CHAPTER

9

MECHANICAL PROPERTIES OF SOLIDS

9.1 ELASTIC AND PLASTIC BEHAVIOUR OF SOLIDS

1. ***Define the terms deforming force, elasticity and plasticity. What are perfectly elastic and perfectly plastic bodies ? Give examples.***

Elastic and plastic behaviour of solids. By a rigid body, we generally mean a hard solid object having a definite shape and size. In reality, solid bodies are not perfectly rigid. They can be stretched, compressed and bent. When an external force is applied, a body may get deformed. When the deforming force is removed, some bodies tend to regain their original size and shape while others do not show any such tendency. Let us define few terms to explain this behaviour of bodies.

Deforming force. If a force is applied on a body which is neither free to move nor free to rotate, the molecules of the body are forced to undergo a change in their relative positions. As a result, the body may undergo a change in length, volume or shape. *A force which changes the size or shape of a body is called a deforming force.*

Elasticity. *If a body regains its original size and shape after the removal of deforming force, it is said to be elastic body and this property is called elasticity.* For example, if we stretch a rubber band and release it, it snaps back to its original length.

Perfectly elastic body. *If a body regains its original size and shape completely and immediately after the removal of deforming force, it is said to be a perfectly elastic body. The nearest approach to a perfectly elastic body is quartz fibre.*

Plasticity. *If a body does not regain its original size and shape even after the removal of deforming force, it is said to be a plastic body and this property is called plasticity.* For example, if we stretch a piece of chewing-gum and release it, it will not regain its original size and shape.

Perfectly plastic body. If *a body does not show any tendency to regain its original size and shape even after the removal of deforming force, it is said to be a perfectly plastic body.* Putty and paraffin wax are nearly perfectly plastic bodies.

For Your Knowledge

▲ No body is perfectly elastic or perfectly plastic. All the bodies found in nature lie between these two limits. When the elastic behaviour of a body decreases, its plastic behaviour increases.

9.2 ELASTIC BEHAVIOUR IN TERMS OF INTERATOMIC FORCES

2. *Give an explanation of the elastic properties of materials in terms of interatomic/intermolecular forces.*

Elastic behaviour in terms of interatomic forces. The atoms in a solid are held together by interatomic forces. The variations of potential energy U and interatomic force F with interatomic separation r are shown in Figs. 9.1(a) and (b) respectively.

When the interatomic separation r is large, the potential energy of the atoms is negative and the interatomic force is *attractive*. At some particular separation r_0, the potential energy becomes minimum and the interatomic force becomes zero. This separation r_0 is called ***normal*** or ***equilibrium separation***.

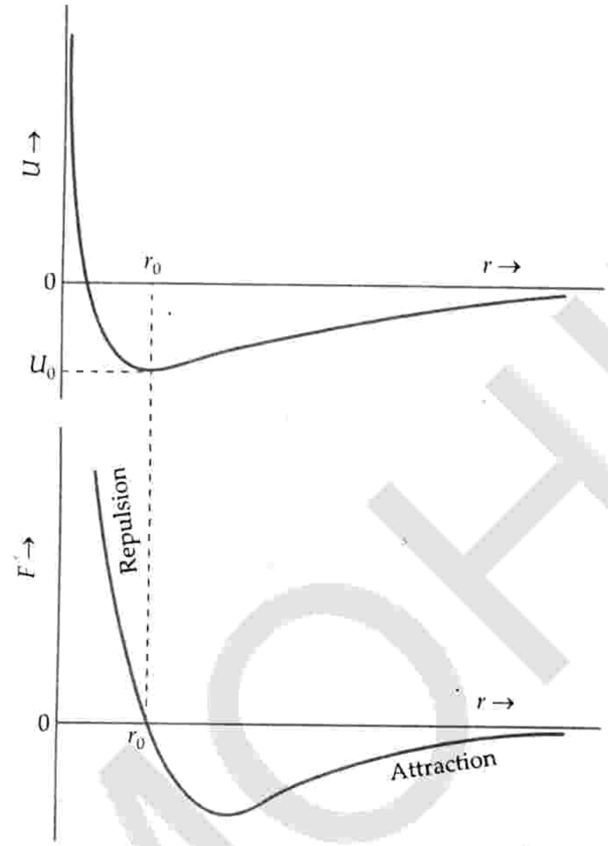


Fig. 9.1. (a) Interatomic potential energy U, (b) Force F, between two identical atoms as a function of interatomic separation r.

When separation reduces below r_0, the potential energy increases steeply and the interatomic force becomes *repulsive*.

Normally, the atoms occupy the positions ($r = r_0$) of minimum potential energy called the positions of stable equilibrium. When a tensile or compressive force is applied on a body, its atoms are pulled apart or pushed closer together to a distance r, greater than or smaller than r_0. When the deforming force is removed, the interatomic forces of attraction / repulsion restore the atoms to their equilibrium positions. The body regains its original size and shape. The stronger the interatomic forces, the smaller will be the displacements of atoms from the equilibrium positions and hence greater is the elasticity (or modulus of elasticity) of the material.

3. *Explain elastic behaviour of solids on the basis of mechanical spring-ball model of a solid.*

Elastic behaviour on the basis of spring-ball model of a solid. The atoms in a solid may be regarded as mass points or small balls connected in three-dimensional space through springs. The springs represent the interatomic forces. This is called spring-ball model of a solid, as shown in Fig. 9.2.

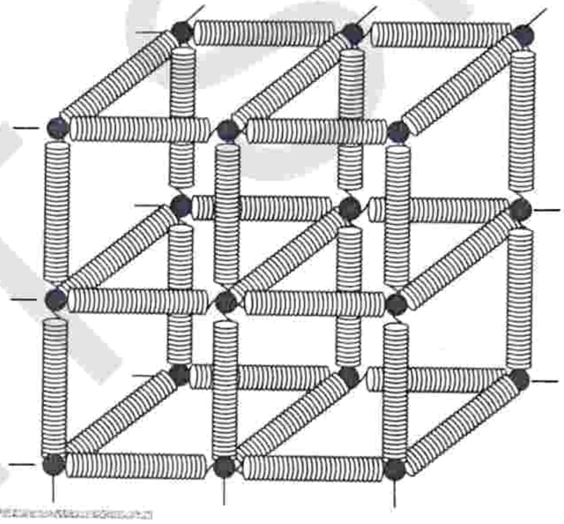


Fig. 9.2 Spring-ball model for explaining elastic behaviour of solids.

Normally, the balls occupy the positions of minimum potential energy or zero interatomic force. When any ball is displaced from its equilibrium position, the various springs connected to it exert a resultant force on this ball. This force tends to bring the ball to its equilibrium position. This explains the elastic behaviour of solid in terms of microscopic nature of the solid.

9.3 STRESS

4. *Define the term stress. Give its units and dimensions. Describe the different types of stress.*

Stress. If a body gets deformed under the action of an external force, then at each section of the body an internal force of reaction is set up which tends to restore the body into its original state. *The internal restoring force set up per unit area of cross-section of the deformed body is called stress.* As the restoring force is equal and opposite to the external deforming force, therefore

$$\text{Stress} = \frac{\text{Applied force}}{\text{Area}} = \frac{F}{A}$$

The SI unit of stress is Nm^{-2} and the CGS unit is dyne cm^{-2}. The dimensional formula of stress is $[ML^{-1}T^{-2}]$.

Types of stress :

(*i*) **Tensile stress.** It is the restoring force set up per unit cross-sectional area of a body when the length of the body increases in the direction of the deforming force. It is also known as *longitudinal stress.*

(*ii*) **Compressional stress.** It is the restoring force set up per unit cross-sectional area of a body when its length decreases under a deforming force.

(*iii*) **Hydrostatic stress.** If a body is subjected to a uniform force from all sides, then the corresponding stress is called *hydrostatic stress.*

(*iv*) **Tangential or Shearing stress.** When a deforming force acts tangentially to the surface of a body, it produces a change in the shape of the body. The tangential force applied per unit area is equal to the tangential stress.

9.4 STRAIN

5. *Define the term strain. Why it has no units and dimensions ? What are different types of strain ?*

Strain. When a deforming force acts on a body, the body undergoes a change in its shape and size. *The ratio of the change in any dimension produced in the body to the original dimension is called strain.*

$$\text{Strain} = \frac{\text{Change in dimension}}{\text{Original dimension}}$$

As strain is the ratio of two like quantities, it has no units and dimensions.

Types of strain :

(*i*) **Longitudinal strain.** *It is defined as the increase in length per unit original length,* when the body is deformed by external forces.

$$\text{Longitudinal strain} = \frac{\text{Change in length}}{\text{Original length}} = \frac{\Delta l}{l}$$

(*ii*) **Volumetric strain.** *It is defined as the change in volume per unit original volume,* when the body is deformed by external forces.

$$\text{Volumetric strain} = \frac{\text{Change in volume}}{\text{Original Volume}} = \frac{\Delta V}{V}$$

(*iii*) **Shear strain.** *It is defined as the angle θ (in radian), through which a face originally perpendicular to the fixed face gets turned on applying tangential deforming force.*

$$\text{Shear strain} = \theta = \tan\theta$$

$$= \frac{\text{Relative displacement between 2 parallel planes}}{\text{Distance between parallel planes}}$$

9.5 ELASTIC LIMIT

6. *What is meant by the term elastic limit ?*

Elastic limit. If a small load is suspended from a wire, its length increases. When the load is removed, the wire regains its original length. But if a sufficiently large force is suspended from the wire, it is found that the wire does not regain its original length after the load is removed. *The maximum stress within which the body regains its original size and shape after the removal of deforming force is called elastic limit.* If the deforming force exceeds the elastic limit, the body acquires a *permanent set* or *deformation* and is said to be *overstrained.*

9.6 HOOKE'S LAW AND MODULUS OF ELASTICITY

7. *State Hooke's law. How can it be verified experimentally ?*

Hooke's law. From experimental investigations, *Robert Hooke,* an English physicist (1635-1703 A.D.), formulated in 1679 a law known after him as Hooke's law which states that *the extension produced in a wire is directly proportional to the load applied.*

In 1807, *Thomos Young* pointed out that the strain is proportional to the extension of the wire and the stress is proportional to the load applied. He, therefore, modified Hooke's law to the more general form as follows :

Within the elastic limit, the stress is directly proportional to strain. Thus within the elastic limit,

$$\text{Stress} \propto \text{Strain}$$

or $$\text{Stress} = \text{Constant} \times \text{Strain}$$

or $$\frac{\text{Stress}}{\text{Strain}} = \text{Constant}$$

The constant of proportionality is called **modulus of elasticity** or **coefficient of elasticity** of the material. Its value depends on the nature of the material of the body and the manner in which it is deformed.

Experimental verification of Hooke's law. As shown in Fig. 9.3, suspend a metallic spring from a rigid support and attach to its lower end a pan and a pointer. Arrange a scale in the vertical position so that a pointer is able to move along it. Read the position of the pointer on the scale when the pan is empty. Place a weight of 50 gram in the pan. Note the position of the pointer on the scale. The difference between the two readings gives the extension produced in the spring by the weight added in the pan.

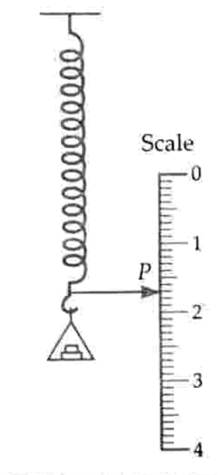


Fig. 9.3 Arrangement for studying Hooke's law.

Increase the weight in the pan in steps of 50 gram and note the corresponding extensions. Plot a graph between the extension of spring and the total load producing it. The graph is a straight line, as shown in Fig. 9.4. This indicates that extension $\propto$ load applied. This verifies Hooke's law.

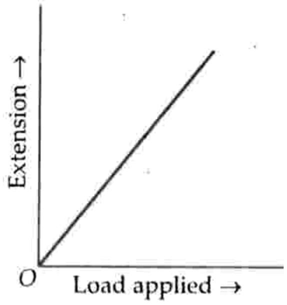


Fig. 9.4 Load-extension graph.

For Your Knowledge

- Like Boyle's law, Hooke's law is one of the earliest quantitative relationships in science.
- Hooke's law is valid only in the linear position of the stress-strain curve. The law is not valid for large values of strains.
- Stress is not a vector quantity since, unlike a force, the stress cannot be assigned a specific direction.
- When a wire, suspended from a ceiling, is stretched by a weight (F) suspended from its lower end, the ceiling exerts a force on the wire equal and opposite to the weight F. But the tension at any cross-section A of the wire is just F and not $2F$. Hence the tensile stress which is equal to the tension per unit area is equal to F/A.

8. *Define modulus of elasticity. Give its units and dimensions. What are different types of moduli of elasticity ?*

Modulus of elasticity. *The modulus of elasticity or coefficient of elasticity of a body is defined as the ratio of stress to the corresponding strain, within the elastic limit.*

$$\text{Modulus of elasticity, } E = \frac{\text{Stress}}{\text{Strain}}$$

The SI unit of modulus of elasticity is Nm^{-2} and its dimensions are $[ML^{-1}T^{-2}]$.

Different types of moduli of elasticity. Corresponding to the three types of strain, we have three important moduli of elasticity :

(*i*) *Young's modulus* (Y), *i.e.*, the modulus of elasticity of length.

(*ii*) *Bulk modulus* (κ), *i.e.*, the modulus of elasticity of volume.

(*iii*) *Modulus of rigidity* or *shear modulus* (η), *i.e.*, modulus of elasticity of shape.

9.7 YOUNG'S MODULUS OF ELASTICITY

9. *Define Young's modulus of elasticity. Give its units and dimensions.*

Young's modulus of elasticity. *Within the elastic limit, the ratio of longitudinal stress to the longitudinal strain is called Young's modulus of the material of the wire.*

As shown in Fig. 9.5, suppose a wire of length l and cross-sectional area A suffers an increase in length Δl under a force F acting along its length l. Then Young's modulus is given by

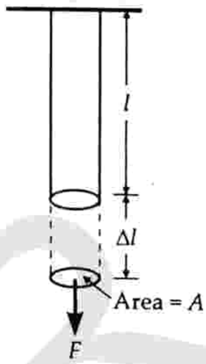


Fig. 9.5 Young's modulus of elasticity.

$$Y = \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}}$$

$$= \frac{F/A}{\Delta l / l}$$

or $$Y = \frac{F}{A} \cdot \frac{l}{\Delta l}.$$

If the wire has a circular cross-section of radius r, then

$$Y = \frac{F}{\pi r^2} \cdot \frac{l}{\Delta l}.$$

If $l = 1$ m, $A = 1\ m^2$ and $\Delta l = 1$ m, then $Y = F$.

Thus, *Young's modulus of elasticity is equal to the force required to extend a wire of unit length and unit area of cross-section by unit amount, i.e., the force required to double the length of the wire.*

Units and dimensions of Y. The SI unit of Young's modulus is Nm^{-2} or pascal (Pa) and its CGS unit is dyne cm^{-2}. The dimensional formula of Y is $[ML^{-1}T^{-2}]$.

9.8 STRESS-STRAIN CURVE FOR A METALLIC WIRE

10. *Explain what happens when the load on a metal wire suspended from a rigid support is gradually increased. Illustrate your answer with a suitable stress-strain graph.*

Stress-strain curve for a metallic wire. Fig. 9.6, shows a stress-strain curve for a metal wire which is gradually being loaded.

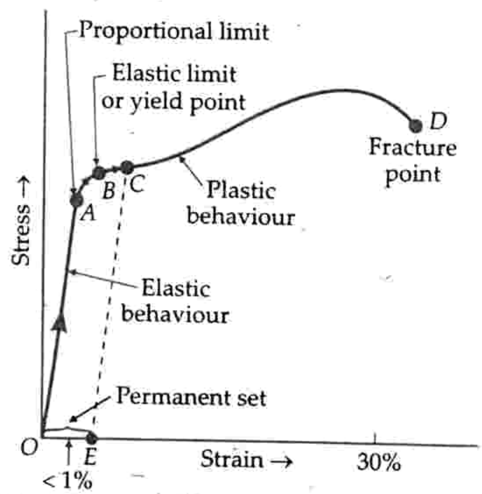


Fig. 9.6 A typical stress-strain curve for a ductile metal.

(*i*) The initial part OA of the graph is a straight line indicating that stress is proportional to strain. Upto the point A, Hooke's law is obeyed. The point A is called the ***proportional limit***. In this region, the wire is perfectly elastic.

(*ii*) After the point A, the stress is not proportional to strain and a curved portion AB is obtained. However, if the load is removed at any point between O and B, the curve is retraced along BAO and the wire attains its original length. The portion OB of the graph is called ***elastic region*** and the point B is called ***elastic limit*** or ***yield point***. The stress corresponding to the yield point is called ***yield strength*** (S_y). Upto point B, the elastic forces of the material are ***conservative*** *i.e.*, when the material returns to its original size, the work done in producing the deformation is completely recovered.

(*iii*) Beyond the point B, the strain increases more rapidly than stress. If the load is removed at any point C, the wire does not come back to its original length but traces dashed line CE. Even on reducing the stress to zero, a residual strain equal to OE is left in the wire. The material is said to have acquired a ***permanent set***. The fact that the stress-strain curve is not retaced on reversing the strain is called ***elastic hysteresis***.

(*iv*) If the load is increased beyond the point C, there is large increase in the strain or the length of the wire. In this region, the constrictions (called necks and waists) develop at few points along the length of the wire and the wire ultimately breaks at the point D, called the ***fracture point***. In the region between B and D, the length of wire goes on increasing even without any addition of load. This region is called *plastic region* and the material is said to undergo ***plastic flow*** or ***plastic deformation***. The stress corresponding to the breaking point is called ***ultimate strength*** or ***tensile strength*** of the material.

9.9 DETERMINATION OF YOUNG'S MODULUS OF THE MATERIAL OF A WIRE

11. *Explain an experiment for the determination of Young's modulus of the material of a wire.*

Experiment to determine the Young's modulus of the material of a wire. A simple experimental arrangement used for the determination of Young's modulus of the material of a wire is shown in Fig. 9.7. It consists of two long straight wires of same length and equal radius suspended side by side from a fixed rigid support. The wire A, called the *reference wire*, carries a main milliammeter scale M and below it a heavy fixed load. This load keeps the wire taught and free from kinks. The wire B, called the *experimental wire*, carries a vernier scale at its bottom. The vernier scale can slide against the main scale attached to the reference wire. A hanger is attached at the lower end of the vernier scale. Slotted half kg weights can be slipped into this hanger.

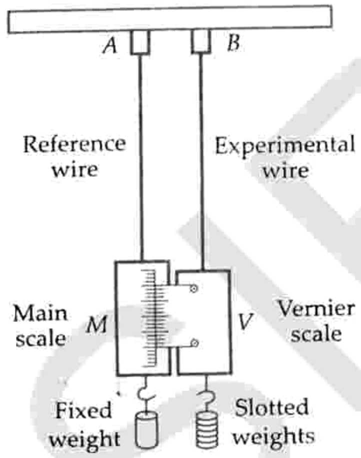


Fig. 9.7 Experimental arrangement for the determination of Young's modulus.

With the help of a screw gauge, the radius of the experimental wire is measured at several places. Let r be the initial average radius and L the initial length of the experimental wire. A small initial load, say 1 kg, is put on the hanger. This keeps the experimental wire straight and kink free. The vernier scale reading is noted. A half kg weight is added to the hanger. The wire is allowed to elongate for a minute. The vernier scale reading is again noted. The difference between the two vernier readings gives the extension produced due to the extra weight added. The weight is gradually increased in few steps and every time we note the extension produced.

A graph is plotted between the load applied and extension produced. It will be a straight line passing through the origin, as shown in Fig. 9.8.

Slope of the load-extension line

$$= \tan\theta = \frac{\Delta L}{Mg}$$

$$\text{Stress} = \frac{Mg}{\pi r^2}$$

$$\text{Strain} = \frac{\Delta L}{L}$$

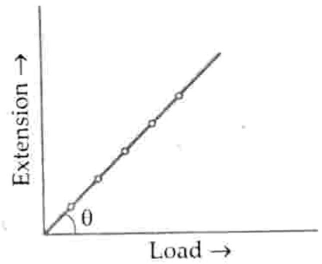


Fig. 9.8 Load-extension graph.

The Young's modulus of the material of the experimental wire will be

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{Mg}{\pi r^2} \cdot \frac{L}{\Delta L} = \frac{L}{\pi r^2 \tan\theta}$$

9.10 CLASSIFICATION OF MATERIALS ON THE BASIS OF STRESS-STRAIN CURVE

12. *Distinguish between ductile and brittle materials on the basis of stress-strain curve.*

(*i*) **Ductile materials.** *The materials which have large plastic range of extension are called ductile materials.* As shown in the stress-strain curve of Fig. 9.9, their fracture point is widely separated from the elastic limit. Such materials undergo an irreversible increase in length before snapping. So they can be drawn into thin wires. For example, copper, silver, iron, aluminium, etc.

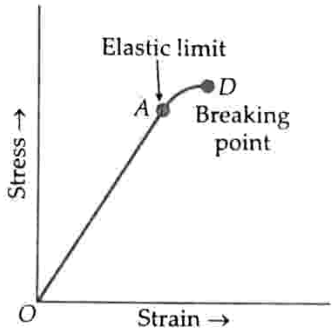


Fig. 9.9 Stress-strain curve for a brittle material.

(*ii*) **Brittle materials.** *The materials which have very small range of plastic extension are called brittle materials.* Such materials break as soon as the stress is increased beyond the elastic limit. Their breaking point lies just close to their elastic limit, as shown in Fig. 9.9. For example, cast iron, glass, ceramics, etc.

13. *Explain malleability on the basis of load-compression curve.*

Malleability. When a solid is compressed, a stage is reached beyond which it cannot recover its original shape after the deforming force is removed. This is the elastic limit (point A') for compression. The solid then behaves like a plastic body. The yield point (B') obtained under compression is called ***crushing point***. After this stage, metals are said to be malleable *i.e.*, they can be hammered or rolled into thin sheets. For example, gold, silver, lead, etc.

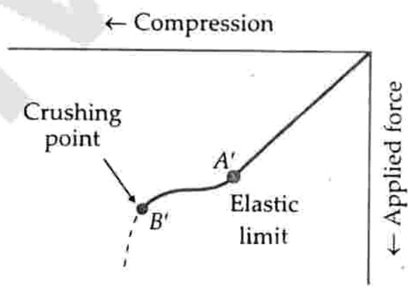


Fig. 9.10 Load-compression curve for a metal.

9.11 ELASTOMERS

14. *What are elastomers ? Give examples. Draw a stress-strain curve for an elastomer.*

Elastomers. *The materials which can be elastically stretched to large values of strain are called elastomers.* For example, rubber can be stretched to several times its original length but still it can regain its original length when the applied force is removed. There is no well defined plastic region, rubber just breaks when pulled beyond a certain limit. Its Young's modulus is very small, about 3×10^5 Nm^{-2} at slow strains. Elastic

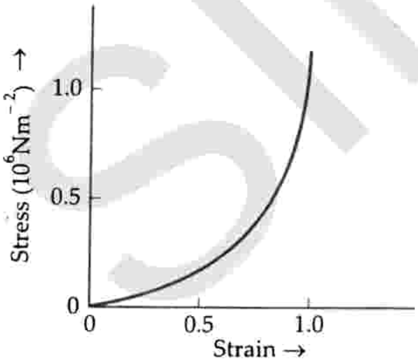


Fig. 9.11 Stress vs. strain curve for the elastic tissue of aorta.

region in such cases is very large, but the material does not obey Hooke's law. In our body, the elastic tissue of aorta (the large blood vessel carrying blood from the heart) is an elastomer, for which the stress-strain curve is shown in Fig. 9.11.

Table 9.1 Young's moduli, ultimate strengths and yield strengths of some materials

Substance	Density ρ (kgm^{-3})	Young's modulus Y ($10^9 Nm^{-2}$)	Ultimate strength S_u ($10^6 Nm^{-2}$)	Yield strength S_y ($10^6 Nm^{-2}$)
Aluminium	2710	70	110	95
Copper	8890	110	400	200
Iron (wrought)	7800-7900	190	330	170
Steel	7860	200	400	250
Glass	2190	65	50	–
Concrete	2320	30	40	–
Wood	525	13	50	–
Bone	1900	9	170	–
Polystyrene	1050	3	48	–

The above table shows that metals have large Young's moduli. Such materials require large forces to produce small changes in length *i.e.*, they are highly elastic. Thus steel is more elastic than copper, brass and aluminium. That is why steel is preferred for making heavy-duty machines and structural designs. On the other hand, the materials like wood, bone, concrete and glass have small Young's moduli.

