^2 \cdot \frac{4Mm}{M^2} \qquad [M + m \approx M]$$

$$= \mathbf{2mv^2}.$$

Competition Section

Work, Energy and Power

GLIMPSES

1. **Work.** Work is said to be done wherever a force acts on a body and the body moves through some distance in the direction of the force. If the force $\vec{F}$ makes angle θ with the direction of displacement $\vec{s}$, then the work done is

$$W = \vec{F} \cdot \vec{s} = Fs \cos \theta$$

Work done is a scalar quantity. It can be positive or negative depending on θ is acute or obtuse. The work done by friction or viscous force on a moving body is negative.

Work done is a scalar quantity.

(*i*) If $\theta = 0°$, $W = Fs$ *i.e.*, work done is maximum.

(*ii*) If $\theta = 90°$, $W = 0$ *i.e.*, work done is zero.

2. **Work done against a variable force.** The work done by a variable force $\vec{F}$ in changing the displacement from $\vec{s_1}$ to $\vec{s_2}$ is

$$W = \int_{s_1}^{s_2} \vec{F} \cdot \vec{ds}$$

= Area under the force-displacement curve.

3. **Units of work.** (*i*) The SI unit of work is joule. One joule of work is said to be done when a force of one newton displaces a body through a distance of one metre in its own direction.

1 joule = 1 newton × 1 metre or 1 J = 1 Nm

(*ii*) The CGS unit is erg. One erg of work is said to be done if a force of one dyne displaces a body through a distance of one centimetre in its own direction.

1 erg = 1 dyne × 1 cm

1 joule = 10^7 erg

4. **Work done in terms of rectangular components.** If

$$\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$$

and $\vec{s} = s_x \hat{i} + s_y \hat{j} + s_z \hat{k}$

then $W = F_x s_x + F_y s_y + F_z s_z$

5. **Newton's third law and work done.** For two bodies, the sum of the mutual forces exerted between the two bodies is zero from Newton's third law,

$$\vec{F_{12}} + \vec{F_{21}} = \vec{0}$$

But the sum of the work done by the two forces need not always cancel, *i.e.*,

$$W_{12} + W_{21} \neq 0.$$

However, it may sometimes be true.

6. **Energy.** It is defined as the ability of a body to do work. It is measured by the amount of work that a body can do. It is a scalar quantity. Like work, SI unit of energy is joule and the CGS unit is erg.

The unit of energy used at the atomic level is electron volt (eV).

$$1\,\text{eV} = 1.6 \times 10^{-19}\ \text{J}$$

$$1\,\text{keV} = 10^3\ \text{eV} = 1.6 \times 10^{-16}\ \text{J}$$

$$1\,\text{MeV} = 10^6\ \text{eV} = 1.6 \times 10^{-13}\ \text{J}$$

7. **Kinetic energy.** It is the energy possessed by a body by virtue of its motion. The K.E. of a body of mass m moving with speed v is

$$K = \frac{1}{2} mv^2 = \frac{p^2}{2m}$$

8. **Work-energy theorem.** The work done by the net force acting on a body is equal to the change in kinetic energy of the body.

$$W = \text{Change in K.E.} = \frac{1}{2} mv^2 - \frac{1}{2} mu^2$$

The *WE* theorem may be regarded as the scalar form of Newton's second law of motion.

9. **Potential energy.** It is the energy possessed by a body by virtue of its position (in a field) or configuration (shape or size).

For a conservative force in one dimension, the potential energy functions $U(x)$ may be defined as

$$F(x) = -\frac{d\,U(x)}{d\,x}$$

or $$\Delta U = U_f - U_i = -\int_{x_i}^{x_f} F(x)\,dx$$

10. **Gravitational potential energy.** It is the energy possessed by a body by virtue of its position above the surface of the earth. The gravitational P.E. of a body of mass m at a height h above the earth's surface is $U = mgh$

11. **Potential energy of a spring.** According to Hooke's law, when a spring is stretched through distance x, the restoring force set up in the spring due to its elasticity is such that

$$F \propto x \quad \text{or} \quad F = -k\,x$$

where k is the force constant or spring constant of the spring. It is the restoring force set up in the spring per unit extension. Its unit is Nm^{-1}. The work done in stretching the spring through distance x will be

$$W = \int_0^x kx\,dx = \frac{1}{2}k\,x^2$$

This work done is stored as potential energy U of the spring.

$$\therefore \quad U = \frac{1}{2}k\,x^2$$

12. **Conservative force.** A force is conservative (*i*) if the work done by the force in displacing a particle from one point to another is independent of the path followed by the particle and (*ii*) if the work done by the force in moving a particle around any closed path is zero. Gravitational force, electrostatic force and elastic force of a spring are all conservative forces.

13. **Non-conservative force.** If the amount of work done in moving an object against a force from one point to another depends on the path along which the body moves, then such a force is called a non-conservative force. Forces of friction and viscosity are non-conservative forces.

14. **Mass-energy equivalence.** According to Einstein, mass can be converted into energy and energy into mass. A mass m is equivalent to energy E given by

$$E = mc^2$$

where c is the speed of light in vacuum and $c = 3 \times 10^8\ ms^{-1}$.

15. **Power.** It is the rate of doing work. It is a scalar quantity.

$$\text{Power} = \frac{\text{Work}}{\text{Time}} \quad \text{or} \quad P = \frac{W}{t}$$

Instantaneous power is given by

$$P = \frac{dW}{dt} = \frac{d}{dt}(\vec{F}\cdot\vec{s}) = \vec{F}\cdot\frac{d\vec{s}}{dt}$$

$$= \vec{F}\cdot\vec{v} = Fv\cos\theta$$

If $\theta = 0°$, then $P = Fv$.

