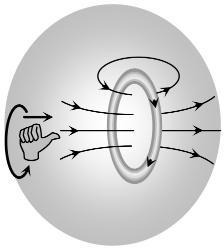
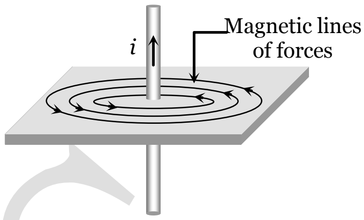
Magnetic Effect of Current



Oersted found that a magnetic field is established around a current carrying conductor.

Magnetic field exists as long as there is current in the wire.

The direction of magnetic field was found to be changed when direction of current was reversed.



Note : ❒ A moving charge produces magnetic as well as electric field, unlike a stationary charge which only produces electric field.

Biot Savart's Law

Biot-Savart's law is used to determine the magnetic field at any point due to a current carrying conductors.

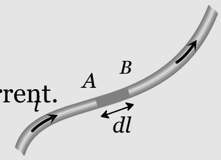
This law is although for infinitesimally small conductors yet it can be used for long conductors. In order to understand the Biot-Savart's law, we need to understand the term current-element.

Current element

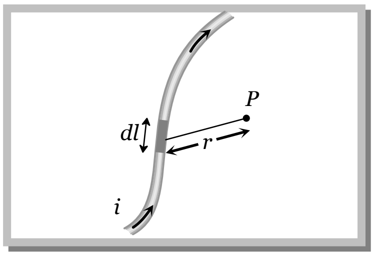
It is the product of current and length of infinitesimal segment of current carrying wire.

The current element is taken as a vector quantity. Its direction is same as the direction of current.

Current element $AB = i\vec{dl}$



In the figure shown below, there is a segment of current carrying wire and P is a point where magnetic field is to be calculated. $i\vec{dl}$ is a current element and r is the distance of the point 'P' with respect to the current element $i\vec{dl}$. According to Biot-Savart Law, magnetic field at point 'P' due to the current element $i\vec{dl}$ is given by the expression, $\boldsymbol{dB = k\dfrac{i\,dl\sin\theta}{r^2}}$ also $B = \int dB = \dfrac{\mu_0 i}{4\pi}.\int \dfrac{dl\sin\theta}{r^2}$



In C.G.S. : $k = 1 \Rightarrow dB = \dfrac{idl\sin\theta}{r^2}$ *Gauss*

In S.I. : $k = \dfrac{\mu_0}{4\pi} \Rightarrow dB = \dfrac{\mu_0}{4\pi}.\dfrac{idl\sin\theta}{r^2}$ *Tesla*

where μ_0 = Absolute permeability of air or vacuum $= 4\pi\times 10^{-7}\dfrac{Wb}{Amp-metre}$. It's other units are $\dfrac{Henry}{metre}$ or $\dfrac{N}{Amp^2}$ or $\dfrac{Tesla-metre}{Ampere}$

(1) **Different forms of Biot-Savarts law**

Vector form	Biot-Savarts law in terms of current density	Biot-savarts law in terms of charge and it's velocity
Vectorially, $d\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{i(d\vec{l} \times \hat{r})}{r^2} = \frac{\mu_0}{4\pi} \cdot \frac{i(d\vec{l} \times \vec{r})}{r^3}$ $\Rightarrow$ Direction of $d\vec{B}$ is perpendicular to both $d\vec{l}$ and $\hat{r}$. This is given by right hand screw rule.	In terms of current density $d\vec{B} = \frac{\mu_0}{4\pi} \frac{\vec{J} \times \vec{r}}{r^3} dV$ where $j = \frac{i}{A} = \frac{idl}{Adl} = \frac{idl}{dV}$ = current density at any point of the element, dV = volume of element	In terms of charge and it's velocity, $d\vec{B} = \frac{\mu_0}{4\pi} q \frac{(\vec{v} \times \vec{r})}{r^3}$ $\because\ id\vec{l} = \frac{q}{dt} d\vec{l} = q\frac{d\vec{l}}{dt} = q\vec{v}$

(2) **Similarities and differences between Biot-Savart law and Coulomb's Law**

(i) The current element produces a magnetic field, whereas a point charge produces an electric field.

(ii) The magnitude of magnetic field varies as the inverse square of the distance from the current element, as does the electric field due to a point charge.

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{id\vec{l} \times \hat{r}}{r^2} \quad \text{Biot-Savart Law} \qquad \vec{F} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} \hat{r} \quad \text{Coulomb's Law}$$

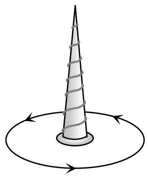
(iii) The electric field created by a point charge is radial, but the magnetic field created by a current element is perpendicular to both the length element $d\vec{l}$ and the unit vector $\hat{r}$.

Direction of Magnetic Field

The direction of magnetic field is determined with the help of the following simple laws :

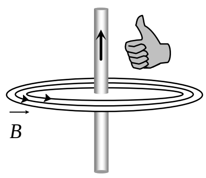
(1) **Maxwell's cork screw rule**

According to this rule, if we imagine a right handed screw placed along the current carrying linear conductor, be rotated such that the screw moves in the direction of flow of current, then the direction of rotation of the thumb gives the direction of magnetic lines of force.

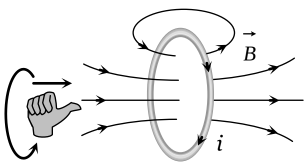


(2) **Right hand thumb rule**

According to this rule if a current carrying conductor is held in the right hand such that the thumb of the hand represents the direction of current flow, then the direction of folding fingers will represent the direction of magnetic lines of force.



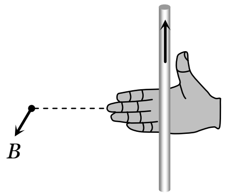
(3) **Right hand thumb rule of circular currents**



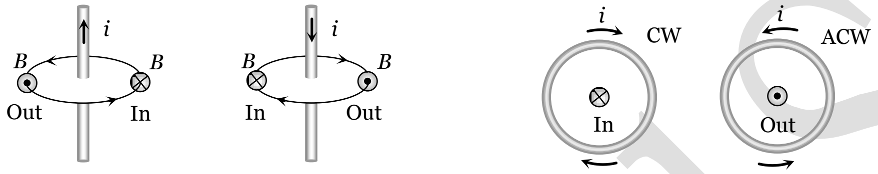
According to this rule if the direction of current in circular conducting coil is in the direction of folding fingers of right hand, then the direction of magnetic field will be in the direction of stretched thumb.

(4) **Right hand palm rule**

If we stretch our right hand such that fingers point towards the point. At which magnetic field is required while thumb is in the direction of current then normal to the palm will show the direction of magnetic field.



Note : ❒ If magnetic field is directed perpendicular and into the plane of the paper it is represented by ⊗ (cross) while if magnetic field is directed perpendicular and out of the plane of the paper it is represented by ⊙ (dot)



In : Magnetic field is away from the observer or perpendicular inwards.

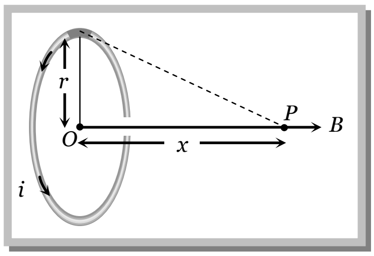
Out : Magnetic field is towards the observer or perpendicular outwards.

Application of Biot-Savarts Law

(1) **Magnetic field due to a circular current**

If a coil of radius r, carrying current i then magnetic field on it's axis at a distance x from its centre given by

$$B_{axis} = \frac{\mu_0}{4\pi} \cdot \frac{2\pi N i r^2}{(x^2 + r^2)^{3/2}}$$; where N = number of turns in coil.



Different cases

***Case* 1 : Magnetic field at the centre of the coil**

(i) At centre $x = 0 \Rightarrow B_{centre} = \frac{\mu_0}{4\pi} \cdot \frac{2\pi N i}{r} = \frac{\mu_0 N i}{2r} = B_{max}$

(ii) For single turn coil $N = 1 \Rightarrow B_{centre} = \frac{\mu_0}{4\pi} \cdot \frac{2\pi i}{r} = \frac{\mu_0 i}{2r}$ (iii) In C.G.S. $\frac{\mu_0}{4\pi} = 1 \Rightarrow B_{centre} = \frac{2\pi i}{r}$

Note : ❒ $B_{centre} \propto N$ (i, r constant), $B_{centre} \propto i$ (N, r constant), $B_{centre} \propto \frac{1}{r}$ (N, i constant)

***Case* 2 : Ratio of B_{centre} and B_{axis}**

The ratio of magnetic field at the centre of circular coil and on it's axis is given by $\frac{B_{centre}}{B_{axis}} = \left(1 + \frac{x^2}{r^2}\right)^{3/2}$

(i) If $x = \pm a,\ B_c = 2\sqrt{2}\, B_a$ $x = \pm\frac{a}{2},\ B_c = \frac{5\sqrt{5}}{8} B_a$ $x = \pm\frac{a}{\sqrt{2}},\ B_c = \left(\frac{3}{2}\right)^{3/2} B_a$

(ii) If $B_a = \frac{B_c}{n}$ then $x = \pm r\sqrt{(n^{2/3} - 1)}$ and if $B_a = \frac{B_c}{\sqrt{n}}$ then $x = \pm r\sqrt{(n^{1/3} - 1)}$

***Case* 3 : Magnetic field at very large/very small distance from the centre**

(i) If $x >> r$ (very large distance) $\Rightarrow B_{axis} = \frac{\mu_0}{4\pi}.\frac{2\pi Nir^2}{x^3} = \frac{\mu_0}{4\pi}.\frac{2NiA}{x^3}$ where $A = \pi r^2$ = Area of each turn of the coil.

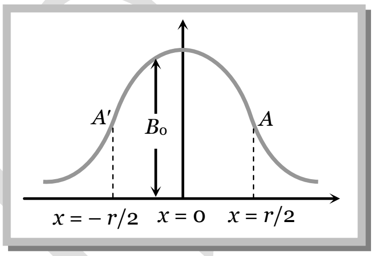
(ii) If $x << r$ (very small distance) $\Rightarrow B_{axis} \neq B_{centre}$, but by using binomial theorem and neglecting higher power of $\frac{x^2}{r^2}$; $B_{axis} = B_{centre}\left(1 - \frac{3}{2}\frac{x^2}{r^2}\right)$

***Case* 4 : *B-x* curve**

The variation of magnetic field due to a circular coil as the distance x varies as shown in the figure.

B varies non-linearly with distance x as shown in figure and is maximum when $x^2 = \text{min} = 0$, *i.e.*, the point is at the centre of the coil and it is zero at $x = \pm\infty$.

Point of inflection (A and A') **:** Also known as points of curvature change or pints of zero curvature.



(i) At these points B varies linearly with $x \Rightarrow \frac{dB}{dx}$ = constant $\Rightarrow \frac{d^2B}{dx^2} = 0$.

(ii) They locates at $x = \pm\frac{r}{2}$ from the centre of the coil.

(iii) Separation between point of inflextion is equal to radius of coil (r)

(iv) Application of points of inflextion is "Hamholtz coils" arrangement.

Note : ❒ The magnetic field at $x = \frac{r}{2}$ is $B = \frac{4\mu_0 Ni}{5\sqrt{5}\,r}$

(2) **Helmholtz coils**

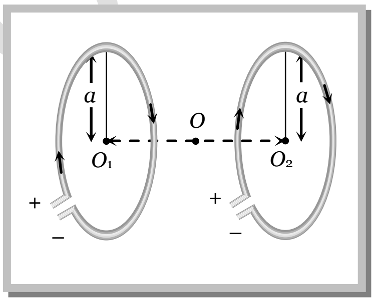
(i) This is the set-up of two coaxial coils of same radius such that distance between their centres is equal to their radius.

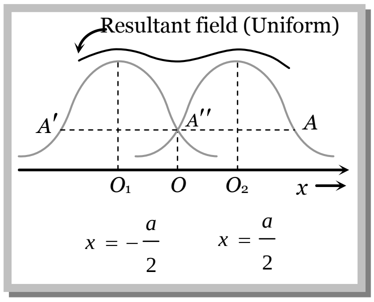
(ii) These coils are used to obtain uniform magnetic field of short range which is obtained between the coils.

(iii) At axial mid point O, magnetic field is given by $B = \frac{8\mu_0 Ni}{5\sqrt{5}R} = 0.716\frac{\mu_0 Ni}{R} = 1.432\,B$, where $B = \frac{\mu_0 Ni}{2R}$

(iv) Current direction is same in both coils otherwise this arrangement is not called Helmholtz's coil arrangement.

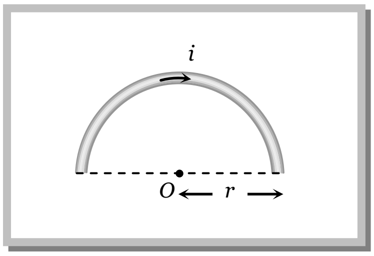
(v) Number of points of inflextion $\Rightarrow$ Three (A, A', A'')



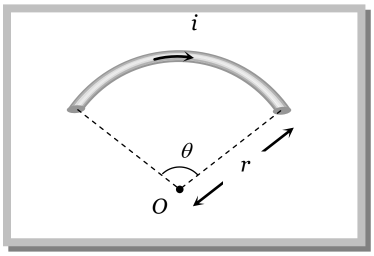


Note : ❒ The device whose working principle based on this arrangement and in which uniform magnetic field is used called as "Halmholtz galvanometer".

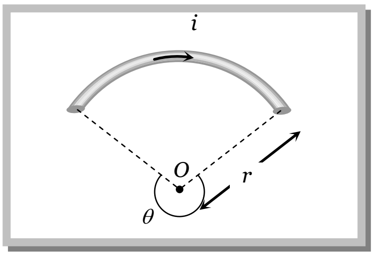
(3) **Magnetic field due to current carrying circular arc :** Magnetic field at centre O



$$B = \frac{\mu_0}{4\pi}.\frac{\pi i}{r} = \frac{\mu_0 i}{4r}$$



$$B = \frac{\mu_0}{4\pi}.\frac{\theta i}{r}$$



$$B = \frac{\mu_0}{4\pi}.\frac{(2\pi - \theta)i}{r}$$

Special results

If magnetic field at the centre of circular coil is denoted by $B_0 \left(= \frac{\mu_0}{4\pi}.\frac{2\pi i}{r}\right)$

Magnetic field at the centre of arc which is making an angle θ at the centre is

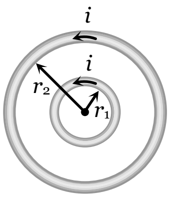
$$B_{arc} = \left(\frac{B_0}{2\pi}\right).\theta$$

Angle at centre	Magnetic field at centre in term of B_0
360° (2π)	B_0
180° (π)	$B_0 / 2$
120° ($2\pi/3$)	$B_0 / 3$
90° ($\pi/2$)	$B_0 / 4$
60° ($\pi/3$)	$B_0 / 6$
30° ($\pi/6$)	$B_0 / 12$

(4) **Concentric circular loops** ($N = 1$)

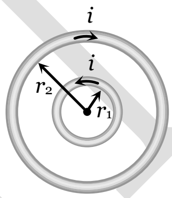
(i) Coplanar and concentric : It means both coils are in same plane with common centre

(a) Current in same direction



$$B_1 = \frac{\mu_0}{4\pi}2\pi i\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

(b) Current in opposite direction



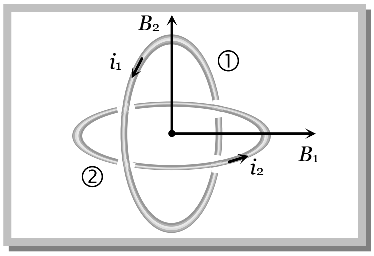
$$B_2 = \frac{\mu_0}{4\pi}2\pi i\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

Note : ❐ $\frac{B_1}{B_2} = \left(\frac{r_2 + r_1}{r_2 - r_1}\right)$

(ii) Non-coplanar and concentric : Plane of both coils are perpendicular to each other

Magnetic field at common centre

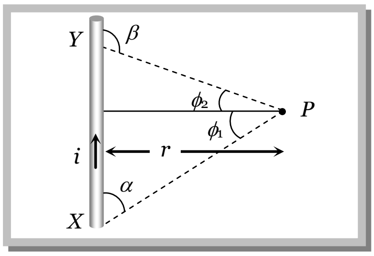
$$B = \sqrt{B_1^2 + B_2^2} = \frac{\mu_0}{2r}\sqrt{i_1^2 + i_2^2}$$



(5) **Magnetic field due to a straight current carrying wire**

Magnetic field due to a current carrying wire at a point P which lies at a perpendicular distance r from the wire as shown is given as

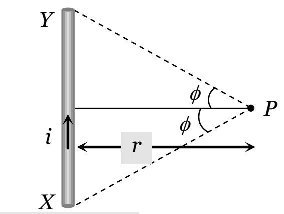
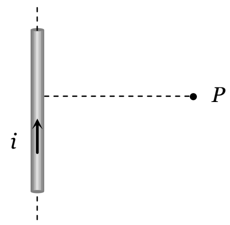
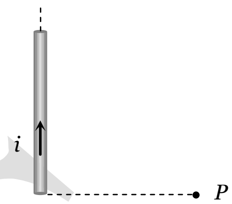
$$B = \frac{\mu_0}{4\pi}.\frac{i}{r}(\sin\phi_1 + \sin\phi_2)$$



From figure $\alpha = (90^o - \phi_1)$ and $\beta = (90^o + \phi_2)$

Hence $B = \frac{\mu_o}{4\pi}.\frac{i}{r}(\cos\alpha - \cos\beta)$

Different cases

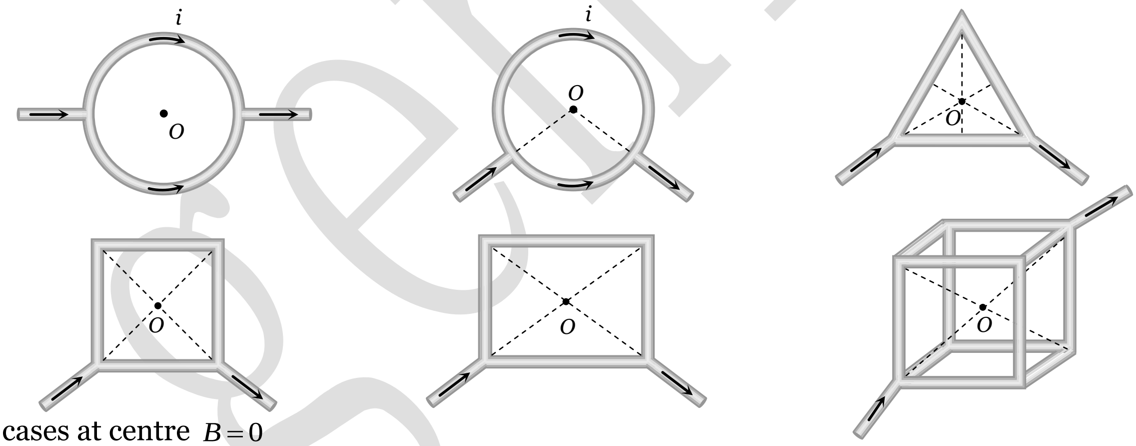
***Case* 1 :** When the linear conductor XY is of finite length and the point P lies on it's perpendicular bisector as shown  $\phi_1 = \phi_2 = \phi$ So $B = \frac{\mu_0}{4\pi}\cdot\frac{i}{r}(2\sin\phi)$	***Case* 2 :** When the linear conductor XY is of infinite length and the point P lies near the centre of the conductor  $\phi_1 = \phi_2 = 90^o$. So, $B = \frac{\mu_0}{4\pi}\frac{i}{r}[\sin 90^o + \sin 90^o] = \frac{\mu_0}{4\pi}\frac{2i}{r}$	***Case* 3 :** When the linear conductor is of infinite length and the point P lies near the end Y or X  $\phi_1 = 90^o$ and $\phi_2 = 0^o$. So, $B = \frac{\mu_0}{4\pi}\frac{i}{r}[\sin 90^o + \sin 0^o] = \frac{\mu_0}{4\pi}\frac{i}{r}$

Note : ❒ When point P lies on axial position of current carrying conductor then magnetic field at P

$B = 0$

❒ The value of magnetic field induction at a point, on the centre of separation of two linear parallel conductors carrying equal currents in the same direction is zero.

(6) **Zero magnetic field :** If in a symmetrical geometry, current enters from one end and exists from the other, then magnetic field at the centre is zero.