Examples based on Young's Modulus

FORMULAE USED

1. Stress $= \dfrac{\text{Force}}{\text{Area}} = \dfrac{F}{A}$
2. Longitudinal strain $= \dfrac{\text{Change in length}}{\text{Original length}} = \dfrac{\Delta l}{l}$
3. Young's modulus $= \dfrac{\text{Longitudinal stress}}{\text{Longitudinal strain}}$ or $Y = \dfrac{F/A}{\Delta l / l} = \dfrac{F}{A} \cdot \dfrac{l}{\Delta l}$
4. Percentage increase in length, $\dfrac{\Delta l}{l} \times 100 = \dfrac{F}{AY} \times 100$

UNITS USED

Force F is in newton, area A in m^2, stress in Nm^{-2}, Young's modulus Y in Nm^{-2} or Pa, strain $\Delta l / l$ has no units

EXAMPLE 1. *The length of a suspended wire increases by 10^{-4} of its original length when a stress of 10^7 Nm^{-2} is applied on it. Calculate the Young's modulus of the material of the wire.* **[Delhi 03C, 05C]**

Solution. Strain $= \dfrac{\Delta l}{l} = 10^{-4}$, stress $= 10^7 \text{ Nm}^{-2}$

Young's modulus,

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{10^7 \text{ Nm}^{-2}}{10^{-4}} = \mathbf{10^{11} \text{ Nm}^{-2}}.$$

EXAMPLE 2. *A uniform wire of steel of length 2.5 m and density 8.0 gcm^{-3} weighs 50 g. When stretched by a force of 10 kgf, the length increases by 2 mm. Calculate Young's modulus of steel.*

Solution. Here $l = 2.5 \text{ m} = 250 \text{ cm}$,

$$\Delta l = 2 \text{ mm} = 0.2 \text{ cm},$$

$$F = 10 \text{ kg f} = 10 \times 9.8 \text{ N} = 10 \times 9.8 \times 10^5 \text{ dyne}$$

Mass = Volume × density

$$\therefore \quad A = \frac{\text{Mass}}{l \times \rho} = \frac{50}{250 \times 8} = 0.025 \text{ cm}^2$$

Young's modulus,

$$Y = \frac{F}{A} \cdot \frac{l}{\Delta l} = \frac{10 \times 9.8 \times 10^5 \times 250}{0.025 \times 0.2}$$

$$= \mathbf{4.9 \times 10^{11} \text{ dyne cm}^{-2}}.$$

EXAMPLE 3. *A structural steel rod has a radius of 10 mm and a length of 1 m. A 100 kN force F stretches it along its length. Calculate (a) the stress, (b) elongation, and (c) strain on the rod. Given that the Young's modulus, Y, of the structural steel is 2.0×10^{11} Nm^{-2}.* **[NCERT]**

Solution. Here $r = 10 \text{ mm} = 0.01 \text{ m}$, $l = 1 \text{ m}$, $F = 100 \text{ kN} = 10^5 \text{ N}$, $Y = 2.0 \times 10^{11} \text{ Nm}^{-2}$

(*a*) Stress $= \dfrac{F}{A} = \dfrac{F}{\pi r^2} = \dfrac{10^5 \text{ N}}{(22/7) \times (0.01 \text{ m})^2}$

$$= \mathbf{3.18 \times 10^8 \text{ Nm}^{-2}}.$$

(*b*) As $Y = \dfrac{F}{A} \cdot \dfrac{l}{\Delta l}$

$\therefore$ Elongation,

$$\Delta l = \frac{F}{A} \cdot \frac{l}{Y} = \frac{3.18 \times 10^8 \times 1}{2.0 \times 10^{11}}$$

$$= 1.59 \times 10^{-3} \text{ m} = \mathbf{1.59 \text{ mm}}.$$

(*c*) Strain $= \dfrac{\Delta l}{l} = \dfrac{1.59 \times 10^{-3} \text{ m}}{1 \text{ m}}$

$$= 1.59 \times 10^{-3} = \mathbf{0.16\%}.$$

EXAMPLE 4. *What is the percentage increase in the length of a wire of diameter 2.5 mm stretched by a force of 100 kg wt? Young's modulus of elasticity of the wire is 12.5×10^{11} dyne cm^{-2}.*

Solution. Given $r = 1.25 \text{ mm} = 0.125 \text{ cm}$,

$$F = 100 \times 9.8 = 980 \text{ N} = 98 \times 10^6 \text{ dyne}$$

$$Y = 12.5 \times 10^{11} \text{ dyne cm}^{-2}$$

As $Y = \dfrac{F}{A} \cdot \dfrac{l}{\Delta l}$ or $\dfrac{\Delta l}{l} = \dfrac{F}{AY} = \dfrac{F}{\pi r^2 Y}$

$\therefore$ The percentage increase in length is

$$\frac{\Delta l}{l} \times 100 = \frac{F \times 100}{\pi r^2 Y} = \frac{98 \times 10^6 \times 7 \times 100}{22 \times (0.125)^2 \times 12.5 \times 10^{11}}$$

$$= 15.965 \times 10^{-2} = \mathbf{0.16\%}.$$

EXAMPLE 5. *The breaking stress for a metal is 7.8×10^9 Nm^{-2}. Calculate the maximum length of the wire made of this metal which may be suspended without breaking. The density of the metal $= 7.8 \times 10^3$ $kg\ m^{-3}$. Take $g = 10$ $N\ kg^{-1}$.* **[Delhi 03]**

Solution. Breaking stress

= Maximum stress that the wire can withstand

$$= 7.8 \times 10^9 \text{ Nm}^{-2}.$$

When the wire is suspended vertically, it tends to break under its own weight. Let its length be l and cross-sectional area A.

Weight of wire $= mg =$ volume × density × $g = Al\rho g$

$$\text{Stress} = \frac{\text{Weight}}{A} = \frac{Al\rho g}{A} = l\rho g$$

For the wire not to break,

$$l\rho g = \text{Breaking stress} = 7.8 \times 10^9\ \text{Nm}^{-2}$$

$$\therefore \quad l = \frac{7.8 \times 10^9}{\rho g} = \frac{7.8 \times 10^9}{7.8 \times 10^3 \times 10} = \mathbf{10^5\ m.}$$

EXAMPLE 6. *A rubber string 10 m long is suspended from a rigid support at its one end. Calculate the extension in the string due to its own weight. The density of rubber is 1.5×10^3 kg m^{-3} and Young's modulus for the rubber is 5×10^6 Nm^{-2}. Take $g = 10$ N kg^{-1}.* **[Delhi 03]**

Solution. Let the area of cross-section of the string be A m^2. Then the weight of the string is

$$W = mg = \text{volume} \times \text{density} \times g$$
$$= 10\ A \times 1.5 \times 10^3 \times 10 = 1.5 \times 10^5 A\ \text{N}$$

Longitudinal stress

$$= \frac{W}{A} = 1.5 \times 10^5\ \text{Nm}^{-2}.$$

As the weight of the string acts on its centre of gravity, so it produces extension only in 5 m length of the string. If Δl be the extension in the string, then

$$\text{Longitudinal strain} = \frac{\Delta l}{l} = \frac{\Delta l}{5}$$

Young's modulus,

$$Y = \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}}$$

or $$5 \times 10^6 = \frac{1.5 \times 10^5}{\Delta l / 5}$$

$$\therefore \quad \Delta l = \frac{1.5 \times 10^5 \times 5}{5 \times 10^6} = \mathbf{0.15\ m.}$$

EXAMPLE 7. *A silica glass rod has a diameter of 1 cm and is 10 cm long. The ultimate strength of glass is 50×10^6 Nm^{-2}. Estimate the largest mass that can be hung from it without breaking it. Take $g = 10$ N kg^{-1}.*

Solution. Radius, $r = \frac{1}{2}$ cm $= 0.5 \times 10^{-2}$ m, ultimate strength $= 50 \times 10^6$ Nm^{-2}.

Let M be the largest mass that can be hung. Then

$$\text{Ultimate strength} = \frac{Mg}{\pi r^2}$$

or $$50 \times 10^6 = \frac{M \times 10}{3.14 \times (0.5 \times 10^{-2})^2}$$

or $$M = \frac{50 \times 10^6 \times 3.14 \times 0.25 \times 10^{-4}}{10}$$

$$= \mathbf{392.5\ kg.}$$

EXAMPLE 8. *A composite wire of uniform diameter 3.0 mm consisting of a copper wire of length 2.2 m and a steel wire of length 1.6 m stretches under a load by 0.7 mm. Calculate the load, given that the Young's modulus for copper is 1.1×10^{11} Pa and for steel is 2.0×10^{11} Pa.* **[NCERT ; Delhi 09]**

Solution. Here $r = \frac{3}{2}$ mm $= 1.5 \times 10^{-3}$ m,

$$l_c = 2.2\ \text{m},\ l_s = 1.6\ \text{m}$$
$$\Delta l_c + \Delta l_s = 0.7\ \text{mm} = 0.7 \times 10^{-3}\ \text{m}$$
$$Y_c = 1.1 \times 10^{11}\ \text{Pa},\ Y_s = 2.0 \times 10^{11}\ \text{Pa}$$

As same load (say F) is being applied on both the wires, which have same area of cross-section A, so stress is same for both wires.

But $$\text{Stress} = \frac{F}{A}$$

$$= \text{Young's modulus} \times \text{strain} = Y \times \frac{\Delta l}{l}$$

Now, Stress on copper wire = Stress on steel wire

$$\therefore \quad Y_c \times \frac{\Delta l_c}{l_c} = Y_s \times \frac{\Delta l_s}{l_s}$$

or $$\frac{\Delta l_c}{\Delta l_s} = \frac{Y_s \times l_c}{Y_c \times l_s} = \frac{2.0 \times 10^{11} \times 2.2}{1.1 \times 10^{11} \times 1.6} = 2.5$$

or $$\Delta l_c = 2.5 \Delta l_s$$

But $$\Delta l_c + \Delta l_s = 0.7 \times 10^{-3}\ \text{m}$$

or $$3.5\ \Delta l_s = 0.7 \times 10^{-3}\ \text{m}$$

or $$\Delta l_s = \frac{0.7 \times 10^{-3}}{3.5} = 2.0 \times 10^{-4}\ \text{m}$$

and $$\Delta l_c = 2.5 \times 2.0 \times 10^{-4} = 5.0 \times 10^{-4}\ \text{m}$$

$$\text{Load},\ F = A \times Y_c \times \frac{\Delta l_c}{l_c} = \pi r^2\ Y_c \times \frac{\Delta l_c}{l_c}$$

$$= \frac{22}{7} \times (1.5 \times 10^{-3})^2 \times 1.1 \times 10^{11} \times \frac{5.0 \times 10^{-4}}{2.2}$$

$$= \mathbf{176.8\ N.}$$

EXAMPLE 9. *The maximum stress that can be applied to the material of a wire used to suspend an elevator is 1.3×10^8 Nm^{-2}. If the mass of the elevator is 900 kg and it moves up with an acceleration of 2.2 ms^{-2}, what is the minimum diameter of the wire ?*

Solution. As the elevator moves up, the tension in the wire is

$$F = mg + ma = m(g + a)$$
$$= 900 \times (9.8 + 2.2) = 10{,}800\ \text{N}$$

$$\text{Stress in the wire} = \frac{F}{A} = \frac{F}{\pi r^2}$$

Clearly, when the stress is maximum, r is minimum.

$\therefore$ Maximum stress $= \dfrac{F}{\pi r_{min}^2}$

or $\quad r_{min}^2 = \dfrac{F}{\pi \times \text{Maximum stress}}$

$= \dfrac{10800}{3.14 \times 1.3 \times 10^8} = 0.2645 \times 10^{-4}$ m

or $\quad r_{min} = 0.5142 \times 10^{-2}$ m

Minimum diameter

$= 2\, r_{min} = 2 \times 0.5142 \times 10^{-2}$

$= \mathbf{1.0284 \times 10^{-2}}$ **m.**

Example 10. *A mass of 100 gram is attached to the end of a rubber string 49 cm long and having an area of cross-section 20 mm². The string is whirled round, horizontally at a constant speed of 40 rps in a circle of radius 51 cm. Find Young's modulus of rubber.*

Solution. When the mass is rotated at the end of the rubber string, the restoring force in the string is equal to the centripetal force.

$\therefore \quad F = mr\omega^2 = mr(2\pi\nu)^2$

$= 100 \times 51 \times (2 \times \pi \times 40)^2$ dyne

Also $\quad l = 49$ cm, $\Delta l = 51 - 49 = 2$ cm,

$A = 20\ \text{mm}^2 = 20 \times 10^{-2}\ \text{cm}^2$

Hence $\quad Y = \dfrac{F}{A} \cdot \dfrac{l}{\Delta l}$

$= \dfrac{100 \times 51 \times 4 \times 9.87 \times 1600 \times 49}{20 \times 10^{-2} \times 2} \quad [\pi^2 = 9.87]$

$= 3.95 \times 10^{10}$ dyne cm^{-2}

$= \mathbf{3.95 \times 10^9\ Nm^{-2}}$.

Example 11. *A uniform heavy rod of weight W, cross-sectional area A and length l is hanging from a fixed support. Young's modulus of the material of the rod is Y. Neglecting the lateral contraction, find the elongation produced in the rod.*

Solution. As shown in Fig. 9.12, consider a small element of thickness dx at distance x from the fixed support. Force acting on the element dx is

$F =$ Weight of length $(l - x)$ of the rod

$= \dfrac{W}{l}(l - x)$

Elongation of the element

$=$ Original length $\times \dfrac{\text{stress}}{Y}$

$= dx \times \dfrac{F/A}{Y} = \dfrac{W}{l\,Ay}(l - x)\,dx$

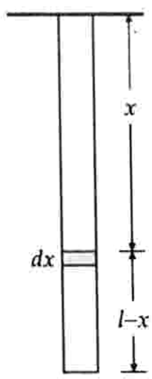


Fig. 9.12

Total elongation produced in the rod

$= \dfrac{W}{l\,AY}\displaystyle\int_0^l (l - x)\,dx = \dfrac{W}{l\,Ay}\left[lx - \dfrac{x^2}{2}\right]_0^l$

$= \dfrac{W}{l\,Ay}\left[l^2 - \dfrac{l^2}{2}\right] = \dfrac{Wl}{2Ay}$.

Example 12. *A steel wire of uniform cross-section of 1 mm² is heated to 70°C and stretched by tying its two ends rigidly. Calculate the change in the tension of the wire when the temperature falls from 70°C to 35°C. Coefficient of linear expansion of steel is 1.1×10^{-5}°C^{-1} and the Young's modulus is 2.0×10^{11} Nm^{-2}.*

Solution. Here $A = 1\ \text{mm}^2 = 10^{-6}\ \text{m}^2$,

$\Delta T = 70 - 35 = 35°$,

$\alpha = 1.1 \times 10^{-5}\ °\text{C}^{-1}$, $Y = 2.0 \times 10^{11}\ \text{Nm}^{-2}$

Increase in length $\Delta l = l\,\alpha\,\Delta T$

$\therefore$ Strain $= \dfrac{\Delta l}{l} = \alpha\,\Delta T = 1.1 \times 10^{-5} \times 35 = 38.5 \times 10^{-5}$

If T is the tension in the wire due to the decrease in temperature, then

Stress $= \dfrac{T}{A} = \dfrac{T}{10^{-6}}$ Nm^{-2}

But $\quad$ Stress $= Y \times$ Strain

$\therefore \quad \dfrac{T}{10^{-6}} = 2.0 \times 10^{11} \times 38.5 \times 10^{-5}$

or $\quad T = 2.0 \times 38.5 = \mathbf{77.0\ N}$.

Example 13. *In a human pyramid in a circus, the entire weight of the balanced group is supported by the legs of a performer who is lying on his back (as shown in Fig. 9.13). The combined mass of all the persons performing the act, and the tables, plaques etc. involved is 280 kg. The mass of the performer lying on his back at the bottom of the pyramid is 60 kg. Each thighbone (femur) of this performer has a length of 50 cm and an effective radius of 2.0 cm. Determine the amount by which each thighbone gets compressed under the extra load.* **[NCERT]**

Fig. 9.13

Solution. Total mass of all the performers, tables, plaques, etc.

$$=280 \text{ kg}$$

Mass of the performer $=60$ kg

Mass supported by the legs of the performer at the bottom of the pyramid,

$$M = 280 - 60 = 220 \text{ kg}$$

Weight of the supported mass,

$$W = Mg = 220 \times 9.8 = 2156 \text{ N}$$

Weight supported by each thighbone of the performer,

$$F = \frac{W}{2} = \frac{1}{2} \times 2156 = 1078 \text{ N}$$

Young's modulus of bone, $Y = 9.4 \times 10^9 \text{ Nm}^{-2}$

Length of each thighbone, $l = 0.5$ m

Radius of a thighbone, $r = 2.0 \text{ cm} = 2 \times 10^{-2}$ m

Cross-sectional area of the thighbone,

$$A = \pi r^2 = 3.14 \times (2 \times 10^{-2})^2$$
$$= 1.26 \times 10^{-3} \text{ m}^2$$

Compression produced in each thighbone of the performer,

$$\Delta l = \frac{F}{A} \cdot \frac{l}{Y} = \frac{1078 \times 0.5}{1.26 \times 10^{-3} \times 9.4 \times 10^9} \text{ m}$$
$$= 4.55 \times 10^{-5} \text{ m} = \mathbf{4.55 \times 10^{-3}} \textbf{ cm.}$$

✖ PROBLEMS FOR PRACTICE

1. A wire increases by 10^{-3} of its length when a stress of $1 \times 10^8 \text{ Nm}^{-2}$ is applied to it. What is the Young's modulus of the material of the wire ? **[Delhi 98]** (**Ans.** 10^{11} Nm^{-2})
2. What force is required to stretch a steel wire 1 cm^2 in cross-section to double its length ? Given $Y = 2 \times 10^{11} \text{ Nm}^{-2}$. (**Ans.** 2×10^7 N)
3. Find the stress to be applied to a steel wire to stretch it by 0.025% of its original length. Y for steel is $9 \times 10^{10} \text{ Nm}^{-2}$. (**Ans.** $2.25 \times 10^7 \text{ Nm}^{-2}$)
4. A steel wire of length 4 m and diameter 5 mm is stretched by 5 kg-wt. Find the increase in its length, if the Young's modulus of steel wire is 2.4×10^{-12} dyne cm^{-2}. **[Delhi 05]** (**Ans.** 0.0041 cm)
5. Two wires made of the same material are subjected to forces in the ratio of 1 : 4. Their lengths are in the ratio 8 : 1 and diameter in the ratio 2 : 1. Find the ratio of their extensions. (**Ans.** 1 : 2)
6. A wire elongates by 9 mm when a load of 10 kg is suspended from it. What is the elongation when its radius is doubled, if all other quantities are same as before ? (**Ans.** 2.25 mm)
7. The breaking stress of aluminium is $7.5 \times 10^7 \text{ Nm}^{-2}$. Find the greatest length of aluminium wire that can hang vertically without breaking. Density of aluminium is $2.7 \times 10^3 \text{ kg m}^{-3}$. (**Ans.** 2.83×10^3 m)
8. A steel wire of length 5.0 m and cross-section $3.0 \times 10^{-5} \text{ m}^2$ stretches by the same amount as a copper wire of length 3.0 m and cross-section $4.0 \times 10^{-5} \text{ m}^2$ under a given load. What is the ratio of Young's modulus of steel to that of copper ? **[Punjab 91]** (**Ans.** 2.22)
9. A stress of 1 kg mm^{-2} is applied to a wire of which Young's modulus is 10^{11} Nm^{-2}. Find the percentage increase in length. (**Ans.** 0.0098%)
10. Two exactly similar wires of steel and copper are stretched by equal forces. If the total elongation is 1 cm, find by how much is each wire elongated ? Given Y for steel $= 20 \times 10^{11}$ dyne cm^{-2} and Y for copper $= 12 \times 10^{11}$ dyne cm^{-2}. (**Ans.** 0.375 cm and 0.625 cm)
11. Two parallel steel wires A and B are fixed to rigid support at the upper ends and subjected to the same load at the lower ends. The lengths of the wires are in the ratio 4 : 5 and their radii are in the ratio 4 : 3. The increase in the length of the wire A is 1 mm. Calculate the increase in the length of the wire B. (**Ans.** 2.22 mm)
12. Two wires of equal cross-section but one made of steel and the other copper are joined end to end. When the combination is kept under tension, the elongation in the two wires is found to be equal. Given Young's moduli of steel and copper are $2.0 \times 10^{11} \text{ Nm}^{-2}$ and $1.1 \times 10^{11} \text{ Nm}^{-2}$. Find the ratio between the lengths of steel and copper wires. (**Ans.** 20 : 11)
13. A lift is tied with thick iron wires and its mass is 1000 kg. If the maximum acceleration of lift is 1.2 ms^{-2} and the maximum safe stress is $1.4 \times 10^8 \text{ Nm}^{-2}$, find the minimum diameter of the wire. Take $g = 9.8 \text{ ms}^{-2}$. (**Ans.** 0.01 m)
14. The length of a metal wire is l_1 when the tension in it is T_1 and l_2 when the tension in it is T_2. Find the original length of the wire. $\left(\textbf{Ans. } \dfrac{T_2 l_1 - T_1 l_2}{T_2 - T_1}\right)$
15. A metal bar of length l and area of cross-section A is rigidly clamped between two walls. The Young's modulus of the material is Y and the coefficient of linear expansion is α. The bar is heated so that its temperature is increased by ΔT. Find the force exerted at the ends of the bar. (**Ans.** $YA\,\alpha\,\Delta T$)

✖ HINTS

1. Stress $= 10^8\ \text{Nm}^{-2}$, Strain $= \dfrac{\Delta l}{l} = 10^{-3}$

Young's modulus, $Y = \dfrac{\text{Stress}}{\text{Strain}} = \dfrac{10^8}{10^{-3}} = \mathbf{10^{11}\ Nm^{-2}}$.

2. Here $l = \Delta l = x$ (say),

$A = 1\ \text{cm}^2 = 10^{-4}\ \text{m}^2,\ Y = 2 \times 10^{11}\ \text{Nm}^{-2}$.

As $Y = \dfrac{F}{A} \cdot \dfrac{l}{\Delta l}$

$$F = \frac{YA \cdot \Delta l}{l} = \frac{2 \times 10^{11} \times 10^{-4} \times x}{x} = \mathbf{2 \times 10^7\ N.}$$

3. Here $\Delta l = \dfrac{0.025}{100} l$ or $\dfrac{\Delta l}{l} = \dfrac{0.025}{100}$

$$\text{Stress} = Y \times \text{strain} = 9 \times 10^{10} \times \frac{0.025}{100}$$

$$= \mathbf{2.25 \times 10^7\ Nm^{-2}}.$$

4. $$\Delta l = \frac{Fl}{\pi r^2 Y} = \frac{(5000 \times 980) \times 400}{3.14 \times (0.025)^2 \times 2.4 \times 10^{12}} = \mathbf{0.0041\ cm.}$$

5. $Y = \dfrac{F}{\pi r^2} \cdot \dfrac{l}{\Delta l}$ or $\Delta l = \dfrac{Fl}{\pi r^2 Y}$

Both wires are of same material, so their Y is same.

$$\therefore \quad \frac{\Delta l_1}{\Delta l_2} = \frac{F_1}{F_2} \cdot \frac{l_1}{l_2} \cdot \frac{r_2^2}{r_1^2} = \frac{1}{4} \times \frac{8}{1} \times \frac{1}{4} = \mathbf{1:2.}$$

6. $Y = \dfrac{F}{\pi r^2} \cdot \dfrac{l}{\Delta l}$ or $r^2 \Delta l = \dfrac{Fl}{\pi Y}$ = a constant

$$\therefore \quad r_1^2 \Delta l_1 = r_2^2\, \Delta l_2$$

Given $\Delta l_1 = 9$ mm, $r_2 = 2r_1$

$$\therefore \ r_1^2 \times 9\ \text{mm} = 4r_1^2 \times \Delta l_2 \quad \text{or} \quad \Delta l_2 = \frac{9}{4} = \mathbf{2.25\ mm.}$$

12. $Y_s = \dfrac{F}{A} \cdot \dfrac{l_s}{\Delta l_s}$ and $Y_c = \dfrac{F}{A} \cdot \dfrac{l_c}{\Delta l_c}$

$$\therefore \quad \frac{Y_s}{Y_c} = \frac{l_s}{l_c} \cdot \frac{\Delta l_c}{\Delta l_s} = \frac{l_s}{l_c} \qquad [\because \Delta l_c = \Delta l_s]$$

or $$\frac{l_s}{l_c} = \frac{Y_s}{Y_c} = \frac{2.0 \times 10^{11}}{1.1 \times 10^{11}} = \mathbf{20:11.}$$

13. Tension in the wire,

$$F = m(g + a) = 1000\,(9.8 + 1.2) = 11{,}000\ \text{N}$$

$$\text{Stress} = \frac{F}{A} = \frac{F}{\pi (d/2)^2} = \frac{4F}{\pi d^2}$$

or $$d^2 = \frac{4F}{\pi \times \text{stress}} = \frac{4 \times 11000 \times 7}{22 \times 1.4 \times 10^8} = 10^{-4}$$

or $$d = 10^{-2}\ \text{m} = \mathbf{0.01\ m.}$$

14. Let l be the original length and A the area of cross-section of the wire.

Change in length in first case $= l_1 - l$

Change in length in second case $= l_2 - l$

$$Y = \frac{T_1}{A} \cdot \frac{l}{l_1 - l} = \frac{T_2}{A} \cdot \frac{l}{l_2 - l}$$

or $$T_1 l_2 - T_1 l = T_2 l_1 - T_2 l$$

or $$l(T_2 - T_1) = T_2 l_1 - T_1 l_2 \quad \text{or} \quad l = \frac{T_2 l_1 - T_1 l_2}{T_2 - T_1}$$

15. Change in length, $\Delta l = l\, \alpha\, \Delta T$

$$\therefore \ Y = \frac{F}{A} \cdot \frac{l}{\Delta l} = \frac{F}{A} \cdot \frac{l}{l\, \alpha\, \Delta T} \quad \text{or} \quad F = YA\, \alpha\, \Delta T.$$

9.12 BULK MODULUS OF ELASTICITY

15. *Define bulk modulus of elasticity. Give its units and dimensions.*

Bulk modulus of elasticity. *Within the elastic limit, the ratio of normal stress to the volumetric strain is called bulk modulus of elasticity.*

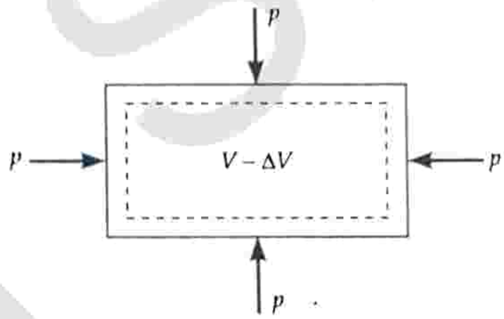


Fig. 9.14 Bulk modulus of elasticity.

Consider a body of volume V and surface area A. Suppose a force F acts uniformly over the whole surface of the body and it decreases the volume by ΔV as shown in Fig. 9.14. Then bulk modulus of elasticity is given by

$$\kappa = \frac{\text{Normal stress}}{\text{Volumetric strain}} = -\frac{F/A}{\Delta V / V}$$

or $$\kappa = -\frac{F}{A} \cdot \frac{V}{\Delta V} = -\frac{pV}{\Delta V}$$

where $p\,(= F/A)$ is the normal pressure. Negative sign shows that the volume decreases with the increase in stress.

Units and dimensions of κ. The SI unit of bulk modulus is Nm^{-2} or Pascal (Pa) and its CGS unit is dyne cm^{-2}. Its dimensional formula is $[\text{ML}^{-1}\text{T}^{-2}]$.

16. *Define the term compressibility. Give its units and dimensions.*

Compressibility. *The reciprocal of the bulk modulus of a material is called its compressibility.*

$$\text{Compressibility} = \frac{1}{\kappa}$$

SI unit of compressibility $= \text{N}^{-1}\text{m}^2$.