16. **Units of power.** The SI unit of power is watt (W).

$$1\text{ watt} = \frac{1\text{ joule}}{1\text{ second}} \quad \text{or} \quad 1\text{ W} = 1\text{ J s}^{-1}$$

The larger units of power are kilowatt (kW) and horse power (hp).

$$1\text{ kW} = 1000\text{ W} \quad \text{and} \quad 1\text{ hp} = 746\text{ W}$$

17. **Kilowatt hour.** It is the commercial unit of electrical energy. One kilowatt hour is the electrical energy consumed by an appliance of 1000 watt in 1 hour.

$$1\text{ kWh} = 3.6 \times 10^6\text{ J}$$

18. **Law of conservation of energy.** It states that energy can neither be created nor destroyed but can only be transformed from one form to another.

19. **Collision.** A collision between two bodies is said to occur if either they physically collide against each other or the path of the motion of one body is influenced by the other.

20. **Elastic collision.** If there is no loss of kinetic energy during a collision, it is called an elastic collision.

21. **Inelastic collision.** If there is a loss of kinetic energy during a collision, it is called an inelastic collision.

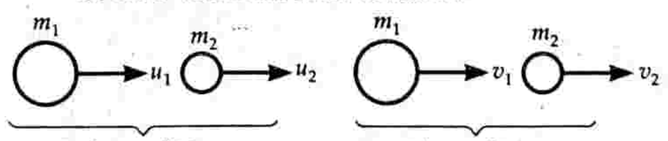
Linear momentum is conserved both in elastic and inelastic collisions.

22. **Perfectly inelastic collision.** It is the collision in which two bodies stick together after the collision.

23. **Head-on collision or one-dimensional collision.** It is a collision in which the colliding bodies move along the same straight line path before and after the collision.

24. **Oblique collision.** If the two bodies do not move along the same straight line path before and after the collision, the collision is said to be oblique collision.

25. **Velocities in one-dimensional elastic collision.** Suppose two bodies of masses m_1 and m_2 moving with velocities u_1 and u_2 $(u_1 > u_2)$ in the same direction suffer head-on elastic collision. Let v_1 and v_2 be their velocities after collision.



Before collision After collision

By the law of conservation of momentum,

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

As K.E. is conserved in an elastic collision, so

$$\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

By solving the above two equations, it can be shown that in an elastic collision,

Velocity of approach = Velocity of separation

or $\quad u_1 - u_2 = v_2 - v_1$

Also, $\quad v_1 = \frac{m_1 - m_2}{m_1 + m_2} \cdot u_1 + \frac{2m_2}{m_1 + m_2} \cdot u_2$

and $\quad v_2 = \frac{2m_1}{m_1 + m_2} \cdot u_1 + \frac{m_2 - m_1}{m_1 + m_2} \cdot u_2$

26. **Coefficient of restitution.** The coefficient of restitution for a collision between two bodies is the ratio of the magnitude of their relative velocity after collision to the magnitude of their relative velocity before the collision.

$$e = -\frac{v_1 - v_2}{u_1 - u_2} = \frac{|v_1 - v_2|}{|u_1 - u_2|}$$

For a perfectly elastic collision, $e = 1$ and for a perfectly inelastic collision, $e = 0$. Hence $0 \le e \le 1$.

For a ball rebounding from a floor,

$$e = \frac{v}{u}$$

where u and v are the magnitudes of the velocities of the ball before and after the collision respectively.

IIT Entrance Exam

MULTIPLE CHOICE QUESTIONS WITH ONE CORRECT ANSWER

1. Two masses of 1 g and 4 g are moving with equal kinetic energy. The ratio of the magnitudes of their momenta is

(a) 4 : 1 (b) $\sqrt{2}$: 1
(c) 1 : 2 (d) 1 : 16 [IIT 80]

2. If a machine is lubricated with oil

(a) the mechanical advantage of the machine increases
(b) the mechanical efficiency of the machine increases
(c) both its mechanical advantage and efficiency increase
(d) its efficiency increases, but its mechanical advantage decreases. [IIT 80]

3. A spring of force-constant k is cut into two pieces such that one piece is double the length of the other. Then the long piece will have a force-constant of

(a) (2/3) k (b) (3/2) k
(c) 3 k (d) 6 k [IIT 99]

4. An ideal spring with spring constant k is hung from the ceiling and a block of mass M is attached to its lower end. The mass is released with the spring initially unstretched. Then the maximum extension in the spring is

(a) $\frac{4Mg}{k}$ (b) $\frac{2Mg}{k}$
(c) $\frac{Mg}{k}$ (d) $\frac{Mg}{2k}$ [IIT 02]

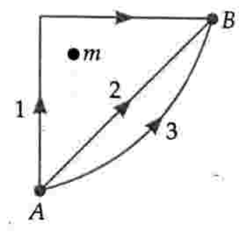
5. A particle of mass m is moving in a circular path of constant radius r such that its centripetal acceleration a_c is varying with time t as $a_c = k^2 r t^2$, where k is a constant. The power delivered to the particle by the force acting on it is

(a) $2\pi m k^2 r^2 t$ (b) $mk^2 r^2 t$
(c) $\frac{(mk^4 r^2 t^5)}{3}$ (d) zero [IIT 94]

6. A wind-powered generator converts wind energy into electrical energy. Assume that the generator converts a fixed fraction of the wind energy intercepted by its blades into electrical energy. For wind speed v, the electrical power output will be proportional to