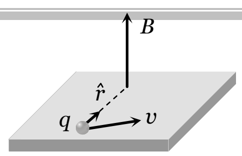
In all cases at centre $B = 0$

Concepts

☞ *If a current carrying circular loop (n = 1) is turned into a coil having n identical turns then magnetic field at the centre of the coil becomes n^2 times the previous field i.e. $B_{(n\ turn)} = n^2\ B_{(single\ turn)}$*

☞ *When a current carrying coil is suspended freely in earth's magnetic field, it's plane stays in **East-West** direction.*

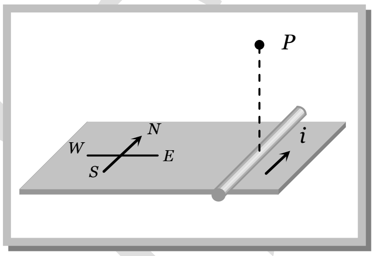
☞ *Magnetic field ($\vec{B}$) produced by a moving charge q is given by $\vec{B} = \frac{\mu_0}{4\pi}\frac{q(\vec{v}\times\vec{r})}{r^3} = \frac{\mu_0}{4\pi}\frac{q(\vec{v}\times\hat{r})}{r^2}$; where v = velocity of charge and v << c (speed of light).*



☞ *If an electron is revolving in a circular path of radius r with speed v then magnetic field produced at the centre of circular path* $B=\frac{\mu_0}{4\pi}.\frac{ev}{r^2}$.

Example

Example: 1 Current flows due north in a horizontal transmission line. Magnetic field at a point *P* vertically above it directed



(a) North wards

(b) South wards

(c) Toward east

(d) Towards west

Solution : (c) By using right hand thumb rule or any other rule which helps to determine the direction of magnetic field.

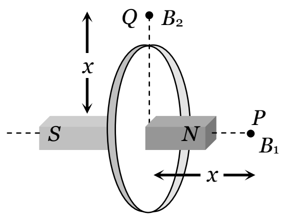
Example: 2 Magnetic field due to a current carrying loop or a coil at a distant axial point *P* is B_1 and at an equal distance in it's plane is B_2 then $\frac{B_1}{B_2}$ is

(a) 2 (b) 1 (c) $\frac{1}{2}$ (d) None of these

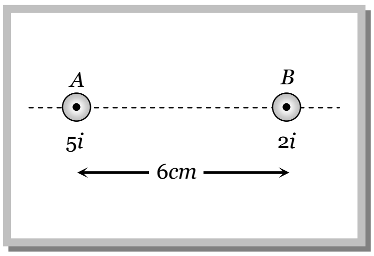
Solution : (a) Current carrying coil behaves as a bar magnet as shown in figure.

We also knows for a bar magnet, if axial and equatorial distance are same then $B_a=2B_e$

Hence, in this equation $\frac{B_1}{B_2}=\frac{2}{1}$



Example: 3 Find the position of point from wire '*B*' where net magnetic field is zero due to following current distribution



(a) 4 *cm*

(b) $\frac{30}{7}$ *cm*

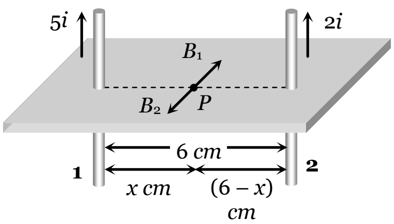
(c) $\frac{12}{7}$ *cm*

(d) 2 *cm*

Solution : (c) Suppose *P* is the point between the conductors where net magnetic field is zero.

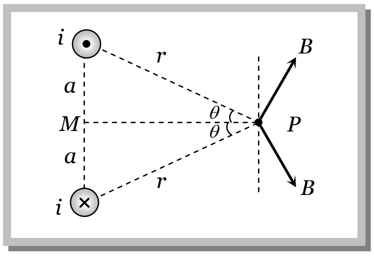
So at *P* |Magnetic field due to conductor 1| = |Magnetic field due to conductor 2|

i.e. $\frac{\mu_0}{4\pi}.\frac{2(5i)}{i}=\frac{\mu_0}{4\pi}.\frac{2(2i)}{(6-x)}\Rightarrow\frac{5}{x}=\frac{9}{6-x}\Rightarrow x=\frac{30}{7}\,cm$



Hence position from $B = 6 - \frac{30}{7} = \frac{12}{7}\,cm$

***Example*: 4** Find out the magnitude of the magnetic field at point P due to following current distribution



(a) $\frac{\mu_0 ia}{\pi r^2}$

(b) $\frac{\mu_0 ia^2}{\pi r}$

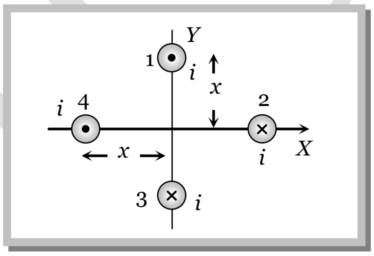
(c) $\frac{\mu_0 ia}{2\pi r^2}$

(d) $\frac{2\mu_0 ia}{\pi r^2}$

Solution : (a) Net magnetic field at P, $B_{net} = 2B\sin\theta$; where B = magnetic field due to one wire at $P = \frac{\mu_0}{4\pi}.\frac{2i}{r}$

and $\sin\theta = \frac{a}{r}$ $\quad\therefore\ B_{net} = 2\times\frac{\mu_0}{4\pi}.\frac{2i}{r}\times\frac{a}{r} = \frac{\mu_0 ia}{\pi r^2}$.

***Example*: 5** What will be the resultant magnetic field at origin due to four infinite length wires. If each wire produces magnetic field 'B' at origin



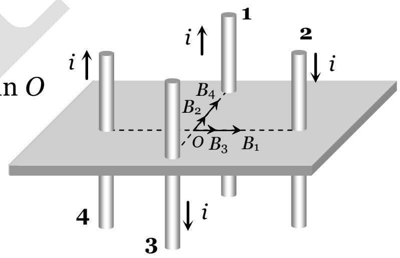
(a) $4B$

(b) $\sqrt{2}\,B$

(c) $2\sqrt{2}\,B$

(d) Zero

Solution : (c) Direction of magnetic field (B_1, B_2, B_3 and B_4) at origin due to wires 1, 2, 3 and 4 are shown in the following figure.



$B_1 = B_2 = B_3 = B_4 = \frac{\mu_0}{4\pi}.\frac{2i}{x} = B$. So net magnetic field at origin O

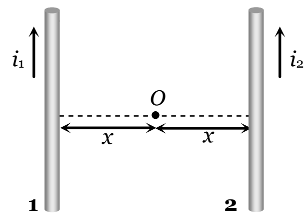
$$B_{net} = \sqrt{(B_1+B_2)^2 + (B_2+B_4)^2}$$

$$= \sqrt{(2B)^2 + (2B)^2} = 2\sqrt{2}B$$

***Example*: 6** Two parallel, long wires carry currents i_1 and i_2 with $i_1 > i_2$. When the currents are in the same direction, the magnetic field at a point midway between the wires is 10 μT. If the direction of i_2 is reversed, the field becomes 30 μT. The ratio i_1/i_2 is

(a) 4 (b) 3 (c) 2 (d) 1

Solution : (c) Initially when wires carry currents in the same direction as shown.

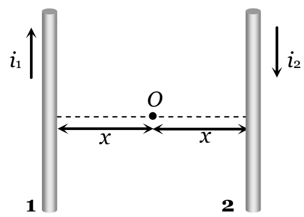


Magnetic field at mid point O due to wires 1 and 2 are respectively

$$B_1 = \frac{\mu_0}{4\pi}.\frac{2i_1}{x} \otimes \text{ and } B_2 = \frac{\mu_0}{4\pi}.\frac{2i_2}{x} \odot$$

Hence net magnetic field at O $\quad B_{net} = \frac{\mu_0}{4\pi}\times\frac{2}{x}(i_1 - i_2)$

$\Rightarrow\ 10\times10^{-6} = \frac{\mu_0}{4\pi}.\frac{2}{x}(i_1 - i_2)$ .....(i)



If the direction of i_2 is reversed then

$$B_1 = \frac{\mu_0}{4\pi}.\frac{2i_1}{x} \otimes \text{ and } B_2 = \frac{\mu_0}{4\pi}.\frac{2i_2}{x} \otimes$$

So $B_{net} = \frac{\mu_0}{4\pi}.\frac{2}{x}(i_1 + i_2) \Rightarrow 30 \times 10^{-6} = \frac{\mu_0}{4\pi}.\frac{2}{x}(i_1 + i_2)$ ......(ii)

Dividing equation (ii) by (i) $\frac{i_1 + i_2}{i_1 - i_2} = \frac{3}{1} \Rightarrow \frac{i_1}{i_2} = \frac{2}{1}$

***Example*: 7** A wire of fixed length is turned to form a coil of one turn. It is again turned to form a coil of three turns. If in both cases same amount of current is passed, then the ratio of the intensities of magnetic field produced at the centre of a coil will be

(a) 9 times of first case (b) $\frac{1}{9}$ times of first case (c) 3 times of first case (d) $\frac{1}{3}$ times of first case

Solution : (a) Magnetic field at the centre of n turn coil carrying current i $B = \frac{\mu_0}{4\pi}.\frac{2\pi n i}{r}$ ......(i)

For single turn $n = 1$ $B = \frac{\mu_0}{4\pi}.\frac{2\pi i}{r}$ .....(ii)

If the same wire is turn again to form a coil of three turns *i.e.* $n = 3$ and radius of each turn $r' = \frac{r}{3}$

So new magnetic field at centre $B' = \frac{\mu_0}{4\pi}.\frac{2\pi(3)}{r'}$ $\Rightarrow B' = 9 \times \frac{\mu_0}{4\pi}.\frac{2\pi i}{r}$ ......(iii)

Comparing equation (ii) and (iii) gives $B' = 9B$.

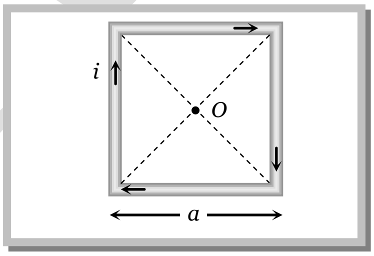
***Example*: 8** A wire in the form of a square of side a carries a current i. Then the magnetic induction at the centre of the square wire is (Magnetic permeability of free space = μ_0)

(a) $\frac{\mu_0 i}{2\pi a}$

(b) $\frac{\mu_0 i\sqrt{2}}{\pi a}$

(c) $\frac{2\sqrt{2}\mu_0 i}{\pi a}$

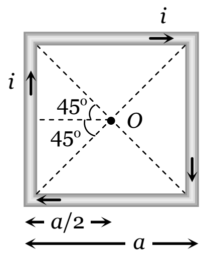
(d) $\frac{\mu_0 i}{\sqrt{2}\pi a}$



Solution : (c) Magnetic field due to one side of the square at centre O

$$B_1 = \frac{\mu_0}{4\pi}.\frac{2i \sin 45^o}{a/2}$$

$$\Rightarrow B_1 = \frac{\mu_0}{4\pi}.\frac{2\sqrt{2}\, i}{a}$$

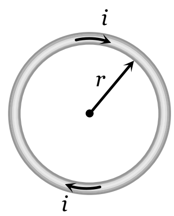


Hence magnetic field at centre due to all side $B_{net} = 4B_1 = \frac{\mu_0(2\sqrt{2}\, i)}{\pi a}$

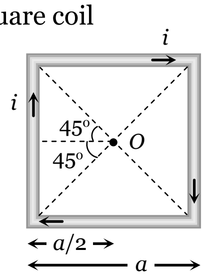
***Example*: 9** The ratio of the magnetic field at the centre of a current carrying circular wire and the magnetic field at the centre of a square coil made from the same length of wire will be

(a) $\frac{\pi^2}{4\sqrt{2}}$ (b) $\frac{\pi^2}{8\sqrt{2}}$ (c) $\frac{\pi}{2\sqrt{2}}$ (d) $\frac{\pi}{4\sqrt{2}}$

Solution : (b) Circular coil



Square coil



Length $L = 2\pi r$

Magnetic field $B = \frac{\mu_0}{4\pi}.\frac{2\pi i}{r} = \frac{\mu_0}{4\pi}.\frac{4\pi^2 i}{r}$

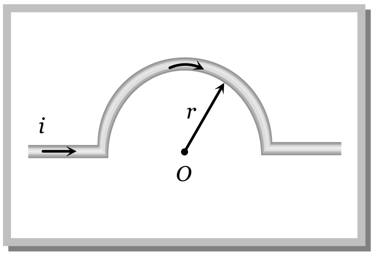
Hence $\frac{B_{circular}}{B_{square}} = \frac{\pi^2}{8\sqrt{2}}$

Length $L = 4a$

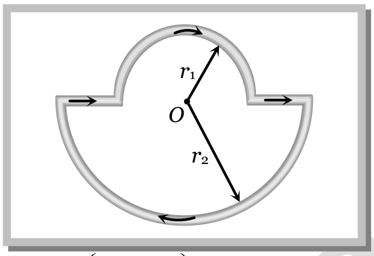
$B = \frac{\mu_0}{4\pi}.\frac{2\sqrt{2}\, i}{a}$ $B = \frac{\mu_0}{4\pi}.\frac{8\sqrt{2}\, i}{a}$

***Example*: 10** Find magnetic field at centre O in each of the following figure

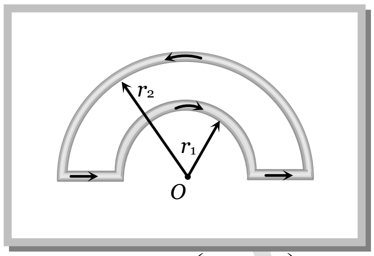
(i)



(ii)



(iii)



(i)

(a) $\frac{\mu_0 i}{r} \otimes$

(b) $\frac{\mu_0 i}{2r} \odot$

(c) $\frac{\mu_0 i}{4r} \otimes$

(d) $\frac{\mu_0 i}{4r} \odot$

(ii)

(a) $\frac{\mu_0 i}{4}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \otimes$

(b) $\frac{\mu_0 i}{4}\left(\frac{1}{r_1} + \frac{1}{r_2}\right) \otimes$

(c) $\frac{\mu_0 i}{4}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \odot$

(d) Zero

(iii)

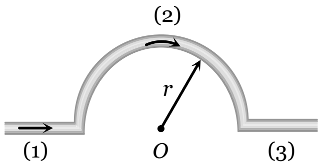
(a) $\frac{\mu_0 i}{4}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \otimes$

(b) $\frac{\mu_0 i}{4}\left(\frac{1}{r_1} + \frac{1}{r_2}\right) \otimes$

(c) $\frac{\mu_0 i}{4}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \odot$

(d) Zero

Solution : (i) (c) Magnetic field at O due to parts 1 and 3, $B_1 = B_3 = 0$

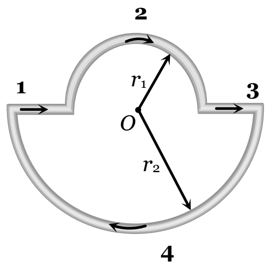


While due to part (2) $B_2 = \frac{\mu_0}{4\pi}.\frac{\pi i}{r} \otimes$

$\therefore$ Net magnetic field at centre O,

$$B_{net} = B_1 + B_2 + B_3 = \frac{\mu_0}{4\pi}.\frac{\pi i}{r} \otimes \Rightarrow B_{net} = \frac{\mu_0 i}{4r} \otimes$$

(ii) (b) $B_1 = B_3 = 0$

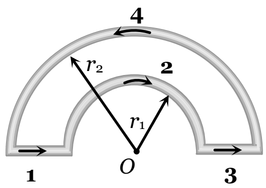


$$B_2 = \frac{\mu_0}{4\pi}.\frac{\pi i}{r_1} \otimes$$

$$B_4 = \frac{\mu_0}{4\pi}.\frac{\pi i}{r_2} \otimes$$

So $B_{net} = B_2 + B_4 = \frac{\mu_0}{4\pi}.\pi i\left(\frac{1}{r_1} + \frac{1}{r_2}\right) \otimes$

(iii) (a) $B_1 = B_3 = 0$



$$B_2 = \frac{\mu_0}{4\pi}.\frac{\pi i}{r_1} \otimes$$

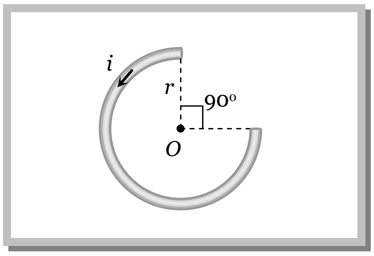
$$B_4 = \frac{\mu_0}{4\pi}.\frac{\pi i}{r_2} \otimes$$

As $|B_2| > |B_4|$

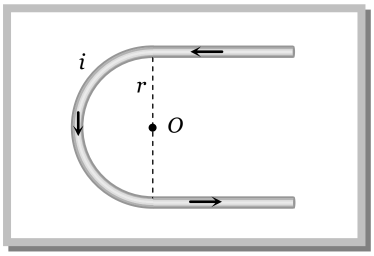
So $B_{net} = B_2 - B_4 \Rightarrow B_{net} = \frac{\mu_0 i}{4}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \otimes$

***Example*: 11** Find magnetic field at centre O in each of each of the following figure

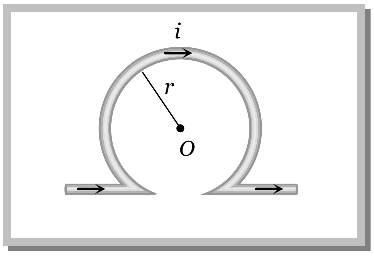
(i)



(ii)



(iii)



(i)	(ii)	(iii)
(a) $\frac{\mu_0 i}{2r} \odot$	(a) $\frac{\mu_0}{2\pi}\frac{i}{r}(\pi - 2) \otimes$	(a) $\frac{\mu_0}{2r}\frac{2i}{r}(\pi + 1) \otimes$
(b) $\frac{\mu_0 i}{2r} \otimes$	(b) $\frac{\mu_0 i}{4\pi}.\frac{i}{r}(\pi + 2) \odot$	(b) $\frac{\mu_0 i}{4r}.\frac{2i}{r}(\pi - 1) \otimes$
(c) $\frac{3\mu_0 i}{8r} \otimes$	(c) $\frac{\mu_0 i}{4r} \otimes$	(c) Zero
(d) $\frac{3\mu_0 i}{8r} \odot$	(d) $\frac{\mu_0 i}{4r} \odot$	(d) Infinite

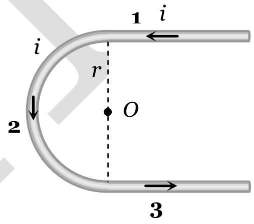
Solution : (i) (d) By using $B = \frac{\mu_0}{4\pi}.\frac{(2\pi - \theta)i}{r} \Rightarrow B = \frac{\mu_0}{4\pi}.\frac{(2\pi - \pi/2)i}{r} = \frac{3\mu_0 i}{8r} \odot$

(ii) (b) Magnetic field at centre O due to section 1, 2 and 3 are respectively

$B_1 = \frac{\mu_0}{4\pi}.\frac{i}{r} \odot$

$B_2 = \frac{\mu_0}{4\pi}.\frac{\pi i}{r} \odot$

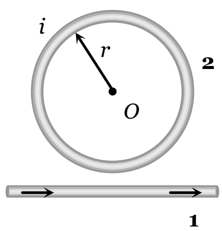
$B_3 = \frac{\mu_0}{4\pi}.\frac{i}{r} \odot$



$\Rightarrow B_{net} = B_1 + B_2 + B_3 = \frac{\mu_0}{4\pi}\frac{i}{r}(\pi + 2) \odot$

(iii) (b) The given figure is equivalent to following figure, magnetic field at O due to long wire (part 1)

$B_1 = \frac{\mu_0}{4\pi}.\frac{2i}{r} \odot$

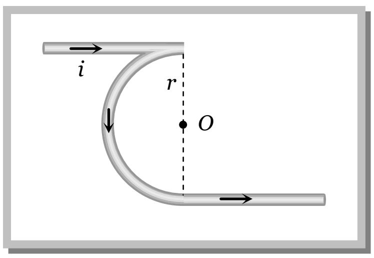


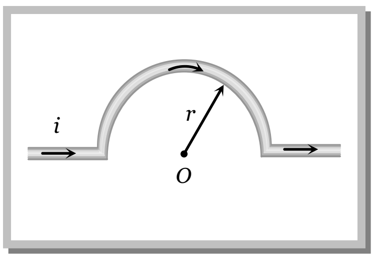
Due to circular coil $B_2 = \frac{\mu_0}{4\pi}.\frac{2\pi i}{r} \otimes$