CGS unit of compressibility $= \text{dyne}^{-1}\text{cm}^2$.

The dimensional formula of compressibility is $[\text{M}^{-1}\text{LT}^2]$.

Table 9.2 Bulk moduli (κ) of some common materials

Material	κ (10^9 Nm^{-2})
Solids	
Aluminium	72
Brass	61
Copper	140
Glass	37
Iron	100
Nickel	260
Steel	160
Liquids	
Water	2.2
Ethanol	0.9
Carbon disulphide	1.56
Glycerine	4.76
Mercury	25
Gases	
Air (at STP)	1.0×10^{-4}

The above table shows that bulk moduli of the solids are in the range of 10^{11} Nm^{-2}, and are about 50 times larger than that of water. Thus solids are least compressible while gases are most compressible. Gases are about a million times more compressible than solids. The solids are incompressible because of tight coupling between the neighbouring atoms. The molecules in liquids are less tightly bound than in liquids. The molecules in gases are very poorly coupled to the neighbouring molecules.

Examples based on Bulk Modulus

Formulae Used

1. Volumetric stress $= \dfrac{F}{A} = p$, the applied pressure
2. Volumetric strain $= \dfrac{\Delta V}{V}$
3. Bulk modulus $= \dfrac{\text{Volumetric stress}}{\text{Volumetric strain}}$

   or $\kappa = -\dfrac{F/A}{\Delta V/V} = -\dfrac{p}{\Delta V/V} = -V\dfrac{p}{\Delta V}$

   Negative sign indicates the decrease in volume with the increase in stress
4. Compressibility $= \dfrac{1}{\kappa} = -\dfrac{\Delta V}{pV}$

Units Used

Bulk modulus κ is in Nm^{-2} and compressibility in $N^{-1}m^2$ or Pa^{-1}.

Example 14. The pressure of a medium is changed from 1.01×10^5 Pa to 1.165×10^5 Pa and change is volume is 10%, keeping temperature constant. Find the bulk modulus of the medium. **[IIT 05]**

Solution. Here : $p = 1.165 \times 10^5 - 1.01 \times 10^5$

$= 0.155 \times 10^5$ Pa

$\dfrac{\Delta V}{V} = 10\% = 0.1$

Bulk modulus of the medium,

$$\kappa = \frac{p}{\Delta V/V} = \frac{0.155 \times 10^5}{0.1} = \mathbf{1.55 \times 10^5 Pa.}$$

Example 15. The average depth of Indian ocean is about 3000 m Calculate the fractional compression $\Delta V/V$, of water at the bottom of the ocean, given that the bulk modulus of water is 2.2×10^9 Nm^{-2}. **[NCERT]**

Solution. Stress = Pressure exerted by a water column of height 3000 m

$= h\rho g = 3000 \text{ m} \times 1000 \text{ kg m}^{-3} \times 10 \text{ ms}^{-2}$

$= \mathbf{3 \times 10^7 \ Nm^{-2}}$

As bulk modulus, $\kappa = \dfrac{\text{Stress}}{\Delta V/V}$

∴ Fractional compression,

$$\frac{\Delta V}{V} = \frac{\text{Stress}}{\kappa} = \frac{3 \times 10^7 \text{ Nm}^{-2}}{2.2 \times 10^9 \text{ Nm}^{-2}}$$

$= 1.36 \times 10^{-2} = \mathbf{1.36\%}$.

Example 16. A sphere contracts in volume by 0.01%, when taken to the bottom of sea 1 km deep. Find the bulk modulus of the material of the sphere. Density of sea water may be taken as 1.0×10^3 kgm^{-3}.

Solution. Here $\dfrac{\Delta V}{V} = \dfrac{0.01}{100}$, $h = 1 \text{ km} = 10^3$ m,

$\rho = 1.0 \times 10^3$ kg m^{-3}

$p = h\rho g = 10^3 \times 1.0 \times 10^3 \times 9.8 = 9.8 \times 10^6$ Nm^{-2}

$$\kappa = \frac{p}{\Delta V/V} = \frac{9.8 \times 10^6 \times 100}{0.01} = \mathbf{9.8 \times 10^{10} \ Nm^{-2}}.$$

Example 17. If the normal density of sea water is 1.00 g cm^{-3}, what will be its density at a depth of 3 km ? Given compressibility of water = 0.0005 per atmosphere, 1 atmospheric pressure = 10^6 dyne cm^{-2}, $g = 980$ cms^{-2}.

Solution. $\kappa = \dfrac{1}{\text{Compressibility}} = \dfrac{1}{0.00005}$

$= 2 \times 10^4$ atm $= 2 \times 10^4 \times 10^6$

$= 2 \times 10^{10}$ dyne cm^{-2}

$p = h\rho g = 3 \times 10^5 \times 1 \times 980$

$= 294 \times 10^6$ dyne cm^{-2}.

[∵ $h = 3$ km $= 3 \times 10^5$ cm, ρ (water) = 1 g cm^{-3}]

As $\kappa = \dfrac{pV}{\Delta V}$

$$\therefore \quad \Delta V = \frac{pV}{\kappa} = \frac{294 \times 10^6 \times 1}{2 \times 10^{10}}$$

$$= 1.47 \times 10^{-2} \text{ cm}^3 \qquad [\because V = 1 \text{ cm}^3]$$

Volume of 1 g of water at a depth of 3 km,

$$V' = V - \Delta V = 1 - 1.47 \times 10^{-2}$$

$$= 0.9853 \text{ cm}^3$$

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} = \frac{1 \text{ g}}{0.9853 \text{ cm}^3}$$

$$= \mathbf{1.0149 \text{ g cm}^{-3}}.$$

EXAMPLE 18. *A solid cube is subjected to a pressure of 5×10^5 Nm^{-2}. Each side of the cube is shortened by 1%. Find volumetric strain and bulk modulus of elasticity of the cube.*

Solution. Let l be the initial length of each side of cube.

Final length of the cube

$$= l - 1\% \text{ of } l = \left(1 - \frac{1}{100}\right) l$$

Initial volume,

$$V_i = l^3 = V \text{ (say)}$$

Final volume,

$$V_f = \left(1 - \frac{1}{100}\right)^3 l^3 = \left(1 - \frac{1}{100}\right)^3 V$$

Change in volume,

$$\Delta V = V_f - V_i = V\left[\left(1 - \frac{1}{100}\right)^3 - 1\right]$$

Volumetric strain

$$= \frac{\Delta V}{V} = \left(1 - \frac{1}{100}\right)^3 - 1 \simeq \left[1 - 3 \times \frac{1}{100}\right] - 1$$

$$[(1-x)^n \simeq 1 - nx \text{ for } x << 1]$$

$$= -\frac{3}{100} = \mathbf{0.03}.$$

Normal stress = Applied pressure = 5×10^5 Nm^{-2}

Bulk modulus,

$$\kappa = \frac{\text{Normal stress}}{\text{Volumetric strain}}$$

$$= \frac{5 \times 10^5}{0.03} = \mathbf{1.67 \times 10^7 \text{ Nm}^{-2}}$$

EXAMPLE 19. *Calculate the pressure required to stop the increase in volume of a copper block when it is heated from 50° to 70°C. Coefficient of linear expansion of copper $= 8.0 \times 10^{-6}\,{}^\circ C^{-1}$ and bulk modulus of elasticity $= 3.6 \times 10^{11}$ Nm^{-2}.*

Solution. When a block of volume V is heated through a temperature of ΔT, the change in volume is

$$\Delta V = \gamma V \Delta T$$

where $\gamma (= 3\alpha)$ is the coefficient of cubical expansion.

$$\therefore \text{ Volume strain } = \frac{\Delta V}{V} = \gamma \Delta T$$

$$\text{Bulk modulus, } \kappa = \frac{p}{\Delta V / V} = \frac{p}{\gamma \Delta T}$$

Pressure, $p = \kappa \gamma \Delta T$

Here $\kappa = 3.6 \times 10^{11}$ Nm^{-2},

$\gamma = 3\alpha = 3 \times 8.0 \times 10^{-6} = 24 \times 10^{-6}\,{}^\circ\text{C}^{-1}$,

$\Delta T = 70 - 50 = 20°\text{C}$

$$\therefore \quad p = 3.6 \times 10^{11} \times 24 \times 10^{-6} \times 20$$

$$= \mathbf{1.728 \times 10^8 \text{ Nm}^{-2}}.$$

PROBLEMS FOR PRACTICE

1. A solid sphere of radius 10 cm is subjected to a uniform pressure $= 5 \times 10^8$ Nm^{-2}. Determine the consequent change in volume. Bulk modulus of the material of the sphere is equal to 3.14×10^{11} Nm^{-2}. (**Ans.** 6.67×10^{-6} m^3)

2. Find the change in volume which 1 m^3 of water will undergo when taken from the surface to the bottom of a lake 100 m deep. Given volume elasticity of water is 22,000 atmosphere. (**Ans.** 4.4×10^{-4} m^3)

3. A solid ball 300 cm in diameter is submerged in a lake at such a depth that the pressure exerted by water is 1.00 kgf cm^{-2}. Find the change in volume of the ball at this depth. κ for material of the ball $= 1.00 \times 10^{13}$ dyne cm^{-2}. (**Ans.** 1.385 cm^3)

4. A spherical ball contracts in volume by 0.0098% when subjected to a pressure of 100 atm. Calculate its bulk modulus. Given 1 atm $= 1.01 \times 10^5$ Nm^{-2}. (**Ans.** 1.033×10^{11} Nm^{-2})

5. What increase in pressure will be needed to decrease the volume of 1.0 m^3 of water by 10 c.c. ? The bulk modulus of water is 0.21×10^{10} Nm^{-2}. (**Ans.** 2.1×10^4 Nm^{-2})

6. Determine the fractional change in volume as the pressure of the atmosphere (1.0×10^5 Pa) around a metal block is reduced to zero by placing the block in vacuum. The bulk modulus for the block is 1.25×10^{11} Nm^{-2}. (**Ans.** 8×10^{-7})

7. Find the density of the metal under a pressure of 20,000 N cm^{-2}. Given density of the metal = 11 g cm^{-3}, bulk modulus of the metal $= 8 \times 10^9$ Nm^{-2}. (**Ans.** 11.28 g cm^{-3})

8. The compressibility of water is 4×10^{-5} per unit atmospheric pressure. What will be the decrease in volume of 100 cm^3 of water under pressure of 100 atmosphere ? (**Ans.** 0.4 cm^3)

9. On taking a solid ball of rubber from the surface to the bottom of a lake of 180 m depth, the reduction of the volume of the ball is 0.1%. The density of water of the lake is 1.0×10^3 kg m^{-3}. Determine the value of the bulk modulus of elasticity of rubber. Take $g = 10$ ms^{-2}. (**Ans.** 1.8×10^9 Nm^{-2})

10. A uniform pressure P is exerted on all sides of a solid cube at temperature t °C. By what amount should the temperature of the cube be raised in order to bring its volume back to the volume it had before the pressure was applied, if the bulk modulus and coefficient of volume expansion of the material are κ and γ respectively ? $\left(\textbf{Ans. } \frac{p}{\gamma \kappa}\right)$

11. A solid sphere of radius R made of a material of bulk modulus κ is surrounded by a liquid in a cylindrical container. A massless piston of area A floats on the surface of the liquid. When a mass M is placed on the piston to compress the liquid, find fractional change in the radius of the sphere. **[IIT 88]** $\left(\textbf{Ans. } \frac{\Delta R}{R} = \frac{Mg}{3 A \kappa}\right)$

✖ HINTS

1. $\Delta V = \frac{pV}{\kappa} = \frac{p \times \frac{4}{3}\pi r^3}{\kappa} = \frac{5 \times 10^8 \times 4 \times 3.14 \times (0.1)^3}{3 \times 3.14 \times 10^{11}}$

$= \mathbf{6.67 \times 10^{-6}\ m^3}$.

2. Here $V = 1\,m^3$, $h = 100$ m, $g = 9.8$ ms^{-2},

ρ (water) = 1000 kg m^{-3}.

$p = h\rho g = 100 \times 1000 \times 9.8 = 9.8 \times 10^5$ Nm^{-2}

$\kappa = 22{,}000$ atm $= 22{,}000 \times 1.013 \times 10^5$ Nm^{-2}

$= 22.286 \times 10^8$ Nm^{-2}

$$\Delta V = \frac{pV}{\kappa} = \frac{9.8 \times 10^5 \times 1}{22.286 \times 10^8} = \mathbf{4.4 \times 10^{-4}\ m^3}.$$

3. Here $\kappa = 1.00 \times 10^{13}$ dyne cm^{-2}, $r = 300/2 = 150$ cm

$V = \frac{4}{3}\pi r^3 = \frac{4}{3} \times 3.14 \times (150)^3 = 1.413 \times 10^7$ cm^3

$p = 1.00$ kg f cm^{-2} = 1000 g f cm^{-2}

$= 1000 \times 980$ dyne cm^{-2}

$$\Delta V = \frac{pV}{\kappa} = \frac{1000 \times 980 \times 1.413 \times 10^7}{1.00 \times 10^{13}} = \mathbf{1.385\ cm^3}.$$

4. Here $\frac{\Delta V}{V} = \frac{0.0098}{100}$,

$p = 100$ atm $= 100 \times 1.01 \times 10^5$ Nm^{-2}

$$\therefore \quad \kappa = p \times \frac{V}{\Delta V} = \frac{100 \times 1.01 \times 10^5 \times 100}{0.0098}$$

$= \mathbf{1.033 \times 10^{11}\ Nm^{-2}}$.

5. Here $V = 1.0\,m^3$,

$\Delta V = 10$ c.c. $= 10 \times 10^{-6}\,m^3 = 10^{-5}\,m^3$

$\kappa = 0.21 \times 10^{10}$ Nm^{-2}

$$\therefore \quad p = \kappa \times \frac{\Delta V}{V} = \frac{0.21 \times 10^{10} \times 10^{-5}}{1.0}$$

$= \mathbf{2.1 \times 10^4\ Nm^{-2}}$.

6. $\frac{\Delta V}{V} = \frac{p}{\kappa} = \frac{1.0 \times 10^5}{1.25 \times 10^{11}} = \mathbf{8 \times 10^{-7}}$.

7. Here $p = 2 \times 10^4$ N $cm^{-2} = 2 \times 10^8$ Nm^{-2},

$\kappa = 8 \times 10^9$ Nm^{-2}

$$\Delta V = \frac{pV}{\kappa} = \frac{2 \times 10^8 \times V}{8 \times 10^9} = \frac{V}{40}$$

Final volume,

$$V' = V - \Delta V = V - \frac{V}{40} = \frac{39V}{40}$$

As the mass of the metal remains constant, so

$m = V\rho = V'\rho'$

or $V \times 11 = \frac{39\,V}{40} \times \rho'$

or $\rho' = \frac{40 \times 11}{39} = \mathbf{11.28\ gcm^{-3}}$.

8. $\kappa = \frac{1}{\text{Bulk modulus}} = \frac{1}{4 \times 10^{-5}} = 0.25 \times 10^5$ atm

$= 0.25 \times 10^5 \times 1.013 \times 10^5$ Nm^{-2}

$= 2.533 \times 10^9$ Nm^{-2}

$V = 100\,cm^3 = 10^{-4}\,m^3$,

$p = 100$ atm $= 100 \times 1.013 \times 10^5$

$= 1.013 \times 10^7$ Nm^{-2}

$$\Delta V = \frac{pV}{\kappa} = \frac{1.013 \times 10^7 \times 10^{-4}}{2.533 \times 10^9}$$

$= 0.4 \times 10^{-6}\,m^3 = \mathbf{0.4\ cm^3}$.

9. Here $h = 180$ m, $\rho = 1.0 \times 10^3$ kgm^{-3}, $g = 10$ ms^{-2}

$p = h\rho g = 180 \times 1.0 \times 10^3 \times 10 = 1.8 \times 10^6$ Nm^{-2}

Volume strain = 0.1% $= \frac{0.1}{100} = 10^{-3}$

$$\kappa = \frac{p}{\text{Volume strain}} = \frac{1.8 \times 10^6}{10^{-3}} = \mathbf{1.8 \times 10^9\ Nm^{-2}}.$$

10. As $\Delta V = \gamma V \Delta T$ $\therefore$ $\frac{\Delta V}{V} = \gamma\, \Delta T$

$$\kappa = \frac{p}{\Delta V / V} = \frac{p}{\gamma \Delta T} \quad \text{or} \quad \Delta T = \frac{p}{\gamma K}.$$

11. When mass M is placed on the piston, the excess pressure, $p = mg/A$. This pressure acts equally from all directions on the sphere. The volume of the sphere decreases due to the decrease in its radius.

As $V = \frac{4}{3}\pi R^3$

$\therefore$ Fractional decrease in volume, $\frac{\Delta V}{V} = 3\frac{\Delta R}{R}$

Also $\kappa = \frac{p}{\Delta V / V}$ or $\frac{\Delta V}{V} = \frac{p}{\kappa} = \frac{Mg}{A\kappa}$

Hence $3\frac{\Delta R}{R} = \frac{Mg}{A\kappa}$ or $\frac{\Delta R}{R} = \frac{Mg}{3A\kappa}$.

9.13 MODULUS OF RIGIDITY OR SHEAR MODULUS

17. *Define modulus of rigidity. Give its units and dimensions.*

Modulus of rigidity or shear modulus. *Within the elastic limit, the ratio of tangential stress to shear strain is called modulus of rigidity.*

As shown in Fig. 9.15, consider a rectangular block whose lower face is fixed and a tangential force F is applied over its upper face of area A. An equal and opposite force F comes into play on its lower fixed face. The two equal and opposite forces form a couple which exerts a torque. As the lower face of the block is fixed, the couple shears the block into a parallelopied by displacing its upper face through distance $AA' = \Delta l$. Let $AB = DC = l$ and $\angle ABA' = \theta$.

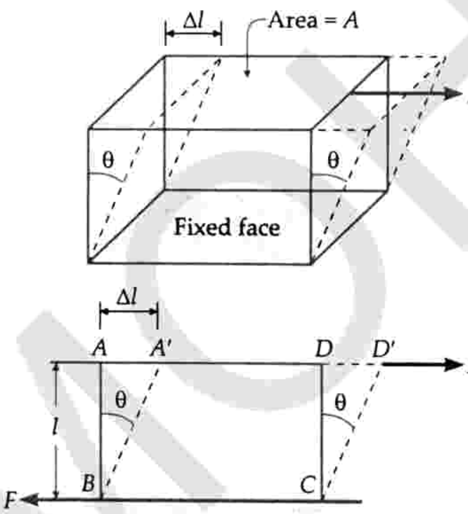


Fig. 9.15 Shear modulus of rigidity

Tangential stress $= \frac{F}{A}$

Shear strain $= \theta \simeq \tan\theta = \frac{AA'}{AB} = \frac{\Delta l}{l}$

The modulus of rigidity is given by

$$\eta = \frac{\text{Tangential stress}}{\text{Shear strain}} = \frac{F/A}{\theta} = \frac{F}{A\theta} = \frac{F}{A}\cdot\frac{l}{\Delta l}$$

Units and dimensions of η. The SI unit of modulus of rigidity is Nm^{-2} and its CGS unit is dyne cm^{-2}. Its dimensional formula is $[ML^{-1}T^{-2}]$.

18. *The shear modulus of a material is always considerably smaller than the Young modulus for it. What does it signify ?*

η of a material is smaller than its Y. This shows that it is easier to slide layers of atoms of solids over one another than to pull them apart or to squeeze them close together.

Table 9.3 Shear moduli (η) of some common materials

Material	η **($10^9 Nm^{-2}$ *or GPa*)**
Aluminium	25
Brass	36
Copper	42
Glass	23
Iron	70
Lead	5.6
Nickel	77
Steel	84
Tungsten	150
Wood	10

If compare the values of η of Table 9.3 with the values of Y of Table 9.1, we see that, in general, the shear modulus is less than Young's modulus. For most of the materials, $\eta = Y/3$.

For Your Knowledge

- Elastic deformations in all bodies become plastic deformations with time.
- As only solids have length and shape, Young's modulus and shear modulus are relevant only for solids.
- As solids, liquids and gases all have volume elasticity, bulk modulus is relevant for all three states of matter.
- Metals have large values of Young's modulus than alloys and elastomers. A material with large Y requires a large force to produce small changes in length.
- Elastic has a different meaning in physics than that in daily life. In daily life, a material which stretches more is said to be more elastic, but it is a misnomer. In physics, a material which stretches to a lesser extent for a given load is considered to be more elastic.

Examples based on Modulus of Rigidity

FORMULAE USED

1. Shearing stress $= \dfrac{\text{Tangential Force}}{\text{Area}} = \dfrac{F}{A}$
2. Shearing strain $= \theta = \dfrac{\Delta l}{l}$
3. Modulus of rigidity $= \dfrac{\text{Shearing stress}}{\text{Shearing strain}}$

or $\quad \eta = \dfrac{F/A}{\theta} = \dfrac{F/A}{\Delta l / l}$

UNITS USED

Modulus of rigidity η is in Nm^{-2} or Pa^{-1}.

EXAMPLE 20. *A cube of aluminium of each side 4 cm is subjected to a tangential (shearing) force. The top face of the cube is sheared through 0.012 cm with respect to the bottom face. Find (i) shearing strain (ii) shearing stress and shearing force. Given $\eta = 2.08 \times 10^{11}$ dyne cm^{-2}.*

Solution. Here $l = 4$ cm, $\Delta l = 0.012$ cm,

$$\eta = 2.08 \times 10^{11} \text{ dyne cm}^{-2}$$

(*i*) Shearing strain,

$$\theta = \frac{\Delta l}{l} = \frac{0.012}{4} = \mathbf{0.003\ rad.}$$

(*ii*) Area of top face

$$= l^2 = (4\text{ cm})^2 = 16 \text{ cm}^2$$

Modulus of rigidity, $\eta = \dfrac{\text{Shearing stress}}{\text{Shearing strain}}$

$\therefore$ Shearing stress

$= \eta \times$ Shearing strain

$= 2.08 \times 10^{11} \times 0.003 = \mathbf{6.24 \times 10^8\ dyne\ cm^{-2}}$.

Shearing force,

F = Shearing stress $\times$ area

$= 6.24 \times 10^8 \times 16 = \mathbf{9.984 \times 10^9\ dyne.}$

EXAMPLE 21. *A square lead slab of side 50 cm and thickness 10 cm is subjected to a shearing force (on its narrow face) of magnitude 9.0×10^4 N. The lower edge is riveted to the floor. How much is the upper edge displaced, if the shear modulus of lead is 5.6×10^9 Pa ?* **[NCERT]**

Solution. Here $l = 50$ cm $= 0.50$ m, $F = 9.0 \times 10^4$ N

$$\eta = 5.6 \times 10^9 \text{ Pa}$$

Area of the face on which force is applied,

$A = 50 \text{ cm} \times 10 \text{ cm}$

$= 500 \text{ cm}^2 = 500 \times 10^{-4} \text{ m}^2 = 0.05 \text{ m}^2$

If Δl is the distance through which the upper edge is displaced relative to the lower fixed edge, then

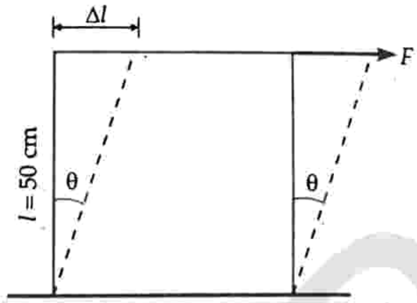


Fig. 9.16

$$\eta = \frac{F/A}{\Delta l / l} = \frac{F}{A} \cdot \frac{l}{\Delta l}$$

or $\quad \Delta l = \dfrac{F}{A} \cdot \dfrac{l}{\eta} = \dfrac{9.0 \times 10^4 \times 0.50}{0.05 \times 5.6 \times 10^9}$

$= \mathbf{1.6 \times 10^{-4}\ m = 0.16\ mm.}$

EXAMPLE 22. *A rubber block 1 cm $\times$ 3 cm $\times$ 10 cm is clamped at one end with its 10 cm side vertical. A horizontal force of 30 N is applied to the free surface. What is the horizontal displacement of the top face ? Modulus of rigidity of rubber $= 1.4 \times 10^5$ Nm^{-2}.*

Solution. Area of the upper face,

$A = 1 \text{ cm} \times 3 \text{ cm} = 3 \times 10^{-4} \text{ m}^2$

$F = 30$ N, $\eta = 1.4 \times 10^5 \text{ Nm}^{-2}$,

$l = 10 \text{ cm} = 0.10$ m

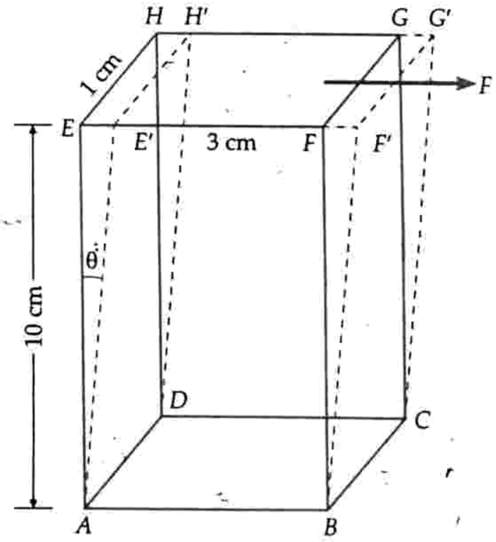


Fig. 9.17

As $\quad \eta = \dfrac{F}{A} \cdot \dfrac{l}{\Delta l}$

$\therefore \quad \Delta l = \dfrac{F}{A} \cdot \dfrac{l}{\eta} = \dfrac{30 \times 0.10}{3 \times 10^{-4} \times 1.4 \times 10^5}$

$= \dfrac{1}{14} = 0.0714 \text{ m} = \mathbf{7.14\ cm.}$

EXAMPLE 23. A 60 kg motor rests on four cylindrical rubber blocks. Each cylinder has a height of 3 cm and a cross-sectional area of 15 cm². The shear modulus for this rubber is 2×10^6 Nm^{-2}. If a sideways force of 300 N is applied to the motor, how far will it move sideways ?