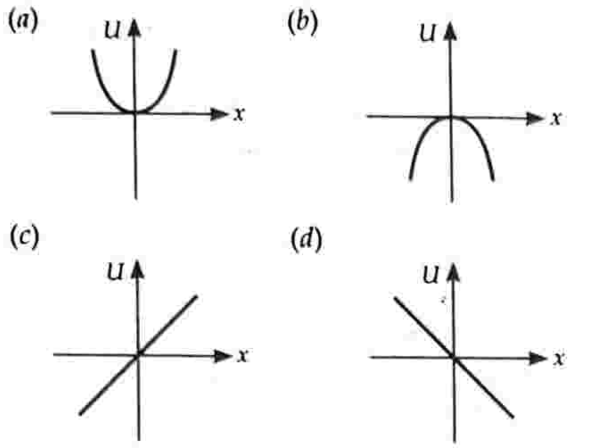
(a) v (b) v^2
(c) v^3 (d) v^4 [IIT 2K]

7. If W_1, W_2 and W_3 represent the work done in moving a particle from A to B along three different paths 1, 2 and 3 respectively (as shown) in the gravitational field of a point mass m, find the correct relation between W_1, W_2 and W_3.

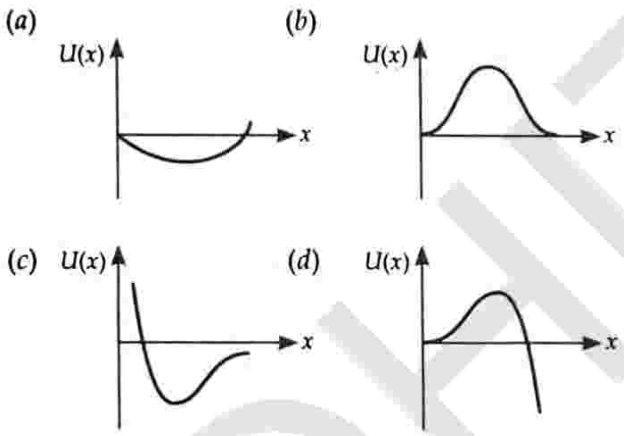


(a) $W_1 > W_2 > W_3$ (b) $W_1 = W_2 = W_3$
(c) $W_1 < W_2 < W_3$ (d) $W_2 > W_1 > W_3$ [IIT 03]

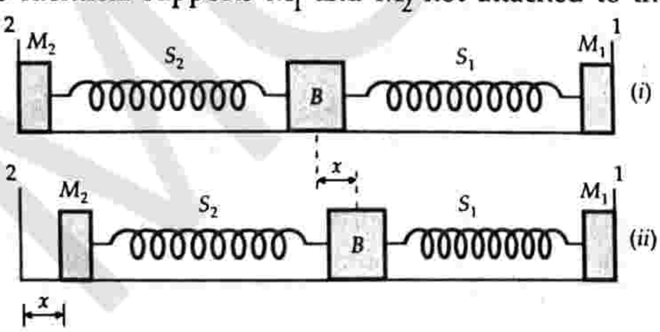
8. A particle is acted by a force $F = kx$, where k is a +ve constant. Its potential energy at $x = 0$ is zero. Which curve correctly represents the variation of potential energy of the block with respect to x? [IIT 04]



9. A particle, which is constrained to move along the x-axis, is subjected to a force in the same direction which varies with the distance x of the particle from the origin as $F(x) = -kx + ax^3$. Here k and a are positive constants. For $x \geq 0$, the functional form of the potential energy $U(x)$ of the particle is [IIT 02]



10. A block (B) is attached to two unstretched springs S_1 and S_2 with spring constants k and $4k$, respectively [see Fig. (*i*)]. The other ends are attached to identical supports M_1 and M_2 not attached to the



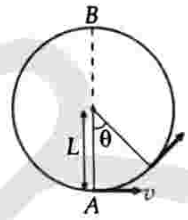
walls. The springs and supports have negligible mass. There is no friction anywhere. The block B is displaced towards wall 1 by a small distance x [see Fig. (*ii*)] and released. The block returns and moves a maximum distance y towards wall 2. Displacements x and y are measured with respect to the equilibrium position of the block B.

The ratio y/x is

(*a*) 4 (*b*) 2

(*c*) $\frac{1}{2}$ (*d*) $\frac{1}{4}$ [IIT 08]

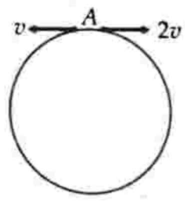
11. A bob of mass M is suspended by a massless string of length L. The horizontal velocity v at position A is just sufficient to make it reach the point B. The angle θ at which the speed of the bob is half of that at A, satisfies



(*a*) $\theta = \frac{\pi}{4}$ (*b*) $\frac{\pi}{4} < \theta < \frac{\pi}{2}$

(*c*) $\frac{\pi}{2} < \theta < \frac{3\pi}{4}$ (*d*) $\frac{3\pi}{4} < \theta < \pi$ [IIT 08]

12. Two small particles of equal masses start moving in opposite directions from a point A in a horizontal circular orbit. Their tangential velocities are v and $2v$, respectively, as shown in the figure. Between collisions, the particles move with constant speeds.

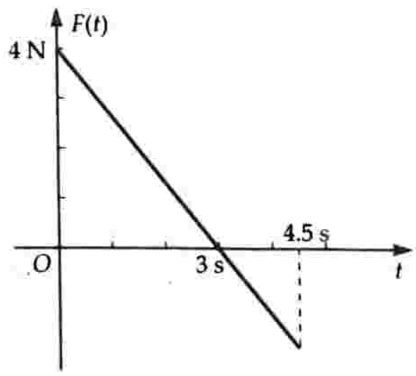


After making how many elastic collisions, other than that at A, these two particles will again reach the point A?