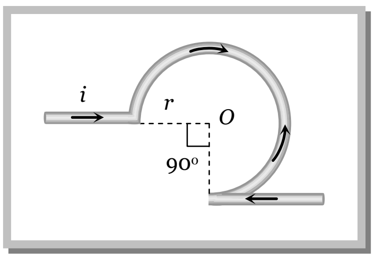
Hence net magnetic field at O

$B_{net} = B_2 - B_1 = \frac{\mu_0}{4\pi}.\frac{2i}{r}(\pi - 1) \otimes$

***Example*: 12** The field B at the centre of a circular coil of radius r is π times that due to a long straight wire at a distance r from it, for equal currents here shows three cases; in all cases the circular part has radius r and straight ones are infinitely long. For same current the field B is the centre P in cases 1, 2, 3 has the ratio**[CPMT 198**





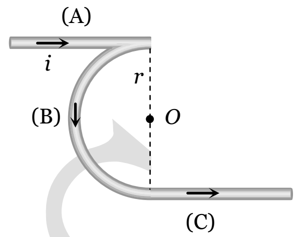


(1) **(2)** **(3)**

(a) $\left(-\frac{\pi}{2}\right):\left(\frac{\pi}{2}\right):\left(\frac{3\pi}{4}-\frac{1}{2}\right)$ (b) $\left(-\frac{\pi}{2}+1\right):\left(\frac{\pi}{2}+1\right):\left(\frac{3\pi}{4}+\frac{1}{2}\right)$

(c) $-\frac{\pi}{2}:\frac{\pi}{2}:\frac{3\pi}{4}$ (d) $\left(-\frac{\pi}{2}-1\right):\left(\frac{\pi}{2}-\frac{1}{2}\right):\left(\frac{3\pi}{4}+\frac{1}{2}\right)$

Solution : (a) ***Case* 1 :** $B_A = \frac{\mu_0}{4\pi}.\frac{i}{r}\otimes$



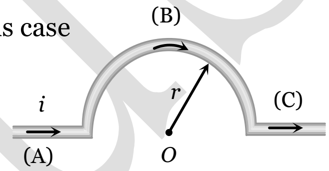
$B_B = \frac{\mu_0}{4\pi}.\frac{i}{r}\odot$

$B_C = \frac{\mu_0}{4\pi}.\frac{i}{r}\odot$

So net magnetic field at the centre of case 1

$B_1 = B_B - (B_A + B_C) \quad \Rightarrow \quad B_1 = \frac{\mu_0}{4\pi}.\frac{\pi i}{r}\odot$ ..... (i)

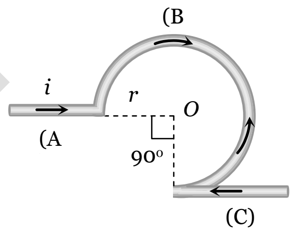
***Case* 2 :** As we discussed before magnetic field at the centre O in this case



$B_2 = \frac{\mu_0}{4\pi}.\frac{\pi i}{r}\otimes$ .....(ii)

***Case* 3 :** $B_A = 0$

$B_B = \frac{\mu_0}{4\pi}.\frac{(2\pi - \pi/2)}{r}\otimes = \frac{\mu_0}{4\pi}.\frac{3\pi i}{2r}\otimes$



$B_C = \frac{\mu_0}{4\pi}.\frac{i}{r}\odot$

So net magnetic field at the centre of case 3

$B_3 = \frac{\mu_0}{4\pi}.\frac{i}{r}\left(\frac{3\pi}{2}-1\right)\otimes$ ....(iii)

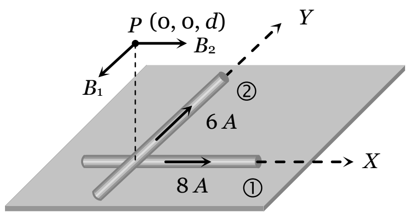
From equation (i), (ii) and (iii) $B_1 : B_2 : B_3 = \pi\odot : \pi\odot : \left(\frac{3\pi}{2}-1\right)\otimes = -\frac{\pi}{2}:\frac{\pi}{2}:\left(\frac{3\pi}{4}-\frac{1}{2}\right)$

***Example*: 13** Two infinite length wires carries currents 8*A* and 6*A* respectively and placed along *X* and *Y*-axis. Magnetic field at a point $P(0,0,d)m$ will be

(a) $\frac{7\mu_0}{\pi d}$ (b) $\frac{10\mu_0}{\pi d}$ (c) $\frac{14\mu_0}{\pi d}$ (d) $\frac{5\mu_0}{\pi d}$

Solution : (d) Magnetic field at *P*

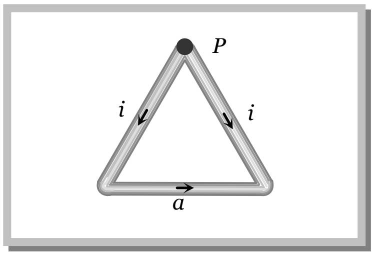
Due to wire 1, $B_1 = \frac{\mu_0}{4\pi}.\frac{2(8)}{d}$



and due to wire 2, $B_2 = \frac{\mu_0}{4\pi}.\frac{2(16)}{d}$

$\therefore \quad B_{net} = \sqrt{B_1^2 + B_2^2} = \sqrt{\left(\frac{\mu_0}{4\pi}.\frac{16}{d}\right)^2 + \left(\frac{\mu_0}{4\pi}.\frac{12}{d}\right)^2} = \frac{\mu_0}{4\pi}\times\frac{2}{d}\times 10 = \frac{5\mu_0}{\pi d}$

***Example*: 14** An equilateral triangle of side '*a*' carries a current *i* then find out the magnetic field at point *P* which is vertex of triangle



(a) $\frac{\mu_0 i}{2\sqrt{3}\pi a} \otimes$

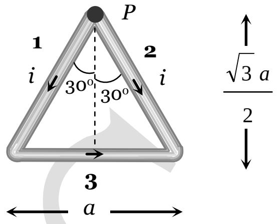
(b) $\frac{\mu_0 i}{2\sqrt{3}\pi a} \odot$

(c) $\frac{2\sqrt{3}\mu_0 i}{\pi a} \odot$

(d) Zero

Solution : (b) As shown in the following figure magnetic field at P due to side 1 and side 2 is zero.

Magnetic field at P is only due to side 3,



which is $B_1 = \frac{\mu_0}{4\pi} \cdot \frac{2i \sin 30^o}{\frac{\sqrt{3}a}{2}} \odot$

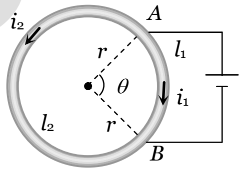
$= \frac{\mu_0}{4\pi} \cdot \frac{2i}{\sqrt{3}a} \odot = \frac{\mu_0 i}{2\sqrt{3}\pi a} \odot$

***Example*: 15** A battery is connected between two points A and B on the circumference of a uniform conducting ring of radius r and resistance R. One of the arcs AB of the ring subtends an angle θ at the centre. The value of, the magnetic induction at the centre due to the current in the ring is **[IIT-JEE 1995]**

(a) Proportional to $2(180^o - \theta)$ (b) Inversely proportional to r

(c) Zero, only if $\theta = 180^o$ (d) Zero for all values of θ

Solution : (d) Directions of currents in two parts are different, so directions of magnetic fields due to these currents are different.



Also applying Ohm's law across AB $\quad i_1 R_1 = i_2 R_2 \Rightarrow i_1 l_1 = i_2 l_2$ .....(i)

Also $B_1 = \frac{\mu_0}{4\pi} \times \frac{i_1 l_1}{r^2}$ and $B_2 = \frac{\mu_0}{4\pi} \times \frac{i_2 l_2}{r^2}$; $\therefore \frac{B_2}{B_1} = \frac{i_1 l_1}{i_2 l_2} = 1$ [Using (i)]

Hence, two field are equal but of opposite direction. So, resultant magnetic induction at the centre is zero and is independent of θ.

***Example*: 16** The earth's magnetic induction at a certain point is $7 \times 10^{-5}\ Wb/m^2$. This is to be annulled by the magnetic induction at the centre of a circular conducting loop of radius 5 *cm*. The required current in the loop is **[MP PET 1999; AIIMS 2000]**

(a) 0.56 *A* (b) 5.6 *A* (c) 0.28 *A* (d) 2.8 *A*

Solution : (b) According to the question, at centre of coil $B = B_H \Rightarrow \frac{\mu_0}{4\pi} \cdot \frac{2\pi i}{r} = B_H$

$\Rightarrow 10^{-7} \times \frac{2\pi i}{(5 \times 1^{-2})} = 7 \times 10^{-5} \Rightarrow i = 5.6\ amp.$

***Example*: 17** A particle carrying a charge equal to 100 times the charge on an electron is rotating per second in a circular path of radius 0.8 *metre*. The value of the magnetic field produced at the centre will be (μ_0 – permeability for vacuum) **[CPMT 1986]**

(a) $\frac{10^{-7}}{\mu_0}$ (b) $10^{-17}\mu_0$ (c) $10^{-6}\mu_0$ (d) $10^{-7}\mu_0$

Solution : (b) Magnetic field at the centre of orbit due to revolution of charge.

$B = \frac{\mu_0}{4\pi} \cdot \frac{2\pi(q\nu)}{r}$; where ν = frequency of revolution of charge

So, $B = \frac{\mu_0}{4\pi} \times \frac{2\pi \times (100e \times 1)}{0.8} \Rightarrow B = 10^{-17}\mu_0$.

***Example*: 18** Ratio of magnetic field at the centre of a current carrying coil of radius R and at a distance of $3R$ on its axis is

(a) $10\sqrt{10}$ (b) $20\sqrt{10}$ (c) $2\sqrt{10}$ (d) $\sqrt{10}$

Solution : (a) By using $\frac{B_{centre}}{B_{axis}} = \left(1 + \frac{x^2}{r^2}\right)^{3/2}$; where $x = 3R$ and $r = R$ $\Rightarrow \frac{B_{centre}}{B_{axis}} = (10)^{3/2} = 10\sqrt{10}$.

***Example*: 19** A circular current carrying coil has a radius R. The distance from the centre of the coil on the axis where the magnetic induction will be $\frac{1}{8}th$ to its value at the centre of the coil, is **[MP PMT 1997]**

(a) $\frac{R}{\sqrt{3}}$ (b) $R\sqrt{3}$ (c) $2\sqrt{3}R$ (d) $\frac{2}{\sqrt{3}}R$

Solution : (b) By using $\frac{B_{centre}}{B_{axis}} = \left(1 + \frac{x^2}{r^2}\right)^{3/2}$, given $r = R$ and $B_{axis} = \frac{1}{8}B_{centre}$

$$\Rightarrow 8 = \left(1 + \frac{x^2}{R^2}\right)^{3/2} \Rightarrow (2)^2 = \left\{\left(1 + \frac{x^2}{R^2}\right)^{1/2}\right\}^3 \Rightarrow 2 = \left(1 + \frac{x^2}{R^2}\right)^{1/2} \Rightarrow 4 = 1 + \frac{x^2}{R^2} \Rightarrow x = \sqrt{3}R$$

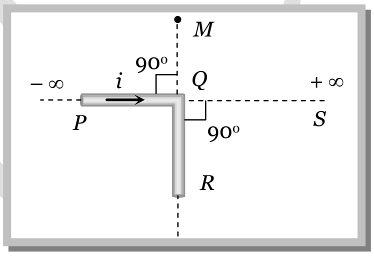
***Example*: 20** An infinitely long conductor PQR is bent to form a right angle as shown. A current I flows through PQR. The magnetic field due to this current at the point M is H_1. Now, another infinitely long straight conductor QS is connected at Q so that the current is $\frac{1}{2}$ in QR as well as in QS, the current in PQ remaining unchanged. The magnetic field at M is now H_2. The ratio H_1 / H_2 is given by

(a) $\frac{1}{2}$

(b) 1

(c) $\frac{2}{3}$

(d) 2



Solution : (c) Magnetic field at any point lying on the current carrying conductor is zero.

Here H_1 = magnetic field at M due to current in PQ

H_2 = magnetic field at M due to R + due to QS + due to PQ $= 0 + \frac{H_1}{2} + H_1 = \frac{3}{2}H_1$

$\therefore \frac{H_1}{H_2} = \frac{2}{3}$

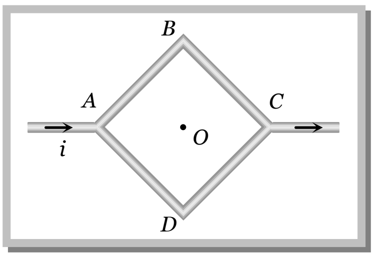
***Example*: 21** Figure shows a square loop $ABCD$ with edge length a. The resistance of the wire ABC is r and that of ADC is $2r$. The value of magnetic field at the centre of the loop assuming uniform wire is

(a) $\frac{\sqrt{2}\,\mu_0 i}{3\pi a} \odot$

(b) $\frac{\sqrt{2}\,\mu_0 i}{3\pi a} \otimes$

(c) $\frac{\sqrt{2}\,\mu_0 i}{\pi a} \odot$

(d) $\frac{\sqrt{2}\,\mu_0 i}{\pi a} \otimes$



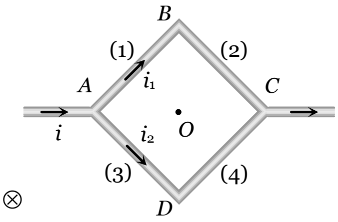
Solution : (b) According to question resistance of wire ADC is twice that of wire ABC. Hence current flows through ADC is half that of ABC *i.e.* $\frac{i_2}{i_1} = \frac{1}{2}$. Also $i_1 + i_2 = i \Rightarrow i_1 = \frac{2i}{3}$ and $i_2 = \frac{i}{3}$

Magnetic field at centre O due to wire AB and BC (part 1 and 2) $B_1 = \frac{\mu_0}{4\pi}.\frac{2i_1 \sin 45^o}{a/2} \otimes = \frac{\mu_0}{4\pi}.\frac{2\sqrt{2}\,i_1}{a} \otimes$

and magnetic field at centre O due to wires AD and DC (*i.e.* part 3 and 4) $B_3 = B_4 = \frac{\mu_0}{4\pi}\frac{2\sqrt{2}\,i_2}{a}\odot$

Also $i_1 = 2i_2$. So $(B_1 = B_2) > (B_3 = B_4)$

Hence net magnetic field at centre O

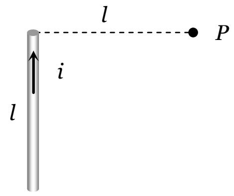


$B_{net} = (B_1 + B_2) - (B_3 + B_4)$

$$= 2\times\frac{\mu_0}{4\pi}.\frac{2\sqrt{2}\times\left(\frac{2}{3}i\right)}{a} - \frac{\mu_0}{4\pi}.\frac{2\sqrt{2}\left(\frac{i}{3}\right)\times 2}{a} = \frac{\mu_0}{4\pi}.\frac{4\sqrt{2}\,i}{3a}(2-1)\otimes = \frac{\sqrt{2}\,\mu_0 i}{3\pi a}\otimes$$

Tricky example: 1

Figure shows a straight wire of length l current i. The magnitude of magnetic field produced by the current at point P is

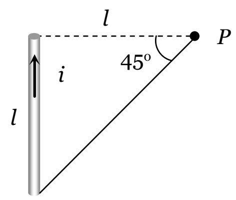


(a) $\frac{\sqrt{2}\mu_0 i}{\pi l}$ (b) $\frac{\mu_0 i}{4\pi l}$ (c) $\frac{\sqrt{2}\mu_0 i}{8\pi l}$ (d) $\frac{\mu_0 i}{2\sqrt{2}\pi l}$

Solution: (c) The given situation can be redrawn as follow.

As we know the general formula for finding the magnetic field due to a finite length wire

$$B = \frac{\mu_0}{4\pi}.\frac{i}{r}(\sin\phi_1 + \sin\phi_2)$$

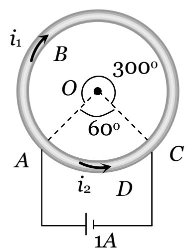


Here $\phi_1 = 0^o$, $\phi = 45^o$

$$\therefore\ B = \frac{\mu_0}{4\pi}.\frac{i}{r}(\sin 0^o + \sin 45^o) = \frac{\mu_0}{4\pi}.\frac{i}{\sqrt{2}l} \Rightarrow B = \frac{\sqrt{2}\mu_0 i}{8\pi l}$$

Tricky example: 2

A cell is connected between the points A and C of a circular conductor $ABCD$ of centre 'O' with angle $AOC = 60°$, If B_1 and B_2 are the magnitudes of the magnetic fields at O due to the currents in ABC and ADC respectively, the ratio $\frac{B_1}{B_2}$ is **[KCET (Engg./ Med.) 1999]**



(a) 0.2

(b) 6

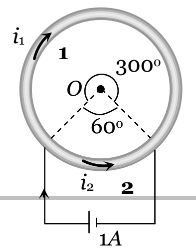
(c) 1

(d) 5

Solution: (c) $B = \frac{\mu_0}{4\pi}.\frac{\theta i}{r}$

$\Rightarrow B \propto \theta i$

$\Rightarrow \frac{B_1}{B_2} = \frac{\theta_1}{\theta_2}\times\frac{i_1}{i_2}$

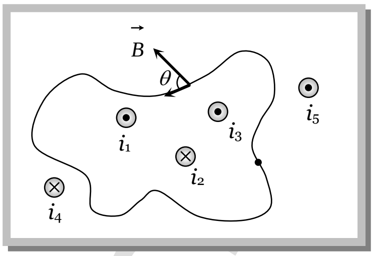


Also $\frac{i_1}{i_2} = \frac{l_2}{l_1} = \frac{\theta_2}{\theta_1}$ Hence $\frac{B_1}{B_2} = \frac{1}{1}$

Amperes Law

Amperes law gives another method to calculate the magnetic field due to a given current distribution.

Line integral of the magnetic field $\vec{B}$ around any closed curve is equal to μ_0 times the net current i threading through the area enclosed by the curve



i.e. $\oint \vec{B} d\vec{l} = \mu_0 \sum i = \mu_0 (i_1 + i_3 - i_2)$

Also using $\vec{B} = \mu_0 \vec{H}$ (where $\vec{H}$ = magnetising field)

$\oint \mu_0 \vec{H}.\vec{dl} = \mu_0 \Sigma i \Rightarrow \oint \vec{H}.\vec{dl} = \Sigma i$

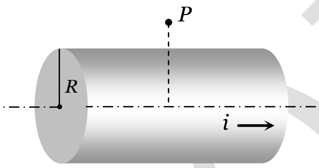
Note : ❒ Total current crossing the above area is $(i_1 + i_3 - i_2)$. Any current outside the area is not included in net current. (Outward ⊙ → +*ve*, Inward ⊗ → – *ve*)

❒ When the direction of current is away from the observer then the direction of closed path is clockwise and when the direction of current is towards the observer then the direction of closed path is anticlockwise.

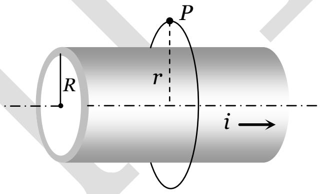
Application of Amperes law

(1) **Magnetic field due to a cylindrical wire**

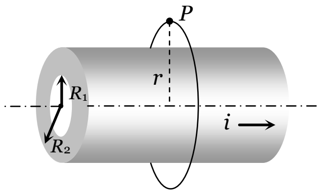
(i) **Outside the cylinder**



Solid cylinder



Thin hollow cylinder



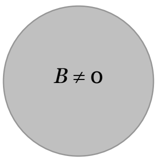
Thick hollow

In all above cases magnetic field outside the wire at P $\oint \vec{B}.\vec{dl} = \mu_0 i \Rightarrow B\oint dl = \mu_0 i \Rightarrow B \times 2\pi r = \mu_0 i \Rightarrow$ $B_{out} = \frac{\mu_0 i}{2\pi r}$

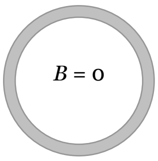
In all the above cases $B_{surface} = \frac{\mu_0 i}{2\pi R}$

(ii) **Inside the cylinder :** Magnetic field inside the hollow cylinder is zero.