Solution. Tangential force on each block,

$$F = (1/4) \times 300 = 75 \text{ N}, \quad l = 3 \text{ cm} = 3 \times 10^{-2} \text{ m},$$

$$A = 15 \text{ cm}^2 = 15 \times 10^{-4} \text{ m}^2, \quad \eta = 2 \times 10^6 \text{ Nm}^{-2}$$

As $\quad \eta = \dfrac{F/A}{\Delta l / l} = \dfrac{F}{A} \cdot \dfrac{l}{\Delta l}$

$\therefore \quad \Delta l = \dfrac{F}{A} \cdot \dfrac{l}{\eta} = \dfrac{75 \times 3 \times 10^{-2}}{15 \times 10^{-4} \times 2 \times 10^6}$

$= 7.5 \times 10^{-4}$ m $=$ **0.075 cm.**

✗ PROBLEMS FOR PRACTICE

1. A metallic cube whose each side is 10 cm is subjected to a shearing force of 100 kg f. The top face is displaced through 0.25 cm with respect to the bottom. Calculate the shearing stress, strain and shear modulus.
   (**Ans.** 9.8×10^4 Nm^{-2}, 0.025 rad, 3.92×10^6 Nm^{-2})

2. An Indian rubber cube of side 7 cm has one side fixed, while a tangential force equal to the weight of 200 kilogram is applied to the opposite face. Find the shearing strain produced and distance through which the strained side moves. Modulus of rigidity for rubber is 2×10^7 dyne cm^{-2}.
   (**Ans.** 0.2 radian, 1.4 cm)

3. A metal cube of side 10 cm is subjected to a shearing stress of 10^4 Nm^{-2}. Calculate the modulus of rigidity if the top of the cube is displaced by 0.05 cm with respect to its bottom. (**Ans.** 2×10^6 Nm^{-2})

4. Two parallel and opposite forces, each 4000 N, are applied tangentially to the upper and lower faces of a cubical metal block 25 cm on a side. Find the angle of shear and the displacement of the upper surface relative to the lower surface. The shear modulus for the metal is 8×10^{10} Nm^{-2}.
   (**Ans.** 8.0×10^{-7} rad, 2.0×10^{-7} m)

✗ HINTS

1. Here $l = 10$ cm $= 0.10$ m, $F = 100$ kg f $= 100 \times 9.8$ N,
   $\Delta l = 0.25$ cm $= 0.25 \times 10^{-2}$ m

   Shearing stress

   $$= \frac{F}{A} = \frac{F}{l^2} = \frac{100 \times 9.8}{0.10 \times 0.10} = \mathbf{9.8 \times 10^4 \ Nm^{-2}}$$

   Shearing strain

   $$= \frac{\Delta l}{l} = \frac{0.25 \times 10^{-2}}{0.10} = \mathbf{0.025 \ rad}$$

   Shear modulus,

   $$\eta = \frac{\text{Shearing stress}}{\text{Shearing strain}} = \frac{9.8 \times 10^4}{0.025} = \mathbf{3.92 \times 10^6 \ Nm^{-2}}.$$

2. Here $l = 7$ cm, $F = 200$ kg f $= 200 \times 1000 \times 981$ dyne,
   $\eta = 2 \times 10^7$ dyne cm^{-2}

   Area of the free face,

   $$A = l^2 = 7 \text{ cm} \times 7 \text{ cm} = 49 \text{ cm}^2$$

   As $\quad \eta = \dfrac{F}{A\theta}$

   $\therefore \quad \theta = \dfrac{F}{A\eta} = \dfrac{200 \times 1000 \times 981}{49 \times 2 \times 10^7} = \mathbf{0.2 \ rad.}$

   $$\Delta l = l\theta = 7 \times 0.2 = \mathbf{1.4 \ cm.}$$

3. Here $l = 10$ cm, $\Delta l = 0.05$ cm

   Shearing stress $= 10^4$ Nm^{-2}

   Shearing strain $= \dfrac{\Delta l}{l} = \dfrac{0.05}{10} = 0.005$

   $\therefore \quad \eta = \dfrac{\text{Shearing stress}}{\text{Shearing strain}} = \dfrac{10^4}{0.005} = \mathbf{2 \times 10^6 \ Nm^{-2}}.$

4. Shearing stress $= \dfrac{F}{l^2} = \dfrac{4000}{0.25 \times 0.25} = 64000$ Nm^{-2}

   Shearing strain,

   $$\theta = \frac{\text{Shearing stress}}{\text{Shear modulus}} = \frac{64000}{8 \times 10^{10}} = \mathbf{8 \times 10^{-7} \ rad.}$$

   $$\Delta l = l\theta = 0.25 \times 8 \times 10^{-7} = \mathbf{2.0 \times 10^{-7} \ m.}$$

9.14 SOME OTHER ELASTIC EFFECTS

19. *What is elastic after effect ? What is its importance ?*

Elastic after effect. The bodies return to their original state on the removal of the deforming force. Some bodies return to their original state immediately after the removal of the deforming force while some bodies take longer time to do so. *The delay in regaining the original state by a body on the removal of the deforming force is called elastic after effect.*

In galvanometers, suspensions made from quartz or phosphor-bronze alloy are used because their elastic after effect is small. On the contrary, a glass fibre takes hours to regain its original state.

20. *What is elastic fatigue ? What is its importance ?*

Elastic fatigue. As shown in Fig. 9.18, in a torsion pendulum, a disc oscillates in a horizontal plane. The elastic twist of the suspension wire provides the restoring torque. During torsional vibrations, the wire is subjected to repeated alternating strains. If we set the wire into torsional vibrations, it will continue vibrating for a long time before its vibrations die out. If it is again made to vibrate, its vibrations will die out in a lesser time. Due to continuous alternating strains, the wire is said to have been *tired* or *fatigued*.

Elastic fatigue is defined as loss in the strength of a material caused due to repeated alternating strains to which the material is subjected.

A hard wire can be broken by bending it repeatedly in opposite directions, as it loses strength due to elastic fatigue. For the same reason, the railway bridges are declared unsafe after a reasonably good period to avoid the risk of a mishap.

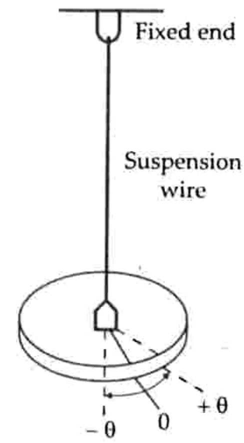


Fig. 9.18 Oscillations of a torsion pendulum.

21. *Describe elastic hysteresis. Mention its few applications.*

Elastic hysteresis. Fig. 9.19 shows the stress-strain curve for a rubber sample when loaded and then unloaded. For increasing load, the stress-strain curve is OAB and for decreasing load, the curve is BCO. *The fact that the stress-strain curve is not retraced on reversing the strain is known as elastic hysteresis.*

The area under the curve OAB represents the work done per unit volume in stretching the rubber. The area under BCO represents the energy given up by rubber on unloading. So the shaded area of the hysteresis loop represents the energy lost as heat during the loading-unloading cycle.

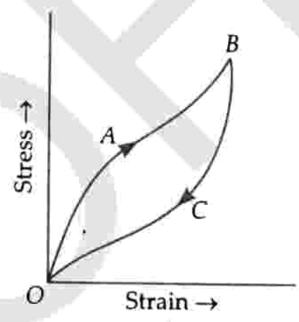


Fig. 9.19

Applications of elastic hysteresis :

(*i*) Car tyres are made with synthetic rubbers having small-area hysteresis loops because a car tyre of such a rubber will not get excessively heated during the journey.

(*ii*) A padding of vulcanized rubber having large-area hysteresis loop is used in shock absorbers between the vibrating system and the flat board. As the rubber is compressed and released during each vibration, it dissipates a large amount of vibrational energy.

9.15 APPLICATIONS OF ELASTICITY

22. *Why is any metallic part of a machinery never subjected to a stress beyond the elastic limit ?*

Any metallic part of a machinery is never subjected to a stress beyond the elastic limit. This is because a stress beyond elastic limit will permanently deform that metallic part.

23. *How is the knowledge of elasticity useful in selecting metal ropes used in cranes for lifting heavy loads ?*

The thickness of metallic ropes used in cranes to lift heavy loads is decided from the knowledge of the elastic limit of the material and the factor of safety. Suppose a crane having steel ropes is required to lift load of ten ton *i.e.*, 10^4 kg. The rope is usually designed for a safety factor of 10 *i.e.*, it should not break even when a load of $10^4 \times 10 = 10^5$ kg is applied to it. If r is the radius of the rope, then

$$\text{Ultimate stress} = \frac{F}{A} = \frac{Mg}{\pi r^2} = \frac{10^5 \times 9.8}{\pi r^2}$$

The ultimate stress should not exceed the elastic limit ($= 30 \times 10^7$ Nm^{-2}) for steel.

$$\therefore \quad \frac{10^5 \times 9.8}{\pi r^2} = 30 \times 10^7 \quad \text{or} \quad r = 0.032 \text{ m} = 3.2 \text{ cm}.$$

A single wire of this much radius would be a rigid rod. For the ease in manufacture and to impart flexibility and strength to the rope, it is always made of a large number of thin wires braided together.

24. *Explain why should the beams used in the construction of bridges have large depth and small breadth.*

Or

Explain why are girders given I *shape.*

The knowledge of elasticity is applied in designing a bridge such that it does not bend too much or break under the load of traffic, the force of wind and under its own weight. Consider a rectangular bar of length l, breadth b and thickness d supported at both ends, as shown in Fig. 9.20. When a load W is suspended at its middle, the bar gets depressed by an amount given by

$$\delta = \frac{Wl^3}{4Y\,bd^3}$$

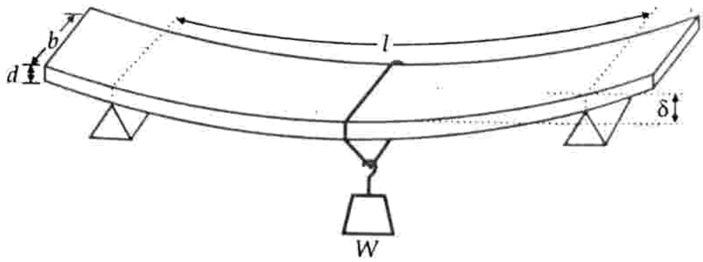


Fig. 9.20

Bending can be reduced by using a material with a large Young's modulus Y. As δ is proportional to d^{-3} and only to b^{-1}, *so depression can be decreased more effectively by increasing the depth d rather than the breadth b*. But a deep bar has a tendency to bend under the weight of a moving traffic, as shown in Fig. 9.21(*b*). This bending is called ***buckling***. Hence a better choice is to have a bar of I-shaped cross-section, as shown in Fig. 9.21(*c*). This section provides a large load bearing surface and enough depth to prevent bending. Also, this shape reduces the weight of the beam without sacrificing its strength and hence reduces the cost.

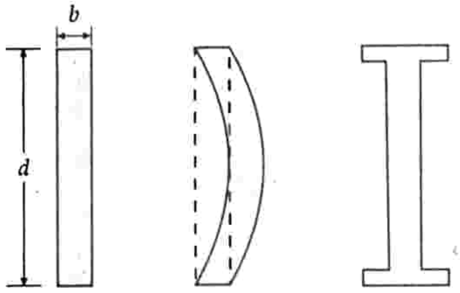


Fig. 9.21. (*a*) Rectangular cross-section of bar, (*b*) Buckling of a deep bar, (*c*) I-shaped cross-section of a bar.

25. *How can the knowledge of elasticity be used to estimate the maximum height of a mountain on earth ?*

The maximum height of mountain on earth depends upon shear modulus of rock. At the base of the mountain, the stress due to all the rock on the top should be less than the critical shear stress at which the rock begins to flow. Suppose the height of the mountain is h and the density of its rock is ρ. Then force per unit area (due to the weight of the mountain) at the base $= h\rho g$. The material at the base experiences this force per unit area in the vertical direction, but sides of the mountain are free. Hence there is a tangential shear of the order of $h\rho g$. The elastic limit for a typical rock is about 3×10^8 Nm^{-2} and its density is 3×10^3 kg m^{-3}. Hence

$$h_{max}\, \rho g = 3 \times 10^8$$

or

$$h_{max} = \frac{3 \times 10^8}{\rho g} = \frac{3 \times 10^8}{3 \times 10^3 \times 9.8}$$

$$\simeq 10{,}000 \text{ m} = 10 \text{ km}$$

This is nearly the height of the Mount Everest. A height greater than this will not be able to withstand the shearing stress due to the weight of the mountain.

26. *Explain why hollow shafts are preferred to solid shafts for transmitting torque.*

A hollow shaft is stronger than a solid shaft made of equal quantity of same material. The torque required to produce unit twist in a solid shaft of radius r, length l and made of material of modulus of rigidity η is given by

$$\tau = \frac{\pi \eta r^4}{2l}$$

The torque required to produce a unit twist in a hollow shaft of internal and external radii r_1 and r_2 is given by

$$\tau' = \frac{\pi \eta (r_2^4 - r_1^4)}{2l}$$

$$\therefore \quad \frac{\tau'}{\tau} = \frac{r_2^4 - r_1^4}{r^4} = \frac{(r_2^2 + r_1^2)(r_2^2 - r_1^2)}{r^4}$$

If the two shafts are made from equal amounts of materials, then

$$\pi r^2 l = \pi (r_2^2 - r_1^2)\, l \quad \text{or} \quad r_2^2 - r_1^2 = r^2$$

$$\therefore \quad \frac{\tau'}{\tau} = \frac{r_2^2 + r_1^2}{r^2}$$

As $\quad r^2 = r_2^2 - r_1^2$

so $\quad r_2^2 + r_1^2 > r^2 \quad$ and hence $\tau' > \tau$.

Thus torque required to twist hollow cylinder through a certain angle is greater than the torque necessary to twist a solid cylinder of same mass, length and material through the same angle. Hence a *hollow shaft is stronger than a solid shaft*. For this reason, electric poles are given hollow structures.

9.16 ELASTIC POTENTIAL ENERGY OF A STRETCHED WIRE

27. *What is meant by elastic potential energy ? Derive an expression for the elastic potential energy of stretched wire. Prove that its elastic energy density is equal to $\frac{1}{2}$ stress × strain.*

Elastic potential energy. When a wire is stretched, interatomic forces come into play which oppose the change. Work has to be done against these restoring forces. The work done in stretching the wire is stored in it as its elastic potential energy.

Expression for elastic potential energy. Suppose a force F applied on a wire of length l increases its length by Δl. Initially, the internal restoring force in the wire is zero. When the length is increased by Δl, the internal force increases from zero to F (= applied force).

$\therefore$ Average internal force for an increase in length Δl of wire

$$= \frac{0 + F}{2} = \frac{F}{2}$$

Work done on the wire is

$$W = \text{Average force} \times \text{increase in length} = \frac{F}{2} \times \Delta l$$

This work done is stored as elastic potential energy U in the wire.

$$\therefore \quad U = \frac{1}{2} F \times \Delta l$$

$$= \frac{1}{2} \text{ Stretching force} \times \text{increase in length}$$

Let A be the area of cross-section of the wire. Then

$$U = \frac{1}{2} \frac{F}{A} \times \frac{\Delta l}{l} \times Al$$

$$= \frac{1}{2} \text{ Stress} \times \text{strain} \times \text{volume of wire}$$

Elastic potential energy per unit volume of the wire or elastic energy density is

$$u = \frac{U}{\text{Volume}}$$

or $$u = \frac{1}{2} \text{ stress} \times \text{strain}$$

But stress = Young's modulus × strain

$$\therefore \quad u = \frac{1}{2} \text{ Young's modulus} \times \text{strain}^2$$

Examples based on Elastic Potential Energy

FORMULAE USED

1. Total P.E. stored in a stretched wire,

$$U = \frac{1}{2} \text{ Stretching force} \times \text{extension} = \frac{1}{2} F \Delta l$$

or $$U = \frac{1}{2} \text{ Stress} \times \text{strain} \times \text{volume of wire}$$

2. P.E. stored per unit volume of a stretched wire,

$$u = \frac{1}{2} \text{ Stress} \times \text{strain}$$

or $$u = \frac{1}{2} \text{ Young's modulus} \times \text{strain}^2$$

UNITS USED

Elastic P.E. is in joule, elastic P.E. per unit volume is in Jm^{-3}.

EXAMPLE 24. *A steel wire of 4.0 m is stretched through 2.0 mm. The cross-sectional area of the wire is 2.0 mm^2. If Young's modulus of steel is 2.0×10^{11} Nm^{-2}, find (i) the energy density of the wire and (ii) the elastic potential energy stored in the wire.*

Solution. Here $l = 4.0$ m,

$\Delta l = 2.0 \text{ mm} = 2.0 \times 10^{-3}$ m,

$A = 2.0 \text{ mm}^2 = 2.0 \times 10^{-6} \text{ m}^2$, $Y = 2.0 \times 10^{11} \text{ Nm}^{-2}$

(*i*) Energy density,

$$u = \frac{1}{2} Y \times (\text{strain})^2 = \frac{1}{2} Y \times \left(\frac{\Delta l}{l}\right)^2$$

$$= \frac{1}{2} \times 2 \times 10^{11} \times \left[\frac{2 \times 10^{-3}}{4.0}\right]^2 = \mathbf{2.5 \times 10^4 \ Jm^{-3}}.$$

(*ii*) Elastic potential energy,

$$U = \text{Energy density} \times \text{volume}$$

$$= u \times A \times l = 2.5 \times 10^4 \times 2.0 \times 10^{-6} \times 4$$

$$= \mathbf{0.2 \ J}.$$

EXAMPLE 25. *Calculate the increase in energy of a brass bar of length 0.2 m and cross-sectional area 1 cm^2 when compressed with a load of 5 kg weight along its length. Young's modulus of brass $= 1.0 \times 10^{11}$ Nm^{-2} and $g = 9.8$ ms^{-2}.*

Solution. Increase in the energy of the bar,

$$U = \frac{1}{2} \times \text{Stretching force} \times \text{extension} = \frac{1}{2} F \times \Delta l$$

As $$Y = \frac{F}{A} \cdot \frac{l}{\Delta l} \quad \therefore \Delta l = \frac{F}{A} \cdot \frac{l}{Y}$$

Hence $$U = \frac{1}{2} F \times \frac{F \cdot l}{AY} = \frac{F^2 l}{2 \ AY}$$

Here $F = 5 \text{ kg wt} = 5 \times 9.8 = 49$ N,

$l = 0.2$ m, $A = 1 \text{ cm}^2 = 10^{-4} \text{ m}^2$,

$Y = 1.0 \times 10^{11} \text{ Nm}^{-2}$

$$\therefore \quad U = \frac{(49)^2 \times 0.2}{2 \times 10^{-4} \times 1.0 \times 10^{11}} = \mathbf{2.4 \times 10^{-5} \ J}.$$

EXAMPLE 26. *When the load on a wire is increased from 3 kg wt to 5 kg wt, the elongation increases from 0.61 mm to 1.02 mm. How much work is done during the extension of the wire ?*

Solution. Work done in stretching the wire through 0.61 mm under the load of 3 kg wt,

$$W_1 = \frac{1}{2} \text{ Stretching force} \times \text{extension}$$

$$= \frac{1}{2} \times 3 \times 9.8 \times 0.61 \times 10^{-3} = 8.967 \times 10^{-3} \text{ J}$$

Work done in stretching the wire through 1.02 mm under the load of 5 kg wt,

$$W_2 = \frac{1}{2} \times 5 \times 9.8 \times 1.02 \times 10^{-3} = 24.99 \times 10^{-3} \text{ J}$$

Hence the work done in stretching the wire from 0.61 mm to 1.02 mm,

$$\Delta W = W_2 - W_1 = (24.99 - 8.967) \times 10^{-3}$$

$$= \mathbf{16.023 \times 10^{-3} \ J}.$$

EXAMPLE 27. *A 40 kg boy whose leg bones are 4 cm^2 in area and 50 cm long falls through a height of 50 cm without breaking his leg bones. If the bones can stand a stress of 0.9×10^8 Nm^{-2}, calculate the Young's modulus for the material of the bone. Take $g = 10$ ms^{-2}.*

Solution. Here $m = 40$ kg, $h = 2$ m, $l = 0.50$ m,

$$A = 4 \times 10^{-4} \text{ m}^2$$

Volume of leg $= Al = 4 \times 10^{-4} \times 0.50$

$$= 2 \times 10^{-4} \text{ m}^3$$

Loss in gravitational P.E.

$$= \text{Gain in elastic P.E. by both legs}$$

$$mgh = 2 \times \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume}$$

$$40 \times 10 \times 2 = 2 \times \frac{1}{2} \times 0.9 \times 10^8 \times \text{strain} \times 2 \times 10^{-4}$$

or $$\text{strain} = \frac{40 \times 10 \times 2}{0.9 \times 2 \times 10^4} = \frac{2}{45}$$

Young's modulus,

$$Y = \frac{\text{stress}}{\text{strain}} = \frac{0.9 \times 10^8 \times 45}{2}$$

$$= \mathbf{2.025 \times 10^9\ Nm^{-2}}.$$

PROBLEMS FOR PRACTICE

1. A steel wire of length 2.0 m is stretched through 2.0 mm. The cross-sectional area of the wire is 4.0 mm^2. Calculate the elastic potential energy stored in the wire in the stretched condition. Young's modulus of steel is 2.0×10^{11} Nm^{-2}. (**Ans.** 0.8 J)
2. If the Young's modulus of steel is 2×10^{11} Nm^{-2}, calculate the work done in stretching a steel wire 100 cm in length and of cross-sectional area 0.03 cm^2 when a load of 20 kg is slowly applied without the elastic limit being reached. (**Ans.** 0.032 J)
3. The limiting stress for a typical human bone is 0.9×10^8 Nm^{-2} while Young's modulus is 1.4×10^{10} Nm^{-2}. How much energy can be absorbed by two legs (without breaking) if each has a typical length of 50 cm and an average cross-sectional area of 5 cm^2 ? (**Ans.** 144.7 J)

HINTS

1. $$\text{Strain} = \frac{\Delta l}{l} = \frac{2.0 \times 10^{-3}}{2.0} = 10^{-3}$$

$$\text{Stress} = Y \times \frac{\Delta l}{l} = 2.0 \times 10^{11} \times 10^{-3} = 2.0 \times 10^8\ Nm^{-2}$$

Volume of the wire

$$= Al = 4.0 \times 10^{-6} \times 2.0 = 8.0 \times 10^{-6}\ m^3$$

$$U = \frac{1}{2} \text{Stress} \times \text{strain} \times \text{volume}$$

$$= \frac{1}{2} \times 2.0 \times 10^8 \times 10^{-3} \times 8.0 \times 10^{-6} = \mathbf{0.8\ J}.$$

2. $$\text{Work done} = \frac{1}{2} \text{Stretching force} \times \text{extension}$$

$$= \frac{1}{2} F\Delta l = \frac{1}{2} F.\frac{Fl}{Ay}$$

$$= \frac{F^2 l}{2\,AY} = \frac{(20 \times 9.8)^2 \times 1.00}{2 \times 0.03 \times 10^{-4} \times 2 \times 10^{11}}$$

$$= \mathbf{0.032\ J}.$$

3. Limiting stress $= 0.9 \times 10^8\ Nm^{-2}$,

$$Y = 1.4 \times 10^{10}\ Nm^{-2}$$

Length of both the legs, $l = 2 \times 50 = 100$ cm $= 1.0$ m,

$A = 5\ cm^2 = 5 \times 10^{-4}\ m^2$

Stretching force,

$$F = \text{Stress} \times \text{area} = 0.9 \times 10^8 \times 5 \times 10^{-4}$$

$$= 4.5 \times 10^4\ N$$

As $$Y = \frac{\text{Stress}}{\Delta l / l}$$

$\therefore$ $$\Delta l = \frac{\text{Stress} \times l}{Y} = \frac{0.9 \times 10^8 \times 1.0}{1.4 \times 10^{10}}$$

$$= 6.43 \times 10^{-3}\ m$$

Elastic P.E.,

$$U = \frac{1}{2} F \times \Delta l = \frac{1}{2} \times 4.5 \times 10^4 \times 6.43 \times 10^{-3}$$

$$= \mathbf{144.7\ J}.$$

9.17 POISSON'S RATIO

28. *Define Poisson's ratio. Write an expression for it. What is the significance of negative sign in this expression ?*

Poisson's ratio. When a wire is loaded, its length increases but its diameter decreases. The strain produced in the direction of applied force is called longitudinal strain and that produced in the perpendicular direction is called lateral strain.

Within the elastic limit, the ratio of lateral strain to the longitudinal strain is called Poisson's ratio.

Suppose the length of the loaded wire increases from l to $l + \Delta l$ and its diameter decreases from D to $D - \Delta D$.

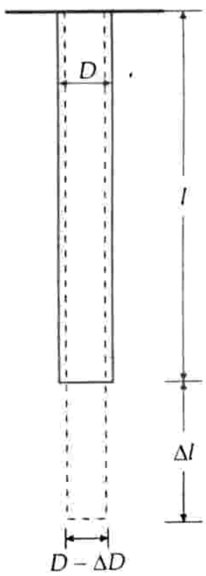


Fig. 9.22 Poisson's ratio.

$$\text{Longitudinal strain} = \frac{\Delta l}{l}$$

$$\text{Lateral strain} = -\frac{\Delta D}{D}$$

Poisson's ratio is

$$\sigma = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

$$= \frac{-\Delta D / D}{\Delta l / l}$$

or $$\sigma = -\frac{l}{D} \cdot \frac{\Delta D}{\Delta l}$$

The negative sign indicates that longitudinal and lateral strains are in opposite sense.

As the Poisson's ratio is the ratio of two strains, it has no units and dimensions.

For Your Knowledge

- For all substances, the theoretical value of σ lies between -1 and $+0.5$. In actual practice, the value of σ lies between 0 and 0.5 for most of the substances.
- **Relations between Y, K, η and σ**

(i) $Y = 3\kappa(1-2\sigma)$ (ii) $Y = 2\eta(1+\sigma)$

(iii) $\sigma = \frac{3\kappa - 2\eta}{6\kappa + 2\eta}$ (iv) $\frac{9}{Y} = \frac{3}{\eta} + \frac{1}{\kappa}$

Examples based on Poisson's Ratio

FORMULA USED

$$\text{Poisson's ratio} = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

or $$\sigma = \frac{\Delta D/D}{\Delta l/l}$$

UNITS USED

Length l and diameter D are in metre, Poisson's ratio σ has no units.