(*a*) 4 (*b*) 3

(*c*) 2 (*d*) 1 [IIT 09]

13. A block of mass 2 kg is free to move along the x-axis. It is at rest and from $t = 0$ onwards it is subjected to a time-dependent force $F(t)$ in the x-direction. The force $F(t)$ varies with t as shown in the figure. The kinetic energy of the block after 4.5 seconds is



(*a*) 4.50 J (*b*) 7.50 J

(*c*) 5.06 J (*d*) 14.06 J [IIT 2010]

MULTIPLE CHOICE QUESTIONS WITH ONE OR MORE THAN ONE CORRECT ANSWER

14. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle. The motion of the particle takes place in a plane. It follows that

(a) its velocity is constant

(b) its acceleration is constant

(c) its kinetic energy is constant

(d) it moves in a circular path. [IIT 87]

15. A force $F = -K(y\hat{i} + x\hat{j})$ (where K is a positive constant) acts on a particle moving in the xy-plane. Starting from the origin, the particle is taken along the positive x-axis to the point $(a, 0)$, and then parallel to the y-axis to the point (a, a). The total work done by the force F on the particle is

(a) $-2Ka^2$ (b) $2Ka^2$

(c) $-Ka^2$ (d) Ka^2 [IIT 98]

16. A body is moved along a straight line by a machine delivering constant power. The distance moved by the body in time t is proportional to

(a) $t^{1/2}$ (b) $t^{3/4}$

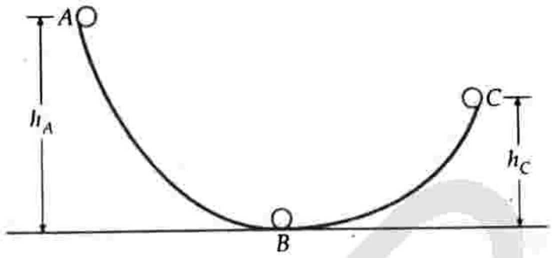
(c) $t^{3/2}$ (d) t^2 [IIT 84]

17. A uniform chain of length L and mass M is lying on a smooth table and one third of its length is hanging vertically down over the edge of the table. If g is acceleration due to gravity, the work required to pull the hanging part on to the table is

(a) MgL (b) $MgL/3$

(c) $MgL/9$ (d) $MgL/18$ [IIT 85]

18. A small ball starts moving from A over a fixed track as shown in the figure. Surface AB has friction. From A to B the ball rolls without slipping. Surface BC is frictionless. K_A, K_B and K_C are kinetic energies of the ball at A, B and C respectively. Then

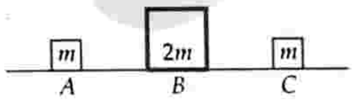


(a) $h_A > h_C$; $K_B > K_C$ (b) $h_A > h_C$; $K_C > K_A$

(c) $h_A = h_C$; $K_B = K_C$ (d) $h_A < h_C$; $K_B > K_C$ [IIT 06]

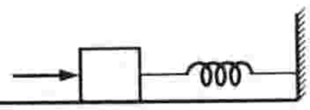
INTEGER ANSWER TYPE

19. Three objects A, B and C are kept in a straight line on a frictionless horizontal surface. These have masses m, $2m$ and m, respectively. The object A moves towards B with a speed 9 m/s and makes an elastic



collision with it. Therefore, B makes completely inelastic collision with C. All motions occur on the same straight line. Find the final speed (in m/s) of the object C. [IIT 09]

20. A block of mass 0.18 kg is attached to a spring of force-constant 2 N/m. The coefficient of friction between the block and the floor is 0.1. Initially the block is at rest and the spring is un-stretched. An



impulse is given to the block as shown in the figure. The block slides a distance of 0.06 m and comes to rest for the first time. The initial velocity of the block in m/s is $v = N/10$. Find N [IIT 2011]

Answers and Explanations

1. (c) $p = \sqrt{2mK}$

For same K, $p \propto \sqrt{m}$

$$\frac{p_1}{p_2} = \sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{1}{4}} = \frac{1}{2} = \mathbf{1:2}.$$

2. (b) When a machine is lubricated with oil, energy wastage against friction decreases.

This increases the mechanical efficiency of the machine.

3. (b) For any spring, kl = constant

Length of longer piece $= \frac{2l}{3}$

$\therefore \quad k' \times \frac{2l}{3} = kl \quad$ or $\quad k' = \left(\frac{3}{2}\right)k.$

4. (*b*) At the maximum extension x of the spring,

Increase in elastic P.E. of the spring

= Loss in P.E. by mass M

or $\quad \frac{1}{2}kx^2 = Mgx$

$\therefore \quad x = \frac{2Mg}{k}.$

5. (*b*) Given $a_c = k^2rt^2$

or $\quad \frac{v^2}{r} = k^2rt^2 \quad$ or $\quad v = krt$

Tangential acceleration,

$$a_t = \frac{dv}{dt} = kr$$

Tangential force on the particle,

$$F_t = ma_t = mkr$$

Power is delivered only by the tangential force not by the radial force.

$\therefore \quad P = F_t . v = mkr \times krt = \mathbf{mk^2r^2t}.$

6. (*e*) Force $= v\frac{dm}{dt} = v\frac{d}{dt}$ (volume × density)

$= v\frac{d}{dt}(Ax \times \rho) = vA\rho\frac{dx}{dt}$

$= A\rho v^2 \quad \left[\frac{dx}{dt} = v\right]$

Power = Force × velocity

$= A\rho v^2 \times v = A\rho v^3$

$\therefore$ Power $\propto v^3$.