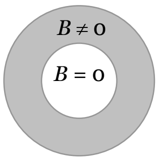
Cross sectional view



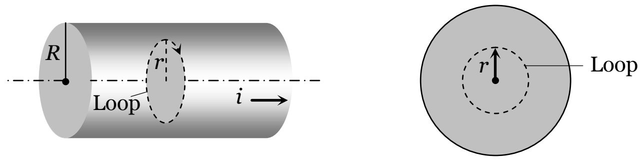
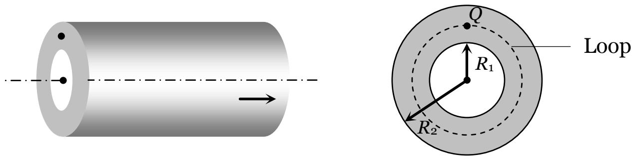
Solid cylinder



Thin hollow cylinder



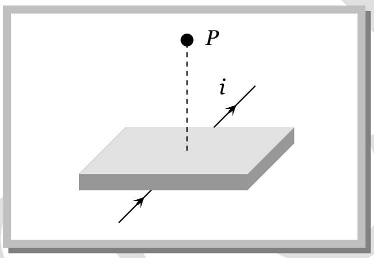
Thick hollow

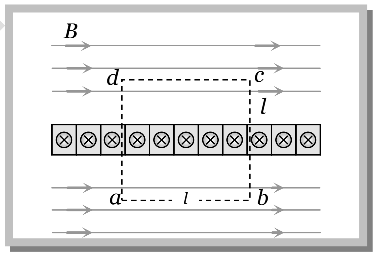
Solid cylinder	Inside the thick portion of hollow cylinder
	
Current enclosed by loop (i') is lesser then the total current (i)	Current enclosed by loop (i') is lesser then the total current (i)
Current density is uniform *i.e.* $J = J' \Rightarrow \frac{i}{A} = \frac{i'}{A'}$	Also $i' = i \times \frac{A'}{A} = i \times \frac{(r^2 - R_1^2)}{(R_2^2 - R_1^2)}$
$\Rightarrow i' = i \times \frac{A'}{A} = i\left(\frac{r^2}{R^2}\right)$	Hence at point Q $\oint \vec{B}.d\vec{l} = \mu_0 i' \Rightarrow B \times 2\pi r = \mu_0 i \times \frac{(r^2 - R_1^2)}{(R_2^2 - R_1^2)}$
Hence at point Q $\oint \vec{B}.d\vec{l} = \mu_0 i' \Rightarrow B \times 2\pi r = \frac{\mu_0 i r^2}{R^2}$	$\Rightarrow B = \frac{\mu_0 i}{2\pi r} \cdot \frac{(r^2 - R_1^2)}{(R_2^2 - R_1^2)}$. If $r = R_1$ (inner surface) $B = 0$
$\Rightarrow B = \frac{\mu_0}{2\pi} \cdot \frac{ir}{R^2}$	If $r = R_2$ (outer surface) $B = \frac{\mu_0 i}{2\pi R_2}$ (max.)

Note : ❒ For all cylindrical current distributions

$B_{axis} = 0$ (min.), $B_{surface}$ = max (distance r always from axis of cylinder), $B_{out} \propto 1/r$.

(2) **Magnetic field due to an infinite sheet carrying current :** The figure shows an infinite sheet of current with linear current density j (A/m). Due to symmetry the field line pattern above and below the sheet is uniform. Consider a square loop of side l as shown in the figure.





According to Ampere's law, $\int_a^b B.dl + \int_b^c B.dl + \int_c^d B.dl + \int_d^a B.dl = \mu_0 i$.

Since $B \perp dl$ along the path $b \to c$ and $d \to a$, therefore, $\int_b^c B.dl = 0$; $\int_d^a B.dl = 0$

Also, $B \,||\, dl$ along the path $a \to b$ and $c \to d$, thus $\int_a^b B.dl + \int_d^a B.dl = 2Bl$

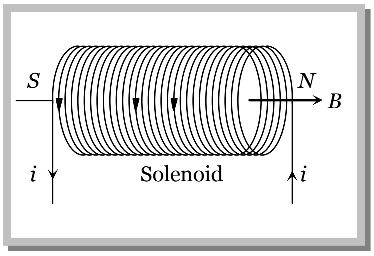
The current enclosed by the loop is $i = jl$

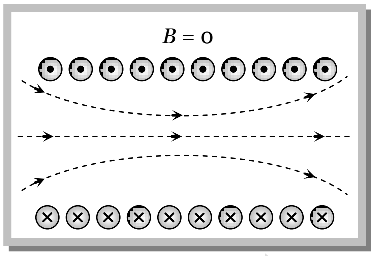
Therefore, according to Ampere's law $2Bl = \mu_0 (jl)$ or $B = \frac{\mu_0 j}{2}$

(3) **Solenoid**

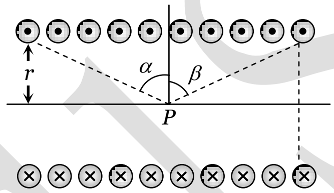
A cylinderical coil of many tightly wound turns of insulated wire with generally diameter of the coil smaller than its length is called a solenoid.

One end of the solenoid behaves like the north pole and opposite end behaves like the south pole. As the length of the solenoid increases, the interior field becomes more uniform and the external field becomes weaker.





A magnetic field is produced around and within the solenoid. The magnetic field within the solenoid is uniform and parallel to the axis of solenoid.



(i) **Finite length solenoid :** If N = total number of turns,

l = length of the solenoid

n = number of turns per unit length $=\frac{N}{l}$

Magnetic field inside the solenoid at point P is given by $B=\frac{\mu_0}{4\pi}(2\pi ni)[\sin\alpha+\sin\beta]$

(ii) **Infinite length solenoid :** If the solenoid is of infinite length and the point is well inside the solenoid *i.e.* $\alpha=\beta=(\pi/2)$.

So $\quad\quad\quad\quad\quad$ $\boldsymbol{B_{in}=\mu_0 ni}$

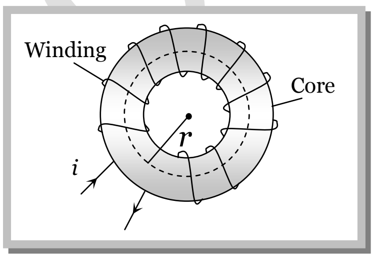
(ii) If the solenoid is of infinite length and the point is near one end *i.e.* $\alpha=0$ and $\beta=(\pi/2)$

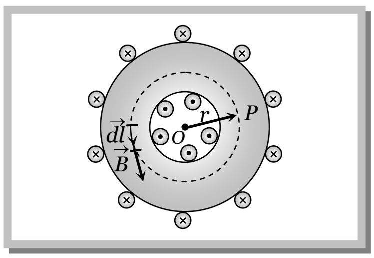
So $\quad\quad\quad\quad\quad$ $\boldsymbol{B_{end}=\frac{1}{2}(\mu_0 ni)}$

Note : ❒ Magnetic field outside the solenoid is zero.

❒ $B_{end}=\frac{1}{2}B_{in}$

(4) **Toroid :** A toroid can be considered as a ring shaped closed solenoid. Hence it is like an endless cylindrical solenoid.





Consider a toroid having n turns per unit length

Let i be the current flowing through the toroid (figure). The magnetic lines of force mainly remain in the core of toroid and are in the form of concentric circles. Consider such a circle of mean radius r. The circular closed path surrounds N loops of wire, each of which carries a current i therefore from $\oint \vec{B}.\vec{dl}=\mu_0 i_{net}$

$$\Rightarrow \; B \times (2\pi r) = \mu_0 Ni \qquad \Rightarrow B = \frac{\mu_0 Ni}{2\pi r} = \mu_o ni \text{ where } n = \frac{N}{2\pi r}$$

For any point inside the empty space surrounded by toroid and outside the toroid, magnetic field B is zero because the net current enclosed in these spaces is zero.

Concepts

- *The line integral of magnetising field ($\vec{H}$) for any closed path called magnetomotive force (MMF). It's S.I. unit is amp.*
- *Ratio of dimension of e.m.f. to MMF is equal to the dimension of resistance.*
- *Biot-Savart law is valid for asymmetrical current distributions while Ampere's law is valid for symmetrical current distributions.*
- *Biot-Savart law is based only on the principle of magnetism while Ampere's laws is based on the principle of electromagnetism.*

Example

***Example*: 22** A long solenoid has 200 *turns per cm* and carries a current of 2.5 *A*. The magnetic field at its centre is [$\mu_0 = 4\pi \times 10^{-7}$ *Wb/m²*] **[MP PET 2000]**

(a) 3.14×10^{-2} *Wb/m²* (b) 6.28×10^{-2} *Wb/m²* (c) 9.42×10^{-2} *Wb/m²* (d) 12.56×10^{-2} *Wb/m²*

Solution : (b) $B = \mu_0 ni = 4\pi \times 10^{-7} \times \frac{200}{10^{-2}} \times 2.5 = 6.28 \times 10^{-2} Wb/m^2$.

***Example*: 23** A long solenoid is formed by winding 20 *turns/cm*. The current necessary to produce a magnetic field of 20 *mili tesla* inside the solenoid will be approximately $\left(\frac{\mu_0}{4\pi} = 10^{-7} \textit{Tesla - metre / ampere}\right)$ **[MP PMT 1994]**

(a) 8.0 *A* (b) 4.0 *A* (c) 2.0 *A* (d) 1.0 *A*

Solution : (a) $B = \mu_0 ni$; where $n = \frac{20}{10}\frac{turn}{cm} = 2000 \frac{turn}{m}$. So, $20 \times 10^{-5} = 4\pi \times 2000 \times i \Rightarrow \; i = 8A$.

***Example*: 24** Two solenoids having lengths L and $2L$ and the number of loops N and $4N$, both have the same current, then the ratio of the magnetic field will be **[CPMT 1994]**

(a) 1 : 2 (b) 2 : 1 (c) 1 : 4 (d) 4 : 1

Solution : (a) $B = \mu_0 \frac{N}{L} i \Rightarrow B \propto \frac{N}{L} \Rightarrow \frac{B_1}{B_2} = \frac{N_1}{N_2} \times \frac{L_2}{L_1} = \frac{N}{4N} \times \frac{2l}{L} = \frac{1}{2}$.

***Example*: 25** The average radius of a toroid made on a ring of non-magnetic material is 0.1 m and it has 500 turns. If it carries 0.5 ampere current, then the magnetic field produced along its circular axis inside the toroid will be

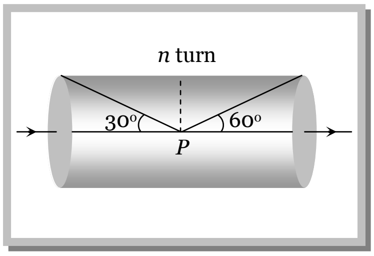
(a) 25×10^{-2} *Tesla* (b) 5×10^{-2} *Tesla* (c) 25×10^{-4} *Tesla* (d) 5×10^{-4} *Tesla*

Solution : (d) $B = \mu_0 ni$; where $n = \frac{N}{2\pi R}$ $\therefore \; B = 4\pi \times 10^{-7} \times \frac{500}{2\pi \times 0.1} \times 0.5 = 5 \times 10^{-4} T$.

***Example*: 26** For the solenoid shown in figure. The magnetic field at point P is

(a) $\frac{\mu_0 ni}{4}(\sqrt{3} + 1)$

(b) $\frac{\sqrt{3}\mu_0 ni}{4}$



(c) $\frac{\mu_0 ni}{2}(\sqrt{3}+1)$

(d) $\frac{\mu_0 ni}{4}(\sqrt{3}-1)$

Solution : (a) $B = \frac{\mu_0}{4\pi}.2\pi ni(\sin\alpha + \sin\beta)$. From figure $\alpha = (90^o - 30^o) = 60^o$ and $\beta = (90^o - 60^o) = 30^o$

$\therefore\ B = \frac{\mu_0 ni}{2}(\sin 60^o + \sin 30^o) = \frac{\mu_0 ni}{4}(\sqrt{3}+1)$.

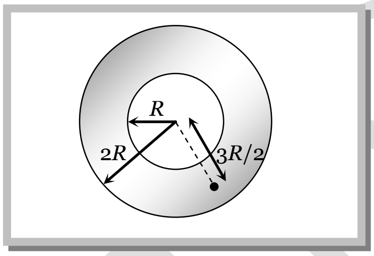
***Example*: 27** Figure shows the cress sectional view of the hollow cylindrical conductor with inner radius 'R' and outer radius '$2R$', cylinder carrying uniformly distributed current along it's axis. The magnetic induction at point 'P' at a distance $\frac{3R}{2}$ from the axis of the cylinder will be

(a) Zero

(b) $\frac{5\mu_0 i}{72\pi R}$

(c) $\frac{7\mu_0 i}{18\pi R}$

(d) $\frac{5\mu_0 i}{36\pi R}$



Solution : (d) By using $B = \frac{\mu_0 i}{2\pi r}\left(\frac{r^2 - a^2}{b^2 - a^2}\right)$ here $r = \frac{3R}{2}$, $a = R$, $ab = 2R \Rightarrow$ $B = \frac{\mu_0 i}{2\pi\left(\frac{3R}{2}\right)} \times \left\{\frac{\left(\frac{3R}{2}\right)^2 - R^2}{(R^2) - R^2}\right\} = \frac{5.\mu_o i}{36\pi r}$.

Tricky example: 3

A winding wire which is used to frame a solenoid can bear a maximum 10 A current. If length of solenoid is 80cm and it's cross sectional radius is 3 cm then required length of winding wire is $(B = 0.2\,T)$

(a) $1.2 \times 10^2\ m$ (b) $4.8 \times 10^2\ m$ (c) $2.4 \times 10^3\ m$ (d) $6 \times 10^3\ m$

Solution : (c) $B = \frac{\mu_0 Ni}{l}$ where N =Total number of turns, l =length of the solenoid

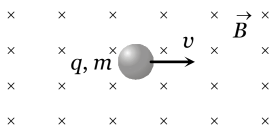
$\Rightarrow\ 0.2 = \frac{4\pi \times 10^{-7} \times N \times 10}{0.8} \Rightarrow\ N = \frac{4 \times 10^4}{\pi}$

Since N turns are made from the winding wire so length of the wire $(L) = 2\pi r \times N\ [2\pi r = \text{length of each turns}]$

$\Rightarrow\ L = 2\pi \times 3 \times 10^{-2} \times \frac{4 \times 10^4}{\pi} = 2.4 \times 10^3\ m$.

Motion of Charged Particle in a Magnetic Field

If a particle carrying a positive charge q and moving with velocity v enters a magnetic field B then it experiences a force F which is given by the expression



$$F = q(\vec{v} \times \vec{B}) \Rightarrow F = qvB \sin\theta$$

Here $\vec{v}$ = velocity of the particle, $\vec{B}$ = magnetic field

(1) **Zero force**

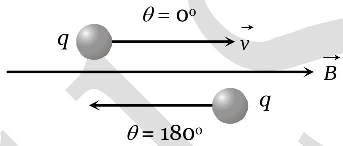
Force on charged particle will be zero (*i.e.* $F = 0$) if

(i) No field *i.e.* $B = 0 \Rightarrow F = 0$

(ii) Neutral particle *i.e.* $q = 0 \Rightarrow F = 0$

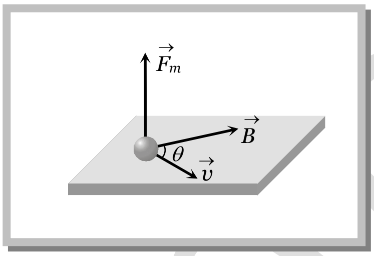
(iii) Rest charge *i.e.* $v = 0 \Rightarrow F = 0$

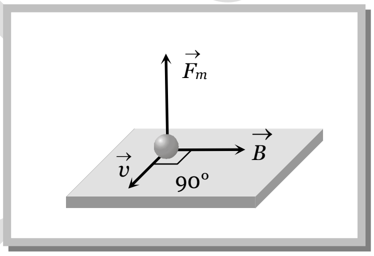
(iv) Moving charge *i.e.* $\theta = 0^o$ or $\theta = 180^o \Rightarrow F = 0$



(2) **Direction of force**

The force $\vec{F}$ is always perpendicular to both the velocity $\vec{v}$ and the field $\vec{B}$ in accordance with Right Hand Screw Rule, through $\vec{v}$ and $\vec{B}$ themselves may or may not be perpendicular to each other.



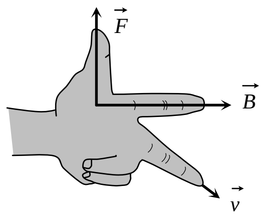


Direction of force on charged particle in magnetic field can also be find by Flemings Left Hand Rule (FLHR).

Here, *First finger* (indicates) $\rightarrow$ Direction of magnetic field

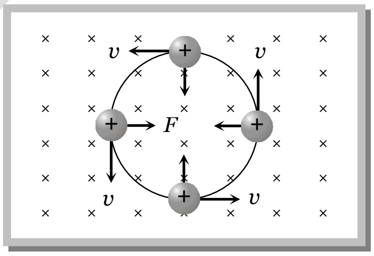
Middle finger $\rightarrow$ Direction of motion of positive charge or direction, opposite to the motion of negative charge.

Thumb $\rightarrow$ Direction of force



(3) **Circular motion of charge in magnetic field**

Consider a charged particle of charge q and mass m enters in a uniform magnetic field B with an initial velocity v perpendicular to the field.



$\theta = 90^o$, hence from $F = qvB \sin\theta$ particle will experience a maximum magnetic force $\boldsymbol{F_{max} = qvB}$ which act's in a direction perpendicular to the motion of charged particle. (By Flemings left hand rule).

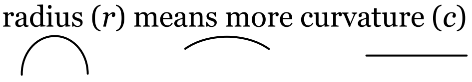
(i) **Radius of the path :** In this case path of charged particle is circular and magnetic force provides the necessary centripetal force *i.e.* $qvB = \frac{mv^2}{r}$ $\Rightarrow$ radius of path $r = \frac{mv}{qB}$

If p = momentum of charged particle and K = kinetic energy of charged particle (gained by charged particle after accelerating through potential difference V) then $p = mv = \sqrt{2mK} = \sqrt{2mqV}$

So $$\boldsymbol{r = \frac{mv}{qB} = \frac{p}{qB} = \frac{\sqrt{2mK}}{qB} = \frac{1}{B}\sqrt{\frac{2mV}{q}}}$$

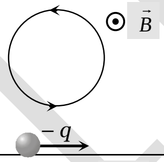
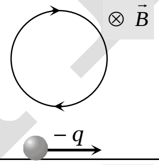
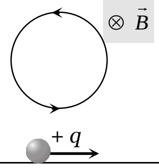
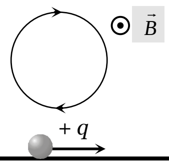
$r \propto v \propto p \propto \sqrt{K}$ *i.e.* with increase in speed or kinetic energy, the radius of the orbit increases.

Note : ❒ Less radius (r) means more curvature (c) *i.e.* $c \propto \frac{1}{r}$



Less : r More : c | More : r Less : c | $r = \infty$ $c = 0$

(ii) **Direction of path :** If a charge particle enters perpendicularly in a magnetic field, then direction of path described by it will be

Type of charge	Direction of magnetic field	Direction of it's circular motion
Negative	Outwards ⊙	 Anticlockwise
Negative	Inward ⊗	 Clockwise
Positive	Inward ⊗	 Anticlockwise
Positive	Outward ⊙	 Clockwise

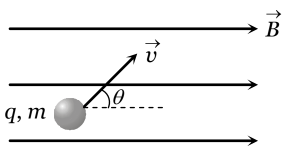
(iii) **Time period** : As in uniform circular motion $v = r\omega$, so the angular frequency of circular motion, called cyclotron or gyro-frequency, will be given by $\omega = \frac{v}{r} = \frac{qB}{m}$ and hence the time period, $T = \frac{2\pi}{\omega} = 2\pi\frac{m}{qB}$

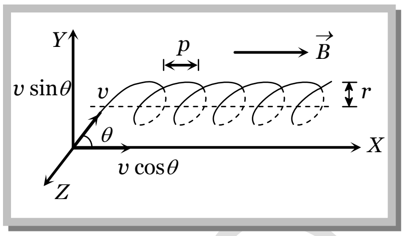
i.e., time period (or frequency) is independent of speed of particle and radius of the orbit and depends only on the field B and the nature, *i.e.*, specific charge $\left(\frac{q}{m}\right)$, of the particle.