EXAMPLE 28. *Determine the Poisson's ratio of the material of a wire whose volume remains constant under an external normal stress.*

Solution. Volume of a wire, $V = \pi \frac{D^2}{4} l$

As volume remains constant, the differentiation of the above equation gives

$$0 = \frac{\pi l}{4} 2D\,dD + \frac{\pi D^2}{4} . dl$$

or $$-2l dD = D dl \quad \text{or} \quad \frac{dD}{D} = -\frac{1}{2}\frac{dl}{l}$$

By definition, Poisson's ratio is

$$\sigma = \frac{-dD/D}{dl/l} = \frac{+\frac{1}{2}\frac{dl}{l}}{dl/l} = \frac{1}{2} = 0.5.$$

EXAMPLE 29. *One end of a nylon rope of length 4.5 m and diameter 6 mm is fixed to a free limb. A monkey weighing 100 N jumps to catch the free end and stays there. Find the elongation of the rope and the corresponding change in diameter. Given Young's modulus of nylon $= 4.8 \times 10^{11}$ Nm^{-2} and Poisson's ratio of nylon = 0.2.*

Solution. Here $l = 4.5$ m,

$$D = 6 \text{ mm} = 6 \times 10^{-3} \text{ m}, \quad F = 100 \text{ N},$$
$$Y = 4.8 \times 10^{11} \text{ Nm}^{-2}, \quad \sigma = 0.2$$

As $$Y = \frac{F}{A}.\frac{l}{\Delta l}$$

$$\therefore \quad \Delta l = \frac{F}{A}.\frac{l}{Y} = \frac{100 \times 4.5}{3.14 \times (3 \times 10^{-3})^2 \times 4.8 \times 10^{11}}$$

$$= \mathbf{3.32 \times 10^{-5} \ m.}$$

Poisson's ratio,

$$\sigma = \frac{\Delta D/D}{\Delta l/l} = \frac{\Delta D}{D}.\frac{l}{\Delta l}$$

$$\therefore \quad \Delta D = \frac{\sigma D \Delta l}{l} = \frac{0.2 \times 6 \times 10^{-3} \times 3.32 \times 10^{-5}}{4.5}$$

$$= \mathbf{8.8 \times 10^{-9} \ m.}$$

EXAMPLE 30. *A material has Poisson's ratio 0.5. If a uniform rod of it suffers a longitudinal strain of 2×10^{-3}, what is the percentage increase in volume ?*

Solution. Longitudinal strain,

$$\frac{\Delta l}{l} = 2 \times 10^{-3}$$

Poisson's ratio, $\sigma = 0.5$

As $$\sigma = \frac{\text{Lateral strain}}{\text{Longitudinal strain}} = \frac{-\Delta R/R}{\Delta l/l}$$

$$\therefore \quad \frac{\Delta R}{R} = -\sigma\frac{\Delta l}{l} = -0.5 \times 2 \times 10^{-3} = -1 \times 10^{-3}$$

Volume of rod,

$$V = \pi R^2 l$$

Percentage increase in volume is

$$\frac{\Delta V}{V} \times 100 = \left(2\frac{\Delta R}{R} + \frac{\Delta l}{l}\right) \times 100$$

$$= [2 \times (-1) \times 10^{-3} + 2 \times 10^{-3}] \times 100 = \mathbf{0.}$$

PROBLEMS FOR PRACTICE

1. Calculate the Poisson's ratio for silver. Given its Young's modulus $= 7.25 \times 10^{10}$ Nm^{-2} and bulk modulus $= 11 \times 10^{10}$ Nm^{-2}. (**Ans.** 0.39)
2. A material has Poisson's ratio 0.2. If a uniform rod of it suffers longitudinal strain 4.0×10^{-3}, calculate the percentage change in its volume. (**Ans.** 0.24%)

HINTS

1. As $Y = 3\kappa(1-2\sigma)$

$$\therefore \quad \sigma = \frac{1}{2}\left(1 - \frac{Y}{3\kappa}\right) = \frac{1}{2}\left(1 - \frac{7.25 \times 10^{10}}{3 \times 11 \times 10^{10}}\right) = \mathbf{0.39}.$$

2. As $$\sigma = -\frac{\Delta R/R}{\Delta l/l}$$

$$\therefore \quad \frac{\Delta R}{R} = -\sigma\frac{\Delta l}{l} = -0.2 \times 4.0 \times 10^{-3}$$
$$= -0.8 \times 10^{-3}$$
$$V = \pi R^2 l$$

$$\therefore \quad \frac{\Delta V}{V} \times 100 = \left(2\frac{\Delta R}{R} + \frac{\Delta l}{l}\right) \times 100$$
$$= [2 \times (-0.8 \times 10^{-3}) + 4.0 \times 10^{-3}] \times 100$$
$$= 2.4 \times 10^{-3} \times 100 = \mathbf{0.24\%}.$$

Very Short Answer Conceptual Problems

Problem 1. What is the nature of intermolecular forces ?

Solution. Generally the intermolecular forces are attractive, but they are repulsive for intermolecular separations less than 10^{-10} m.

Problem 2. What is the origin of interatomic force ?

Solution. Interatomic force arises due to the electrostatic interaction between the nuclei of two atoms, their electron clouds and between the nucleus of one atom and the electron cloud of the other atom.

Problem 3. What is the origin of intermolecular force ?

Solution. Intermolecular force arises due to the electrostatic interaction between the opposite charged ends of molecular dipoles.

Problem 4. Are the intermolecular forces involved in the formation of liquids and solids different in nature ? If yes, how ?

Solution. Yes. The intermolecular forces involved in the formation of liquids are attractive in nature while in the formation of solids, the repulsive intermolecular forces are more important.

Problem 5. State the factors due to which three states of matter differ from each other.

Solution. The three states of matter differ from each other due to the following *two* factors :

(*i*) The different magnitudes of interatomic and intermolecular forces.

(*ii*) The degree of random thermal motion of the atoms and the molecules of a substance depending on temperature.

Problem 6. What do you mean by long range order in a crystalline structure ?

Solution. Long range order means that similar patterns of atoms or molecules repeat over a large distance in a crystal.

Problem 7. What is the important structural difference between crystalline and glassy solids ?

Solution. In crystalline solids the atoms or molecules are arranged in a definite and long range order, but in glassy solids there exists no such long range order in the arrangement of atoms or molecules.

Problem 8. Amorphous solids do not melt at a sharp temperature, rather these have softening range. Explain this observation.

Solution. All bonds in an amorphous solid are not equally strong. When the solid is heated, weaker bonds get ruptured at lower temperatures and the stronger ones at higher temperatures. So the solid first softens and then finally melts.

Problem 9. In what respect, the behaviour of glassy solids is similar to that of the liquids ?

Solution. In case of glassy solids, the orderly arrangement of atoms is limited to a very short range and in this respect they are similar to liquids.

Problem 10. Why do crystalline solids have well defined geometrical external shapes ?

Solution. This is because the atoms and molecules are arranged in a definite geometrical repeating manner throughout the body of the crystal.

Problem 11. Amorphous solids are not true solids. Why and what are they called then ?

Solution. Like liquids, amorphous solids have disordered arrangement of atoms or molecules. The molecules of a liquid are free to move but the molecules of an amorphous solid are almost fixed at their positions *i.e.*, amorphous solids are rigid due to their high viscosity. That is why, we say amorphous solids are super-cooled liquids of high viscosity.

Problem 12. Our knowledge about crystalline solids is better than amorphous solids. Why ?

Solution. As crystalline solids possess long range and regular arrangement of atoms, hence their behaviour can be easily understood.

Problem 13. Crystalline solids are called true solids. Why ?

Solution. This is because crystalline solids have well defined, regularly repeated three-dimensional arrangement of atoms or molecules.

Problem 14. What is a perfectly elastic body ? Give an example.

Solution. If, on removal of deforming force, a body completely regains its original configuration, then it is said to be perfectly elastic. For example, quartz.

Problem 15. What is a perfectly plastic body ? Give an example.

Solution. If, on removal of deforming force, a body does not regain its original configuration even a little, then it is said to be perfectly plastic. For example, putty.

Problem 16. No material is perfectly elastic. Why ?

Solution. All materials undergo a change in their original state, howsoever small it may be, after the removal of deforming force. Hence, there is no such material which is perfectly elastic.

Problem 17. When does a body acquire a permanent set ?

Solution. When the deforming force exceeds the elastic limit, the body acquires the permanent set.

Problem 18. A thick wire is suspended from a rigid support, but no load is attached to its free end. Is this wire under stress ?

Solution. Yes, the wire is under stress due to its own weight.

Problem 19. State the two factors on which the modulus of elasticity depends.

Solution. The modulus of elasticity depends upon (*i*) nature of the material and (*ii*) type of stress used in producing the strain.

Problem 20. Is it possible to double the length of a metallic wire by applying a force over it ?

Solution. No, it is not possible because within elastic limit strain is only of the order of 10^{-3}. Wires actually break much before it is stretched to double the length.

Problem 21. Is elastic limit a property of the material of the wire ?

Solution. No. It also depends on the radius of the wire.

Problem 22. Stress and pressure are both forces per unit area. Then in what respect does stress differ from pressure ?

Solution. Pressure is the external force per unit area, while stress is the internal restoring force which comes into play in a deformed body acting transversely per unit area of the body.

Problem 23. Among solids, liquids and gases, which can have all the three moduli of elasticity ?

Solution. Only solids. Liquids and gases have only bulk modulus.

Problem 24. Among solids, liquids and gases, which possess the greatest bulk modulus ?

Solution. Solids.

Problem 25. Which type of elasticity is involved in the following cases ?

(*i*) Compressing of gas (*ii*) Compressing a liquid (*iii*) Stretching a wire (*iv*) Tangential push on the upper face of a block.

Solution. (*i*) Bulk modulus (*ii*) Bulk modulus (*iii*) Young's modulus (*iv*) Modulus of rigidity.

Problem 26. What does the slope of stress versus strain graph give ?

Solution. The slope of stress-strain gives modulus of elasticity.

Problem 27. How does Young's modulus change with the rise of temperature ?

Solution. Young's modulus decreases with the rise of temperature.

Problem 28. Write copper, steel, glass and rubber in the order of increasing coefficient of elasticity.

Solution. Rubber < glass < copper < steel.

Problem 29. Which is more elastic—water or air ?

Solution. Water is more elastic than air. Air can be easily compressed while water is incompressible and bulk modulus is reciprocal of compressibility.

Problem 30. Why are springs made of steel and not of copper ? *[Himachal 03]*

Solution. Young's modulus of steel is greater than that of copper. So steel spring is stretched lesser than a copper spring under the same deforming force. Moreover, steel returns to its original state more quickly than copper on the removal of deforming force.

Problem 31. In stretching a wire, we have to perform work. Why ?

Solution. When a wire is stretched, interatomic forces of attraction come into play. In order to stretch the wire, work has to be done against these forces.

Problem 32. What happens to the work done in stretching a wire ?

Solution. The work done in stretching a wire is stored in it as elastic potential energy.

Problem 33. Two identical springs of steel and copper are equally stretched. On which more work will have to be done ?

Solution. Young's modulus of steel is greater than that of copper. In order to produce same extension, large force will have to be applied on the steel spring than that on the copper spring. Hence more work will be done on the steel spring.

Problem 34. If two identical springs of steel and copper are pulled by applying equal forces, then in which case more work will have to be done ?

Solution. Steel spring will be stretched to a lesser extent. Now more work will be done on the copper spring.

Problem 35. Why does a wire get heated when it is bent back and forth ?

Solution. When a wire is bent back and forth, its deformations are beyond elastic limit. The work done against interatomic forces is no longer stored totally in the form of potential energy. The crystalline structure of the wire gets affected and work done is converted into heat energy.

Problem 36. A hard wire is broken by bending it repeatedly in alternating directions. Why ?

Solution. When the wire is subjected to repeated alternating strains, the strength of its material decreases and the wire breaks.

Problem 37. Why is the longer side of cross-section of girder used as depth ?

Solution. Depression, $\delta = \dfrac{WL^3}{4\,bd^3}$

Clearly, the depression of the girder will be small when depth d is large, because $\delta \propto d^{-3}$.

Problem 38. The ratio stress/strain remains constant for a small deformation. What happens to this ratio if deformation is made very large ?

Solution. When the deforming force exceeds the elastic limit, the strain increases more rapidly than stress. Hence the ratio of stress/strain decreases.

Problem 39. Why are electric poles given hollow structure ?

Solution. This is because a hollow shaft is stronger than a solid shaft made from the same and equal amount of material.

Problem 40. The Young's modulus of a wire of length L and radius r is Y. If the length is reduced to $L/2$ and radius $r/4$, what will be its Young's modulus ? *[Central Schools 04]*

Solution. Young's modulus is a material constant. It is not affected by the change in dimensions of the wire. It will remain equal to Y.

Problem 41. A wire fixed at the upper end stretches by length l by applying a force F. What is the work done in stretching the wire ? *[AIEEE 04]*

Solution. Work done in stretching the wire,

$$W = \frac{1}{2}\text{Stretching force} \times \text{increase in length} = \frac{1}{2}Fl.$$

Problem 42. A wire suspended vertically from one of its ends is stretched by attaching a weight of 200 N to the lower end. The weight stretches the wire by 1 mm. Find the elastic energy stored in the wire. *[AIEEE 03]*

Solution. Here $F = 200\text{ N}$, $l = 1\text{ mm} = 10^{-3}\text{ m}$

Elastic potential energy stored in the wire,

$$U = \frac{1}{2}Fl = \frac{1}{2} \times 200 \times 10^{-3} = \mathbf{0.1\ J.}$$

Problem 43. If S is the stress and Y is Young's modulus of the material of a wire, what is the energy stored in the wire per unit volume in terms of S and Y ? *[AIEEE 05]*

Solution. Elastic potential energy stored per unit volume

$$u = \frac{1}{2}\text{Stress} \times \text{Strain} = \frac{1}{2} \times \text{Stress} \times \frac{\text{Stress}}{Y} = \frac{S^2}{2Y}.$$

Problem 44. Following are the graphs of elastic materials. Which one corresponds to that of brittle material ? *[Central Schools 08]*

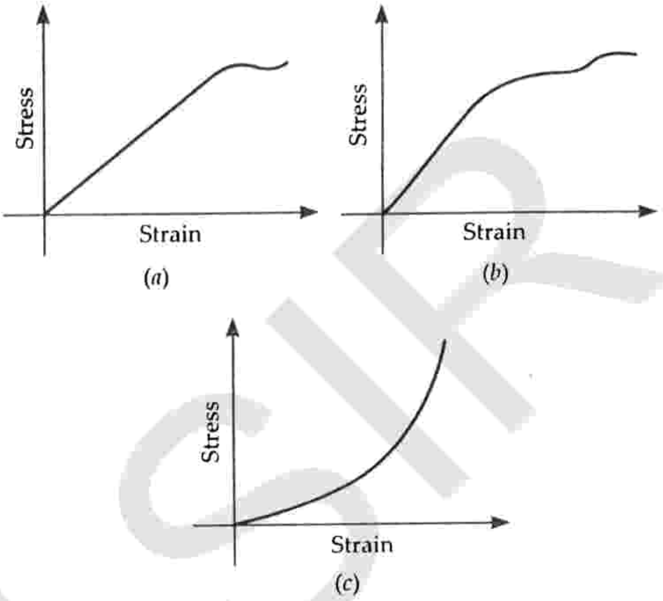


Fig. 9.23

Solution. Graph (*a*) represents a brittle material as it indicates a very small plastic range of extension.

Problem 45. A wire stretches by a certain amount under a load. If the load and radius both are increased to four times, find the stretch caused in the wire. *[Chandigarh 08]*

Solution. Young's modulus, $Y = \frac{W}{A} \times \frac{L}{l}$

$\therefore$ Elongation, $l = \frac{WL}{AY} = \frac{WL}{\pi r^2 Y}$

When both load and radius are increased to four times, the elongation becomes

$$l' = \frac{4W \times L}{\pi(4r)^2 Y} = \frac{WL}{4\pi r^2 Y} = \frac{l}{4}.$$

Short Answer Conceptual Problems

Problem 1. In the diagram a graph between the intermolecular force F acting between the molecules of a solid and the distance r between them is shown. Explain the graph.

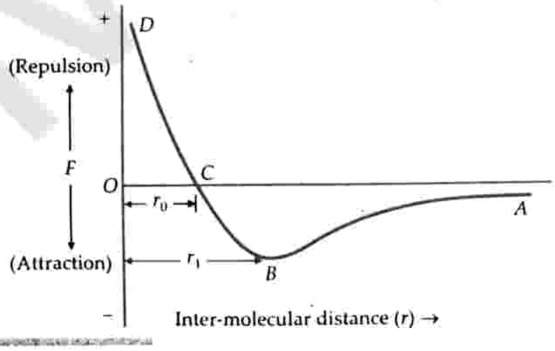


Fig. 9.24

Solution. (*i*) As the intermolecular distance r decreases, the force of attraction between the molecules increases.

(*ii*) When the distance decreases to r_1, the force of attraction is maximum.

(*iii*) As the distance further decreases, the attractive force goes on decreasing and when the distance decreases to r_0, the force becomes zero. When the distance decreases below r_0, the molecules begin to repel and the repulsive force increases rapidly.

Problem 2. Crystalline solids have sharp melting points. Amorphous solids do not melt at a sharp temperature ; rather these have a softening range of temperature. Explain.

Solution. All bonds in a crystalline solid are equally strong. When the solid is heated, these bonds ruptured at the same temperature. So crystalline solids have sharp melting points.

On the other hand, all bonds in an amorphous solid are not equally strong. When the solid is heated, weaker bonds break at lower temperatures and the stronger ones at higher temperatures. So the solid first softens and then finally melts, *i.e.*, the amorphous solids do not have sharp melting points.

Problem 3. Which is more elastic—rubber or steel ? Explain. *[Delhi 96 ; Himachal 03, 05, 07C]*

Solution. Consider two rods of steel and rubber, each having length l and area of cross-section A. If they are subjected to the same deforming force F, then the extension Δl_s produced in the steel rod will be less than the extension Δl_r in the rubber rod, *i.e.*, $\Delta l_s < \Delta l_r$. Now

$$Y_s = \frac{F}{A} \cdot \frac{l}{\Delta l_s} \quad \text{and} \quad Y_r = \frac{F}{A} \cdot \frac{l}{\Delta l_r}$$

$$\therefore \quad \frac{Y_s}{Y_r} = \frac{\Delta l_r}{\Delta l_s}$$

As $\Delta l_s < \Delta l_r$, so $Y_s > Y_r$

i.e., Young's modulus for steel is greater than that of rubber. Hence *steel is more elastic than rubber.*

Problem 4. The stress-strain graph for a metal wire is shown in Fig. 9.25. Up to the point *E*, the wire returns to its original state *O* along the curve *EPO* when it is gradually unloaded. Point *B* corresponds to the fracture of the wire.

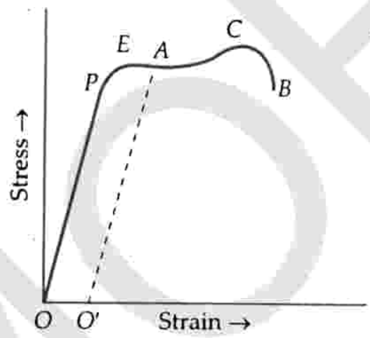


Fig. 9.25

(*a*) **Up to which point on the curve is Hooke's law obeyed ? This point is sometimes called "Proportionality limit".**

(*b*) **Which point on the curve corresponds to elastic limit and yield point of the wire ?**

(*c*) **Indicate the elastic and plastic regions of the stress-strain graph.**

(*d*) **Describe what happens when the wire is loaded up to a stress corresponding to the point *A* on the graph, and then unloaded gradually. In particular, explain the dotted curve.**

(*e*) **What is peculiar about the portion of the stress-strain graph from *C* to *B* ? Up to what stress can the wire be subjected without causing fracture ?** *[Delhi 08]*

Solution. (*a*) Hooke's law is obeyed upto the point *P*, because upto this point, stress $\propto$ strain.

(*b*) Point *E* corresponds to elastic limit because the wire returns to original state *O* along *EPO* if it is gradually unloaded.

(*c*) The elastic region is from *O* to *E* and the plastic region is from *E* to *B*.

(*d*) Upto point *P*, stress is proportional to strain. Between *P* and *E*, strain increases more rapidly than stress and Hooke's law is not obeyed. When the wire is unloaded at any point *A* beyond *E*, the wire does not retrace the curve *AEPO* but follows the dashed curve *AO'*. When the stress becomes zero, a residual strain *OO'* is left in the wire.

(*e*) Between *C* and *B*, the wire virtually flows out, *i.e.*, the strain increases even when the wire is being unloaded. Fracture takes place at point *B*. The stress can be applied to the value corresponding to the point *C* without causing fracture.

Problem 5. Two different types of rubber are found to have the stress-strain curves as shown in Fig. 9.26.

(*a*) **In which significant ways do these curves differ from the stress-strain curve of a metal wire shown in Fig. 9.26 ?**

(*b*) **A heavy machine is to be installed in a factory. To absorb vibrations of the machine, a block of rubber is placed between the machinery and the floor. Which of the two rubbers *A* and *B* would you prefer to use for this purpose ? Why ?**

(*c*) **Which of the two rubber materials would you choose for a car tyre ?** *[Central Schools 07]*

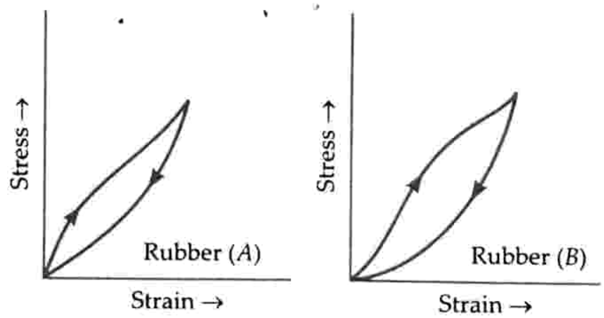


Fig. 9.26

Solution. (*a*) The stress-strain curves for rubber differ from the stress-strain curve for a metal in following respects :

(*i*) Hooke's law is not obeyed even for small stresses.

(*ii*) There is no permanent set (residual strain) even for large stresses.

(*iii*) There is large elastic region for both types of rubber.

(*iv*) Neither material retraces the curve during unloading. Thus both materials exhibit elastic hysteresis.

(*b*) The area of the hysteresis loop is proportional to the energy dissipated by the material as heat when the material undergoes loading and unloading. A material for which the hysteresis loop has larger area would absorb more energy when subjected to vibrations. Therefore to absorb vibrations, we would prefer rubber *B*.

(*c*) In car tyre, the energy dissipation must be minimised to avoid excessive heating of the car tyre.

As rubber *A* has smaller hysteresis loop area (and hence smaller energy loss), so it is preferred to *B* for a car tyre.

Problem 6. Read each of the statements below carefully and state, with reasons, if it is true or false.

(*a*) When a material is under tensile stress, the restoring forces are caused by interatomic attraction while under compressional stress, the restoring forces are due to inter-atomic repulsion.

(*b*) A piece of rubber under an ordinary stress can display 1000% strain : yet when unloaded returns to its original length. This shows that the elastic restoring forces in a rubber piece are strictly conservative.

(*c*) Elastic restoring forces are strictly conservative only when Hooke's law is obeyed.

Solution. (*a*) **True.** In tensile stress, the interatomic separation becomes greater than equilibrium separation and the interatomic forces are attractive. In compressional stress, the interatomic separation becomes less than r_0 and the interatomic forces are repulsive.

(*b*) **False.** As the piece of rubber returns to its original length when unloaded, it is a case of elastic hysteresis in which there is some loss of energy. This signifies non-conservative forces.

(*c*) **False.** Even if the stress-strain curves are non-linear, the elastic forces are conservative as long as loading and unloading curves are identical.

Problem 7. Two wires of different materials are suspended from a rigid support. They have the same length and diameter and carry the same load at their free ends. (*a*) Will the stress and strain in each wire be the same ? (*b*) Will the extension in both wires be the same ?

Solution. (*a*) Stress in both the wires is the same as both the wires have the same diameter and carry the same load at their free ends. Strain will be different in the two wires as the wires are of different materials, even though the stress is the same.

(*b*) Because the original lengths of the two wires are equal and strains produced in them are different, hence extensions in the two wires will not be same.

Problem 8. A cable is replaced by another of the same length and material but of twice the diameter. (*a*) How does this affect its elongation under a given load ? (*b*) How many times will be the maximum load it can now support without exceeding the elastic limit ?

Solution. (*a*) Young's modulus,

$$Y = \frac{Mgl}{\pi r^2 \,.\, \Delta l} = \frac{Mgl}{\pi\left(\frac{D}{2}\right)^2 .\, \Delta l} = \frac{4\,Mgl}{\pi D^2 \,.\, \Delta l}$$

where D is the diameter of the wire.

Elongation, $\Delta l = \dfrac{4\,Mgl}{\pi\, D^2 Y}$ *i.e.,* $\Delta l \propto \dfrac{1}{D^2}$

Clearly, if the diameter is doubled, the elongation will become one-fourth.

(*b*) Also load, $Mg = \dfrac{\pi D^2 \,.\, \Delta l \,.\, Y}{4l}$ *i.e.,* $Mg \propto D^2$

Clearly, if the diameter is doubled, the wire can support 4 times the original load.

Problem 9. Two wires of same length and material but of different radii are suspended from a rigid support. Both carry the same load. Will the stress, strain and extension in them be same or different ?

Solution. Let r_1 and r_2 be the radii of the two wires.

(*i*) Stress $= \dfrac{F}{A} = \dfrac{F}{\pi r^2}$. For same load F, $\dfrac{(\text{stress})_1}{(\text{stress})_2} = \dfrac{r_2^2}{r_1^2}$

(*ii*) Strain, $\dfrac{\Delta l}{l} = \dfrac{F}{AY} = \dfrac{F}{\pi r^2\, Y}$

For the two wires F and Y are same, so $\dfrac{(\text{strain})_1}{(\text{strain})_2} = \dfrac{r_2^2}{r_1^2}$

(*iii*) Extension, $\Delta l = \dfrac{F}{A} \cdot \dfrac{l}{Y} = \dfrac{F}{\pi r^2} \cdot \dfrac{l}{Y}$

For the two wires F, L and Y are same, so $\dfrac{(\Delta l)_1}{(\Delta l)_2} = \dfrac{r_2^2}{r_1^2}$

Hence stress, strain and extension are all different for the two wires.