7. (*b*) Gravitational force is a conservative force. Work done against such a force does not depend on the path followed.

$\therefore \quad W_1 = W_2 = W_3.$

8. (*b*) For a conservative force,

$$F = -\frac{dU}{dt}$$

$\therefore \quad \int_0^{U(x)} dU = -\int_0^x F\,dx = -\int_0^x kx\,dx$

As $\quad U(0) = 0$

So, $\quad U(x) = -\frac{kx^2}{2}$

Thus, the U-x graph is a parabola, symmetric about U-axis, lying below x-axis with its vertex at the origin. Hence the correct option is (*b*).

9. (*d*) $\quad F = -\frac{dU}{dx}$

or $\quad dU = -F\,dx$

$\therefore \quad U(x) = -\int_0^x (-kx + ax^3)dx$

or $\quad U(x) = \frac{kx^2}{2} - \frac{ax^4}{4} = \frac{x^2}{2}\left(k - \frac{ax^2}{2}\right)$

Clearly, $U(x) = 0$ at $x = 0$ and $x = \sqrt{\frac{2k}{a}}$

For $x > \sqrt{\frac{2k}{a}}$, $U(x)$ will be negative.

At $x = 0$, $\quad F = -\frac{dU}{dx} = 0$

i.e., slope of U-x graph is zero at $x = 0$.

Hence, the most appropriate option is (*d*).

10. (*c*) By conservation of energy,

$$\frac{1}{2}kx^2 = \frac{1}{2}(4k)y^2$$

or $\quad \frac{y^2}{x^2} = \frac{1}{4} \quad \therefore \quad \frac{y}{x} = \frac{1}{2}.$

11. (*d*) By conservation of energy,

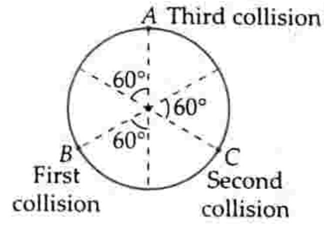
$$\frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{v}{2}\right)^2 + mgh$$

or $\quad \frac{1}{2}m.5gL = \frac{1}{2}m\frac{5gL}{4} + mgL(1-\cos\theta)$

$\frac{5}{2} = \frac{5}{8} + 1 - \cos\theta \quad$ or $\quad \cos\theta = -\frac{7}{8}$

Hence $\quad \frac{3\pi}{4} < \theta < \pi.$

12. (*c*) After the collision the velocities of the two particles get exchanged. After the first collision the particles will be at position B and after the second collision the particles will be at position C. After this collision, the two particles will return to position A.



13. (*c*)

Area under F-t graph = Change in momentum

$\therefore \quad \Delta p = \frac{1}{2} \times 4 \times 3 - \frac{1}{2} \times 1.5 \times \frac{4}{3} \times 1.5 = 4.5 \text{ kgms}^{-1}$

Kinetic energy

$$= \frac{p^2}{2m} = \frac{4.5^2}{2 \times 2} = \mathbf{5.06\ J.}$$

14. (*c*), (*d*) For a particle in uniform circular motion, the directions of both velocity and acceleration vectors change continuously but their magnitudes remain unchanged. The kinetic energy is not affected. Hence (*c*) and (*d*) are correct options.

15. (*c*) Position vector of point $(a, 0)$,

$$\vec{r_1} = a\hat{i} + 0\hat{j}$$

Position vector of point (a, a),

$$\vec{r_2} = a\hat{i} + a\hat{j}$$

Displacement vector,

$$\vec{r} = \vec{r_2} - \vec{r_1} = (a\hat{i} + a\hat{j}) - (a\hat{i} + 0\hat{j})$$
$$= a\hat{j}$$

Also, $\vec{F} = -k(y\hat{i} + x\hat{j})$

$\therefore$ $W = \vec{F} \cdot \vec{r} = -k(y\hat{i} + x\hat{j}) \cdot a\hat{j} = -kax$

As $x = a$, so

$$W = -\mathbf{K}\boldsymbol{a}^{\mathbf{2}}.$$

16. (*c*) Refer answer to NCERT Exercise 6.10 on page 6.53.

17. (*d*) Refer to the solution of Problem 13 on page 6.46.

18. (*a*), (*c*)

Total energy at A, $E_A = K_A + mgh_A$

Total energy at B, $E_B = K_B$

Total energy at C, $E_C = K_C + mgh_C$

By conservation of energy,

$$E_A = E_B = E_C$$

Now, $E_A = E_B \Rightarrow K_B > K_C$ ...(*i*)

Consider $E_A = E_C$

or $K_A + mgh_A = K_C + mgh_C$

or $$h_A - h_C = \frac{K_C - K_A}{mg}$$

For $h_A > h_C$; $K_C > K_A$

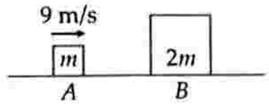
Also, $h_A > h_C$ and from (*i*), $K_B > K_C$

$\therefore$ Option (*a*) is correct.

19.

0	0	0	4

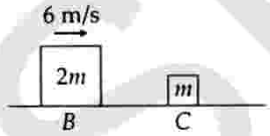
For elastic collision between A and B,



$$v_B = \frac{2m}{m+2m} \times 9 + \frac{2m - m}{2m + m} \times 0$$

$$= \mathbf{6\ m/s}$$

For inelastic collision between B and C,



$$2m \times 6 + m \times 0 = (2m + m)v$$

$$v = \mathbf{4\ m/s}.$$

20.