(4) **Motion of charge on helical path**

When the charged particle is moving at an angle to the field (other than 0°, 90°, or 180°).

In this situation resolving the velocity of the particle along and perpendicular to the field, we find that the particle moves with constant velocity $v\cos\theta$ along the field (as no force acts on a charged particle when it moves parallel to the field) and at the same time it is also moving with velocity $v\sin\theta$ perpendicular to the field due to which it will describe a circle (in a plane perpendicular to the field) of radius. $r = \dfrac{m(v\sin\theta)}{qB}$





Time period and frequency do not depend on velocity and so they are given by $T = \dfrac{2\pi m}{qB}$ and $\nu = \dfrac{qB}{2\pi m}$

So the resultant path will be a *helix* with its axis parallel to the field $\vec{B}$ as shown in figure in this situation.

The *pitch* of the *helix*, (*i.e.*, linear distance travelled in one rotation) will be given by $p = T(v\cos\theta) = 2\pi\dfrac{m}{qB}(v\cos\theta)$

Note : ❒ 1 rotation $\equiv 2\pi \equiv T$ and 1 pitch $\equiv 1\,T$

❒ Number of pitches ≡ Number of rotations ≡ Number of repetition = Number of helical turns

❒ If pitch value is p, then number of pitches obtained in length l given as

Number of pitches $= \dfrac{l}{p}$ and time reqd. $t = \dfrac{l}{v\cos\theta}$

Some standard results

& Ratio of radii of path described by proton and α-particle in a magnetic field (particle enters perpendicular to the field)

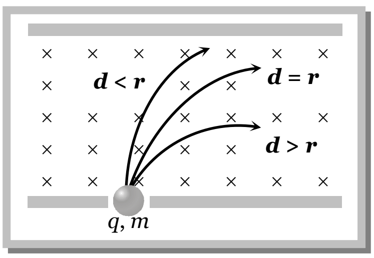
Constant quantity	Formula	Ratio of radii	Ratio of curvature (c)
v - same	$r = \dfrac{mv}{qB} \Rightarrow r \propto \dfrac{m}{q}$	$r_p : r_\alpha = 1:2$	$c_p : c_R = 2:1$
p - same	$r = \dfrac{p}{qB} \Rightarrow r \propto \dfrac{1}{q}$	$r_p : r_\alpha = 2:1$	$c_p : c_R = 1:2$
k - same	$r = \dfrac{\sqrt{2mk}}{qB} \Rightarrow r \propto \dfrac{\sqrt{m}}{q}$	$r_p : r_\alpha = 1:1$	$c_p : c_R = 1:1$
V - same	$r \propto \sqrt{\dfrac{m}{q}}$	$r_p : r_\alpha = 1:\sqrt{2}$	$c_p : c_R = \sqrt{2}:1$

& **Particle motion between two parallel plates** $(\vec{v} \perp \vec{B})$

(i) To strike the opposite plate it is essential that $d < r$.

(ii) Does not strike the opposite plate $d > r$.

(iii) To touch the opposite plate $d = r$.



(iv) To just not strike the opposite plate $d \geq r$.

(v) To just strike the opposite plate $d \leq r$.

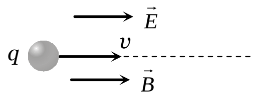
(5) **Lorentz force**

When the moving charged particle is subjected simultaneously to both electric field $\vec{E}$ and magnetic field $\vec{B}$, the moving charged particle will experience electric force $\vec{F}_e = q\vec{E}$ and magnetic force $\vec{F}_m = q(\vec{v} \times \vec{B})$; so the net force on it will be $\boldsymbol{\vec{F} = q[\vec{E} + (\vec{v} \times \vec{B})]}$. Which is the famous 'Lorentz-force equation'.

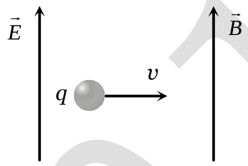
Depending on the directions of $\vec{v}, \vec{E}$ and $\vec{B}$ following situations are possible

(i) **When $\vec{v}, \vec{E}$ and $\vec{B}$ all the three are collinear :** In this situation as the particle is moving parallel or antiparallel to the field, the magnetic force on it will be zero and only electric force will act and so $\vec{a} = \frac{\vec{F}}{m} = \frac{q\vec{E}}{m}$

The particle will pass through the field following a straight line path (parallel field) with change in its speed. So in this situation speed, velocity, momentum kinetic energy all will change without change in direction of motion as shown



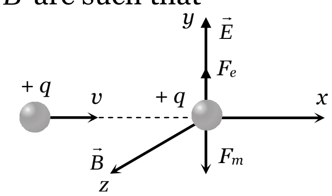
(ii) **When $\vec{E}$ is parallel to $\vec{B}$ and both these fields are perpendicular to $\vec{v}$ then :** $\vec{F}_e$ is perpendicular to $\vec{F}_m$ and they cannot cancel each other. The path of charged particle is curved in both these fields.



(iii) **$\vec{v}$, $\vec{E}$ and $\vec{B}$ are mutually perpendicular :** In this situation if $\vec{E}$ and $\vec{B}$ are such that

$$\vec{F} = \vec{F}_e + \vec{F}_m = 0 \;\; i.e., \;\; \vec{a} = (\vec{F}/m) = 0$$

as shown in figure, the particle will pass through the field with same velocity.



And in this situation, as $F_e = F_m$ *i.e.*, $qE = qvB \;\; v = E/B$

This principle is used in 'velocity-selector' to get a charged beam having a specific velocity.

Note : ❒ From the above discussion, conclusion is as follows

❒ If E = 0, B = 0, so F = 0.

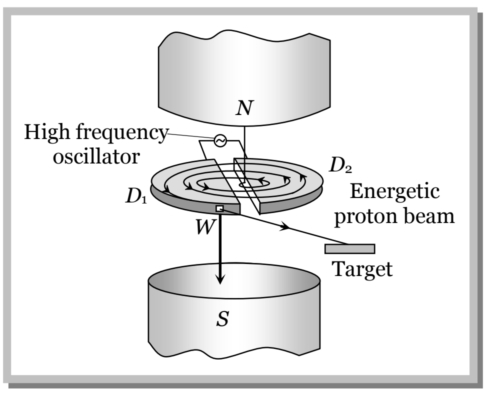
❒ If E = 0, $B \neq$ 0, so F may be zero (if $\theta = 0^o$ or 180^o).

❒ If $E \neq$ 0, $B \neq$ 0, so F = 0 (if $|\vec{F}_e| = |\vec{F}_m|$ and their directions are opposite)

❒ If $E \neq$ 0, B = 0, so $F \neq$ 0 (because $\vec{v} \neq$ constant).

Cyclotron

Cyclotron is a device used to accelerated positively charged particles (like, α-particles, deutrons *etc.*) to acquire enough energy to carry out nuclear disintegration *etc.* t is based on the fact that the electric field accelerates a charged particle and the magnetic field keeps it revolving in circular orbits of constant frequency. Thus a small potential difference would impart if



enormously large velocities if the particle is made to traverse the potential difference a number of times.

It consists of two hollow D-shaped metallic chambers D_1 and D_2 called dees. The two dees are placed horizontally with a small gap separating them. The dees are connected to the source of high frequency electric field. The dees are enclosed in a metal box containing a gas at a low pressure of the order of 10^{-3} *mm* mercury. The whole apparatus is placed between the two poles of a strong electromagnet NS as shown in fig. The magnetic field acts perpendicular to the plane of the dees.

Note : ❒ The positive ions are produced in the gap between the two dees by the ionisation of the gas. To produce proton, hydrogen gas is used; while for producing alpha-particles, helium gas is used.

(1) **Cyclotron frequency :** Time taken by ion to describe q semicircular path is given by $t = \frac{\pi r}{v} = \frac{\pi m}{qB}$

If T = time period of oscillating electric field then $T = 2t = \frac{2\pi m}{qB}$ the cyclotron frequency $\nu = \frac{1}{T} = \frac{Bq}{2\pi m}$

(2) **Maximum energy of position :** Maximum energy gained by the charged particle $E_{\max} = \left(\frac{q^2B^2}{2m}\right)r^2$

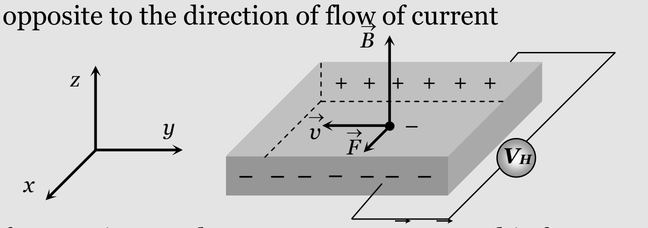
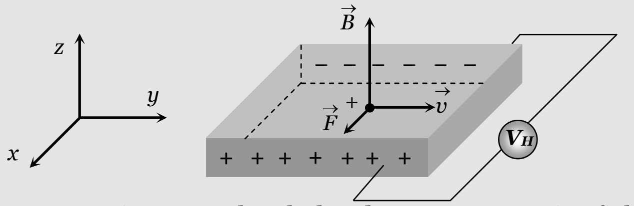
where r_0 = maximum radius of the circular path followed by the positive ion.

Note : ❒ Cyclotron frequency is also known as magnetic resonance frequency.

❒ Cyclotron can not accelerate electrons because they have very small mass.

Hall effect : The Phenomenon of producing a transverse emf in a current carrying conductor on applying a magnetic field perpendicular to the direction of the current is called Hall effect.

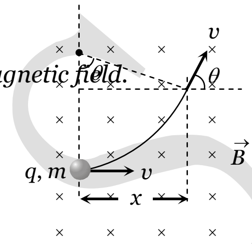
Hall effect helps us to know the nature and number of charge carriers in a conductor.

Negatively charged particles	**Positively charged particles**
Consider a conductor having electrons as current carriers. The electrons move with drift velocity $\vec{v}$ opposite to the direction of flow of current  force acting on electron $F_m = -e(\vec{v} \times \vec{B})$. This force acts along x-axis and hence electrons will move towards face (2) and it becomes negatively charged.	Let the current carriers be positively charged holes. The hole move in the direction of current  Force acting on the hole due to magnetic field $\vec{F}_m = +e(\vec{v} \times \vec{B})$ force acts along x-axis and hence holes move towards face (2) and it becomes positively charged.

Concepts

- *The energy of a charged particle moving in a uniform magnetic field does not change because it experiences a force in a direction, perpendicular to it's direction of motion. Due to which the speed of charged particle remains unchanged and hence it's K.E. remains same.*
- *Magnetic force does no work when the charged particle is displaced while electric force does work in displacing the charged particle.*
- *Magnetic force is velocity dependent, while electric force is independent of the state of rest or motion of the charged particle.*
- *If a particle enters a magnetic field normally to the magnetic field, then it starts moving in a circular orbit. The point at which it enters the magnetic field lies on the circumference. (Most of us confuse it with the centre of the orbit)*
- ***Deviation of charged particle in magnetic field :** If a charged particle (q, m) enters a uniform magnetic field $\vec{B}$ (extends upto a length x) at right angles with speed v as shown in figure.*

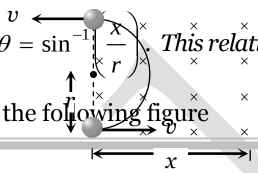
  The speed of the particle in magnetic field does not change. But it gets deviated in the magnetic field.



Deviation in terms of time t; $\theta = \omega t = \left(\frac{Bq}{m}\right)t$

Deviation in terms of length of the magnetic field ; $\theta = \sin^{-1}\left(\frac{x}{r}\right)$. *This relation can be used only when* $x \le r$.

For x > r, the deviation will be 180° as shown in the following figure



Example

***Example*: 28** Electrons move at right angles to a magnetic field of 1.5×10^{-2} *Tesla* with a speed of 6×10^{27} m/s. If the specific charge of the electron is 1.7×10^{11} *Coul/kg*. The radius of the circular path will be **[BHU 2003]**

(a) 2.9 *cm* (b) 3.9 *cm* (c) 2.35 *cm* (d) 3 *cm*

Solution : (c) $r = \frac{mv}{qB} \Rightarrow \frac{v}{(q/m).B} = \frac{6 \times 10^{27}}{17 \times 10^{11} \times 1.5 \times 10^{-2}} = 2.35 \times 10^{-2}\ m = 2.35\ cm.$

***Example*: 29** An electron (mass $= 9 \times 10^{-31}$ *kg*. charge $= 1.6 \times 10^{-19}$ *coul.*) whose kinetic energy is 7.2×10^{-18} *joule* is moving in a circular orbit in a magnetic field of 9×10^{-5} *weber* $/m^2$. The radius of the orbit is **[MP PMT 2002]**

(a) 1.25 *cm* (b) 2.5 *cm* (c) 12.5 *cm* (d) 25.0 *cm*

Solution : (d) $r = \frac{\sqrt{2mK}}{qB} = \frac{\sqrt{2 \times q \times 10^{-31} \times 7.2 \times 10^{-8}}}{1.6 \times 10^{-19} \times q \times 10^{-5}} = 0.25\ cm = 25\ cm$.

***Example*: 30** An electron and a proton enter a magnetic field perpendicularly. Both have same kinetic energy. Which of the following is true **[MP PET 1999]**

(a) Trajectory of electron is less curved (b) Trajectory of proton is less curved

(c) Both trajectories are equally curved (d) Both move on straight line path

Solution : (b) By using $r = \frac{\sqrt{2mk}}{qB}$; For both particles $q \to$ same, $B \to$ same, $k \to$ same

Hence $r \propto \sqrt{m} \Rightarrow \frac{r_e}{r_p} = \sqrt{\frac{m_e}{m_p}}$ $\because m_p > m_e$ so $r_p > r_e$

Since radius of the path of proton is more, hence it's trajectory is less curved.

***Example*: 31** A proton and an α – particles enters in a uniform magnetic field with same velocity, then ratio of the radii of path describe by them

(a) 1 : 2 (b) 1 : 1 (c) 2 : 1 (d) None of these

Solution : (b) By using $r = \frac{mv}{qB}$; $v \to$ same, $B \to$ same $\Rightarrow r \propto \frac{m}{2} \Rightarrow \frac{r_p}{r_\alpha} = \frac{m_p}{m_\alpha} \times \frac{q_\alpha}{q_p} = \frac{m_p}{4m_p} \times \frac{2q_p}{q_p} = \frac{1}{2}$

***Example*: 32** A proton of mass m and charge $+e$ is moving in a circular orbit of a magnetic field with energy 1*MeV*. What should be the energy of α-particle (mass = 4 m and charge = +2e), so that it can revolve in the path of same radius **[BHU 1997]**

(a) 1 *MeV* (b) 4 *MeV* (c) 2 *MeV* (d) 0.5 *MeV*

Solution : (a) By using $r = \frac{\sqrt{2mK}}{qB}$; $r \to$ same, $B \to$ same $\Rightarrow K \propto \frac{q^2}{m}$

Hence $\frac{K_\alpha}{K_p} = \left(\frac{q_\alpha}{q_p}\right)^2 \times \frac{m_p}{m_\alpha} = \left(\frac{2q_p}{q_p}\right)^2 \times \frac{m_p}{4m_p} 1 \Rightarrow K_\alpha = K_p = 1\,meV.$

***Example*: 33** A proton and an α – particle enter a uniform magnetic field perpendicularly with the same speed. If proton takes $25\,\mu\,\text{sec}$ to make 5 revolutions, then the periodic time for the α – particle would be **[MP PET 1993]**

(a) $50\,\mu$ sec (b) $25\,\mu$ sec (c) $10\,\mu$ sec (d) $5\,\mu$ sec

Solution : (c) Time period of proton $T_p = \frac{25}{5} = 5\,\mu\,\text{sec}$

By using $T = \frac{2\pi m}{qB} \Rightarrow \frac{T_\alpha}{T_p} = \frac{m_\alpha}{m_p} \times \frac{q_p}{q_\alpha} = \frac{4m_p}{m_p} \times \frac{q_p}{2q_p} \Rightarrow T_\alpha = 2T_p = 10\,\mu$ sec .

***Example*: 34** A particle with 10^{-11} *coulomb* of charge and 10^{-7} *kg* mass is moving with a velocity of 10^8 *m/s* along the *y*-axis. A uniform static magnetic field *B* = 0.5 *Tesla* is acting along the *x*-direction. The force on the particle is

[MP PMT 1997]

(a) 5×10^{-11} *N* along $\hat{i}$ (b) 5×10^3 *N* along $\hat{k}$ (c) 5×10^{-11} *N* along $-\hat{j}$ (d) 5×10^{-4} *N* along $-\hat{k}$

Solution : (d) By using $\vec{F} = q(\vec{v} \times \vec{B})$; where $\vec{v} = 10\hat{j}$ and $\vec{B} = 0.5\hat{i}$

$\Rightarrow \vec{F} = 10^{-11}(10^8\hat{j} \times 0.5\hat{i}) = 5 \times 10^{-4}(\hat{j} \times \hat{i}) = 5 \times 10^{-4}(-\hat{k})$ *i.e.*, $5 \times 10^{-4}\,N$ along $-\hat{k}$.

***Example*: 35** An electron is moving along positive *x*-axis. To get it moving on an anticlockwise circular path in *x-y* plane, a magnetic filed is applied

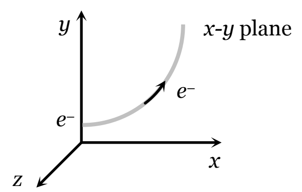
(a) Along positive *y*-axis (b) Along positive *z*-axis

(c) Along negative *y*-axis (d) Along negative *z*-axis

Solution : (a) The given situation can be drawn as follows

According to figure, for deflecting electron in *x-y* plane, force must be acting an it towards *y*-axis.

Hence according to Flemings left hand rule, magnetic field directed along positive y – axis.



***Example*: 36** A particle of charge -16×10^{-18} coulomb moving with velocity 10 *m/s* along the *x*-axis enters a region where a magnetic field of induction *B* is along the *y*-axis, and an electric field of magnitude 10^4 *V/m* is along the negative *z*-axis. If the charged particle continuous moving along the *x*-axis, the magnitude of *B* is **[AIEEE 2003]**

(a) $10^{-3}\ Wb/m^2$ (b) $10^3\ Wb/m^2$ (c) $10^5\ Wb/m^2$ (d) $10^{16}\ Wb/m^2$

Solution : (b) Particles is moving undeflected in the presence of both electric field as well as magnetic field so it's speed

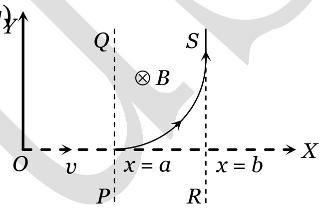
$$v = \frac{E}{B} \Rightarrow B = \frac{E}{v} = \frac{10^4}{10} = 10^3\, Wb/m^2.$$

***Example*: 37** A particle of mass *m* and charge *q* moves with a constant velocity *v* along the positive *x* direction. It enters a region containing a uniform magnetic field *B* directed along the negative *z* direction extending from *x* = *a* to *x* = *b*. The minimum value of *v* required so that the particle can just enter the region *x* > *b* is

[IIT-JEE (Screening) 2002]

(a) *qbB/m* (b) *q*(*b* – *a*)*B/m* (c) *qaB/m* (d) *q*(*b*+*a*)*B*/2*m*

Solution : (b) As shown in the following figure, the $z-$axis points out of the paper and the magnetic fields is directed into the paper, existing in the region between *PQ* and *RS*. The particle moves in a circular path of radius *r* in the magnetic field. It can just enter the region $x > b$ for $r \geq (b-a)$



Now $r = \dfrac{mv}{qb} \geq (b-a)$

$$\Rightarrow v \geq \frac{q(b-a)B}{m} \Rightarrow v_{\min} = \frac{q(b-a)B}{m}.$$

***Example*: 38** At a certain place magnetic field vertically downwards. An electron approaches horizontally towards you and enters in this magnetic fields. It's trajectory, when seen from above will be a circle which is

(a) Vertical clockwise (b) Vertical anticlockwise

(c) Horizontal clockwise (d) Horizontal anticlockwise

Solution : (c) By using Flemings left hand rule.