Problem 10. A uniform plank of Young's modulus *Y* is moved over a smooth horizontal surface by a constant horizontal force *F*. The area of transverse section of the plank is *A*. Find the compressive strain on the plank in the direction of the force.

Solution. As the force at the other end of the plank is zero, so the average stretching force $= \dfrac{F + 0}{2} = \dfrac{F}{2}$

$\therefore$ Stress $= \dfrac{F}{2A}$

Strain $= \dfrac{\text{Stress}}{Y} = \dfrac{1}{2}\dfrac{F}{AY}$.

Problem 11. What are the factors which affect the elasticity of a material ?

Solution. The following factors affect the elasticity of a material :

(*i*) ***Hammering and rolling.*** In both of these processes, the crystal grains are broken into small units and the elasticity of the material increases.

(*ii*) ***Annealing.*** This process results in the formation of larger crystal grains and elasticity of the material decreases.

(*iii*) ***Presence of impurities.*** Depending on the nature of the impurity, the elasticity of a material can be increased or decreased.

(*iv*) ***Temperature.*** Elasticity of most of the materials decreases with the increase in temperature. The elasticity of in var is not affected by temperature.

Problem 12. Elasticity has a different meaning in physics than that in daily life. Comment.

Solution. In daily life, a body is said to be elastic if a large deformation or strain is produced on applying a given stress on it. In physics, elasticity is the property of the material of a body by virtue of which it opposes any change in its size or shape when a stress is applied on it. Thus a body will be more elastic if a small strain is produced on applying a given stress on it.

Problem 13. Why a spring balance does not give correct measurement, when it has been used for a long time ? *[Himachal 05C, 07C]*

Solution. When a spring balance has been used for a long time, it develops an elastic fatigue, the spring of such a balance takes longer time to recover its original configuration and therefore it does not give correct measurement.

Problem 14. Why the bridges are declared unsafe after long use ? *[Himachal 03, 07C]*

Solution. During its long use, a bridge suffers alternating strains continuously. Consequently, the elastic strength of the bridge gets reduced. After a long time, the bridge develops elastic fatigue and there occurs a permanent change in its structure. This permanent change ultimately leads the bridge to collapse. In order to avoid this event, the bridges are declared unsafe after long use.

Problem 15. Two identical solid balls, one of ivory and the other of wet-clay, are dropped from the same height on the floor. Which will rise to a greater height after striking the floor and why ?

Solution. The ball which is more elastic rises to a greater height after striking the floor. Ivory is more elastic than wet-clay. Hence the ball of ivory will rise to a greater height. In fact, the ball of wet-clay will not rise at all, it will get flattened.

Problem 16. The breaking force for a wire is *F*. What will be the breaking force for (*a*) two parallel wires of the same size (*b*) for a single wire of double the thickness ?

Solution. (*a*) When two wires of same size are suspended in parallel, a force *F* equal to the breaking force will act on each wire if a breaking force of 2*F* is applied on the parallel combination.

(*b*) $F = \frac{YA\,\Delta l}{l} = \frac{Y \cdot \pi r^2\,\Delta l}{l}$ *i.e.,* $F \propto r^2$

Thus for a single wire of double the thickness, the breaking force will be **4*F***.

Problem 17. Graphite consists of planes of carbon atoms. Between atoms in the planes there are only weak forces. What kind of elastic properties do you expect from graphite ?

Solution. Due to weak attractive forces between carbon atoms of different planes, it is easier to produce a large shearing strain by moving one plane of atoms over the other with the application of a small tangential stress. Now

$$\text{Modulus of rigidity, } \eta = \frac{\text{Tangential stress}}{\text{Shear strain}}$$

Hence graphite should possess a small modulus of rigidity.

Problem 18. Why does modulus of elasticity of most of the materials decrease with the increase of temperature ?

Solution. As the temperature increases, the inter-atomic forces of attraction become weaker. For given stress, a larger strain or deformation is produced at a higher temperature. Hence the modulus of elasticity (stress/strain) decreases with the increase of temperature.

HOTS

Problems on Higher Order Thinking Skills

Problem 1. *A wire elongates by l mm when a load W is hanged from it. If the wire goes over a pulley and two weights W each are hung at the two ends, what will be the elongation of the wire in mm ?* **[AIEEE 06]**

Solution. Young's modulus, $Y = \frac{W}{A} \times \frac{L}{l}$

$\therefore$ Elongation, $l = \frac{WL}{AY}$

On either side of the wire, tension is *W* but length is $l/2$.

$\therefore$ Elongation produced along either side $= l/2$ mm

Total elongation produced $= l/2 + l/2 = \boldsymbol{l}$ **mm.**

Problem 2. *A wire is cut to half its original length. (a) How would it affect the elongation under a given load ? (b) How does it affect the maximum load it can support without exceeding the elastic limit ?*

Solution. Young's modulus, $Y = \frac{F}{A} \cdot \frac{l}{\Delta l}$

(*a*) $\Delta l = \frac{F}{A} \cdot \frac{l}{Y}$ *i.e.,* $\Delta l \propto l$

So when the wire is cut half to its original length, extension is halved.

(*b*) Maximum load, $F = \frac{YA\,\Delta l}{l}$

Here Y and A are constant. When the wire is cut to half its original length, there is no change on the value of $\Delta l / l$. Hence there is no effect on the maximum load.

Problem 3. *A bar of cross-section A is subjected to equal and opposite tensile forces at its ends. Consider a plane section of the bar whose normal makes an angle θ with the axis of the bar.*

(*a*) *What is the tensile stress on this plane ?*

(*b*) *What is the shearing stress on this plane ?*

(*c*) *For what value of θ is the tensile stress maximum ?*

(*d*) *For what value of θ is the shearing stress maximum ?*

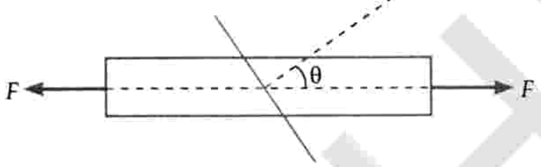


Fig. 9.27

Solution. (*a*) The resolved part of F along the normal is the tensile force on this plane and the resolved part parallel to the plane is the shearing force on the plane.

$$\text{Tensile stress} = \frac{\text{Force}}{\text{Area}} = \frac{F\cos\theta}{A\sec\theta} = \frac{F}{A}\cos^2\theta$$

[∵ Area of plane section = $A \sec \theta$]

(*b*) Shearing stress

$$= \frac{\text{Force}}{\text{Area}} = \frac{F\cos\theta}{A\sec\theta} = \frac{F}{A}\sin\theta\cos\theta = \frac{F}{2A}\sin 2\theta.$$

(*c*) Tensile stress will be maximum when $\cos^2\theta$ is maximum, *i.e.,* $\cos\theta = 1$ or $\theta = \mathbf{0°}$.

(*d*) Shearing stress will be maximum when $\sin 2\theta$ is maximum, *i.e.,* $\sin 2\theta = 1$ or $2\theta = 90°$ or $\theta = \mathbf{45°}$.

Problem 4. *The graph (Fig. 9.28) shows the extension (Δl) of a wire of length 1 m suspended from the top of a roof at one end with a load W connected to the other end. If the cross-sectional area of the wire is $10^{-6}\,m^2$, calculate the Young's modulus of the material of the wire.* **[IIT Screening 03]**

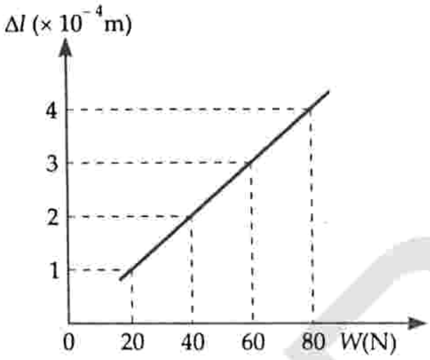


Fig. 9.28

Solution. When $W = 20$ N, $\Delta l = 1 \times 10^{-4}$ m

$$\therefore \quad Y = \frac{F}{A} \cdot \frac{l}{\Delta l} = \frac{W}{A} \cdot \frac{l}{\Delta l} = \frac{20}{10^{-6}} \times \frac{1}{1 \times 10^{-4}}$$

$$= \mathbf{2 \times 10^{11}\ Nm^{-2}}.$$

Problem 5. *A metallic wire is stretched by suspending weight from it. If α is the longitudinal strain and Y is the Young's modulus, show that elastic potential energy per unit volume is given by $\frac{1}{2}Y\alpha^2$.* **[Roorkee 82]**

Solution. Elastic P.E. per unit volume,

$$u = \frac{1}{2}\,\text{stress} \times \text{strain}$$

But $\quad Y = \frac{\text{Stress}}{\text{Strain}} = \frac{\text{Stress}}{\alpha} \quad$ [∵ Strain = α]

$\therefore$ Stress $= Y\alpha$

Hence $\quad u = \frac{1}{2}Y\alpha \times \alpha = \frac{1}{2}Y\alpha^2.$

Problem 6. *A copper wire of negligible mass, 1 m length and cross-sectional area $10^{-6}\,m^2$ is kept on a smooth horizontal table with one end fixed. A ball of mass 1 kg is attached to the other end. The wire and the ball are rotating with an angular velocity of 20 rad s^{-1}. If the elongation in the wire is 10^{-3} m, obtain the Young's modulus. If on increasing the angular velocity to 100 rad s^{-1}, the wire breaks down, obtain the breaking stress.* **[Roorkee 92]**

Solution. When the ball is rotated at the end of copper wire, restoring force in the wire is equal to the centripetal force on the ball.

Centripetal force,

$$F = mr\omega^2 = ml\omega^2 \quad [\because r = l]$$

As $\quad Y = \frac{F}{A} \cdot \frac{l}{\Delta l} \quad$ or $\quad F = \frac{YA\,\Delta l}{l}$

$\therefore \quad \frac{YA\,\Delta l}{l} = ml\omega^2 \quad$ or $\quad Y = \frac{ml^2\omega^2}{A\,\Delta l}$

But $m = 1$ kg, $l = 1$ m, $\omega = 20$ rad s^{-1}, $A = 10^{-6}$ m^2, $\Delta l = 10^{-3}$ m

$$\therefore \quad Y = \frac{1 \times (1)^2 \times (20)^2}{10^{-6} \times 10^{-3}} = \mathbf{4 \times 10^{11}\ Nm^{-2}}.$$

Breaking force

$$= mr\omega_{max}^2 = m\,l\,\omega_{max}^2 = 1 \times 1 \times (100)^2 = 10^4 \text{ N}$$

Breaking stress

$$= \frac{\text{Breaking force}}{\text{Area}} = \frac{10^4}{10^{-6}} = \mathbf{10^{10}\ Nm^{-2}}.$$

Problem 7. *A load of 31.4 kg is suspended from a wire of radius 10^{-3} m and density 9×10^3 kg m^{-3}. Calculate the change in temperature of the wire if 75% of the work done is converted into heat. The Young's modulus and the specific heat of the material of the wire are 9.8×10^{10} Nm^{-2} and 490 J kg^{-1} K^{-1} respectively.* **[Roorkee 93]**

Solution. As $Y = \frac{F}{A} \cdot \frac{l}{\Delta l}$ $\therefore$ $\Delta l = \frac{F}{A} \cdot \frac{l}{Y}$

Work done,

$$W = \frac{1}{2} \text{ Stretching force} \times \text{extension} = \frac{1}{2} F \Delta l$$

$$= \frac{1}{2} F \times \frac{F}{A} \frac{l}{Y} = \frac{F^2 l}{2\,AY}$$

Let ΔT be the rise in temperature when 75% of the work done changes into heat. Then

$$0.75\,W = mc\,\Delta T$$

where $m (= A\,l\,\rho)$ is the mass of the wire and c its specific heat.

$$\therefore\quad 0.75 \times \frac{F^2 l}{2\,AY} = Al\rho\,c\,\Delta T \quad \text{or} \quad \Delta T = \frac{0.75\,F^2}{2\,A^2\,Y\,\rho c}$$

But $F = Mg = 31.4 \times 9.8$ N, $A = 3.14 \times 10^{-6}$ m^2,

$Y = 9.8 \times 10^{10}$ Nm^{-2}, $\rho = 9 \times 10^3$ kg m^{-3},

$c = 490$ J kg^{-1} K^{-1}

$$\therefore \Delta T = \frac{0.75 \times (31.4 \times 9.8)^2}{2 \times (3.14 \times 10^{-6})^2 \times 9.8 \times 10^{10} \times 9 \times 10^3 \times 490}$$

$$= \mathbf{\frac{1}{120}\ K.}$$

Problem 8. *A light rod of length 2 m is suspended horizontally by means of two vertical wires of equal lengths tied to its ends. One of the wires is made of steel and is of cross-section $A_1 = 0.1$ cm^2 and the other is of brass and is of cross-section $A_2 = 0.2$ cm^2. Find out the position along the rod at which a weight must be suspended to produce (i) equal stresses in both wires, (ii) equal strains in both wires. For steel, $Y = 20 \times 10^{10}$ Nm^{-2} and for brass $Y = 10 \times 10^{10}$ Nm^{-2}.* **[IIT]**

Solution. The situation is shown in Fig. 9.29. Let AB be the rod of length 2 m. Suppose a weight W is hung at C at distance x from A. Let T_1 and T_2 be the tensions in the steel and brass rods respectively.

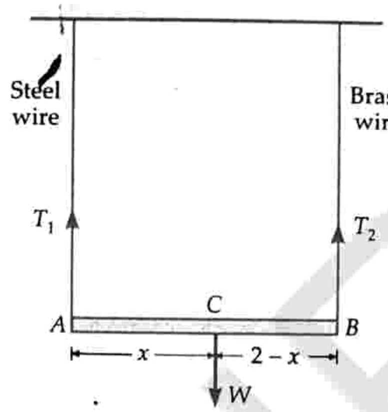


Fig. 9.29

(*i*) Stress in steel wire $= \frac{T_1}{A_1}$,

Stress in brass wire $= \frac{T_2}{A_2}$

As both the stresses are equal, so

$$\therefore\quad \frac{T_1}{A_1} = \frac{T_2}{A_2} \quad \text{or} \quad \frac{T_1}{T_2} = \frac{A_1}{A_2} = \frac{0.1}{0.2} = \frac{1}{2}$$

Now moments about C are equal as the system is in equilibrium

$$\therefore\quad T_1\,x = T_2\,(2 - x) \quad \text{or} \quad \frac{T_1}{T_2} = \frac{2 - x}{x}$$

or $$\frac{1}{2} = \frac{2 - x}{x} \qquad \left(\because \frac{T_1}{T_2} = \frac{1}{2}\right)$$

$$x = 4 - 2x$$

$$\therefore\quad 3x = 4 \quad \text{or} \quad x = \frac{4}{3} = \mathbf{1.33\ m.}$$

(*ii*) Now Y = Stress/Strain

$\therefore$ Strain = Stress/Y

Strain in steel wire $= \frac{T_1 / A_1}{Y_1}$

Strain in brass wire $= \frac{T_2 / A_2}{Y_2}$

Now $$\frac{T_1}{A_1\,Y_1} = \frac{T_2}{A_2\,Y_2}$$

$$\therefore\quad \frac{T_1}{T_2} = \frac{A_1\,Y_1}{A_2\,Y_2} = \frac{0.1 \text{ cm}^2 \times 20 \times 10^{10} \text{ Nm}^{-2}}{0.2 \text{ cm}^2 \times 10 \times 10^{10} \text{ Nm}^{-2}}$$

$$= 1$$

Again, $$T_1 x = T_2\,(2 - x) \quad \text{or} \quad \frac{T_1}{T_2} = \frac{2 - x}{x}$$

or $$1 = \frac{2 - x}{x} \qquad \left[\because \frac{T_1}{T_2} = 1\right]$$

$$\therefore\quad x = 2 - x$$

or $$2x = 2 \quad \text{or} \quad x = \mathbf{1\ m.}$$

Problem 9. *A thin rod of negligible mass and area of cross-section $4 \times 10^{-6}\ m^2$, suspended vertically from one end has a length of 0.5 m at 100°C. The rod is cooled at 0°C, but prevented from contracting by attaching a mass at the lower end. Find (i) this mass and (ii) the energy stored in the rod. Given for this rod, $Y = 10^{11}\ Nm^{-2}$, coefficient of linear expansion $= 10^{-5}\ K^{-1}$ and $g = 10\ ms^{-2}$.* **[IIT 97]**

Solution. Here

$$A = 4 \times 10^{-6}\ \text{m}^2, \quad l = 0.5\ \text{m},$$
$$\Delta T = 100 - 0 = 100°\text{C} = 100\ \text{K}$$
$$Y = 10^{11}\ \text{Nm}^{-2}, \quad \alpha = 10^{-5}\ \text{K}^{-1}$$

Change in length,

$$\Delta l = l\,\alpha\,\Delta T = 0.5 \times 10^{-5} \times 100 = 5 \times 10^{-4}\ \text{m}$$

As $$Y = \frac{\text{Stress}}{\Delta l / l}$$

$$\text{Stress} = Y\,\frac{\Delta l}{l} = Y \times \alpha\,\Delta T = 10^{11} \times 10^{-5} \times 100 = 10^8\ \text{Nm}^{-2}$$

Stretching force,

$$F = \text{Stress} \times \text{area} = 10^8 \times 4 \times 10^{-6} = 4 \times 10^2\ \text{N}$$

But $F = Mg \quad \therefore \quad M = 4 \times 10^2$

or $$M = \frac{4 \times 10^2}{g} = \frac{4 \times 10^2}{10} = \mathbf{40\ kg.}$$

(*ii*) Energy stored in the rod

$$= \frac{1}{2} F \times \Delta l = \frac{1}{2} \times 4 \times 10^2 \times 5 \times 10^{-4} = \mathbf{0.1\ J.}$$

Problem 10. *A wire of cross-sectional area A is stretched horizontally between two clamps located at a distance 2l metres from each other. A weight W kg is suspended from the midpoint of the wire. If the vertical distance through which the mid-point of the wire moves down be x < l, then find (i) the strain produced in the wire. (ii) the stress in the area. (iii) If Y is the Young's modulus of wire, then find the value of x.*

Solution. The situation is shown in Fig. 9.30. The increase in length of the wire when it is pulled down into shape *BOC* is

$$\Delta l = BO + OC - 2l = 2\,BO - 2l = 2\,(l^2 + x^2)^{1/2} - 2l$$
$$= 2l\left(1 + \frac{x^2}{l^2}\right)^{1/2} - 2l = 2l\left[1 + \frac{x^2}{2l^2}\right] - 2l = \frac{x^2}{l}$$

$$\therefore \text{Strain} = \frac{\Delta l}{2l} = \frac{x^2}{2l^2}.$$

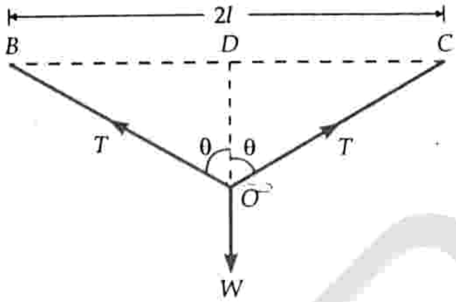


Fig. 9.30

(*ii*) If *T* is the tension in the string, then

$$2T\cos\theta = W \quad \text{or} \quad T = \frac{W}{2\cos\theta}$$

Now $$\cos\theta = \frac{x}{OB} = \frac{x}{\sqrt{l^2 + x^2}} = \frac{x}{l\left[1 + \frac{x^2}{l^2}\right]^{1/2}} = \frac{x}{l\left[1 + \frac{x^2}{2l^2}\right]}$$

As $\frac{x^2}{2l^2} << 1$, so $1 + \frac{x^2}{2l^2} = 1$

$$\therefore \cos\theta = \frac{x}{l} \quad \text{and} \quad T = \frac{W}{2\,(x/l)} = \frac{Wl}{2x}$$

$$\text{Stress} = \frac{T}{A} = \frac{Wl}{2Ax}.$$

$$(iii)\ Y = \frac{\text{Stress}}{\text{Strain}} = \frac{Wl}{2Ax} \times \frac{2l^2}{x^2} = \frac{Wl^3}{Ax^3}$$

or $$x = l\left[\frac{W}{YA}\right]^{1/3}.$$

Problem 11. *A stone of 0.5 kg mass is attached to one end of a 0.8 m long aluminium wire of 0.7 mm diameter and suspended vertically. The stone is now rotated in a horizontal plane at a rate such that the wire makes an angle of 85° with the vertical. Find the increase in the length of the wire. The Young's modulus of aluminium $= 7 \times 10^{10}\ Nm^{-2}$, $\sin 85° = 0.9962$, $\cos 85° = 0.0872$)* **[Roorkee 90]**

Solution. As shown in Fig. 9.31, let the stone be rotated in a circle of radius *r* with speed *v*. Then the forces acting on the stone are

(*i*) its weight *mg* acting vertically downwards,

(*ii*) centrifugal force mv^2/r in horizontal direction, and

(*iii*) tension *T* in the wire.

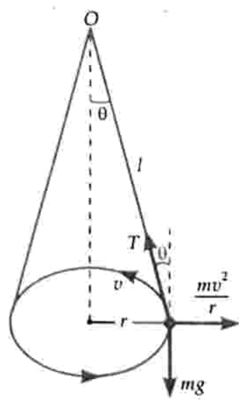


Fig. 9.31

Resolving T in horizontal and vertical directions, we get

$$T \cos\theta = mg \quad ...(i)$$

and $$T \sin\theta = \frac{mv^2}{r} \quad ...(ii)$$

$$\therefore \quad T = \frac{mg}{\cos\theta} = \frac{mg}{\cos 85°} = \frac{0.5 \times 9.8}{0.0872} = \textbf{56.19 N.}$$

As $$Y = \frac{T/A}{\Delta l / l} = \frac{T}{A} \cdot \frac{l}{\Delta l}$$

$$\therefore \quad \Delta l = \frac{Tl}{YA} = \frac{56.19 \times 0.8}{7 \times 10^{10} \times \pi \times (0.35 \times 10^{-3})^2}$$

$$= 1.67 \times 10^{-3} \text{ m} = \textbf{1.67 mm.}$$

Problem 12. *Two rods of different materials but of equal cross-sections and lengths (1.0 m each) are joined to make a rod of length 2.0 m. The metal of one rod has coefficient of linear thermal expansion 10^{-5} °C^{-1} and Young's modulus 3×10^{10} Nm^{-2}. The other metal has the values 2×10^{-5} °C^{-1} and 10^{10} Nm^{-2} respectively. How much pressure must be applied to the ends of the composite rod to prevent its expansion when the temperature is raised by 100°C ?*

[Roorkee 91]

Solution. When the temperature is raised by 100°C, the extensions in the two rods are

$$\Delta l_1 = \alpha_1 \, l \, \Delta T = 10^{-5} \times 1.0 \times 100 = 10^{-3} \text{ m}$$

$$\Delta l_2 = \alpha_2 \, l \, \Delta T = 2 \times 10^{-5} \times 1.0 \times 100 = 2 \times 10^{-3} \text{ m}$$

Tensions produced in the rods are

$$T_1 = Y_1 \, A . \frac{\Delta l_1}{l} = \frac{3 \times 10^{-10} \times A \times 10^{-3}}{1.0}$$

$$= 3 \times 10^7 \, A \text{ newton}$$

$$T_2 = Y_2 \, A . \frac{\Delta l_2}{l} = \frac{10^{10} \times A \times 2 \times 10^{-3}}{1.0}$$

$$= 2 \times 10^7 \, A \text{ newton}$$

where A = area of cross-section of each rod.

Total force needed to be applied at the ends to prevent expansion

$$= T_1 + T_2 = 5 \times 10^7 \, A \text{ newton}$$

$$\therefore \quad \text{Pressure} = \frac{\text{Force}}{\text{Area}} = \frac{5 \times 10^7 A}{A} = \mathbf{5 \times 10^7 \ Nm^{-2}}.$$

Guidelines to NCERT Exercises

9.1. *A steel wire of length 4.7 m and cross-section 3.0×10^{-5} m^2 stretches by the same amount as a copper wire of length 3.5 m and cross-section 4.0×10^{-5} m^2 under a given load. What is the ratio of the Young's modulus of steel to that of copper ?*

Ans. For steel : $l = 4.7$ m, $A = 3.0 \times 10^{-5}$ m^2

For copper : $l = 3.5$ m, $A = 4.0 \times 10^{-5}$ m^2

Applied force F and extension Δl are same for both wires.

$\therefore$ Young's modulus of steel,

$$Y_S = \frac{Fl}{A . \Delta l} = \frac{F \times 4.7}{3.0 \times 10^{-5} \times \Delta l}$$

Young's modulus of copper,

$$Y_C = \frac{Fl}{A . \Delta l} = \frac{F \times 3.5}{4.0 \times 10^{-5} \times \Delta l}$$

$$\frac{Y_S}{Y_C} = \frac{F \times 4.7}{3.0 \times 10^{-5} \times \Delta l} \times \frac{4.0 \times 10^{-5} \times \Delta l}{F \times 3.5}$$

$$= \mathbf{1.79}.$$

9.2. *Fig. 9.32 shows the stress-strain curve for a given material. What are (a) Young's modulus and (b) approximate yield strength for this material ?*

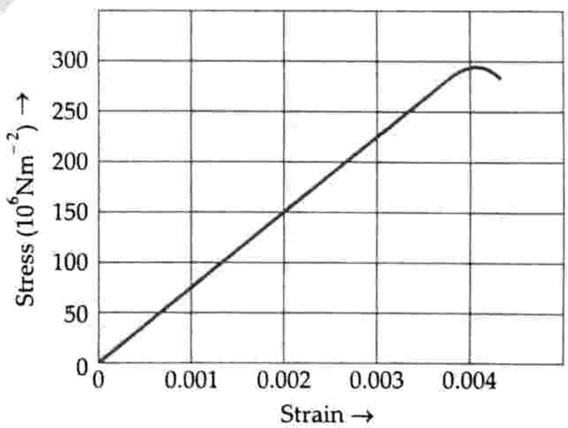


Fig. 9.32

Ans. (*a*) From the given graph for a stress of 150×10^6 Nm^{-2}, the strain is 0.002.

$\therefore$ Young's modulus,

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{150 \times 10^6 \text{ Nm}^{-2}}{0.002}$$

$$= \mathbf{7.5 \times 10^{10} \ Nm^{-2}}.$$

(*b*) Near the bend of the curve, the stress is nearly 300×10^6 Nm^{-2}.