0	0	0	4

Loss in K.E. of the block

= Gain in P.E. of the spring

+ Work done against friction

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2 + \mu mgx$$

or $$\frac{1}{2} \times 0.18v^2 = \frac{1}{2} \times 2 \times (0.06)^2 + 0.1 \times 0.18 \times 10 \times 0.06$$

or $$0.09v^2 = 36 \times 10^{-4} \times 108 \times 10^{-4} = 144 \times 10^{-4}$$

or $$v = \frac{12 \times 10^{-2}}{0.3} = \frac{4}{10}$$

Given $$v = \frac{N}{10}$$

$\therefore$ $N = \mathbf{4}$.

AIEEE

1. A force $\vec{F} = (5\hat{i} + 3\hat{j} + 2\hat{k})N$ is applied over a particle which displaces it from its origin to the point $\vec{r} = (2\hat{i} - \hat{j})m$. The work done on the particle (in joule) is

(*a*) −7 (*b*) +7

(*c*) +10 (*d*) +13 [AIEEE 04]

2. An athlete in the olympic games covers a distance of 100 m in 10 s. His kinetic energy can be estimated in the range

(*a*) 200 J – 500 J (*b*) 2×10^5 J – 3×10^5 J

(*c*) 20,000 J – 50,000 J (*d*) 2000 J – 5000 J

[AIEEE 08]

3. A particle of mass 100 g is thrown vertically upwards with a speed of 5 ms^{-1}. The work done by the force of gravity during the time the particle goes up is

(*a*) 1.25 J (*b*) 0.5 J
(*c*) –0.5 J (*d*) –1.25 J [AIEEE 06]

4. A ball whose kinetic energy is *E*, is projected at an angle of 45° to the horizontal. The kinetic energy of the ball at the highest point of its flight will be

(*a*) E (*b*) $E/\sqrt{2}$
(*c*) $E/2$ (*d*) zero [AIEEE 02]

5. A particle is projected at an angle of 60° to the horizontal with a kinetic energy *E*. The kinetic energy at the highest point is

(*a*) E (*b*) $E/4$
(*c*) $E/2$ (*d*) zero [AIEEE 07]

6. A particle moves in a straight line with retardation proportional to its displacement. Its loss of kinetic energy for any displacement *x* is proportional to

(*a*) x^2 (*b*) e^x
(*c*) x (*d*) $\log_e x$ [AIEEE 04]

7. The potential energy of a 1 kg particle free to move along the X-axis is given by

$$V(x) = \frac{x^4}{4} - \frac{x^2}{2} \text{ (in joule)}$$

The total mechanical energy of the particle is 2 J. Then the maximum speed (in ms^{-1}) is

(*a*) $1/\sqrt{2}$ (*b*) 2
(*c*) $3/\sqrt{2}$ (*d*) $\sqrt{2}$ [AIEEE 06]

8. Consider the following *two* statements :

(A) Linear momentum of a system of particles is zero.

(B) Kinetic energy of a system of particles is zero.

Then

(*a*) *A* does not imply *B* and *B* does not imply *A*
(*b*) *A* implies *B* but *B* does not imply *A*
(*c*) *A* does not imply *B* but *B* implies *A*
(*d*) *A* implies *B* and *B* implies *A*. [AIEEE 03]

9. A spherical ball of mass 20 kg is stationary at the top of a hill of height 100 m. It rolls down a smooth surface to the ground, then climbs up another hill of height 30 m and finally rolls down to a horizontal base at a height of 20 m above the ground. The velocity attained by the ball is

(*a*) 10 ms^{-1} (*b*) $10\sqrt{30}$ ms^{-1}
(*c*) 40 ms^{-1} (*d*) 20 ms^{-1} [AIEEE 05]

10. A uniform chain of length 2 m is kept on a table such that a length of 60 cm hangs freely from the edge of the table. The total mass of the chain is 4 kg. What is the work done in pulling the entire chain on the table ?

(*a*) 7.2 J (*b*) 3.6 J
(*c*) 120 J (*d*) 1,200 J [AIEEE 04]

11. A mass of *M* kg is suspended by a weightless string. The horizontal force that is required to displace it, until the string makes an angle 45° with the initial vertical direction, is

(*a*) $Mg/\sqrt{2}$ (*b*) $(\sqrt{2}-1)Mg$
(*c*) $(\sqrt{2}+1)Mg$ (*d*) $\sqrt{2}\,Mg$ [AIEEE 06]

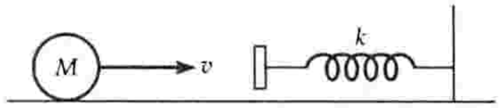
12. A spring of spring constant $5 \times 10^3\ Nm^{-1}$ is stretched initially by 5 cm from the unstretched position. Then the work required to stretch it further by another 5 cm is

(*a*) 12.50 Nm (*b*) 18.75 Nm
(*c*) 25.00 Nm (*d*) 6.25 Nm [AIEEE 03]

13. A spring of force constant 800 Nm^{-1} has an extension of 5 cm. The work done in extending it from 5 cm to 15 cm is

(*a*) 16 J (*b*) 8 J
(*c*) 32 J (*d*) 24 J [AIEEE 02]

14. The block of mass *M* moving on the frictionless horizontal surface collides with the spring of spring



constant *k* and compresses it by length *L*. The maximum momentum of the block after collision is

(*a*) $\frac{ML^2}{k}$ (*b*) $\frac{kL^2}{2M}$
(*c*) $\sqrt{Mk}\,L$ (*d*) zero [AIEEE 05]

15. A 2 kg block slides on a horizontal floor with a speed of 4 ms^{-1}. It strikes an uncompressed spring and compresses it, till the block is motionless. The force of kinetic friction is 15 N and spring constant is 10,000 Nm^{-1}.