***Example*: 39** When a charged particle circulates in a normal magnetic field, then the area of it's circulation is proportional to

(a) It's kinetic energy (b) It's momentum

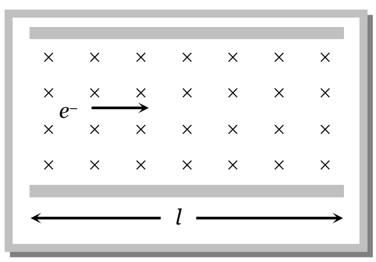
(c) It's charge (d) Magnetic fields intensity

Solution : (a) $r = \dfrac{\sqrt{2mK}}{qB}$ and $A = Aq^2 \Rightarrow A = \dfrac{\pi(2mK)}{q^2b^2} \Rightarrow A \propto K.$

***Example*: 40** An electron moves straight inside a charged parallel plate capacitor at uniform charge density σ. The space between the plates is filled with constant magnetic field of induction $\vec{B}$. Time of straight line motion of the electron in the capacitor is

(a) $\dfrac{e\sigma}{\varepsilon_0 lB}$

(b) $\dfrac{\varepsilon_0 lB}{\sigma}$



(c) $\frac{e\sigma}{\varepsilon_0 B}$

(d) $\frac{\varepsilon_0 B}{e\sigma}$

Solution : (b) The net force acting on the electron is zero because it moves with constant velocity, due to it's motion on straight line.

$$\Rightarrow \vec{F}_{net} = \vec{F}_e + \vec{F}_m = 0 \Rightarrow |\vec{F}_e| = |\vec{F}_m| \Rightarrow eE = evB \Rightarrow ve = \frac{E}{B} = \frac{\sigma}{\varepsilon_0 B} \qquad \left[E = \frac{\sigma}{\varepsilon_o}\right]$$

$\therefore$ The time of motion inside the capacitor $t = \frac{l}{v} = \frac{\varepsilon_0 lB}{\sigma}$.

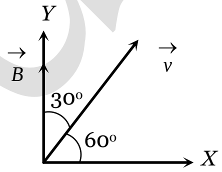
Example: 41 A proton of mass $1.67 \times 10^{-27}\ kg$ and charge $1.6 \times 10^{-19}\ C$ is projected with a speed of $2 \times 10^6\ m/s$ at an angle of 60° to the *X*-axis. If a uniform magnetic field of 0.104 *Tesla* is applied along *Y*-axis, the path of proton is

(a) A circle of radius = 0.2 m and time period $\pi \times 10^{-7}\ s$

(b) A circle of radius = 0.1 m and time period $2\pi \times 10^{-7}\ s$

(c) A helix of radius = 0.1 m and time period $2\pi \times 10^{-7}\ s$

(d) A helix of radius = 0.2 m and time period $4\pi \times 10^{-7}\ s$



Solution : (b) By using $r = \frac{mv\sin\theta}{qB} \Rightarrow r = \frac{1.67 \times 15^{27} \times 2 \times 10^6 \times \sin 30°}{1.6 \times 10^{-19} \times 0.104} = 0.1m$

and it's time period $T = \frac{2\pi m}{qB} = \frac{2 \times \pi \times 9.1 \times 10^{-31}}{1.6 \times 10^{-19} \times 0.104} = 2\pi \times 10^{-7}$ sec.

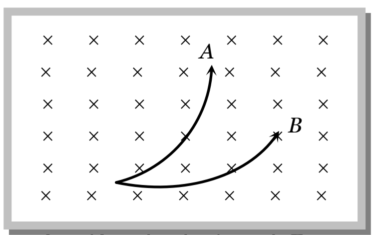
Example: 42 A charge particle, having charge q accelerated through a potential difference V enter a perpendicular magnetic field in which it experiences a force F. If V is increased to 5V, the particle will experience a force

(a) F (b) $5F$ (c) $\frac{F}{5}$ (d) $\sqrt{5}F$

Solution : (d) $\frac{1}{2}mv^2 = qV \Rightarrow v = \sqrt{\frac{2qV}{m}}$. Also $F = qvB$

$\Rightarrow F = qB\sqrt{\frac{2qV}{m}}$ hence $F \propto \sqrt{V}$ which gives $F' = \sqrt{5}F$.

Example: 43 The magnetic field is downward perpendicular to the plane of the paper and a few charged particles are projected in it. Which of the following is true **[CPMT 1997]**



(a) A represents proton and B and electron

(b) Both A and B represent protons but velocity of A is more than that of B

(c) Both A and B represents protons but velocity of B is more than that of A

(d) Both A and B represent electrons, but velocity of B is more than that of A

Solution : (c) Both particles are deflecting in same direction so they must be of same sign.(*i.e.,* both A and B represents protons)

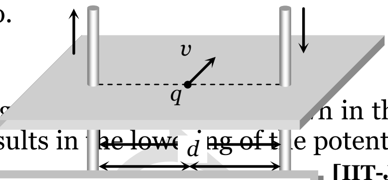
By using $r = \frac{mv}{qB} \Rightarrow r \propto v$

From given figure radius of the path described by particle B is more than that of A. Hence $v_B > v_A$.

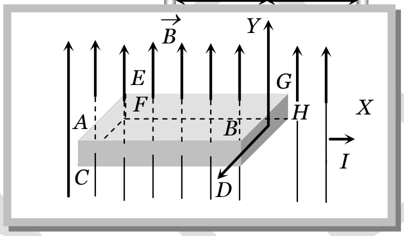
***Example*: 44** Two very long straight, particle wires carry steady currents i and $-i$ respectively. The distance between the wires is d. At a certain instant of time, a point charge q is at a point equidistant from the two wires, in the plane of the wires. It's instantaneous velocity $\vec{v}$ is perpendicular to this plane. The magnitude of the force due to the magnetic field acting on the charge at this instant is

(a) $\frac{\mu_0 iqv}{2\pi d}$ (b) $\frac{\mu_0 iqv}{\pi d}$ (c) $\frac{2\mu_0 iqv}{\pi d}$ (d) Zero

Solution : (d) According to gives information following figure can be drawn, which shows that direction of magnetic field is along the direction of motion of charge so net on it is zero.



***Example*: 45** A metallic block carrying current i is subjected to a uniform ma n in the figure. The moving charges experience a force F given by .... which results in he lowe ng of t e potential of the face .... Assume the speed of the carriers to be v **[IIT-JEE 1996]**



(a) $eVB\hat{k}$, $ABCD$

(b) $eVB\hat{k}$, $ABCD$

(c) $-eVB\hat{k}$, $ABCD$

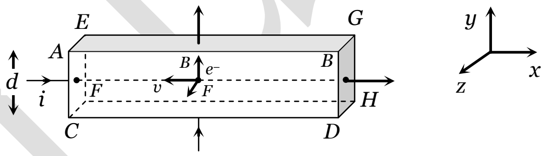
(d) $-eVB\hat{k}$, $EFGH$

Solution : (c) As the block is of metal, the charge carriers are electrons; so for current along positive x-axis, the electrons are moving along negative x-axis, *i.e.* $\vec{v} = -vi$

and as the magnetic field is along the y-axis, *i.e.* $\vec{B} = B\hat{j}$

so $\vec{F} = q(\vec{v} \times \vec{B})$ for this case yield $\vec{F} = (-e)[-v\hat{i} \times B\hat{j}]$

i.e., $\vec{F} = evB\hat{k}$ [As $\hat{i} \times \hat{j} = \hat{k}$]



As force on electrons is towards the face $ABCD$, the electrons will accumulate on it an hence it will acquire lower potential.

Tricky example: 4

An ionised gas contains both positive and negative ions. If it is subjected simultaneously to an electric field along the $+ve$ x-axis and a magnetic field along the $+z$ direction then **[IIT-JEE (Screening**

(a) Positive ions deflect towards $+y$ direction and negative ions towards $-y$ direction

(b) All ions deflect towards $+y$ direction

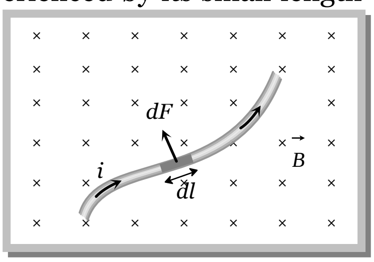
(c) All ions deflect towards $-y$ direction

(d) Positive ions deflect towards $-y$ direction and negative ions towards $+y$ direction.

Solution : (c) As the electric field is switched on, positive ion will start to move along positive x-direction and negative ion along negative x-direction. Current associated with motion of both types of ions is along positive x-direction. According to Flemings left hand rule force on both types of ions will be along negative y-direction.

Force on a Current Carrying Conductor in Magnetic Field

In case of current carrying conductor in a magnetic field force experienced by its small length element is $d\vec{F} = i d\vec{l} \times \vec{B}$; $i d\vec{l}$ = current element $d\vec{F} = I(d\vec{l} \times \vec{B})$



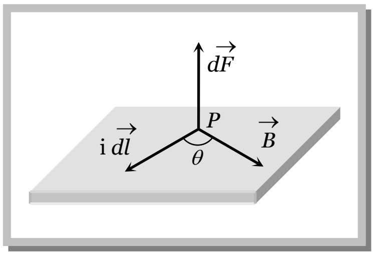
Total magnetic force $\vec{F} = \int d\vec{F} = \int i(d\vec{l} \times \vec{B})$

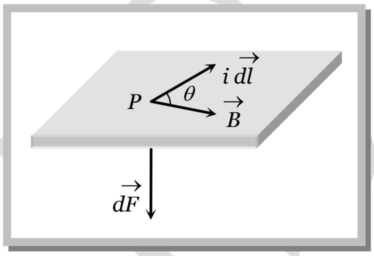
If magnetic field is uniform *i.e.*, $\vec{B}$ = constant

$$\vec{F} = i\left[\int d\vec{l}\right] \times \vec{B} = i(\vec{L}' \times \vec{B})$$

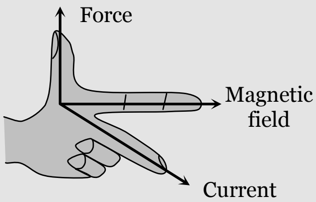
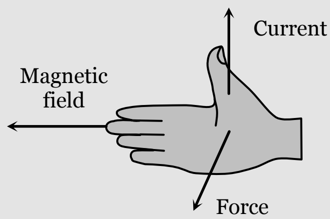
$\int d\vec{l} = \vec{L} =$ vector sum of all the length elements from initial to final point. Which is in accordance with the law of vector addition is equal to length vector $\vec{L}'$ joining initial to final point.

(1) **Direction of force :** The direction of force is always perpendicular to the plane containing $id\vec{l}$ and $\vec{B}$ and is same as that of cross-product of two vectors $(\vec{A} \times \vec{B})$ with $\vec{A} = i\,d\vec{l}$.

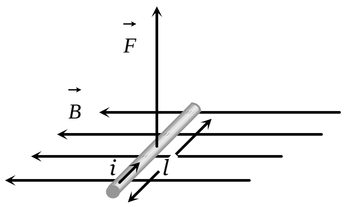




The direction of force when current element $id\vec{l}$ and $\vec{B}$ are perpendicular to each other can also be determined by applying either of the following rules

Fleming's left-hand rule	Right-hand palm rule
Stretch the fore-finger, central finger and thumb left hand mutually perpendicular. Then if the fore-finger points in the direction of field $\vec{B}$ and the central in the direction of current i, the thumb will point in the direction of force 	Stretch the fingers and thumb of right hand at right angles to each other. Then if the fingers point in the direction of field $\vec{B}$ and thumb in the direction of current i, then normal to the palm will point in the direction of force 

(2) **Force on a straight wire :** If a current carrying straight conductor (length l) is placed in an uniform magnetic field (B) such that it makes an angle θ with the direction of field then force experienced by it is $F = Bil\sin\theta$

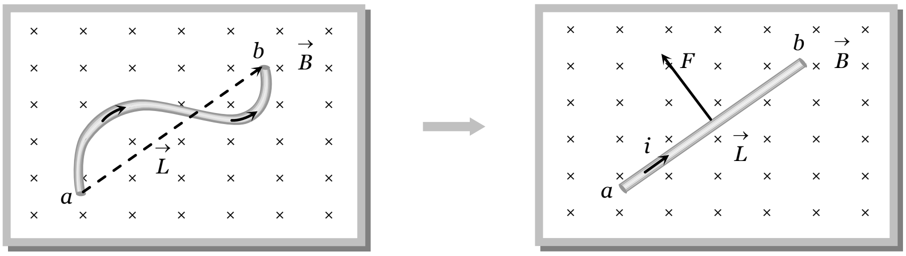


If $\theta = 0^o$, $F = 0$

If $\theta = 90^o$, $F_{\max} = Bil$

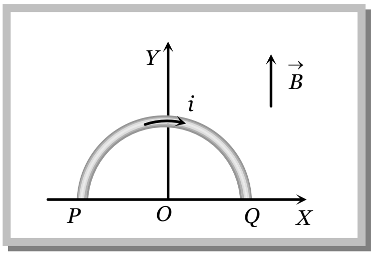
(3) **Force on a curved wire**

The force acting on a curved wire joining points a and b as shown in the figure is the same as that on a straight wire joining these points. It is given by the expression $\vec{F} = i\vec{L} \times \vec{B}$



Specific Example

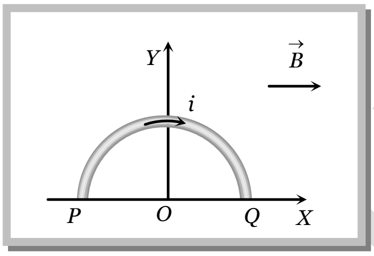
The force experienced by a semicircular wire of radius R when it is carrying a current i and is placed in a uniform magnetic field of induction B as shown.



$\vec{L}' = 2R\hat{i}$ and $\vec{B} = B\hat{i}$

So by using $\vec{F} = i(\vec{L}' \times \vec{B})$ force on the wire

$\vec{F} = i(2R)(B)(\hat{i} \times \hat{i}) \Rightarrow \vec{F} = 0$

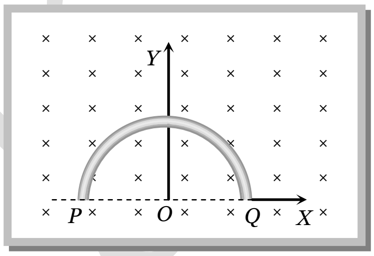


$\vec{L}' = 2R\hat{i}$ and $\vec{B} = B\hat{j}$

$\vec{F} = i \times 2BR(\hat{i} \times \hat{j})$

$\vec{F} = 2BiR\hat{k}$ i.e. $F = 2BiR$

(perpendicular to paper outward)



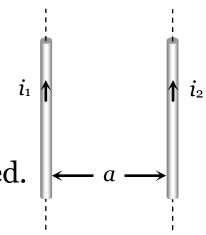
$\vec{L}' = 2R\hat{i}$ and $\vec{B} = B(-\hat{k})$

$\therefore\ \vec{F} = i \times 2BR(+\hat{j})$

$F = 2BiR$ (along Y-axis)

Force Between Two Parallel Current Carrying Conductors

When two long straight conductors carrying currents i_1 and i_2 placed parallel to each other at a distance 'a' from each other. A mutual force act between them when is given as

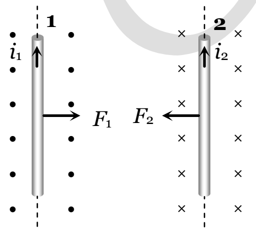


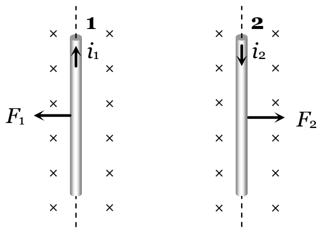
$$F_1 = F_2 = F = \frac{\mu_0}{4\pi} \cdot \frac{2i_1 i_2}{a} \times l$$

where l is the length of that portion of the conductor on which force is to be calculated.

Hence force per unit length $\frac{F}{l} = \frac{\mu_0}{4\pi} \cdot \frac{2i_1 i_2}{a} \frac{N}{m}$ or $\frac{F}{l} = \frac{2i_1 i_2}{a} \frac{dyne}{cm}$

Direction of force : If conductors carries current in same direction, then force between them will be attractive. If conductor carries current in opposite direction, then force between them will be repulsive.



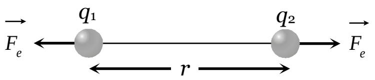


Note : ❒ If a = 1m and in free space $\frac{F}{l} = 2 \times 10^{-7}\, N/m$ then $i_1 = i_2 = 1 Amp$ in each identical wire.

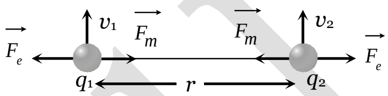
By this concept S.I. unit of Ampere is defined. This is known as **Ampere's law**.

Force Between Two Moving Charges

If two charges q_1 and q_2 are moving with velocities v_1 and v_2 respectively and at any instant the distance between them is r, then



Stationary charges



Moving charges

Magnetic force between them is $F_m = \frac{\mu_0}{4\pi} \cdot \frac{q_1 q_2 v_1 v_2}{r^2}$ ..... (i)

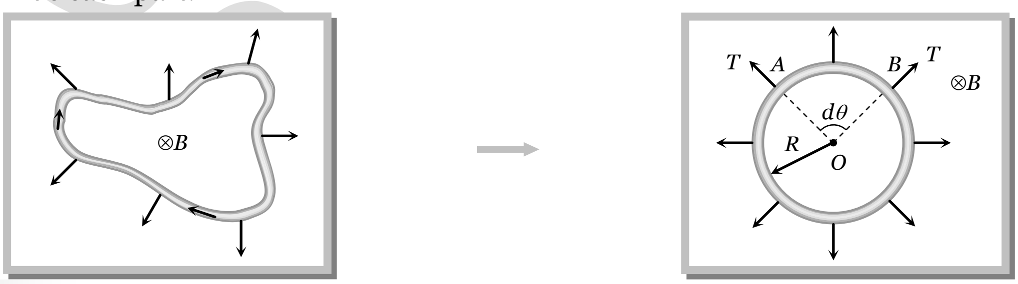
and Electric force between them is $F_e = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}$ ..... (ii)

From equation (i) and (ii) $\frac{F_m}{F_e} = \mu_0 \varepsilon_0 v^2$ but $\mu_0 \varepsilon_0 = \frac{1}{c^2}$; where c is the velocity light in vacuum. So $\frac{F_m}{F_e} = \left(\frac{v}{c}\right)^2$

If $v << c$ then $F_m << F_e$

Standard Cases for Force on Current Carrying Conductors

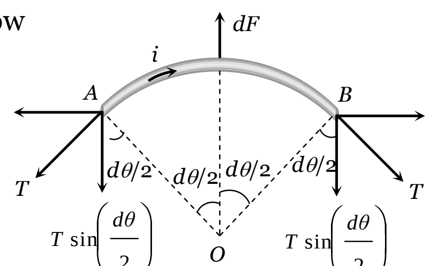
***Case* 1 :** When an arbitrary current carrying loop placed in a magnetic field ($\perp$ to the plane of loop), each element of loop experiences a magnetic force due to which loop stretches and open into circular loop and tension developed in it's each part.



Specific example

In the above circular loop tension in part A and B.

In balanced condition of small part AB of the loop is shown below

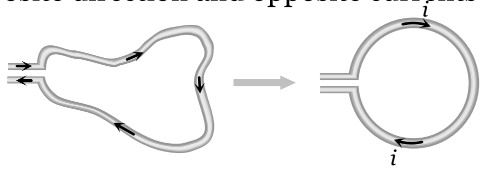


$$2T\sin\frac{d\theta}{2} = dF = Bidl \Rightarrow 2T\sin\frac{d\theta}{2} = BiRd\theta$$

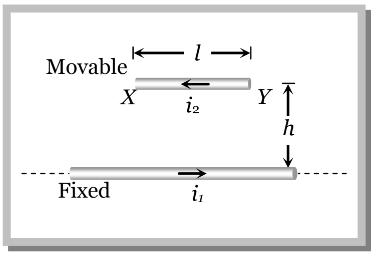
If $d\theta$ is small so, $\sin\frac{d\theta}{2} \approx \frac{d\theta}{2} \Rightarrow 2T.\frac{d\theta}{2} = BiRd\theta$

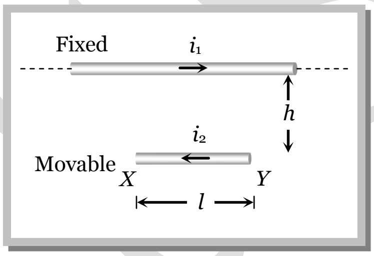
$$T = BiR,\ \text{if } 2\pi R = L \text{ so } T = \frac{BiL}{2\pi}$$

Note : ❒ If no magnetic field is present, the loop will still open into a circle as in it's adjacent parts current will be in opposite direction and opposite currents repel each other.