$\therefore$ Approximate yield strength of the material

$$= \mathbf{3 \times 10^8 \ Nm^{-2}}.$$

9.3. *The stress-strain graphs for materials A and B are shown in Fig. 9.33.*

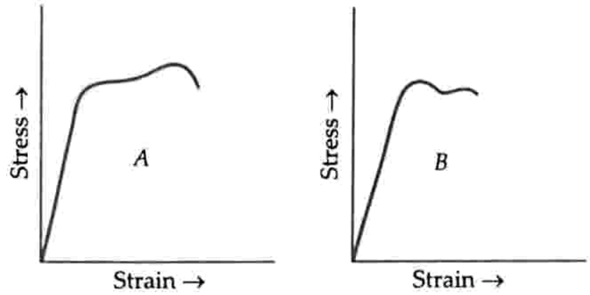


Fig. 9.33

The graphs are drawn to the same scale.

(*a*) *Which of the material has greater Young's modulus ?*

(*b*) *Which material is more ductile ?*

(*c*) *Which is more brittle ?*

(*d*) *Which of the two is stronger material ?*

[Delhi 09 ; Central Schools 12]

Ans. (*a*) Young's modulus $= \dfrac{\text{Stress}}{\text{Strain}} =$ slope of stress-strain graph.

As the slope of stress-strain graph for material *A* is higher than that for material *B*, so **material *A* has greater Young's modulus than *B*.**

(*b*) **Material *A* is more ductile than *B*,** because it has larger range of plastic extension between its elastic limit and fracture point.

(*c*) **Material *B* is more brittle than *A*,** because its plastic range of extension is very small.

(*d*) **Material *A* is stronger than *B*,** because it can withstand greater stress before breaking.

9.4. *Read each of the statements below carefully and state, with reasons, if it is true or false.*

(*a*) *The modulus of elasticity of rubber is greater than that of steel.*

(*b*) *The stretching of a coil is determined by its shear modulus.*

Ans. (*a*) **False.** When steel and rubber are subjected to the same deforming force, less extension and hence less strain is produced in the steel than the rubber and as

$$Y = \frac{\text{Stress}}{\text{Strain}}$$

so *Y* is more in case of steel than in the case of rubber.

(*b*) **True.** When the coil is stretched, there is no change in the length or the volume of the wire used in the coil. There is only a change in the shape of the spring, so shear modulus is involved.

9.5. *Two wires of diameter 0.25 cm, one made of steel and other made of brass are loaded as shown in Fig. 9.34. The unloaded length of steel wire is 1.5 m and that of brass wire is 1.0 m. Young's modulus of steel is 2.0×10^{11} Pa and that of brass is 0.91×10^{11} Pa. Compute the elongations of steel and brass wires. (1 Pa = 1 Nm^{-2}).*

Ans. *For steel wire :*

$$l_1 = 1.5 \text{ m}, \quad r_1 = \frac{0.25}{2} \text{ cm} = 0.125 \times 10^{-2} \text{ m}$$

$$F_1 = 6 + 4 = 10 \text{ kg f} = 10 \times 9.8 \text{ N}$$

$$Y_1 = 2.0 \times 10^{11} \text{ Pa}$$

As $$Y = \frac{F}{A} \cdot \frac{l}{\Delta l} = \frac{F}{\pi r^2} \cdot \frac{l}{\Delta l}$$

$$\Delta l = \frac{F}{\pi r^2} \cdot \frac{l}{Y}$$

$$\therefore \Delta l_1 = \frac{F_1}{\pi r_1^2} \cdot \frac{l_1}{Y_1}$$

$$= \frac{10 \times 9.8 \times 1.5}{3.14 \times (0.125 \times 10^{-2})^2 \times 2.0 \times 10^{11}}$$

$$= \mathbf{1.5 \times 10^{-4} \text{ m}}.$$

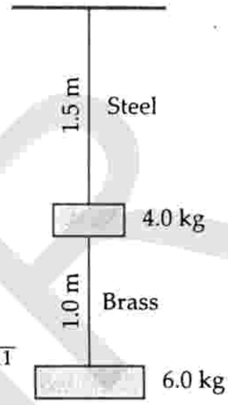


Fig. 9.34

For brass wire :

$$l_2 = 1.0 \text{ m}, \quad r_2 = 0.125 \times 10^{-2} \text{ m}$$

$$F_1 = 6 \text{ kg f} = 6 \times 9.8 \text{ N}, \quad Y_2 = 0.91 \times 10^{11} \text{ Pa}$$

$$\therefore \quad \Delta l_2 = \frac{F_2}{\pi r_2^2} \cdot \frac{l_2}{Y_2}$$

$$= \frac{6 \times 9.8 \times 1.0}{3.14 \times (0.125 \times 10^{-2})^2 \times 0.91 \times 10^{11}}$$

$$= \mathbf{1.3 \times 10^{-4} \text{ m}}.$$

9.6. *The edge of an aluminium cube is 10 cm long. One face of the cube is firmly fixed to a vertical wall. A mass of 100 kg is then attached to the opposite face of the cube. The shear modulus of aluminium is 25 G Pa. What is the vertical deflection of this face ? (1 Pa = 1 Nm^{-2}).*

Ans. Area of the face on which force is applied,

$$A = 10 \text{ cm} \times 10 \text{ cm} = 100 \text{ cm}^2$$

$$= 100 \times 10^{-4} \text{ m}^2 = 10^{-2} \text{ m}^2$$

$$F = \text{L.I.} = 100 \times 10 = 1000 \text{ N},$$

$$\eta = 25 \text{ G Pa} = 25 \times 10^9 \text{ Pa}$$

$$l = 10 \text{ cm} = 0.10 \text{ m}$$

As $$\eta = \frac{F/A}{\Delta l / l} = \frac{F}{A} \cdot \frac{l}{\Delta l}$$

$$\therefore \quad \Delta l = \frac{F}{A} \cdot \frac{l}{\eta} = \frac{1000 \times 0.10}{10^{-2} \times 25 \times 10^9} = \mathbf{4 \times 10^{-7} \text{ m}}.$$

9.7. *Four identical hollow cylindrical columns of mild steel support a big structure of mass 50,000 kg. The inner and outer radii of each column are 30 cm and 40 cm respectively. Assuming the load distribution to be uniform, calculate the compressional strain of each column. The Young's modulus of steel is 2.0×10^{11} Pa.*

Ans. Here $r_1 = 30 \text{ cm} = 0.3 \text{ m}$,

$$r_2 = 40 \text{ cm} = 0.4 \text{ m}, \quad Y = 2.0 \times 10^{11} \text{ Pa}$$

As the load is uniformly distributed among the four columns, hence the load on each column

$$= \frac{50,000}{4} \text{ kg} = 12500 \text{ kg}$$

$$\therefore \quad F = 12500 \times 9.8 \text{ N}$$

Also A = Area of cross-section of each column

$$= \pi r_2^2 - \pi r_1^{\,2} = \pi (r_2^2 - r_1^2)$$

$$= \frac{22}{7}[(0.4)^2 - (0.3)^2] = \frac{22}{7} \times 0.07 = 0.22 \text{ m}^2$$

$\therefore$ Compressional strain

$$= \frac{\text{Stress}}{\text{Young's modulus}} = \frac{F/A}{Y} = \frac{F}{AY}$$

$$= \frac{12500 \times 9.8}{0.22 \times 2.0 \times 10^{11}} = \mathbf{2.8 \times 10^{-6}}.$$

9.8. *A piece of copper having a rectangular cross-section of 15.2 mm × 19.1 mm is pulled in tension with 44,500 N force, producing only elastic deformation. Calculate the resulting strain.*

Ans. Here $F = 44,\ 500$ N,

$$A = 15.2 \text{ mm} \times 19.1 \text{ mm}$$

$$= 15.2 \times 19.1 \times 10^{-6} \text{ m}^2$$

For copper,

$$Y = 1.2 \times 10^{11} \text{ Nm}^{-2}$$

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{F}{A \times \text{strain}}$$

$$\therefore \quad \text{Strain} = \frac{F}{AY} = \frac{44500}{15.2 \times 19.1 \times 10^{-6} \times 1.2 \times 10^{11}}$$

$$= \mathbf{0.001277}.$$

9.9. *A steel cable with a radius of 1.5 cm supports a chairlift at a ski area. If the maximum stress is not to exceed 10^8 Nm^{-2}, what is the maximum load the cable can support ?*

Ans. Maximum stress

$$= \frac{\text{Maximum load}}{\text{Area of cross - section}} = \frac{\text{Maximum load}}{\pi r^2}$$

$\therefore$ Maximum load $= \pi r^2 \times$ Maximum stress

$$= 3.142 \times (1.5 \times 10^{-2})^2 \times 10^8 \text{ N}$$

$$= \mathbf{7.07 \times 10^4 \ N}.$$

9.10. *A rigid bar of mass 15 kg is supported symmetrically by three wires each 2.0 m long. Those at each end are of copper and the middle one is of iron. Determine the ratios of their diameters if each is to have the same tension.*

Ans. Let T be the tension in each wire. As the bar is supported symmetrically by the three wires, the increase in length Δl of each wire should be same.

Now $\quad Y = \frac{T}{A} \frac{l}{\Delta l}$

For all wires, we have same l, Δl and T.

Hence $\quad Y \propto \frac{1}{A} \quad$ or $\quad A \propto \frac{1}{Y}$

or $\quad \frac{\pi D^2}{4} \propto \frac{1}{Y} \quad$ or $\quad D \propto \frac{1}{\sqrt{Y}}$

$$\therefore \quad \frac{D_{\text{copper}}}{D_{\text{iron}}} = \sqrt{\frac{Y_{\text{iron}}}{Y_{\text{copper}}}} = \sqrt{\frac{1.9 \times 10^{11} \text{ Pa}}{1.1 \times 10^{11} \text{ Pa}}} = \mathbf{1.3.}$$

9.11. *A 14.5 kg mass, fastened to the end of a steel wire of unstretched length 1.0 m, is whirled in a vertical circle with an angular velocity of 2 rev/s at the bottom of the circle. The cross-sectional area of the wire is 0.005 cm^2. Calculate the elongation of the wire when the mass is at the lowest point of its path.*

Ans. The centripetal force at the lowest point is given by

$$mr\omega^2 = T - mg$$

where T is the tension in the wire when the mass is at the lowest point.

$\therefore$ Tension,

$$T = mg + mr\omega^2 = m[g + r(2\pi v)^2]$$

$$= 14.5[9.8 + 1.0 \times 4 \times \pi^2 \times (2)^2]$$

$$= 14.5[9.8 + 16 \times 9.87] = 14.5 \times 167.72$$

$$= 2431.94 \text{ N.}$$

Now $\quad Y = \frac{\text{Stress}}{\text{Strain}} = \frac{F/A}{\Delta l / l} = \frac{T/A}{\Delta l / l}$

$$\therefore \quad \Delta l = \frac{Tl}{AY} = \frac{2431.94 \times 1.0}{0.065 \times 10^{-4} \times 2 \times 10^{11}}$$

$$= \mathbf{1.87 \times 10^{-3} \ m.}$$

9.12. *Compute the bulk modulus of water from the following data : Initial volume = 100.0 litre, pressure increase = 100.0 atm, final volume = 100.5 litre (1 atm = 1.013×10^5 Pa).*

Ans. Here $p = 100 \text{ atm} = 100 \times 1.013 \times 10^5$ Pa

Initial volume,

$$V = 100.0 \text{ litre} = 100.0 \times 10^{-3} \text{ m}^3$$

Final volume,

$$V' = 100.5 \text{ litre} = 100.5 \times 10^{-3} \text{ m}^3$$

Increase in volume,

$$\Delta V = (100.5 - 100.0) \times 10^{-3} = 0.5 \times 10^{-3} \text{ m}^3$$

Bulk modulus of water,

$$\kappa = \frac{pV}{\Delta V} = \frac{100 \times 1.013 \times 10^5 \times 100.0 \times 10^{-3}}{0.5 \times 10^{-3}}$$

$$= \mathbf{2.026 \times 10^9 \ Pa}.$$

9.13. *What is the density of ocean water at a depth, where the pressure is 80.0 atm, given that its density at the surface is 1.03×10^3 kgm^{-3} ? Compressibility of water $= 45.8 \times 10^{-11}$ Pa^{-1}. Given 1 atm $= 1.013 \times 10^5$ Pa.*

Ans. Compressibility

$$= \frac{1}{\kappa} = 45.8 \times 10^{-11} \text{ Pa}^{-1}$$

Change in pressure,

$$p = 80 - 1 = 79 \text{ atm} = 79 \times 1.013 \times 10^5 \text{ Pa}$$

Density at the surface, $\rho = 1.03 \times 10^3 \text{ kgm}^{-3}$

As $\kappa = \dfrac{p}{\Delta V / V}$

$\therefore \quad \dfrac{\Delta V}{V} = p \times \dfrac{1}{\kappa}$

$$= 79 \times 1.013 \times 10^5 \times 45.8 \times 10^{-11}$$

$$= 3.665 \times 10^{-3}$$

Also, $\dfrac{\Delta V}{V} = \dfrac{V - V'}{V} = \dfrac{M/\rho - M/\rho'}{M/\rho} = 1 - \dfrac{\rho}{\rho'}$

or $\dfrac{\rho}{\rho'} = 1 - \dfrac{\Delta V}{V}$.

or $\rho' = \dfrac{\rho}{1 - \dfrac{\Delta V}{V}} = \dfrac{1.03 \times 10^3}{1 - 3.665 \times 10^{-3}}$

$$= \frac{1.03 \times 10^3}{0.996} = \mathbf{1.034 \times 10^3 \text{ kgm}^{-3}}.$$

9.14. *Compute the fractional change in volume of a glass slab, when subjected to a hydraulic pressure of 10 atm.*

Ans. Here $p = 10 \text{ atm} = 10 \times 1.013 \times 10^5 \text{ Nm}^{-2}$, $\kappa = 37 \times 10^9 \text{ Nm}^{-2}$

Bulk modulus, $\kappa = \dfrac{pV}{\Delta V}$

Fractional change in volume of glass slab,

$$\frac{\Delta V}{V} = \frac{p}{\kappa} = \frac{10 \times 1.013 \times 10^5}{37 \times 10^9} = \mathbf{2.74 \times 10^{-5}}$$

9.15. *Determine the volume contraction of a solid copper cube, 10 cm on an edge, when subjected to a hydraulic pressure of 7.0×10^6 Pa.*

Ans. Original volume

$$V = (10 \text{ cm})^3 = 1000 \text{ cm}^3$$

$$= 1000 \times 10^{-6} \text{ m}^3 = 10^{-3} \text{ m}^3$$

Pressure, $p = 7.0 \times 10^6$ Pa

For copper, $\kappa = 140 \times 10^9$ Pa

Bulk modulus, $\kappa = \dfrac{pV}{\Delta V}$

Volume contraction of copper cube,

$$\Delta V = \frac{pV}{\kappa} = \frac{7.0 \times 10^6 \times 10^{-3}}{140 \times 10^9}$$

$$= 0.05 \times 10^{-6} \text{ m}^3 = \mathbf{0.05 \text{ cm}^3}.$$

9.16. *How much should the pressure on a litre of water be changed to compress it by 0.10% ?*

Ans. Here $V = 1 \text{ litre} = 10^{-3} \text{m}^3$,

$$\frac{\Delta V}{V} = 0.10\% = \frac{0.10}{100} = 0.001$$

For water, $\kappa = 2.2 \times 10^9 \text{ Nm}^{-2}$

As $\kappa = \dfrac{pV}{\Delta V}$

$\therefore \quad p = \dfrac{\kappa \, \Delta V}{V} = 2.2 \times 10^9 \times 0.001$

$$= \mathbf{2.2 \times 10^6 \text{ Nm}^{-2}}.$$

9.17. *Anvils made of single crystals of diamond, with the shape as shown in Fig. 9.35, are used to investigate behaviour of materials under very high pressures. Flat faces at the narrow end of the anvil have a diameter of 0.5 mm, and the wide ends are subjected to a compressional force of 50,000 N. What is the pressure at the tip of the anvil ?*

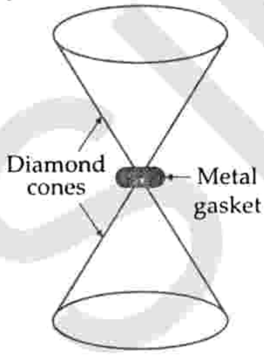


Fig. 9.35

Ans. Here $r = 0.25 \text{ mm} = 0.25 \times 10^{-3}$ m

$$F = 50{,}000 \text{ N}$$

Pressure at the tip of the anvil

$$= \frac{\text{Force}}{\text{Area}} = \frac{F}{\pi r^2} = \frac{50{,}000}{3.14 \times (0.25 \times 10^{-3})^2}$$

$$= \mathbf{2.55 \times 10^{11} \text{ Nm}^{-2}}.$$

9.18. *A rod of length 1.05 m having negligible mass is supported at its ends by two wires of steel (wire A) and aluminium (wire B) of equal lengths as shown in Fig. 9.36. The cross-sectional areas of wires A and B are 1.0 mm² and 2.0 mm² respectively. At what point along the rod should a mass m be suspended in order to produce (a) equal stresses and (b) equal strains in both steel and aluminium wires ?*

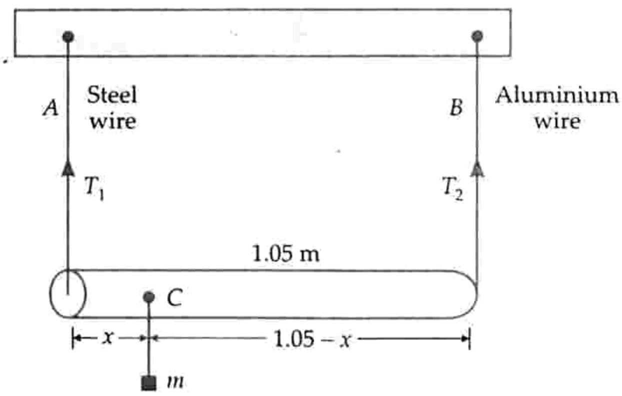


Fig. 9.36

Ans. Suppose the mass m is suspended at distance x from the wire A. Let T_1 and T_2 be the tensions in the steel and aluminium wires respectively.

(*a*) Stress in steel wire $= \dfrac{T_1}{A_1}$

Stress in aluminium wire $= \dfrac{T_2}{A_2}$

As both the stresses are equal, so

$$\frac{T_1}{A_1} = \frac{T_2}{A_2} \quad \text{or} \quad \frac{T_1}{T_2} = \frac{A_1}{A_2} = \frac{1.0\ \text{mm}^2}{2.0\ \text{mm}^2} = \frac{1}{2}$$

Now the moments about point C are equal because the system is in equilibrium.

$$\therefore \quad T_1 x = T_2 (1.05 - x)$$

or $$\frac{T_1}{T_2} = \frac{1.05 - x}{x} \quad \text{or} \quad \frac{1}{2} = \frac{1.05 - x}{x}$$

or $$x = 2.10 - 2x$$

or $$x = \mathbf{0.7\ m} \qquad \text{(from steel wire)}$$

(*b*) Strain $= \dfrac{\text{Stress}}{\text{Young's modulus}}$

$\therefore$ Strain in steel wire $= \dfrac{T_1 / A_1}{Y_1} = \dfrac{T_1}{A_1 Y_1}$

Strain in aluminium wire $= \dfrac{T_2}{A_2 Y_2}$

For the two strains to be equal,

$$\frac{T_1}{A_1 Y_1} = \frac{T_2}{A_2 Y_2}$$

or $$\frac{T_1}{T_2} = \frac{A_1 Y_1}{A_2 Y_2} = \frac{1.0\ \text{mm}^2 \times 200 \times 10^9\ \text{Pa}}{2.0\ \text{mm}^2 \times 70 \times 10^9\ \text{Pa}} = \frac{10}{7}$$

Again, $$T_1 x = T_2 (1.05 - x)$$

or $$\frac{T_1}{T_2} = \frac{1.05 - x}{x} \quad \text{or} \quad \frac{10}{7} = \frac{1.05 - x}{x}$$

or $$10x = 7.35 - 7x$$

or $$x = \frac{7.35}{17} = \mathbf{0.43\ m} \qquad \text{(from steel wire)}$$

9.19. *A mild steel wire of length 1.0 m and cross-sectional area* 0.50×10^{-2} *cm*^{2} *is stretched, well within its elastic limit, horizontally between two pillars. A mass of 100 g is suspended from the mid-point of wire. Calculate the depression at the mid-point.*

Ans. The situation is shown in Fig. 9.37. The increase in length of the wire when it is pulled down into shape BOC is

$$\Delta l = BO + OC - 2l = 2BO - 2l$$

$$= 2(l^2 + x^2)^{1/2} - 2l = 2l\left(1 + \frac{x^2}{l^2}\right)^{1/2} - 2l$$

$$= 2l\left(1 + \frac{x^2}{2l^2}\right) - 2l = \frac{x^2}{l}$$

$$\therefore \quad \text{Strain} = \frac{\Delta l}{2l} = \frac{x^2}{2l^2}$$

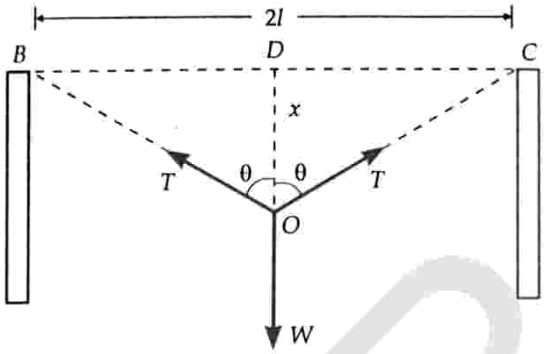


Fig. 9.37

Let T be the tension in the wire. Equating the vertical components of the forces, we get

$$2T\cos\theta = W \quad \text{or} \quad T = \frac{W}{2\cos\theta}$$

Now,

$$\cos\theta = \frac{x}{OB} = \frac{x}{\sqrt{l^2 + x^2}} = \frac{x}{l\left[1 + \dfrac{x^2}{l^2}\right]^{1/2}} = \frac{x}{l\left[1 + \dfrac{x^2}{2l^2}\right]}$$

As $\dfrac{x^2}{2l^2} << 1$ so $1 + \dfrac{x^2}{2l^2} \simeq 1$

$$\therefore \quad \cos\theta = \frac{x}{l} \quad \text{and} \quad T = \frac{W}{2(x/l)} = \frac{Wl}{2x}$$

$$\text{Stress} = \frac{T}{A} = \frac{Wl}{2Ax}$$

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{Wl}{2Ax} \times \frac{2l^2}{x^2} = \frac{Wl^3}{Ax^3}$$

or $$x = l\left[\frac{W}{YA}\right]^{1/3} = \frac{1.0}{2}\left[\frac{0.100 \times 9.8}{2 \times 10^{11} \times 0.5 \times 10^{-6}}\right]^{1/3}$$

$$= 0.5\,(9.8 \times 10^{-6})^{1/3} = 0.5 \times 2.14 \times 10^{-2}$$

$$= \mathbf{1.07 \times 10^{-2} = 1.07\ cm.}$$

9.20. *Two strips of metal are riveted together at their ends by four rivets, each of diameter 6.0 mm. What is the maximum tension that can be exerted by the riveted strip if the shearing stress on the rivet is not to exceed* 2.3×10^9 *Pa ? Assume that each rivet is to carry one quarter of the load.*

Ans. Let the tension exerted by riveted strip = F

This tension would provide shearing force on the *four* rivets, which share it equally.

$\therefore$ Shearing force on each rivet $= \dfrac{F}{4}$

and shearing stress on each rivet $= \dfrac{F/4}{A} = \dfrac{F}{4A}$

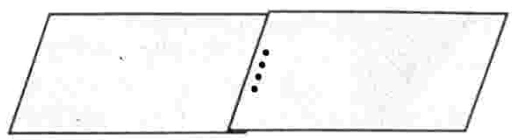


Fig. 9.38

As the maximum shearing stress on each rivet is given to be 2.3×10^9 Pa, so we have

$$\frac{F_{max}}{4A} = 2.3 \times 10^9$$

or $$F_{max} = 4A \times 2.3 \times 10^9 = 4 \times \pi r^2 \times 2.3 \times 10^9$$

$$= 4 \times \frac{22}{7} \times (3.0 \times 10^{-3})^2 \times 2.3 \times 10^9$$

$$= 260.2 \times 10^3 \text{ N} = \mathbf{260\ kN}.$$

9.21. *The Marina trench is located in the Pacific Ocean and at one place it is nearly eleven km beneath the surface of water. The water pressure at the bottom of the trench is about 1.1×10^8 Pa. A steel ball of initial volume 0.32 m^3 is dropped into the ocean and falls to the bottom of the trench. What is the change in the volume of the ball when it reaches to the bottom ?*

Ans. Here $p = 1.01 \times 10^8$ Pa, $V = 0.32$ m^3

For steel, $\kappa = 160 \times 10^9$ Pa

As $$\kappa = \frac{p}{\Delta V / V}$$

$$\therefore \quad \Delta V = \frac{pV}{\kappa} = \frac{1.01 \times 10^8 \times 0.32}{160 \times 10^9}$$

$$= \mathbf{2.02 \times 10^{-4}\ m^3}.$$

Text Based Exercises

Type A : Very Short Answer Questions **1 Mark Each**

1. Among the interatomic and intermolecular forces, which are the stronger ones ? How much ?
2. Write a relation between interatomic force and potential energy.
3. Write an expression showing the dependence of potential energy on interatomic separation *r*.
4. Among the three states of matter : solid, liquid and gas ; which has got its own shape ?
5. Give another name for amorphous solids.
6. What is an isotropic medium ?
7. What is an anisotropic solid ?
8. What type of solids : crystalline or amorphous, are anisotropic ?
9. Give one example each of isotropic and anisotropic substance ?
10. What is the meaning of word 'amorphous' ?
11. Which state of the solid is more stable : crystalline or amorphous ?
12. Give an example of semi-crystalline solid.
13. What is a deforming force ? **[Himachal 07]**
14. What is restoring force ? **[Himachal 07]**
15. Are the elastic restoring forces conservative in nature ?
16. Define stress and strain. **[Delhi 95, 05C]**
17. Define elastic limit. **[Meghalaya 99]**
18. Define yield point.
19. Define Young's modulus of elasticity. **[Delhi 03]**
20. Write dimensional formula of Young's modulus.
21. What is the value of Young's modulus for a perfectly rigid body ?
22. What is the limitation of Hooke's law ?
23. Why any metallic part of a mechinery is never subjected to a stress beyond the elastic limit of the material ?
24. Define bulk modulus of elasticity. Give its units and dimensions.
25. What is the reciprocal of bulk modulus known as ?
26. Give the SI unit and dimensions of compressibility ?
27. What is the value of bulk modulus for an incompressible liquid ?
28. What is the value of modulus of rigidity for an incompressible liquid ? **[Central Schools 09]**
29. What a Poisson's ratio ? Does it have any unit ?
30. Young's modulus of the material of a wire is *Y*. On pulling the wire by a force *F*, the increase in its length is *x*. What will be the potential energy of the stretched wire ?
31. Define modulus of rigidity. What is its SI unit ?
32. What is breaking stress for a wire of unit cross-section called ?
33. What is elastic after effect ?
34. What is elastic fatigue ?
35. What is elastic hysteresis ?
36. If the length of a wire increases by 1 mm under 1 kg wt, what will be the increase under

    (*i*) 2 kg wt (*ii*) 100 kg wt ?