The spring compresses by

(*a*) 5.5 cm (*b*) 2.5 cm
(*c*) 11.0 cm (*d*) 8.5 cm [AIEEE 07]

16. A body of mass *m* accelerates uniformly from rest to v_1 in time t_1. The instantaneous power delivered to the body as a function of time *t* is

(a) $\frac{mv_1 t}{t_1}$ (b) $\frac{mv_1^2 t}{t_1^2}$

(c) $\frac{mv_1 t^2}{t_1}$ (d) $\frac{mv_1^2 t}{t_1}$ [AIEEE 04, 05]

17. A body is moved along a straight line by a machine delivering a constant power. The distance moved by the body in time t is proportional to

(a) $t^{3/4}$ (b) $t^{3/2}$

(c) $t^{1/4}$ (d) $t^{1/2}$ [AIEEE 03]

18. If mass-energy equivalence is taken into account, when water is cooled to form ice, the mass of water should

(a) increase (b) remain unchanged

(c) decrease

(d) first increase then decrease [AIEEE 02]

19. A bomb of mass 16 kg at rest explodes into two pieces of masses 4 kg and 12 kg. The velocity of the 12 kg mass is $4\,\text{ms}^{-1}$. The kinetic energy of the other mass is

(a) 192 J (b) 96 J

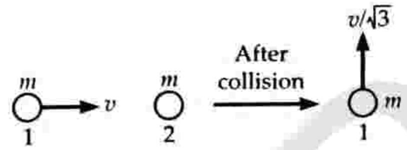
(c) 144 J (d) 288 J [AIEEE 06]

20. A block of mass 0.50 kg is moving with a speed of $2.00\,\text{ms}^{-1}$ on a smooth surface. It strikes another mass of 1.00 kg and then move together as a single body. The energy loss during the collision is

(a) 0.16 J (b) 1.00 J

(c) 0.67 J (d) 0.34 J [AIEEE 08]

21. A mss m moves with a velocity v and collides elastically with another identical mass at rest. After collision the first mass moves with velocity $v/\sqrt{3}$ in a



direction perpendicular to the initial direction of motion. Find the speed of the second mass after collision.

(a) v (b) $\sqrt{3}\,v$

(c) $2v/\sqrt{3}$ (d) $v/\sqrt{3}$ [AIEEE 05]

22. **Statement – 1.** Two particles moving in the same direction do not lose all their energy in a completely inelastic collision.

Statement – 2. Principle of conservation of momentum holds true for all kinds of collision.

(a) Statement – 1 is true, Statement – 2 is false

(b) Statement – 1 is true, Statement – 2 is true ; Statement – 2 is the correct explanation of Statement – 1

(c) Statement – 1 is true, Statement – 2 is true ; Statement – 2 is *not* the correct explanation of Statement – 1

(d) Statement – 1 is false, Statement – 2 is true [AIEEE 2010]

Answers and Explanations

1. (b) $W = \vec{F}\cdot\vec{r} = (5\hat{i}+3\hat{j}+2\hat{k})\cdot(2\hat{i}-\hat{j})$

$= 5\times 2 + 3(-1) + 2\times 0 = \mathbf{+7\,J}.$

2. (d) Average speed of the athlete,

$$v = \frac{s}{t} = \frac{100}{10} = 10\ \text{ms}^{-1}$$

Assuming the mass of the athelete to 60 kg, his average K.E. would be

$$K = \frac{1}{2}\times 60\times(10)^2 = \mathbf{3000\ J}.$$

3. (d) Work done by the force of gravity

= Loss in K.E. of the body

$$= \frac{1}{2}m(v^2-u^2) = \frac{1}{2}\times\frac{100}{1000}(0^2-5^2)\,\text{J}$$

$= \mathbf{-1.25\ J}.$

4. (c) Refer to the solution of Problem 11 on page 6.50.

5. (b) Kinetic energy at the highest point,

$$E' = \frac{1}{2}m(u\cos 60^\circ)^2 = \frac{1}{2}mu^2\times\frac{1}{4} = \frac{E}{4}.$$

6. (a) Refer to the solution of Problem 12 on page 6.50.

7. (c) Potential energy,

$$V(x) = \frac{x^4}{4} - \frac{x^2}{2}\ \text{joule}$$

For maxima or minima,

$$\frac{dV}{dx} = 0$$

$$x^3 - x = 0$$

or $x = 0, -1, 1$

Now $\frac{d^2V}{dx^2} = 3x^2 - 1$

At $x = 0$, $\frac{d^2V}{dx^2} = -1$

At $x = \pm 1$ $\frac{d^2V}{dx^2} = 2 > 0$

Hence P.E. is minimum at $x = \pm 1$.

$$V_{min} + K_{max} = \text{Total energy}$$

$$\left(\frac{1}{4} - \frac{1}{2}\right) + K_{max} = 2$$

or $$K_{max} = 2 + \frac{1}{4} = \frac{9}{4}$$

$$\frac{1}{2}mv^2 = \frac{9}{4}$$

or $$v = \sqrt{\frac{9}{2m}} = \sqrt{\frac{9}{2 \times 1}} = \frac{3}{\sqrt{2}}\ \mathbf{ms^{-1}}.$$

8. (*c*) When the linear momentum of a system of particles is zero, the velocities of the individual particles may not be zero. Then the kinetic energy of the system of particles may be non-zero. Thus *A* does not imply *B*.