***Case* 2 : Equilibrium of a current carrying conductor :** When a finite length current carrying wire is kept parallel to another infinite length current carrying wire, it can suspend freely in air as shown below



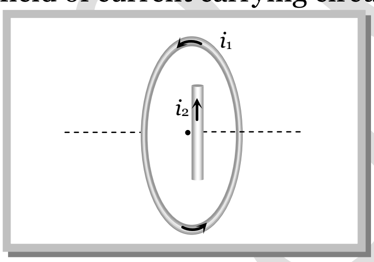


In both the situations for equilibrium of XY it's downward weight = upward magnetic force *i.e.* $mg = \frac{\mu_0}{4\pi}.\frac{2i_1i_2}{h}.l$

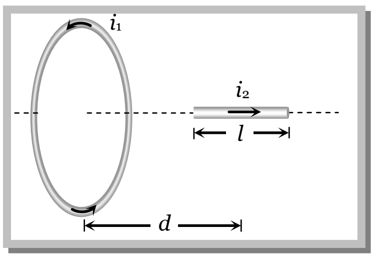
Note : ❒ In the first case if wire XY is slightly displaced from its equilibrium position, it executes SHM and it's time period is given by $T = 2\pi\sqrt{\frac{h}{g}}$.

❒ If direction of current in movable wire is reversed then it's instantaneous acceleration produced is $2g\downarrow$.

***Case* 3 : Current carrying wire and circular loop :** If a current carrying straight wire is placed in the magnetic field of current carrying circular loop.

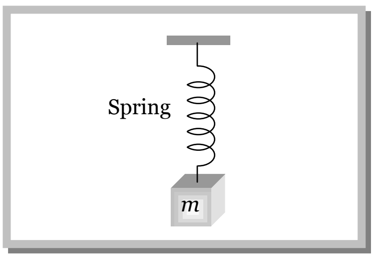


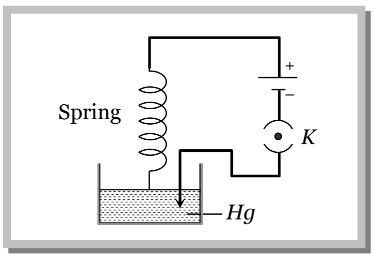
Wire is placed in the perpendicular magnetic field due to coil at it's centre, so it will experience a maximum force $F = Bil = \frac{\mu_0 i_1}{2r}\times i_2 l$



wire is placed along the axis of coil so magnetic field produced by the coil is parallel to the wire. Hence it will not experience any force.

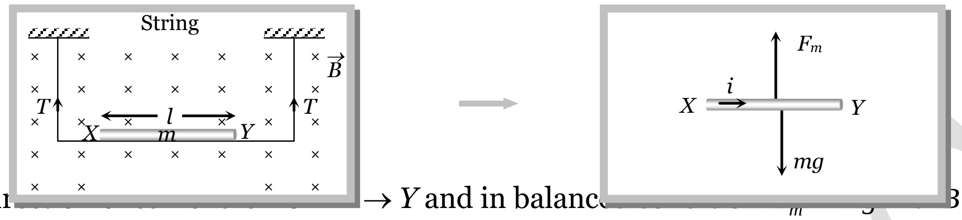
***Case* 4 : Current carrying spring :** If current is passed through a spring, then it will contract because current will flow through all the turns in the same direction.





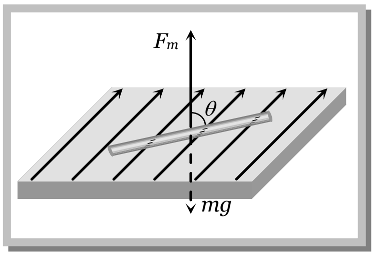
***Case* 5 : Tension less strings :** In the following figure the value and direction of current through the conductor XY so that strings becomes tensionless?

Strings becomes tensionless if weight of conductor XY balanced by magnetic force (F_m).



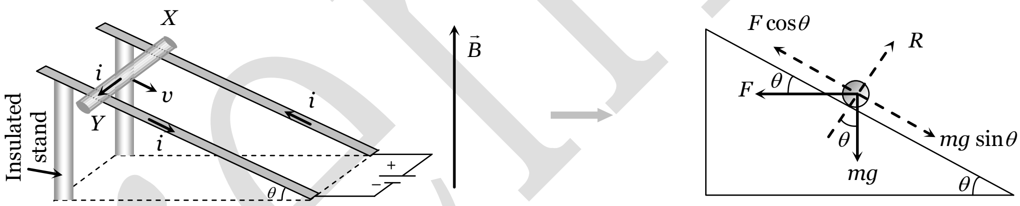
Hence di … $\rightarrow Y$ and in balanc… $Bil = mg \Rightarrow i = \dfrac{mg}{Bl}$

***Case* 6 :** A current carrying conductor floating in air such that it is making an angle θ with the direction of magnetic field, while magnetic field and conductor both lies in a horizontal plane.



In equilibrium $mg = Bil\sin\theta \Rightarrow i = \dfrac{mg}{Bl\sin\theta}$

***Case* 7 : Sliding of conducting rod on inclined rails :** When a conducting rod slides on conducting rails.

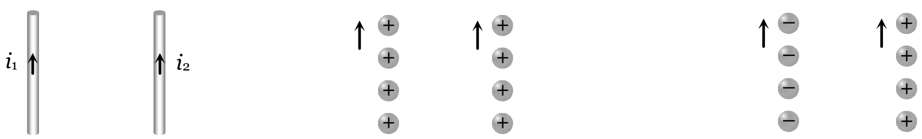


In the following situation conducting rod (X, Y) slides at constant velocity if

$$F\cos\theta = mg\sin\theta \Rightarrow Bil\cos\theta = mg\sin\theta \Rightarrow B = \frac{mg}{il}\tan\theta$$

Concepts

- *Electric force is an absolute concept while magnetic force is a relative concept for an observer.*
- *The nature of force between two parallel charge beams decided by electric force, as it is dominator. The nature of force between two parallel current carrying wires decided by magnetic force.*



$F_{net} = F_m$ only

$F_e \rightarrow$ repulsion
$F_m \rightarrow$ attraction
$F_{net} \rightarrow$ repulsion (Due to this force these beams diverges)

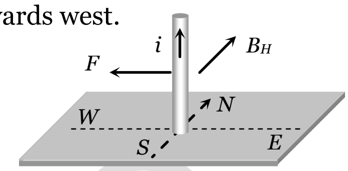
$F_e \rightarrow$ attraction
$F_m \rightarrow$ repulsion
$F_{net} \rightarrow$ attraction (Due to this force these beams converges)

Example

Example: 46 A vertical wire carrying a current in the upward direction is placed in a horizontal magnetic field directed towards north. The wire will experience a force directed towards

(a) North (b) South (c) East (d) West

Solution : (d) By applying Flemings left hand rule, direction of force is found towards west.



Example: 47 3 *A* of current is flowing in a linear conductor having a length of 40 *cm*. The conductor is placed in a magnetic field of strength 500 *gauss* and makes an angle of 30º with the direction of the field. It experiences a force of magnitude

(a) $3 \times 10^4\ N$ (b) $3 \times 10^2\ N$ (c) $3 \times 10^{-2}\ N$ (d) $3 \times 10^{-4}\ N$

Solution : (c) By using $F = Bil\sin\theta \Rightarrow F = (500 \times 10^{-4}) \times 0.4 \times \sin 30^\circ \Rightarrow 3 \times 10^{-2}\ N$.

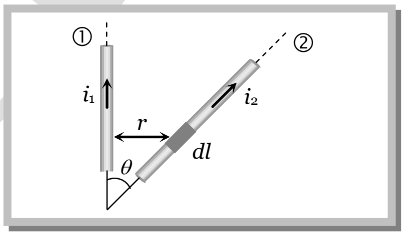
Example: 48 Wires 1 and 2 carrying currents t_1 and t_2 respectively are inclined at an angle θ to each other. What is the force on a small element *dl* of wire 2 at a distance of *r* from 1 (as shown in figure) due to the magnetic field of wire 1 **[AIEEE 2002]**

(a) $\frac{\mu_0}{2\pi r} i_1, i_2 dl \tan\theta$

(b) $\frac{\mu_0}{2\pi r} i_1, i_2\, dl \sin\theta$

(c) $\frac{\mu_0}{2\pi r} i_1, i_2\, dl \cos\theta$

(d) $\frac{\mu_0}{4\pi r} i_1, i_2\, dl \sin\theta$



Solution : (c) Length of the component *dl* which is parallel to wire (1) is *dl* cos *θ*, so force on it

$$F = \frac{\mu_0}{4\pi} \cdot \frac{2i_1 i_2}{r}(dl\cos\theta) = \frac{\mu_0 i_1 i_2 dl\cos\theta}{2\pi r}.$$

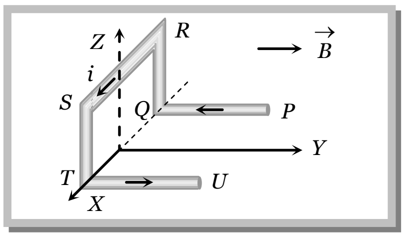
Example: 49 A conductor *PQRSTU*, each side of length *L*, bent as shown in the figure, carries a current *i* and is placed in a uniform magnetic induction *B* directed parallel to the positive *Y*-axis. The force experience by the wire and its direction are

(a) 2*iBL* directed along the negative Z-axis

(b) 5*iBL* directed along the positive *Z*-axis

(c) *iBL* direction along the positive Z-axis

(d) 2*iBL* directed along the positive Z-axis

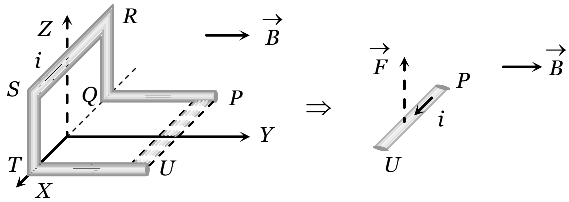


Solution : (c) As *PQ* and *UT* are parallel to *Q*, therefore $F_{PQ} = F_{UT} = 0$

The current in *TS* and *RQ* are in mutually opposite direction. Hence, $F_{TS} - F_{RQ} = 0$

Therefore the force will act only on the segment *SR* whose value is *Bil* and it's direction is +*z*.

Alternate method :



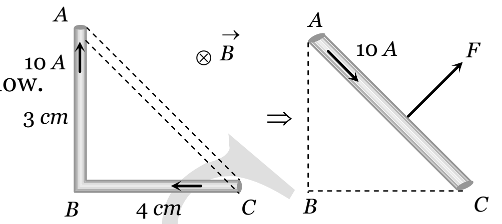
The given shape of the wire can be replaced by a straight wire of length l between P and U as shown below

Hence force on replaced wire PU will be $F = Bil$

and according to *FLHR* it is directed towards $+z$-axis

Example: 50 A conductor in the form of a right angle ABC with $AB = 3cm$ and $BC = 4\ cm$ carries a current of $10\ A$. There is a uniform magnetic field of $5T$ perpendicular to the plane of the conductor. The force on the conductor will be

(a) $1.5\ N$ (b) $2.0\ N$ (c) $2.5\ N$ (d) $3.5\ N$



Solution : (c) According to the question figure can be drawn as shown below.

Force on the conductor ABC = Force on the conductor AC

$= 5 \times 10 \times (5 \times 10^{-2})$

$= 2.5\ N$

Example: 51 A wire of length l carries a current i along the X-axis. A magnetic field exists which is given as $\vec{B} = B_0(\hat{i} + \hat{j} + \hat{k})\ T$. Find the magnitude of the magnetic force acting on the wire

(a) $B_0 il$ (b) $B_0 i l \times \sqrt{2}$ (c) $2B_0 i l$ (d) $\frac{1}{\sqrt{2}} \times B_0 i l$

Solution : (b) By using $\vec{F} = i(\vec{l} \times \vec{B}) \Rightarrow \vec{F} = i[l\hat{i} \times B_0(\hat{i} + \hat{j} + \hat{k})] = B_0 il[\hat{i} \times (\hat{i} + \hat{j} + \hat{k})]$

$\Rightarrow \quad \vec{F} = B_0 il[\hat{i} \times \hat{i} + \hat{i} \times \hat{j} + \hat{i} \times \hat{k}] = B_0 il[\hat{k} - \hat{j}] \quad \{\hat{i} \times \hat{i} = 0, \hat{i} \times \hat{j} = \hat{k}, \hat{i} \times \hat{k} = -\hat{j}\}$

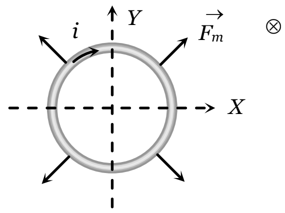
It's magnitude $F = \sqrt{2} B_0 il$

Example: 52 A conducting loop carrying a current i is placed in a uniform magnetic field pointing into the plane of the paper as shown. The loop will have a tendency to **[IIT-JEE (Screening) 2003]]**

(a) Contract (b) Expand

(c) Move towards $+ve$ x-axis (d) Move towards $-ve$ x-axis

Solution : (b) Net force on a current carrying loop in uniform magnetic field is zero. Hence the loop can't translate. So, options (c) and (d) are wrong. From Flemings left hand rule we can see that if magnetic field is perpendicular to paper inwards and current in the loop is clockwise (as shown) the magnetic force $\vec{F_m}$ on each element of the loop is radially outwards, or the loops will have a tendency to expand.



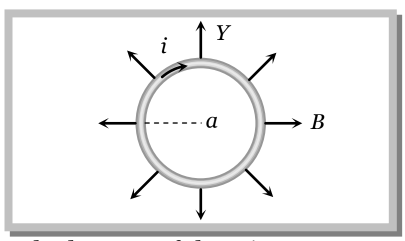
Example: 53 A circular loop of radius a, carrying a current i, is placed in a two-dimensional magnetic field. The centre of the loop coincides with the centre of the field. The strength of the magnetic field at the periphery of the loop is B. Find the magnetic force on the wire

(a) $\pi\, i\, a\, B$

(b) $4\pi\, i\, a\, B$

(c) Zero

(d) $2\pi\, i\, a\, B$

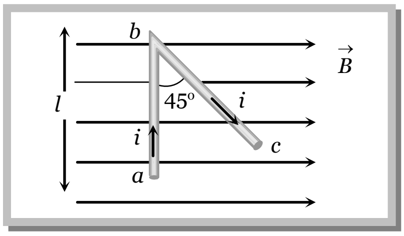


Solution : (d) The direction of the magnetic force will be vertically downwards at each element of the wire.

Thus $F = Bil = Bi\,(2\pi a) = 2\pi iaB$.

Example: 54 A wire abc is carrying current i. It is bent as shown in fig and is placed in a uniform magnetic field of magnetic induction B. Length $ab = l$ and $\angle\, abc = 45^o$. The ratio of force on ab and on bc is

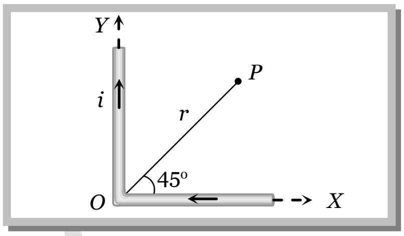
(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}$ (c) 1 (d) $\frac{2}{3}$



Solution : (c) Force on portion *ab* of wire $F_1 = Bil \sin 90^o = Bil$

Force on portion *bc* of wire $F_2 = Bi\left(\frac{l}{\sqrt{2}}\right)\sin 45^o = Bil$. So $\frac{F_1}{F_2} = 1$.

***Example*: 55** Current *i* flows through a long conducting wire bent at right angle as shown in figure. The magnetic field at a point *P* on the right bisector of the angle *XOY* at a distance *r* from *O* is



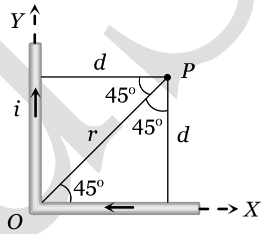
(a) $\frac{\mu_0 i}{\pi r}$

(b) $\frac{2\mu_0 i}{\pi r}$

(c) $\frac{\mu_0 i}{4\pi r}(\sqrt{2}+1)$

(d) $\frac{\mu_0}{4\pi}.\frac{2i}{r}(\sqrt{2}+1)$

Solution : (d) By using $B = \frac{\mu_0}{4\pi}.\frac{i}{r}(\sin\phi_1 + \sin\phi_2)$, from figure $d = r\sin 45^o = \frac{r}{\sqrt{2}}$



Magnetic field due to each wire at *P* $B = \frac{\mu_0}{4\pi}.\frac{i}{(r/\sqrt{2})}(\sin 45^o + \sin 90^o)$

$= \frac{\mu_0}{4\pi}.\frac{i}{r}(\sqrt{2}+1)$

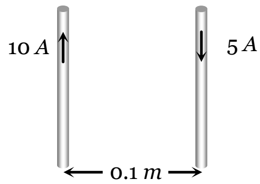
Hence net magnetic field at *P* $B_{net} = 2\times\frac{\mu_0}{4\pi}.\frac{i}{r}(\sqrt{2}+1) = \frac{\mu_0}{2\pi}.\frac{i}{r}(\sqrt{2}+1)$

***Example*: 56** A long wire A carries a current of 10 *amp*. Another long wire *B*, which is parallel to *A* and separated by 0.1 *m* from *A*, carries a current of 5 *amp*. in the opposite direction to that in *A*. What is the magnitude and nature of the force experienced per unit length of *B* [$\mu_0 = 4\pi\times 10^{-7}$ *weber/amp – m*]

(a) Repulsive force of $10^{-4}\ N/m$ (b) Attractive force of $10^{-4}\ N/m$

(c) Repulsive force of $2\pi\times 10^{-5}\ N/m$ (d) Attractive force of $2\pi\times 10^{-5}\ N/m$

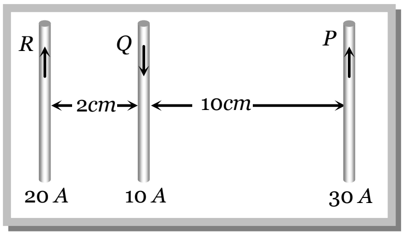
Solution : (a) By using $\frac{F}{l} = \frac{\mu_0}{4\pi}.\frac{2i_1 i_2}{a}$



$\Rightarrow \frac{F}{l} = 10^{-7}\times\frac{2\times 10\times 5}{0.1} = 10^{-4}\ N$

Wires are carrying current in opposite direction so the force will be repulsive.

***Example*: 57** Three long, straight and parallel wires carrying currents are arranged as shown in figure. The force experienced by 10 *cm* length of wire *Q* is **[MP PET 1997]**



(a) 1.4×10^{-4} *N* towards the right
(b) 1.4×10^{-4} *N* towards the left
(c) 2.6×10^{-4} *N* to the right
(d) 2.6×10^{-4} *N* to the left

Solution : (a) Force on wire *Q* due to *R* ; $F_R = 10^{-7}\times\frac{2\times 20\times 10}{(2\times 10^{-2})}\times(10\times 10^{-2}) = 2\times 10^{-4}\ m$ (Repulsive)

Force on wire *Q* due to *P* ; $F_P = 10^{-7}\times 2\times\frac{10\times 30}{(10\times 10^{-2})}\times(10\times 10^{-2}) = 0.6\times 10^{-4}\ N$ (Repulsive)

Hence net force $F_{net} = F_R - F_P = 2 \times 10^{-4} - 0.6 \times 10^{-4} = 1.4 \times 10^{-4}\,N$ (towards right *i.e.* in the direction of $\overrightarrow{F_R}$.

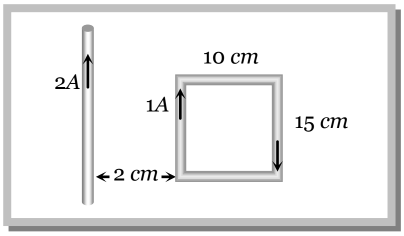
***Example*: 58** What is the net force on the coil **[DCE 2000]**

(a) $25 \times 10^{-7}\,N$ moving towards wire

(b) $25 \times 10^{-7}\,N$ moving away from wire

(c) $35 \times 10^{-7}\,N$ moving towards wire

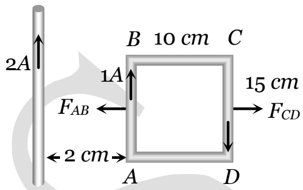
(d) $35 \times 10^{-7}\,N$ moving away from wire



Solution : (a) Force on sides *BC* and *CD* cancel each other.