37. What will happen to the potential energy of the atoms of a solid when it is compressed ? What happens when a wire is stretched ?

38. A spiral spring is stretched by a force. What type of strain is produced in it ?

39. State Hooke's law. **[Himachal 03, 05 ; Delhi 05]**

40. Which of the two forces-deforming or restoring force is responsible for elastic behaviour of substance. **[Himachal 01]**

Answers

1. Interatomic forces are 50 to 100 times stronger than the intermolecular forces.

2. $F = -\frac{dU}{dr}$

3. $U(r) = \frac{A}{r^n} - \frac{B}{r^m}$

   For most of the substances, exponents n and m are 12 and 6 respectively.

4. Solid has its own shape.

5. Amorphous solids are also called glassy solids.

6. Any medium which has the same physical properties in all directions is called an isotropic medium.

7. If a solid has different physical properties (thermal, electrical, mechanical and optical) in different directions, then it is said to be an anisotropic solid.

8. Crystalline solids are anisotropic.

9. Glass is an isotropic substance and quartz is anisotropic substance.

10. The word amorphous means without any form.

11. Crystalline state is more stable.

12. Polyethylene.

13. A force which produces a change in the size or shape of a body is called deforming force.

14. The intermolecular force developed within a body due to relative molecular displacements is called restoring force.

15. The elastic restoring forces are conservative only when the loading and unloading curves coincide.

16. *Stress* is the internal restoring force set up per unit area of a deformed body.

    $$\text{Stress} = \frac{\text{Force}}{\text{Area}}$$

    Strain. When external deforming forces act upon a body, the fractional change produced in the body is called strain.

    $$\text{Strain} = \frac{\text{Change in any dimension}}{\text{Original dimension}}$$

17. That maximum stress within which a body completely regains its original size and shape after the removal of the deforming force is called elastic limit.

18. The stage of a material when it yields to the deforming force and goes on increasing in length even when the load is kept constant is called yield point.

19. Young's modulus of elasticity is defined as the ratio of the stress to the longitudinal strain, within the elastic limit.

20. $[ML^{-1}T^{-2}]$.

21. Infinite.

22. Hooke's law is obeyed upto the proportionality limit of the material.

23. When a metallic part is subjected to a stress beyond the elastic limit, a permanent deformation is set up in it.

24. Bulk modulus of elasticity is defined as the ratio of tangential stress to shear strain within the elastic limit. Its SI unit is Nm^{-2} and its dimensional formula is $[ML^{-1}T^{-2}]$.

25. Compressibility.

26. The SI unit of compressibility is $N^{-1}m^2$ and its dimensional formula is $[M^{-1}LT^2]$.

27. Infinite.

28. Zero.

29. The ratio of the lateral strain to the longitudinal strain is called Poisson's ratio. It has no units.

30. P.E. of stretched wire $= \frac{1}{2} Fx$.

31. Modulus of rigidity is the ratio of tangential stress to the shearing strain within the elastic limit. Its SI unit is Nm^{-2}.

32. Tensile strength.

33. The delay on the part of the body in regaining its original configuration on the removal of the deforming force is called the elastic after effect.

34. Elastic fatigue is defined as the loss in the strength of a material caused due to repeated alternating strains to which the material is subjected.

35. The fact that the stress-strain curve is not retraced on reversing the strain (for a material like rubber) is called elastic hysteresis.

36. (*i*) 2 mm

(*ii*) The wire will break, because the stretching force is increased beyond breaking force.

37. Potential energy increases in both cases.

38. Shear strain.

39. Refer to point 15 of Glimpses.

40. Restoring force.

Type B : Short Answer Questions

2 or 3 Marks Each

1. Distinguish between elasticity and plasticity.
2. State Hooke's law and hence define modulus of elasticity. **[Meghalaya 96]**
3. Define stress and strain and derive their units. What is Hooke's law ? Write its one limitation. **[Delhi 04]**
4. Which is more elastic – iron or rubber ? Why ? **[Delhi 96 ; Himachal 05, 07]**
5. Define the terms stress and strain and also state their SI units. Draw the stress versus strain graph for a metallic wire, when stretched upto the breaking point. **[Himachal 06]**
6. Represent graphically the variation of extension with load in an elastic body. On the graph mark :
   (*a*) Hooke's law region. (*b*) Elastic limit.
   (*c*) Yield point. (*d*) Breaking point. **[Delhi 99]**
7. Define the terms Young's modulus, bulk modulus and modulus of rigidity. Also give their units. **[Himachal 06, 07]**
8. What are elastomers ? Draw a stress-strain graph for an elastomer.
9. Define elastic limit and elastic fatigue. What are ductile and brittle substances ? **[Himachal 06]**
10. What is elastic after effect ? What is its importance ?
11. Describe elastic hysteresis. Mention its two applications.
12. Explain how is the knowledge of elasticity useful in selecting metal ropes used in cranes for lifting heavy loads.
13. Explain why should the beams used in the construction of bridges have large depth and small breadth.

    Or

    Why are girders given I shape ?
14. Show that the maximum height of any mountain on the earth cannot exceed 10 km.
15. Explain why hollow shafts are preferred to solid shafts for transmitting torque.
16. What is elastic potential energy ? Prove that the work done by a stretching force to produce certain tension in a wire is

    $$W = \frac{1}{2}\text{ Stretching force} \times \text{extension}.$$
17. Derive an expression for energy stored in a wire due to extension. **[Chandigarh 07]**
18. Define Poisson's ratio. Write an expression for it. What is the significance of negative sign in this expression ?

Answers

1. Refer answer to Q. 1 on page 9.1.
2. Refer answer to Q.7 on page 9.3 and Q. 8 on page 9.4.
3. Refer to points 12, 13 and 15 of Glimpses.
4. Refer to the solution of Problem 3 on page 9.26.
5. Refer to points 12 and 13 of Glimpses and see Fig. 9.6 on page 9.4.
6. See Fig. 9.6 on page 9.4.
7. Refer to points 17, 18, 19 and 20 of Glimpses.
8. Refer answer to Q. 14 on page 9.6.
9. Refer to points 14, 25, 26 and 30 of Glimpses.
10. Refer answer to Q. 19 on page 9.17.
11. Refer answer to Q. 21 on page 9.18.
12. Refer answer to Q. 23 on page 9.18.
13. Refer answer to Q. 24 on page 9.18.
14. Refer answer to Q. 25 on page 9.19.
15. Refer answer to Q. 26 on page 9.19.
16. Refer answer to Q. 27 on page 9.19.
17. Refer answer to Q.27 on page 9.19.
18. Refer answer to Q. 28 on page 9.21.

Type C : Long Answer Questions

5 Marks Each

1. What is interatomic force ? Discuss the variation of interatomic force with the interatomic separation. **[Himachal 02]**
2. Define the term elasticity. Give an explanation of the elastic properties of materials in terms of interatomic forces ?
3. State Hooke's law. How can it be verified experimentally ?
4. Define Young's modulus of elasticity. Describe an experiment for the determination of Young's modulus of the material of a wire.
5. Define Young's modulus, bulk modulus and modulus of rigidity. Write mathematical expressions for these moduli. What is compressibility.
6. Discuss stress vs. strain graph, explaining clearly the terms elastic limit, permanent set, elastic hysteresis and tensile strength. **[Delhi 06]**
7. Describe stress-strain relationship for a loaded steel wire and hence explain the terms elastic limit, yield point, tensile strength. **[Chandigarh 03]**
8. On the basis of stress-strain curves, distinguish between ductile, brittle and malleable materials.
9. Derive an expression for the elastic potential energy stored in a stretched wire under stress. Define the terms elastic after effect and elastic fatigue. **[Himachal 07C]**

Answers

1. Refer to point 1 of Glimpses and answer to Q. 2 on page 9.2.
2. Refer answer to Q. 2 on page 9.2.
3. Refer answer to Q. 7 on page 9.3.
4. Refer answer to Q. 11 on page 9.5.
5. Refer to points 17, 18, 19 and 21 of Glimpses.
6. Refer answer to Q. 10 on page 9.4.
7. Refer answer to Q. 10 on page 9.4.
8. Refer answer to Q. 12 and Q. 13 on page 9.6.
9. Refer answer to Q.27 on page 9.19 and refer to points 29 and 30 of Glimpses.

Competition Section

Mechanical Properties of Solids

GLIMPSES

1. **Interatomic force.** It is the force between the atoms of a molecule. It arises due to the electrostatic interaction between the nuclei of two atoms, their electron clouds and between the nucleus of one atom and the electron cloud of the other atom.
2. **Intermolecular forces.** It is the force acting between the two molecules of a substance due to electrostatic interaction between their oppositely charged ends. Such forces operate over distances of 10^{-9} m and are weaker than interatomic forces.
3. **Solids.** A solid is a large accumulation of ($\sim 10^{23}$) atoms or molecules. It has definite shape and size. The solids we come across in daily life can be classified into three groups :
   (*i*) Crystalline solids.
   (*ii*) Amorphous solids.
   (*iii*) Semi-crystalline solids.
4. **Crystalline solids.** Those solids in which the atoms or molecules are arranged in a regular and repeated geometrical pattern are called crystalline solids. Such solids are bounded by flat surfaces, are anisotropic, have sharp melting points and have long range order in their structure.

   Examples. Rock salt, quartz, mica, calcite, diamond, etc.
5. **Amorphous solids.** These are the solids in which the atoms or molecules are not arranged according to certain definite geometrical order, *i.e.*, the atoms or molecules are arranged in a random order. Such solids are isotropic, do not have flat surfaces and their melting points are not sharp. They are super-cooled liquids.

   Examples. Glass, rubber, cellulose, bitumen, bone and many plastics.
6. **Semi-crystalline solids.** These are the solids in which the crystalline phase is inter-dispersed in the amorphous phase, *i.e.*, in which crystalline and amorphous phases co-exist. Polyethylene and protein are such solids.
7. **Deforming force.** A force which changes the size and shape of body is called deforming force.
8. **Elasticity.** The property by virtue of which a body regains its original size and shape after the removal of deforming force is called elasticity.
9. **Perfectly elastic body.** If a body regains its original size and shape completely and immediately after the removal of deforming force, it is said to be perfectly elastic body. The nearest approach to a perfectly elastic body is quartz fibre.
10. **Plasticity.** The property by virtue of which a body does not regain its original size and shape even after the removal of the deforming force is called plasticity.
11. **Perfectly plastic body.** If a body does not show any tendency to regain its original size and shape even after the removal of deforming force, it is said to be perfectly plastic body. Putty and paraffin wax are nearly perfectly elastic bodies.
12. **Stress.** The restoring force set up per unit area of a deformed body is called stress.

$$\text{Stress} = \frac{\text{Restoring force}}{\text{Area}} = \frac{\text{Applied force}}{\text{Area}} = \frac{F}{A}$$

The SI unit of stress is Nm^{-2} and the CGS unit is dyne cm^{-2}. Its dimensional formula is $[ML^{-1}T^{-2}]$.

Stress is of *two* types :

(*i*) Normal stress

(*ii*) Tangential stress.

13. **Strain.** The ratio of the change in any dimension produced in the body to the original dimension is called strain.

$$\text{Strain} = \frac{\text{Change in dimension}}{\text{Original dimension}}$$

As strain is the ratio of two like quantities, it has no units and dimensions. Strain is of *three* types :

(*i*) **Longitudinal strain.** It is defined as the ratio of change in length to the original length.

$$\text{Longitudinal strain} = \frac{\text{Change in length}}{\text{Original length}} = \frac{\Delta l}{l}$$

(*ii*) **Volumetric strain.** It is defined as the ratio of the change in volume to the original volume.

$$\text{Volumetric strain} = \frac{\text{Change in volume}}{\text{Original volume}} = \frac{\Delta V}{V}.$$

(*iii*) **Shear strain.** It is defined as the angle θ (in radian) through which a face originally perpendicular to the fixed face gets turned on applying tangential deforming force.

$$\text{Shear strain} = \theta \simeq \tan\theta$$

$$= \frac{\text{Relative displacement between two} \parallel \text{planes}}{\text{Distance between} \parallel \text{planes}}$$

$$= \frac{\Delta l}{l}$$

14. **Elastic limit.** The maximum stress upto which stress is proportional to strain is called elastic limit.

15. **Hooke's law.** It states that within the elastic limit, stress is proportional to strain.

$$\text{Stress} \propto \text{Strain}$$

or $\quad \text{Stress} = E \times \text{strain}$

The constant E is called modulus of elasticity of the material of the body.

16. **Modulus of elasticity or coefficient of elasticity.** It is defined as the ratio of stress to the corresponding strain, within the elastic limit.

It is of *three* types :

(*i*) Young's modulus

(*ii*) Bulk modulus

(*iii*) Modulus of rigidity.

17. **Young's modulus of elasticity.** It is defined as the ratio of longitudinal stress to the longitudinal strain within the elastic limit.

It is given by

$$Y = \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}} = \frac{F/A}{\Delta l/l} = \frac{F}{A}\cdot\frac{l}{\Delta l}$$

18. **Bulk modulus of elasticity.** It is defined as the ratio of normal stress to volumetric strain within the elastic limit.

It is given by

$$\kappa = \frac{\text{Normal stress}}{\text{Volumetric strain}} = -\frac{F/A}{\Delta V/V}$$

$$= -\frac{p}{\Delta V/V} = -\frac{pV}{\Delta V}.$$

The negative sign shows that the volume decreases with the increase in stress.

19. **Modulus of rigidity.** It is defined as the ratio of tangential stress to shear strain within the elastic limit.

It is given by

$$\eta = \frac{\text{Tangential stress}}{\text{Shear strain}} = \frac{F/A}{\theta} = \frac{F/A}{\Delta l/l} = \frac{F}{A}\cdot\frac{l}{\Delta l}$$

20. **Units of moduli of elasticity.** As strain is a pure ratio, the unit of elasticity is same as that of stress. So SI unit of Y, κ or η is Nm^{-2} and the CGS unit is dyne cm^{-2}.

21. **Compressibility.** The reciprocal of the bulk modulus of a material is called its compressibility.

$$\text{Compressibility} = 1/\kappa$$

The SI unit of compressibility is $N^{-1}\ m^2$ or Pa^{-1}.

22. **Yield point.** The stress beyond which a solid flows is called yield point. For example, a paste of flour and water flows under its own weight.

23. **Breaking stress.** The stress corresponding to which a wire breaks is called breaking stress.

Breaking force = Breaking stress × area of cross-section of the wire.

24. **Plastic region.** The region of stress-strain curve between the elastic limit and the breaking point is called plastic region.

25. **Ductile materials.** The materials which have large plastic range of extension are called ductile materials. Such materials can be drawn into thin wires.

Examples. Copper, silver, iron, aluminium, etc.

26. **Brittle materials.** The materials which have very small range of plastic extension are called brittle materials. Such materials break as soon as the stress is increased beyond the elastic limit.

Examples. Cast iron, glass, ceramics, etc.

27. **Malleable metals.** The metals which can be hammered or rolled into thin sheets are called malleable metals.

Examples. Gold, silver, lead, etc.

28. **Elastomers.** The materials which can be elastically stretched to large values of strain are called elastomers. They have large elastic region but do not obey Hooke's law.

Examples. Rubber and elastic tissue of aorta.

29. **Elastic after effect.** The delay in regaining the original state by a body on the removal of deforming force is called elastic after effect. This effect is minimum for quartz and phosphor bronze and maximum for glass fibre.

30. **Elastic fatigue.** It is defined as the loss in the strength of a material caused due to repeated alternating strains to which the material is subjected.

31. **Elastic hysteresis.** The fact that the stress-strain curve is not retraced on reversing the strain (for a material like rubber) is called elastic hysteresis.

32. **Poisson's ratio.** When the length of a loaded wire increases, its diameter decreases. The ratio of the lateral strain to the longitudinal strain is called Poisson's ratio. It is given by

$$\sigma = \frac{\text{Lateral strain}}{\text{Longitudinal strain}} = -\frac{\Delta D / D}{\Delta l / l}$$

Poisson's ratio σ is a pure number. It has no units or dimensions.

33. **Elastic potential energy in a stretched wire.** The work done against the internal restoring forces in stretching a wire is stored as its elastic potential energy.

It is given by

$$U = \frac{1}{2}\text{ Stretching force} \times \text{extension}$$
$$= \frac{1}{2} F\,\Delta l$$
$$= \frac{1}{2} \cdot \frac{F}{A} \cdot \frac{\Delta l}{l} \cdot Al$$
$$= \frac{1}{2}\text{ Stress} \times \text{strain} \times \text{volume of the wire}$$

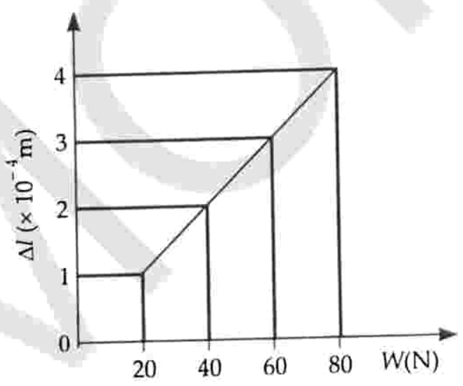
P.E. stored per unit volume of the wire or elastic energy density is

$$u = \frac{U}{V} = \frac{1}{2}\text{ Stress} \times \text{strain}$$
$$= \frac{1}{2}\text{ Young's modulus} \times \text{strain}^2.$$

IIT Entrance Exam

MULTIPLE CHOICE QUESTIONS WITH ONE CORRECT ANSWER

1. The adjacent graph shows the extension (Δl) of a wire of length 1 m suspended from the top of a roof at one end and with a load W connected to the other end.



If the cross-sectional area of the wire is $10^{-6}\,\text{m}^2$, calculate the Young's modulus of the material of the wire

(*a*) $2 \times 10^{11}\,\text{N/m}^2$

(*b*) $2 \times 10^{-11}\,\text{N/m}^2$

(*c*) $3 \times 10^{-12}\,\text{N/m}^2$

(*d*) $2 \times 10^{-13}\,\text{N/m}^2$ [IIT 04]

2. The following four wires are made of the same material. Which of these will have the largest extension, when the same tension is applied ?

(*a*) length = 50 cm, diameter = 0.5 mm

(*b*) length = 100 cm, diameter = 1 mm

(*c*) length = 200 cm, diameter = 2 mm

(*d*) length = 300 cm, diameter = 3 mm [IIT 81]

3. A wire of length L, and cross-sectional area A is made of a material of Young's modulus Y. If the wire is stretched by an amount x, the work done is

(*a*) $YAx^2/2L$ (*b*) YAx^2/L

(*c*) $YAx/2L$ (*d*) YAx^2L [IIT 87]

4. The pressure of a medium is changed from 1.01×10^5 Pa to 1.65×10^5 Pa and change in volume is 10% keeping temperature constant. The bulk modulus of the medium is

(*a*) 204.8×10^5 Pa (*b*) 102.4×10^5 Pa

(*c*) 51.2×10^5 Pa (*d*) 1.55×10^5 Pa [IIT 05]

5. A given quantity of an ideal gas is at pressure P and absolute temperature T. The isothermal bulk modulus of the gas is

(*a*) $2P/3$ (*b*) P

(*c*) $3P/2$ (*d*) $2P$ [IIT 98]

Answers and Explanations

1. (*a*) When $W = 20$ N, $\Delta l = 1\times 10^{-4}$ m

$$Y = \frac{F}{A}\cdot\frac{l}{\Delta l} = \frac{W}{A}\cdot\frac{l}{\Delta l}$$
$$= \frac{20}{10^{-6}}\times\frac{1}{1\times 10^{-4}}$$
$$= \mathbf{2\times 10^{11}\ Nm^{-2}}.$$

2. (*a*) $\Delta l = \frac{F}{A}\cdot\frac{l}{Y} = \frac{4F}{\pi D^2}\cdot\frac{l}{Y}$

$\therefore\quad \Delta l \propto \frac{l}{D^2}$

In (*a*), $\frac{l}{D^2} = \frac{50}{(0.05)^2} = 2\times 10^4\ \text{cm}^{-1}$

In (*b*), $\frac{l}{D^2} = \frac{100}{(0.1)^2} = 10^4\ \text{cm}^{-1}$

In (*c*), $\frac{l}{D^2} = \frac{200}{(0.2)^2} = 5\times 10^3\ \text{cm}^{-1}$

In (*d*), $\frac{l}{D^2} = \frac{300}{(0.3)^2} = 3.3\times 10^3\ \text{cm}^{-1}$

Hence Δl is maximum in (*a*).

3. (*a*) W = Average force × increase in length

$$= \frac{1}{2}F\times x = \frac{1}{2}\frac{F}{A}\cdot\frac{L}{x}\cdot\frac{Ax^2}{L}$$
$$= \frac{1}{2}Y\cdot\frac{Ax^2}{L} = \mathbf{\frac{YAx^2}{2L}}.$$

4. (*d*) Bulk modulus $= \frac{\Delta P}{\Delta V/V}$

$$= \frac{(1.165-1.01)\times 10^5}{10/100}$$
$$= \mathbf{1.55\times 10^5\ Pa}.$$

5. (*b*) Isothermal bulk modulus of a gas

= Pressure of the gas = $\boldsymbol{P}$.

AIEEE

1. A wire elongates by l mm when a load W is hanged from it. If the wire goes over a pulley and two weights W each are hung at the two ends, the elongation of the wire will be (in mm)

(*a*) $l/2$ (*b*) l

(*c*) $2l$ (*d*) zero [AIEEE 06]

2. A wire fixed at the upper end stretches by length l by applying a force F. The work done in stretching is

(*a*) $F/2l$ (*b*) Fl

(*c*) $2Fl$ (*d*) $Fl/2$

[AIEEE 04]

3. A wire suspended vertically from one of its ends is stretched by attaching a weight of 200 N to the lower end. The weight stretches the wire by 1 mm. Then the elastic energy stored in the wire is

(*a*) 0.2 J (*b*) 10 J

(*c*) 20 J (*d*) 0.1 J [AIEEE 03]

4. If S is stress and Y is Young's modulus of material of a wire, the energy stored in the wire per unit volume is

(*a*) $2Y/S$ (*b*) $S/2Y$

(*c*) $2S^2Y$ (*d*) $\frac{S^2}{2Y}$

[AIEEE 05]

5. Two wires are made of the same material and have the same volume. However wire 1 has cross-sectional area A and wire 2 has cross-sectional area $3A$. If the length of wire 1 increases by Δx on applying force F, how much force is needed to stretch wire 2 by the same amount ?

(*a*) F (*b*) $4F$

(*c*) $6F$ (*d*) $9F$ [AIEEE 09]

6. The potential energy function for the force between two atoms in a diatomic molecule is approximately given by

$$U(x) = \frac{a}{x^{12}} - \frac{b}{x^6},$$

where a and b are constants and x is the distance between the atoms. If the dissociation energy of the molecule is $D = [U(x=\infty) - U_{\text{at equilibrium}}]$, D is

(*a*) $\frac{b^2}{6a}$ (*b*) $\frac{b^2}{2a}$

(*c*) $\frac{b^2}{12a}$ (*d*) $\frac{b^2}{4a}$

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Answers and Explanations

1. (*b*) Refer to the solution of Problem 1 on page 9.28.

2. (*d*) $W = \frac{1}{2}$ Stretching force × increase in length $= \frac{1}{2} \boldsymbol{Fl}$.

3. (*d*) Here $F = 200$ N, $l = 1$ mm $= 10^{-3}$ m

$$U = \frac{1}{2} Fl = \frac{1}{2} \times 200 \times 10^{-3} = \mathbf{0.1\ J}.$$

4. (*d*) Elastic P.E. stored per unit volume,

$$u = \frac{1}{2} \times \text{Stress} \times \text{Strain}$$

$$= \frac{1}{2} \times \text{Stress} \times \frac{\text{Stress}}{Y} = \frac{\mathbf{S^2}}{\mathbf{2Y}}.$$

5. (*d*)

$$Y = \frac{F}{A} \cdot \frac{l}{\Delta l} = \frac{F}{A^2} \frac{Al}{\Delta x} = \frac{FV}{A^2 \Delta x}$$

$$\therefore \quad F = \frac{Y \Delta x}{V} A^2 \quad \text{or} \quad F \propto A^2$$

$$F' = (3A)^2$$

$$\frac{F'}{F} = 9$$

$$F' = \mathbf{9F}.$$

6. (*d*)

$$U = \frac{a}{x^{12}} - \frac{b}{x^6}$$

$$F = -\frac{dU}{dx} = \frac{12a}{x^{13}} - \frac{6b}{x^7}$$

At equilibrium,

$$F = \frac{12a}{x^{13}} - \frac{6b}{x^7} = 0$$

$$\therefore \quad x^6 = \frac{2a}{b}$$

$$\Rightarrow \quad x = \left(\frac{2a}{b}\right)^{1/6}$$

$$U(x = \infty) = 0$$

$$U_{\text{equilibrium}} = \frac{a}{\left(\frac{2a}{b}\right)^2} - \frac{b}{\left(\frac{2a}{b}\right)} = -\frac{b^2}{4a}$$

$$\therefore \quad D = U(x = \infty) - U_{\text{equilibrium}}$$

$$= 0 - \left(-\frac{b^2}{2a}\right) = \frac{\mathbf{b^2}}{\mathbf{4a}}$$