When the kinetic energy of a system of particles is zero, then kinetic energy and hence velocity of each particle is zero, As such, the linear momentum of the system of particles is also zero. Thus *B* implies *A*.

Hence the correct option is (*c*).

9. (*c*) Total energy at 100 m height

= Total energy at 20 m height

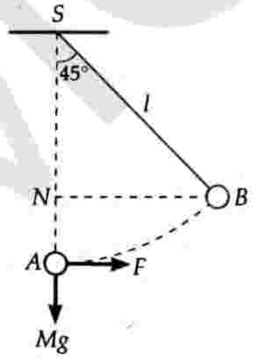
$$mgh_1 = mgh_2 + \frac{1}{2}mv^2$$

$$v = \sqrt{2g(h_1 - h_2)} = \sqrt{2 \times 10 \times (100 - 20)}$$

$$= \mathbf{40\ ms^{-1}}.$$

10. (*b*) Refer to the solution of Example 6 on page 6.4.

11. (*b*)



By conservation of energy,

Work done by force F

= Change in P.E. of mass M

$$F \times BN = Mg \times AN$$

$$F \times l \sin 45° = Mg \times (l - l\cos 45°)$$

$$F = Mg\left(\frac{1}{\sin 45°} - \cot 45°\right)$$

$$= Mg(\sqrt{2} - 1)$$

12. (*b*) $W = \frac{1}{2}k(x_f^2 - x_i^2)$

$$x_i = 5 \text{ cm} = 0.05 \text{ m}$$

$$x_f = 5 + 5 = 10 \text{ cm} = 0.10 \text{ m}$$

$\therefore$ $$W = \frac{1}{2} \times 5 \times 10^3(0.10^2 - 0.05^2) = \mathbf{18.75\ J}.$$

13. (*b*) $W = \frac{1}{2} \times 800(0.15^2 - 0.05^2)$

$$= 400 \times 0.20 \times 0.10 = \mathbf{8\ J}.$$

14. (*c*) When the spring gets compressed by length L, K.E. lost by mass M

= P.E. stored in the compressed spring

$$\frac{1}{2}Mv_{max}^2 = \frac{1}{2}kL^2$$

$\therefore$ $$v_{max} = \sqrt{\frac{k}{M}}L$$

Maximum momentum of the block,

$$p_{max} = Mv_{max} = \sqrt{\mathbf{Mk}}\,\mathbf{L}.$$

15. (*a*) Suppose the spring gets compressed by length x. Then

Initial K.E. of the block

= P.E. stored in the spring

+ Work done against friction

$$\frac{1}{2} \times 2 \times 4^2 = \frac{1}{2} \times 10000 \times x^2 + 15x$$

or $$5000x^2 + 15x - 16 = 0$$

or $$x = 0.055 \text{ m} = \mathbf{5.5\ cm}.$$

16. (*b*) Refer to the solution of Problem 13 on page 6.50.

17. (*b*) Refer to the solution of Problem 14 on page 6.46.

18. (*a*) The heat energy possessed by water gets converted into mass, when ice is formed. This increases mass.

19. (*d*) By conservation of momentum,

$$16 \times 0 = 4 \times v + 12 \times 4$$

$$v = -12 \text{ ms}^{-1}$$

K.E. of 4 kg mass

$$= \frac{1}{2}mv^2 = \frac{1}{2} \times 4 \times 12^2 = \mathbf{288\ J}.$$

20. (*c*) Momentum after collision

= Momentum before collision

$$(m_1 + m_2)v = m_1 u_1 + m_2 u_2$$

$$(0.50 + 1.00)v = 0.50 \times 2.00 + 1.00 \times 0$$

$$v = \frac{1}{1.5} = \frac{2}{3}\ \text{ms}^{-1}$$

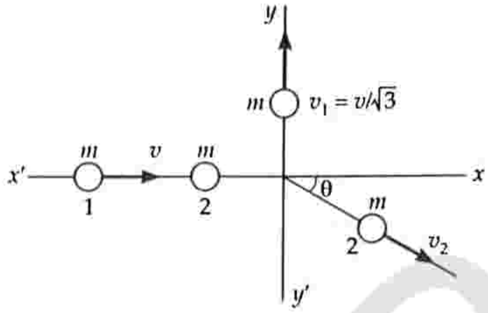
Loss of energy,

$$= \frac{1}{2} m_1 u_1^2 - \frac{1}{2}(m_1 + m_2)v^2$$

$$= \frac{1}{2} \times 0.5 \times 2^2 - \frac{1}{2}(0.50 + 1.0)\left(\frac{2}{3}\right)^2$$

$$= 1 - \frac{1}{3} = \mathbf{0.67\ J.}$$

21. (*c*) The situation is shown in the figure.



Considering conservation of momentum along x-axis,

$$mv + m \times 0 = m \times 0 + m \times v_2 \cos\theta$$

or $$v_2 \cos\theta = v \quad \text{...(i)}$$

Along y-axis, we have

$$m \times 0 + m \times 0 = m \times \frac{v}{\sqrt{3}} + m(-v_2 \sin\theta)$$

or $$v_2 \sin\theta = \frac{v}{\sqrt{3}} \quad \text{...(ii)}$$

Squaring and adding (*i*) and (*ii*), we get

$$v_2^2 \cos^2\theta + v_2^2 \sin^2\theta = v^2 + \left(\frac{v}{\sqrt{3}}\right)^2$$

or $$v_2^2 = \frac{4v^2}{3} \quad \text{or} \quad v_2 = \frac{\mathbf{2v}}{\sqrt{\mathbf{3}}}.$$

22. (*b*) If the particles moving in same direction lose all their energy, final momentum will become zero, whereas initial momentum is not zero.