Force on side *AB* $F_{AB} = 10^{-7} \times \dfrac{2 \times 2 \times 1}{2 \times 10^{-2}} \times 15 \times 10^{-2} = 3 \times 10^{-6}\,N$

Force on side *CD* $F_{AB} = 10^{-7} \times \dfrac{2 \times 2 \times 1}{12 \times 10^{-2}} \times 15 \times 10^{-2} = 0.5 \times 10^{-6}\,N$



Hence net force on loop = $F_{AB} - F_{CD} = 25 \times 10^{-7}\,N$ (towards the wire).

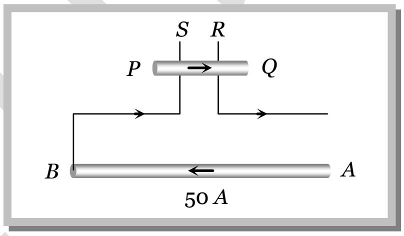
***Example*: 59** A long wire *AB* is placed on a table. Another wire *PQ* of mass 1.0 *g* and length 50 *cm* is set to slide on two rails *PS* and *QR*. A current of 50*A* is passed through the wires. At what distance above *AB*, will the wire *PQ* be in equilibrium

(a) 25 *mm*

(b) 50 *mm*

(c) 75 *mm*

(d) 100 *mm*



Solution : (a) Suppose in equilibrium wire *PQ* lies at a distance *r* above the wire *AB*

Hence in equilibrium $mg = Bil \Rightarrow mg = \dfrac{\mu_0}{4\pi}\left(\dfrac{2i}{r}\right) \times il \Rightarrow 10^{-3} \times 10 = 10^{-7} \times \dfrac{2 \times (50)^2}{r} = 0.5 \Rightarrow r = 25\,mm$

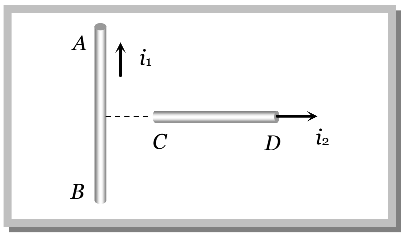
***Example*: 60** An infinitely long, straight conductor *AB* is fixed and a current is passed through it. Another movable straight wire *CD* of finite length and carrying current is held perpendicular to it and released. Neglect weight of the wire

(a) The rod *CD* will move upwards parallel to itself

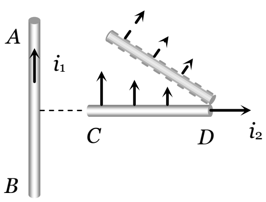
(b) The rod *CD* will move downward parallel to itself

(c) The rod *CD* will move upward and turn clockwise at the same time

(d) The rod *CD* will move upward and turn anti –clockwise at the same time



Solution : (c) Since the force on the rod *CD* is non-uniform it will experience force and torque. From the left hand side it can be seen that the force will be upward and torque is clockwise.



Tricky example: 5

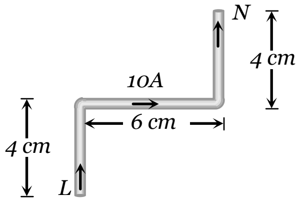
A current carrying wire LN is bent in the from shown below. If wire carries a current of 10 A and it is placed in a magnetic field a $5T$ which acts perpendicular to the paper outwards then it will experience a force

(a) Zero

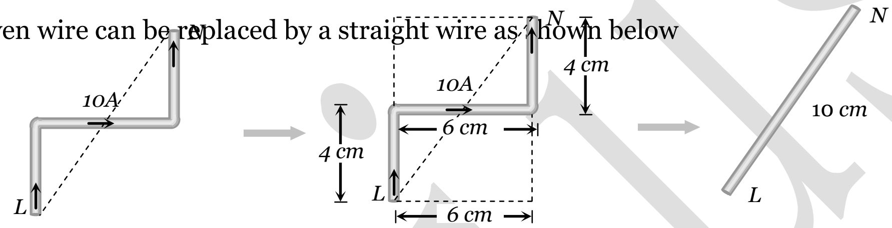
(b) 5 N

(c) 30 N

(d) 20 N



Solution : (b) The given wire can be replaced by a straight wire as shown below



Hence force experienced by the wire $F = Bil = 5 \times 10 \times 0.1 = 5\ N$

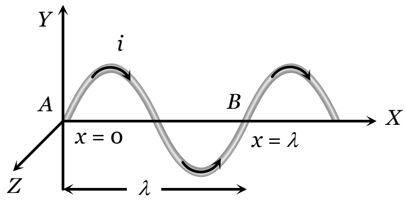
Tricky example: 6

A wire, carrying a current i, is kept in $X - Y$ plane along the curve $y = A\sin\left(\frac{2\pi}{\lambda}x\right)$. A magnetic field B exists in the Z-direction find the magnitude of the magnetic force on the portion of the wire between $x = 0$ and $x = \lambda$

(a) $i\lambda B$ (b) Zero (c) $\frac{i\lambda B}{2}$ (d) $3/2i\lambda B$

Solution : (a) The given curve is a sine curve as shown below.

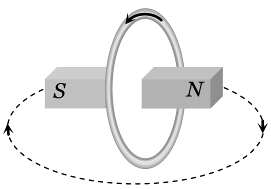
The given portion of the curved wire may be treated as a straight wire AB of length λ which experiences *a* magnetic force $F_m = Bi\lambda$

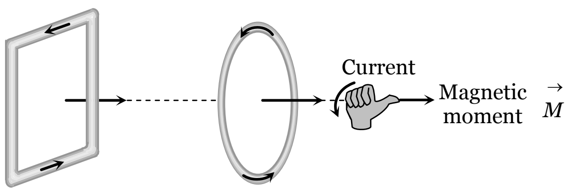


Current Loop As a Magnetic Dipole

A current carrying circular coil behaves as a bar magnet whose magnetic moment is $M = NiA$; *Where* N = Number of turns in the coil, i = Current through the coil and A = Area of the coil

Magnetic moment of a current carrying coil is a vector and it's direction is given by right hand thumb rule

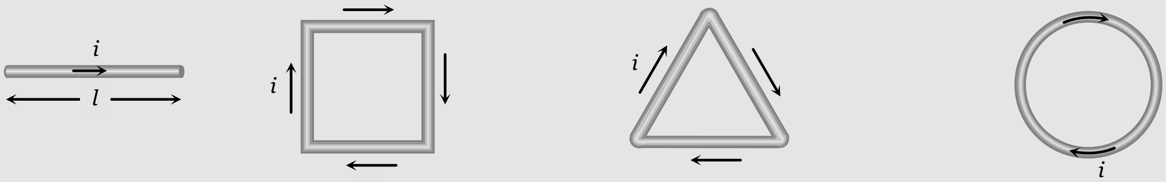




Specific examples

A given length constant current carrying straight wire moulded into different shaped loops. as shown

Linear	Square	Equilateral	Circle



	$l = 4a$	$l = 3a$	$l = 2\pi r$
	$A = a^2$	$A = \frac{\sqrt{3}}{4}a^2$	$A = \pi r^2$
	$M = ia^2 = \frac{il^2}{16}$	$M = i\left(\frac{\sqrt{3}}{4}a^2\right) = \frac{\sqrt{3}\, il^2}{36}$	$M = i(\pi r^2) = \frac{il^2}{4\pi}$ ← max.

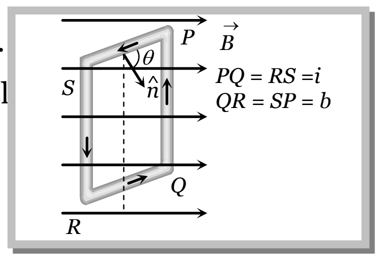
Note : ❒ For a given perimeter circular shape have maximum area. Hence maximum magnetic moment.

❒ For a any loop or coil $\vec{B}$ and $\vec{M}$ are always parallel.

Behaviour of Current loop In a Magnetic Field

(1) **Torque**

Consider a rectangular current carrying coil *PQRS* having *N* turns and area *A*, placed in a uniform field *B*, in such a way that the normal $(\hat{n})$ to the coil makes an angle θ with the direction of *B*. the coil experiences a torque given by $\boldsymbol{\tau = NBiA \sin\theta}$. Vectorially $\vec{\tau} = \vec{M} \times \vec{B}$



(i) τ is zero when $\theta = 0$, *i.e.*, when the plane of the coil is perpendicular to the field.

(ii) τ is maximum when $\theta = 90^\circ$, *i.e.*, the plane of the coil is parallel to the fiel

$\Rightarrow \tau_{max} = NBiA$

The above expression is valid for coils of all shapes.

(2) **Workdone**

If coil is rotated through an angle θ from it's equilibrium position then required work. $W = MB(1 - \cos\theta)$. It is maximum when $\theta = 180^\circ \Rightarrow W_{max} = 2\,MB$

(3) **Potential energy**

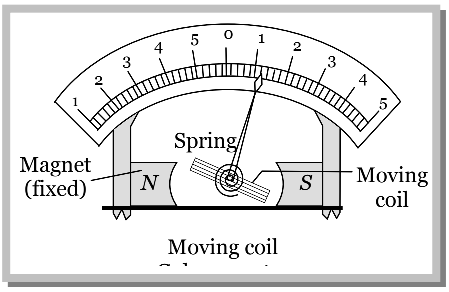
Is given by $U = -MB\cos\theta \Rightarrow U = \vec{M}.\vec{B}$

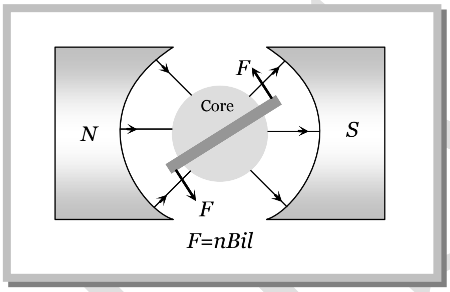
Note : ❒ Direction of $\vec{M}$ is found by using Right hand thumb rule according to which curl the fingers of right hand in the direction of circulation of conventional current, then the thumb gives the direction of $\vec{M}$.

❒ Instruments such as electric motor, moving coil galvanometer and tangent galvanometers *etc.* are based on the fact that a current-carrying coil in a uniform magnetic field experiences a torque (or couple).

Moving coil galvanometer

In a moving coil galvanometer the coil is suspended between the pole pieces of a strong horse-shoe magnet. The pole pieces are made cylinderical and a soft iron cylinderical core is placed within the coil without touching it. This makes the field radial. In such a field the plane of the coil always remains parallel to the field. Therefore $\theta = 90^o$ and the deflecting torque always has the maximum value.





$$\tau_{\text{def}} = NBiA \qquad \text{......(i)}$$

coil deflects, a restoring torque is set up in the suspension fibre. If α is the angle of twist, the restoring torque is

$$\tau_{\text{rest}} = C\alpha \qquad \text{.....(ii)}$$ where C is the torsional constant of the fibre.

When the coil is in equilibrium.

$$NBiA = C\alpha \Rightarrow i = \frac{C}{NBA}\alpha \Rightarrow i = K\alpha,$$

Where $K = \frac{C}{NBA}$ is the galvanometer constant. This linear relationship between i and α makes the moving coil galvanometer useful for current measurement and detection.

Current sensitivity : The current sensitivity of a galvanometer is defined as the deflection produced in the galvanometer per unit current flowing through it.

$$S_i = \frac{\alpha}{i} = \frac{NBA}{C}$$

Thus in order to increase the sensitivity of a moving coil galvanometer, N, B and A should be increased and C should be decreased.

Quartz fibres can also be used for suspension of the coil because they have large tensile strength and very low value of k.

Voltage sensitivity (S_V) : Voltage sensitivity of a galvanometer is defined as the deflection produced in the galvanometer per unit applied to it.

$$S_V = \frac{\alpha}{V} = \frac{\alpha}{iR} = \frac{S_i}{R} = \frac{NBA}{RC}$$

Concepts

☞ *The field in a moving coil galvanometer radial in nature in order to have a linear relation between the current and the deflection.*

☞ *A rectangular current loop is in an arbitrary orientation in an external magnetic field. No work required to rotate the loop* *about an axis perpendicular to it's plane.*

☞ *Moving coil galvanometer can be made ballistic by using a non-conducting frame (made of ivory or bamboo) instead of a metallic frame.*

Example

Example: 61 A circular coil of radius 4 *cm* and 20 turns carries a current of 3 *ampere*. It is placed in a magnetic field of 0.5 *T*. The magnetic dipole moment of the coil is **[MP PMT 2001]**

(a) 0.60 $A\text{-}m^2$ (b) 0.45 $A\text{-}m^2$ (c) 0.3 $A\text{-}m^2$ (d) 0.15 $A\text{-}m^2$

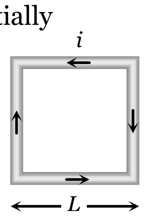
Solution : (c) $M = niA \Rightarrow M = 20 \times 3 \times \pi (4 \times 10^{-2})^2 = 0.3\ A\text{-}m^2$.

Example: 62 A steady current *i* flows in a small square loop of wire of side *L* in a horizontal plane. The loop is now folded about its middle such that half of it lies in a vertical plane. Let $\overrightarrow{\mu_1}$ and $\overrightarrow{\mu_2}$ respectively denote the magnetic moments due to the current loop before and after folding. Then **[IIT-JEE 1993]**

(a) $\overrightarrow{\mu_2} = 0$ (b) $\overrightarrow{\mu_1}$ and $\overrightarrow{\mu_2}$ are in the same direction

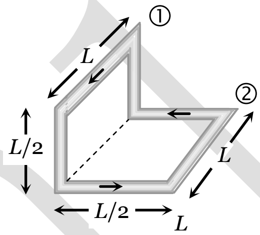
(c) $\dfrac{|\overrightarrow{\mu_1}|}{|\overrightarrow{\mu_2}|} = \sqrt{2}$ (d) $\dfrac{|\overrightarrow{\mu_1}|}{|\overrightarrow{\mu_2}|} = \left(\dfrac{1}{\sqrt{2}}\right)$

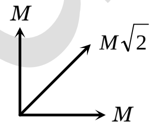
Solution : (c) Initially



$\mu_1 = iL^2$

Finally

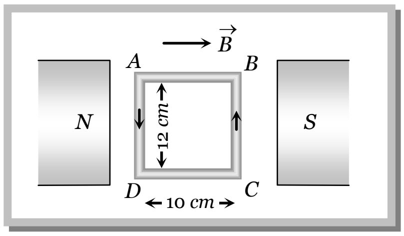




$$M = \text{magnetic moment due to each part} = i\left(\frac{L}{2}\right) \times L = \frac{iL^2}{2} = \frac{\mu_1}{2}$$

$$\therefore\ \mu_2 = M\sqrt{2} = \frac{\mu_1}{2} \times \sqrt{2} = \frac{\mu_1}{\sqrt{2}}$$

Example: 63 A coil of 50 turns is situated in a magnetic field b = 0.25*weber*/m^2 as shown in figure. A current of 2*A* is flowing in the coil. Torque acting on the coil will be



(a) 0.15 *N*

(b) 0.3 *N*

(c) 0.45 *N*

(d) 0.6 *N*

Solution : (b) Since plane of the coil is parallel to magnetic field. So $\theta = 90^o$

Hence $\tau = NBiA \sin 90^o = NBiA = 50 \times 0.25 \times 2 \times (12 \times 10^{-2} \times 10 \times 10^{-2}) = 0.3\ N$.

Example: 64 A circular loop of area 1 cm^2, carrying a current of 10 *A*, is placed in a magnetic field of 0.1 *T* perpendicular to the plane of the loop. The torque on the loop due to the magnetic field is

(a) Zero (b) 10^{-4} *N-m* (c) 10^{-2} *N-m* (d) 1 *N-m*

Solution : (a) $\tau = NBiA \sin\theta$; given $\theta = 0$ so $\tau = 0$.

Example: 65 A circular coil of radius 4 *cm* has 50 turns. In this coil a current of 2 *A* is flowing. It is placed in a magnetic field of 0.1 *weber*/m^2. The amount of work done in rotating it through 180° from its equilibrium position will be **[CPMT 1977]**

(a) 0.1 *J* (b) 0.2 *J* (c) 0.4 (d) 0.8 *J*

Solution : (a) Work done in rotating a coil through an angle θ from it's equilibrium position is $W = MB(1 - \cos\theta)$ where $\theta = 180^o$ and $M = 50 \times 2 \times \pi(4 \times 10^{-2}) = 50.24 \times 10^{-2}\ A\text{-}m^2$. Hence $W = 0.1\ J$

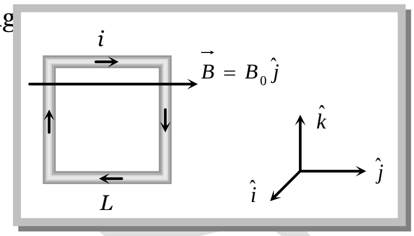
***Example*: 66** A wire of length L is bent in the form of a circular coil and current i is passed through it. If this coil is placed in a magnetic field then the torque acting on the coil will be maximum when the number of turns is

(a) As large as possible (b) Any number (c) 2 (d) 1

Solution : (d) $\tau_{\max} = MB$ or $\tau_{\max} = ni\pi a^2 B$. Let number of turns in length l is n so $l = n(2\pi a)$ or $a = \dfrac{l}{2\pi n}$

$$\Rightarrow \tau_{\max} = \frac{ni\pi B l^2}{4\pi^2 n^2} = \frac{l^2 iB}{4\pi n_{\min}} \Rightarrow \tau_{\max} \propto \frac{1}{n_{\min}} \Rightarrow n_{\min} = 1$$

***Example*: 67** A square coil of N turns (with length of each side equal L) carrying current i is placed in a uniform magnetic field $\vec{B} = B_0\hat{j}$ as shown in figure. What is the torque acting



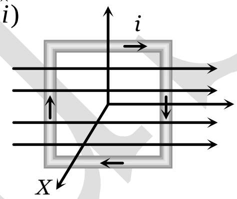
(a) $+B_0 NiL^2\hat{k}$

(b) $-B_0 NiL^2\hat{k}$

(c) $+B_0 NiL^2\hat{j}$

(d) $-B_0 NiL^2\hat{j}$

Solution : (b) The magnetic field is $\vec{B} = B_0\hat{j}$ and the magnetic moment $\vec{m} = i\vec{A} = -i(NL^2\hat{i})$



The torque is given by $\vec{\tau} = \vec{m} \times \vec{B}$

$= -iNL^2\hat{i} \times B_0\hat{j} = -iNB_0L^2\hat{i} \times \hat{j}$

$= -iNB_0L^2\hat{k}$

***Example*: 68** The coil of a galvanometer consists of 100 *turns* and effective area of 1 *square cm*. The restoring couple is 10^{-8} *N-m rad*. The magnetic field between the pole pieces is 5 *T*. The current sensitivity of this galvanometer will be **[MP PMT 1997]**

(a) 5×10^4 *rad/μ amp* (b) 5×10^{-6} *per amp* (c) 2×10^{-7} *per amp* (d) 5 *rad./μ amp*

Solution : (d) Current sensitivity $(S_i) = \dfrac{\theta}{i} = \dfrac{NBA}{C} \Rightarrow \dfrac{\theta}{i} = \dfrac{100 \times 5 \times 10^{-4}}{10^{-8}} = 5\ rad/\mu\ amp$.

***Example*: 69** The sensitivity of a moving coil galvanometer can be increased by **[SCRA 2000]]**

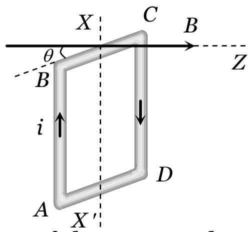
(a) Increasing the number of turns in the coil (b) Decreasing the area of the coil

(c) Increasing the current in the coil (d) Introducing a soft iron core inside the coil

Solution : (a) Sensitivity $(S_i) = \dfrac{NBA}{C} \Rightarrow S_i \propto N$.

Tricky example: 7

The square loop *ABCD*, carrying a current *i*, is placed in uniform magnetic field *B*, as shown. The loop can rotate about the axis *XX'*. The plane of the loop makes and angle $\theta\ (\theta < 90^o)$ with the direction of *B*. Through what angle will the loop rotate by itself before the torque on it becomes zero



(a) θ

(b) $90^o - \theta$

(c) $90^o + \theta$

(d) $180^o - \theta$

Solution : (c) In the position shown, *AB* is outside and *CD* is inside the plane of the paper. The Ampere force on *AB* acts into the paper. The torque on the loop will be clockwise, as seen from above. The loop must rotate through an angle $(90^o + \theta)$ before the plane of the loop becomes normal to the direction of *B* and the torque becomes zero.